Sheet (2)- Random Variables

1- A coin is tossed n times. Let the random variable Y be the difference between the number of heads and the number of tails.
   a. Describe the sample space of Y, S_Y
   b. Find the equivalent event for the event Y=0
   c. Find the equivalent event for the event Y ≤ k for k a positive integer

2- A dart is thrown onto a square b units wide. Assume that the dart is equally likely to fall anywhere in the square. Let the random variable Z be given by the sum of the two coordinates of the point where the dart lands.
   a. Describe the sample space for Z, S_Z
   b. Find the region in the square corresponding to the event Z ≤ z for -∞ < x < ∞
   c. Find P[Z ≤ z]

3- The cdf of the random variable X is given by

\[ F_X(x) = \begin{cases} 
1/3 + 2/3 (x+1)^2 & \text{if } -1 \leq x \leq 0 \\
0 & \text{if } x < -1 
\end{cases} \]

Find the probability of the events A {X > 1/3}, B= {\mid X \mid \geq 1}, C= \{\mid X-1/3\mid <1 \}, D= \{X<0\}.

4- A continuous random variable X has cdf

\[ F_X(x) = \begin{cases} 
0 & \text{if } x \leq -\pi/2 \\
c(1+\sin(x)) & \text{if } -\pi/2 < x < \pi/2 \\
1 & \text{if } x \geq \pi/2 
\end{cases} \]

- Find c
- Plot F_x(x)
5- A random variable X has the pdf shown below
   a. Find \( f_x(x) \)
   b. Find the cdf of X
   c. Find \( b \) such that \( P[|X|<b]=1/2 \)

6- A communication channel accepts an arbitrary voltage input \( v \) and outputs a voltage \( Y=v+N \), where \( N \) is a Gaussian random variable with mean 0 and variance \( \sigma^2 = 1 \). Suppose that the channel is used to transmit binary information as follows:
   
   To transmit 0  input -1
   To transmit 1  input +1

   The receiver decides a 0 was sent if the voltage is negative and a 1 otherwise. Find the probability of the receiver making an error if a 0 was sent; if a 1 was sent.

Web site:
http://www.aast.edu/~khedr/Courses/Undergraduate/Communication%20system%20II/index.htm