Sheet (8)- Random Processes -I

1- 
   a. Random process X(t, f) = sin (2πft)

   \[ P(f) = \begin{cases} 
   1/w & 0 \leq f \leq w \\
   0 & \text{elsewhere} 
   \end{cases} \]

   Show that X(t, f) is non-stationary

   b. Random process X(t, a) = a cos (2πft)

   \[ P(a) = \begin{cases} 
   1 & 0 \leq a \leq 1 \\
   0 & \text{elsewhere} 
   \end{cases} \]

   Determine whether X(t, a) is stationary or not and check its ergodicity

   c. Random process = a cos (2πft + θ)

   \[ P(\theta) = \begin{cases} 
   1/2\pi & 0 \leq \theta \leq 2\pi \\
   0 & \text{elsewhere} 
   \end{cases} \]

   Determine whether X(t, θ) is stationary or not and check its ergodicity

2- A random process X(t) is defined by

   \[ X(t) = A \cos(2\pi ft) \]

   Where A is a gaussian distributed random variable of zero mean and variance \( \sigma_A^2 \). This random process is applied to an ideal integrator, producing the output

   \[ Y(t) = \int_0^t X(\tau) \, d\tau \]

   (a) Determine the probability density function of the output Y(t) at a particular time \( t_k \)
   (b) Determine whether or not Y(t) is stationary and if so whether or not Y(t) is ergodic
3- A random process Y(t) consists of a dc component of $\sqrt{3}/2$ volts, a periodic component g(t) and a random component X(t). The autocorrelation function of Y(t) is shown below.

(a) What is the average power of the periodic component g(t) ?
(b) What is the average power of the random component X(t) ?

4- The power spectral density of a random process X(t) is shown below.

(a) Determine and sketch the autocorrelation function $R_x(\tau)$ of X(t)
(b) What is the dc power contained in X(t)?
(c) What is the ac power contained in X(t)?