Communication Systems II

Noise in CW modulation
FM Modulation
FM Receivers

- FM receivers are also of superheterodyne type.
- Typical frequencies for commercial FM radio are:
  - RF carrier range: 88-108 MHz
  - Mid band frequency of IF section: 10.7 MHz
  - IF bandwidth: 0.2 MHz

The amplitude limiter removes amplitude variations by clipping the modulated wave at the IF section output.

The resulting rectangular wave is rounded off by a bandpass filter which suppresses harmonics of the carrier frequency.
FM Receivers

- The discriminator consists of two components
  - A slope network or differentiator with purely imaginary transfer function that varies linearly with frequency.
    - Produces a hybrid modulation of amplitude and frequency
  - An envelope detector that recovers the amplitude variation and thus reproduces the message signal.

- The post detection filter removes the out of band components of the noise at the discriminator output and thus keeps the output noise as small as possible.
FM Receiver Model

- $W(t)$ is modeled as white Gaussian noise with zero mean and PSD of $N_0/2$.
- FM signal of center frequency $f_c$ and $BW = B_T$

![Diagram of FM Receiver Model]

- IF filter is assumed IBPF with bandwidth $B_T$
  - We can use the narrow band noise representation in terms of its inphase and quadrature components.
Noise in FM reception

- The NBN at the IF output is defined as
  \[ n(t) = n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t) \]

- Equivalently
  \[ n(t) = r(t) \cos(2\pi f_c t + \psi(t)) \]
  \[ r(t) = \sqrt{n_c^2(t) + n_s^2(t)} \]
  \[ \psi(t) = \tan^{-1} \frac{n_s(t)}{n_c(t)} \]

- \( r(t) \) is Rayleigh distributed and phase is uniformly distributed over \( 2\pi \)
Noise in FM reception

The FM signal at the IF output is

\[ s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t m(t)dt) \]

\[ \phi(t) = 2\pi k_f \int_0^t m(t)dt \]

\[ s(t) = A_c \cos(2\pi f_c t + \phi(t)) \]

The total signal at the output of IF section is

- \( X(t) = s(t) + n(t) \)

\[ x(t) = A_c \cos(2\pi f_c t + \phi(t)) + r(t) \cos(2\pi f_c t + \psi(t)) \]
Noise in FM reception

- Represent $x(t)$ by means of a phasor diagram

$$x(t) = A_c \cos(2\pi f_c t + \phi(t)) + r(t) \cos(2\pi f_c t + \psi(t))$$

- The relevant phase $\theta(t)$ can be calculated

$$\theta(t) = \phi(t) + \tan^{-1}\left\{ \frac{r(t) \sin(\psi(t) - \phi(t))}{A_c + r(t) \cos(\psi(t) - \phi(t))} \right\}$$
Noise in FM reception

- Envelope of $x(t)$ is not of interest to us
- The output of the ideal discriminator will be proportional to $\theta'(t)/2\pi$

$$\theta(t) = \phi(t) + \tan^{-1} \left\{ \frac{r(t)\sin(\psi(t) - \phi(t))}{A_c + r(t)\cos(\psi(t) - \phi(t))} \right\}$$

$$\therefore \theta(t) \approx \phi(t) + \frac{r(t)}{A_c} \sin(\psi(t) - \phi(t))$$

$$\theta(t) \approx 2\pi k_f \int_0^t m(t)\,dt + \frac{r(t)}{A_c} \sin(\psi(t) - \phi(t))$$

- Assumptions
  - Carrier to noise ratio is large compared with unity

$v(t)$ discriminator output $= \frac{1}{2\pi} \theta'(t)$

$v(t) = k_f m(t) + n_d(t)$

where $n_d(t) = \frac{1}{2\pi} \frac{d}{dt} \left\{ \frac{r(t)}{A_c} \sin(\psi(t) - \phi(t)) \right\}$
Noise in FM reception

The noise component can be simplified further to

\[ n_d(t) \approx \frac{1}{2\pi} \frac{d}{dt} \left\{ \frac{r(t)}{A_c} \sin(\psi(t)) \right\} \]

However

\[ n_s(t) = r(t) \sin(\psi(t)) \]

\[ n_d(t) \approx \frac{1}{2\pi} \frac{d}{dt} \left\{ \frac{n_s(t)}{A_c} \right\} \]

The power in the output message is \( k_f^2P \) where \( P \) is power in transmitted message

Required to find the power in the noise component

- Derivative in time \( \rightarrow j2\pi f \) in frequency

\[ \therefore S_{n_d}(f) = \frac{f^2}{A_c^2} S_{n_s}(f) \]
Noise in FM reception

With the Equivalent IF filter has ideal response.

- It follows that the narrow-band noise \( n(t) \) will have a PSD similar in shape to the transfer function of the IF filter
- Therefore the quadrature component of the NBN will have the ideal low pass characteristic.
- The corresponding PSD of \( n_d \) will be

\[
S_{n_d}(f) = \frac{f^2}{A_c^2} S_n(f)
\]

\[
S_{n_d}(f) = \begin{cases} 
\frac{N_0 f^2}{A_c^2}, & |f| \leq \frac{B_T}{2} \\
0, & \text{elsewhere}
\end{cases}
\]
Noise in FM reception

- The discriminator output is followed by a low pass filter with bandwidth equal to the bandwidth of the message $W$.
  - $W \gg B_T/2$
  - Therefore the noise component $n_d(t)$ will have rejection region.
  - The corresponding PSD of $n_d$ after LPF will be

\[
S_{n_{out}}(f) = \begin{cases} 
\frac{N_0 f^2}{A_c^2}, & \text{if } |f| \leq W \\
0, & \text{elsewhere}
\end{cases}
\]
Noise in FM reception

The average output noise power is determined by integrating the PSD $S_{\text{out}}$ from $-W$ to $W$

$$\text{average power of output noise} = \frac{N_o}{A_c^2} \int_{-W}^{W} f^2 df$$

$$= \frac{2N_o}{3A_c^2} W^3$$

Note that the average output noise power is inversely proportional to the carrier power $A_c^2/2$

$$\text{SNR}_{c,FM} = \frac{A_c^2}{2N_o W}$$

$$\text{SNR}_{o,FM} = \frac{3A_c^2 k_f^2 P}{2N_o W^3}$$

$$\text{Figure of Merit} = \frac{3k_f^2 P}{W^2}$$
Consider the case of single tone modulation with maximum frequency deviation $\Delta f$.

\[ s(t) = A_c \cos(2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t)) \]

\[ m(t) = \frac{\Delta f}{k_f} \cos(2\pi f_m t) \]

\[ P = \frac{\Delta f^2}{2k_f^2} \]

\[ SNR_{o,FM} = \frac{3A_c^2 \Delta f^2}{4N_o W^3} = \frac{3A_c^2}{4N_o W} \beta^2 \]

\[ FoM = \frac{3}{2} \beta^2 \]
Narrow Band FM

- **NBFM** $\implies \beta < 1/3$

$$s_{NBFM}(t) = A_c \cos(2\pi f_c t) - A_c \sin(2\pi f_c t)[k_f \int_0^t m(t)dt]$$

$$P = \frac{A_c^2}{2}$$

$$R_{NBFM}(\tau) = \frac{A_c^2}{2} \cos(2\pi f_c \tau) + \frac{A_c^2}{2} \cos(2\pi f_c \tau) R_{\phi_m}(\tau)$$

$$\phi_m = \begin{cases} k_p m(t), & PM \\ k_f \int_0^t m(t)dt, & FM \end{cases}$$

$$S_m(f) = \begin{cases} k_p^2 S_m(f) \\ k_f^2 S_m(f) \\ \frac{1}{(2\pi f)^2} \end{cases}$$

- Received signal

$$x(t) = A_c \cos(2\pi f_c t) - A_c \sin(2\pi f_c t)[k_f \int_0^t m(t)dt] + w(t)$$

$$SNR_{c,NBFM} = \frac{A_c^2}{2N_0 W_{ch}}$$

*after IF section*

$$SNR_{NBFM} = \frac{A_c^2}{2N_0 2W}$$

$$P = \frac{A_c^2}{2}$$

$$SNR_{o,FM} = \frac{3A_c^2 k_f^2 P}{2N_0 W^3}$$

*Figure of Merit* $= \frac{3k_f^2 P}{W^2} = \frac{3}{2} \beta^2$, $\beta \ll 1$