

## Linear Antenna Arrays

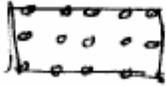
array antenna : a configuration of multiple radiating elements arranged in space to produce a directional radiation pattern. ✓

### advantages

- ① many small antennas to replace single large antenna  
so the mechanical problems associated with large antenna can be avoided. (reflector example)
- ② offer a unique capability of electronic scanning of the main beam. (without changing the antenna configuration) by changing the phase of (I) in each element. (phased array) ✓

### types

linear array :  lie along a straight line

planar array :  in rectangular area

Conformal array :

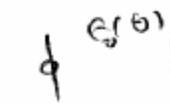
### disadvantages

- ① Complexity of the network required to feed the elements.
- ② BW limitations
- ③ mutual coupling.

## array Radiation pattern & array Factor

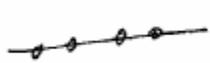


is determined by the type of individual elements used, their orientations, their positions in space & amplitude & phase of the current feeding them.



by considering, each element  $\rightarrow$  isotropic point source  $\rightarrow$  the resulting radiation pattern is called (Array factor).

AF



then

the array RP can be obtained by pattern multiplication

Total pattern = one element pattern  $\times$  Array factor

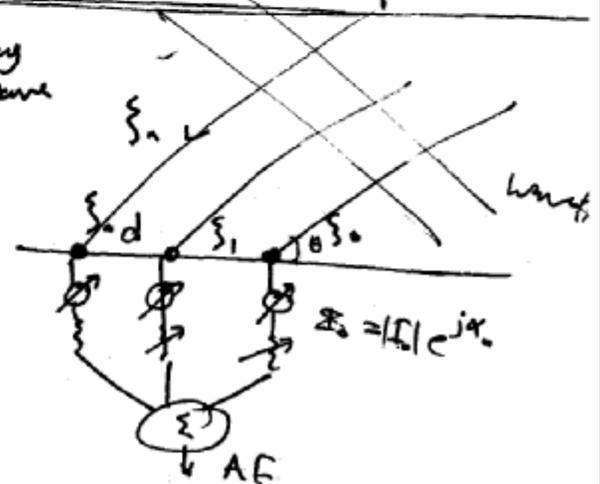
$\beta n d \cos \theta = \xi_n =$  spatial phase shift delay of incoming plane wave

$$AF = \sum_{n=1}^N I_n e^{j \xi_n}$$

$$= \sum_{n=1}^N |I_n| e^{j \xi_n} e^{j \alpha_n}$$

$$\xi_n = \xi_n + \alpha_n$$

$I_n =$  Complex currents of elements with amplitude ( $|I_n|$ ) phase ( $\alpha_n$ )

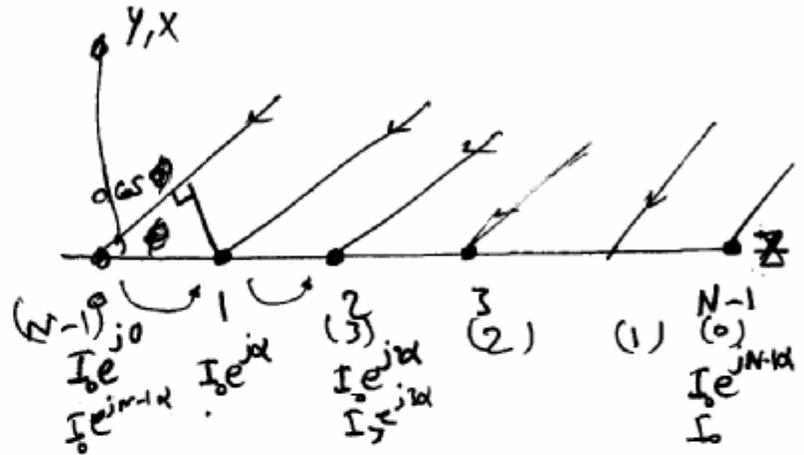


Uniform Linear, equally spaced array

the Array factor

① uniform array: the elements current amplitudes are identical

$$I_0 = I_1 = \dots = I_{N-1} \quad \text{(uniform N element array)}$$

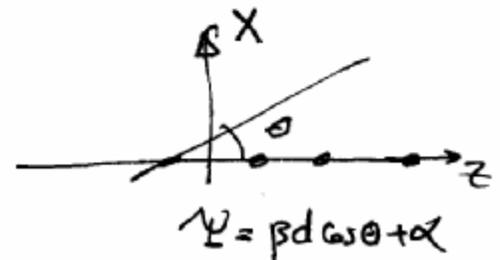


"Equally spaced Linear array" of isotropic point source

②  $\alpha$ : the phase shift between each ~~element~~ two successive elements is  $\alpha$ : "inter-element phase shift"

$$\Psi = \text{total phase shift} = \beta d \cos \theta + \alpha$$

(due to spacing between element)      (due to element current phase shift)



$$AF = \sum_{n=0}^{N-1} I_n e^{jn(\beta d \cos \phi + \alpha)} = \sum_{n=0}^{N-1} I_n e^{jn\psi}$$

$$AF = \sum_{n=0}^{N-1} I_n e^{jn\psi} = I_0 (1 + e^{j\psi} + \dots + e^{j(N-1)\psi}) \quad \text{--- } \textcircled{1}$$

$$AF e^{j\psi} = I_0 (e^{j\psi} + \dots + e^{jN\psi}) \quad \text{--- } \textcircled{2}$$

subtracting  $\textcircled{2}$  from  $\textcircled{1}$

$$\therefore AF (1 - e^{j\psi}) = I_0 (1 - e^{jN\psi}) \quad \checkmark$$

$$\therefore AF = \frac{I_0 (1 - e^{jN\psi})}{(1 - e^{j\psi})} = \frac{I_0 e^{j\frac{N\psi}{2}} (e^{j\frac{N\psi}{2}} - e^{-j\frac{N\psi}{2}})}{e^{j\frac{\psi}{2}} (e^{j\frac{\psi}{2}} - e^{-j\frac{\psi}{2}})}$$

$$AF = I_0 e^{j\frac{(N-1)\psi}{2}} \frac{\sin \frac{N\psi}{2}}{\sin \frac{\psi}{2}} \Rightarrow \boxed{I_0 \frac{\sin \frac{N\psi}{2}}{\sin \frac{\psi}{2}}}$$

the phase factor could be neglected  
(unless the array o/p is combined with another array)

$$AF = \frac{I_0 \sin \frac{N\psi}{2}}{\sin \frac{\psi}{2}}$$

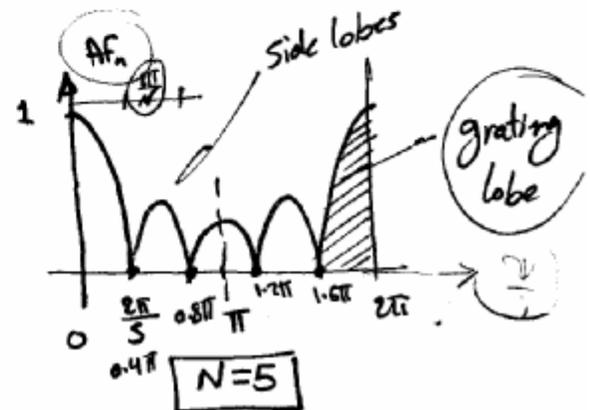
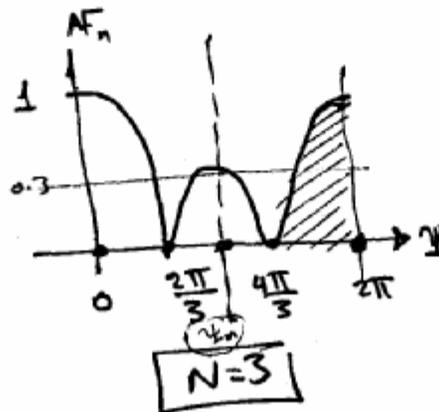
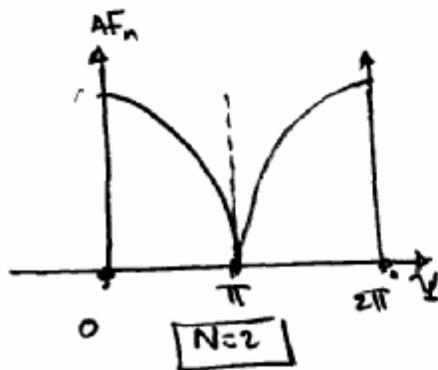
the expression is maximum for  $\psi = 0$  & the maximum value =  $I_0 N$

$$AF = \sum_{n=0}^{N-1} I_0 (1 + 1 + \dots + 1) = NI_0$$

$$\therefore f(\psi) = AF|_n = \frac{\sin \frac{N\psi}{2}}{N \sin \frac{\psi}{2}}$$

normalized array factor for an N element, uniform equally spaced array UESLA

Examples



The AF( $\psi$ ) has the following properties

① AF is symmetrical around the Lines  $\psi=0$  &  $\psi=\pi$   
& repeated each  $2\pi$

② Maximum at  $\psi=0$  &  $2\pi$

③ the secondary maxima of AF occurs at  $\psi_m$  where  $\sin\left(\frac{N\psi_m}{2}\right) = 1$

ie. the first secondary maximum occurs at  $\frac{N\psi_m}{2} = \pm\left(\frac{2k+1}{2}\right)\pi$   $k=1,2,3,\dots$

$\psi_{m1} \Rightarrow \frac{\psi_{m1}}{2} = \frac{3\pi}{2N}$   $\psi_{m1} = \frac{3\pi}{N}$

④ the AF Zeros at  $\psi_z$

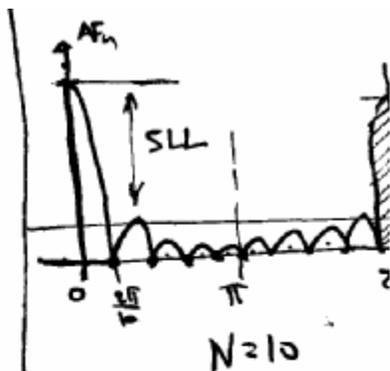
1st zero at  $\psi_{z1} = \frac{2\pi}{N}$

$\therefore \frac{N\psi_z}{2} = \pm k\pi$   $k=1,2,3,\dots$

$\psi_z = \pm \frac{2k\pi}{N}$

⑤ AS N increases, the main lobe narrows

⑥ AS N  $\sim$ , there are more side lobes & SLL decrease



$SLL = \frac{|\text{max. value of largest SL}|}{|\sim \sim \sim \text{main lobe}|}$

N=5	12db
N=20	13db
N >>	13.3db

~~fact~~

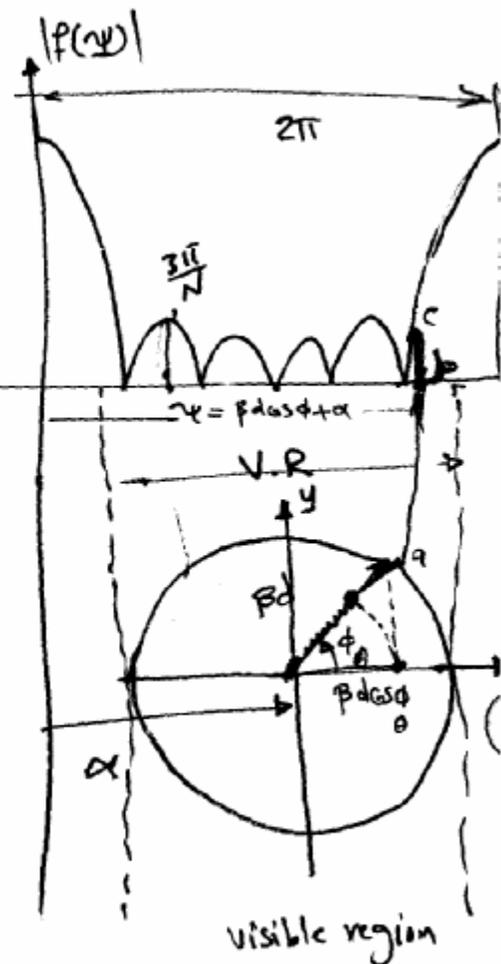
Construction technique to find the AF as a function of  $(\theta$  or  $\phi$ ) polar angle

(3)

- ① AF plot  $|F(\psi)|$ .
- ② draw a circle of radius  $\psi = \beta d$  below the  $F(\psi)$  with its center at  $(\psi = \alpha)$  below
- ③ drop ~~the~~ lines from zeros & peaks to intersect the circle

note as  $\theta \rightarrow 0 < \phi < \pi$   
 $\alpha - \beta d \leq \psi < \alpha + \beta d$

- ④ Connect these line to the circle center and draw the equivalent value of the AF.



the Visible region

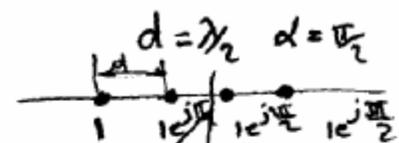
the region of interest of the AF is known as

$0 < \phi < \pi$   
 $\alpha - \beta d < \psi < \alpha + \beta d$

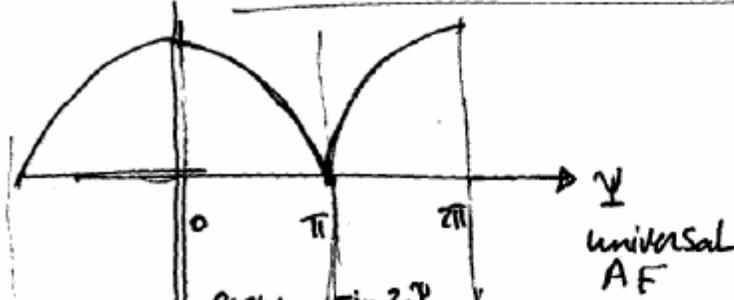
$N = 7$   
 $\alpha = 0$   $d = \frac{\lambda}{2}$

Example 8

plot the polar plot of the AF for two elements ~~separated by~~  $2$   $4$  elements UESLA



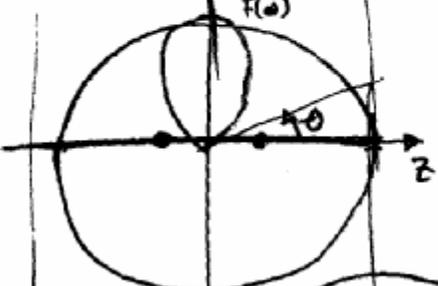
(a)



$$f(\psi) = \frac{\sin \frac{2\psi}{2}}{2 \sin \frac{\psi}{2}} = \cos \frac{\psi}{2}$$

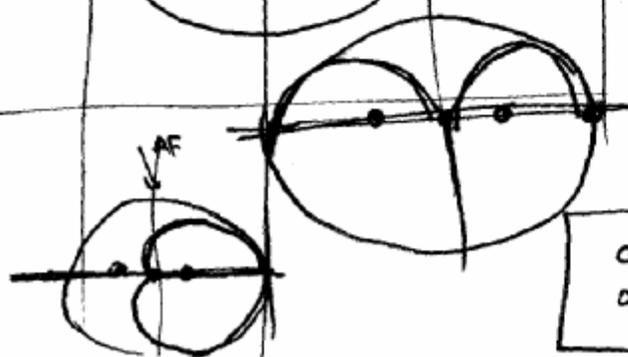
universal AF

(b)



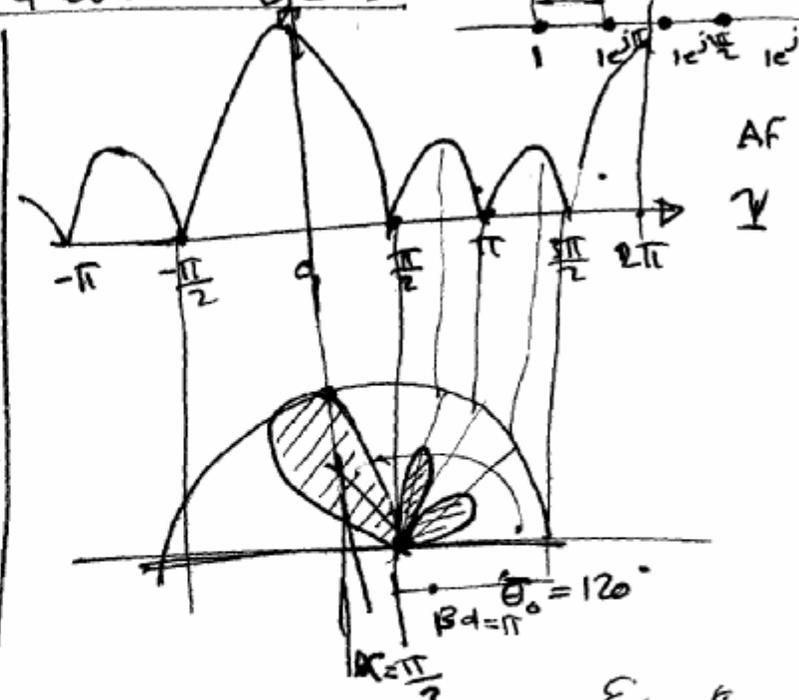
$$d = \lambda/2 \quad \beta d = \pi \\ \alpha = 0 \quad \text{Example 1}$$

(c)



$$d = \lambda/2 \quad \beta d = \pi \\ \alpha = \pi$$

$$d = \lambda/4 \quad \beta d = \pi/2 \\ \alpha = -\pi/2$$



$$N = 4$$

$$d = \lambda/2 \\ \alpha = \pi/2$$

Example  
 $N = 5$   
 $\alpha = \pi/11$   
 $d = \lambda$

# Broad side & End fire Arrays

4

## Case 1: Broadside Arrays

all array elements are in phase

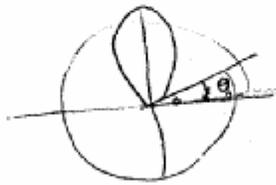
$$\alpha = 0$$

Broadside

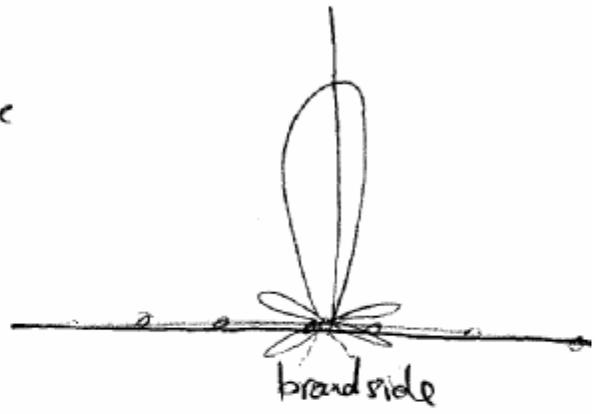
the principal maximum occurs at  $\psi = 0$

$$\& \text{ thus } \beta d \cos \theta_m = 0$$

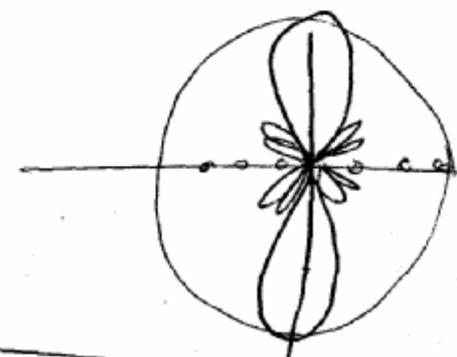
$$\theta_m = \pm \frac{\pi}{2}$$



which means main lobe  $\perp$  array line



Example  $N=6$   
 $\alpha=0$   
 $d=0.5$



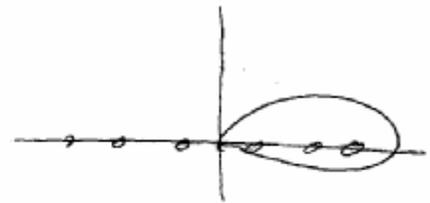
Case II: End fire Array

$$\alpha = -\beta d$$

End fire array

$$\Psi = 0 \quad \beta d \cos \theta_m - \beta d = 0$$

$$\theta_m = 0$$



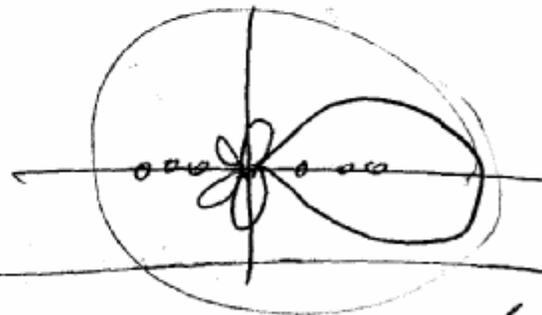
the main lobe in the direction of Array line

Example

$$N = 6$$

$$\alpha = -0.5\pi$$

$$d = 0.2\lambda$$



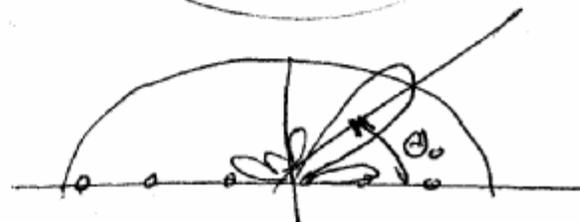
Case II: Electronic Scanning Array

Electronic phase shift

$$\alpha = -\beta d \cos \theta_0$$

$$\Psi = \beta d \cos \theta_0 + \alpha = 0$$

$$\theta_0 = \cos^{-1} \frac{-\alpha}{\beta d}$$



Example

① Main lobe direction

$$AF_n = \frac{\sin \frac{N\psi}{2}}{N \sin \frac{\psi}{2}}$$

$$\psi = \alpha + \beta d \cos \theta$$

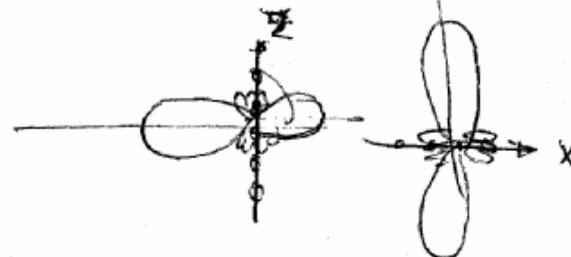
has 2 main lobes

Main lobe @  $\psi = 0$

① broadside Array

$$0 = \psi = \alpha + \beta d \cos \theta_m$$

$$\theta_m = 90^\circ \quad \therefore \boxed{\alpha = 0} \quad \text{all elements have the same phase}$$

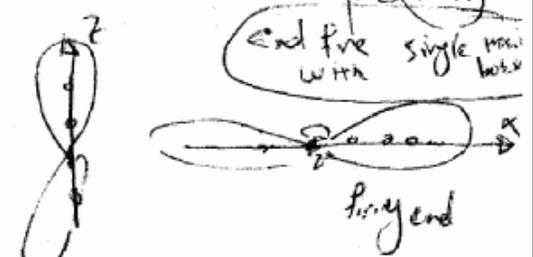


Pan beam

② End fire array in one direction

$$\theta_m = 0 \quad \psi = \alpha + \beta d \cos 0 \Rightarrow \boxed{\alpha = -\beta d}$$

or Pan  
End fire with single main lobe



single end

③ Electronic scanning array

$$\boxed{\alpha = -\beta d \cos \theta_0}$$

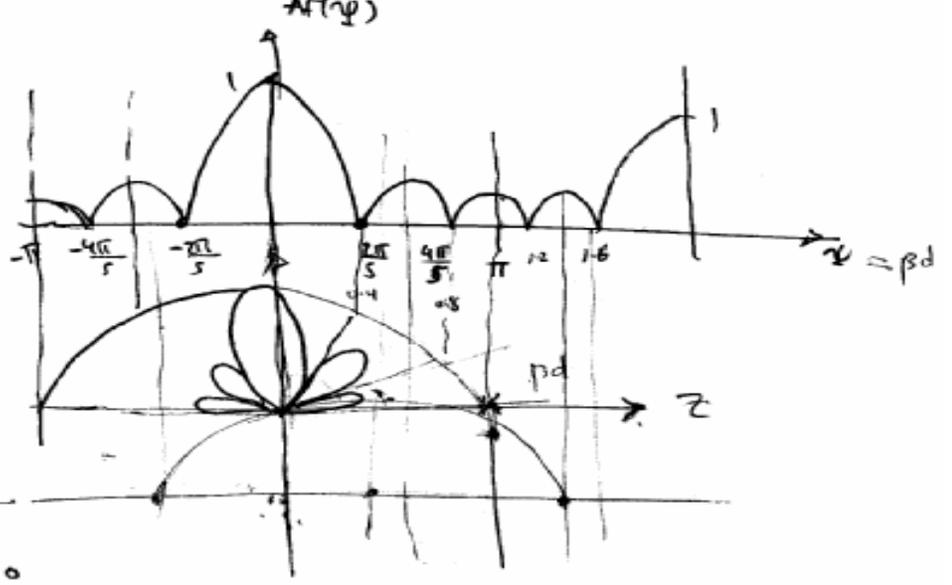


(i)  $d = \lambda/2$   
 $\beta d = \frac{2\pi}{\lambda} \frac{\lambda}{2} = \pi$

$\alpha = 0$

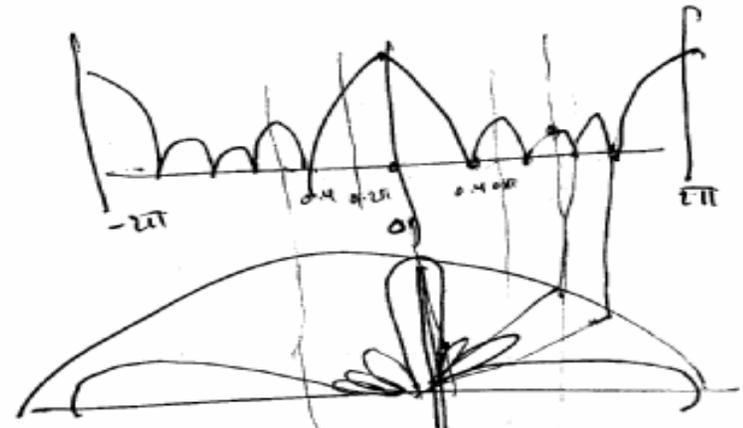
broadside

$\psi = \alpha + \beta d \cos \theta = 0$   
 $\alpha = 0$   
 $\theta = 90^\circ$



(ii)

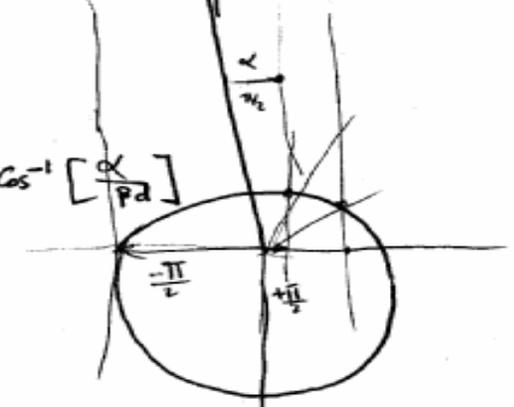
$d = \lambda \Rightarrow \beta d = 2\pi$   
 $\alpha = 0$



(iv)

$\theta_0 = 45^\circ$   
 $d = \lambda/2 \quad \beta d = \pi$   
 $\psi = \alpha + \beta d \cos \theta = 0$   
 $\alpha = -\beta d \cos \theta = -\frac{\pi}{\sqrt{2}}$   
 $\alpha = -\frac{\pi}{\sqrt{2}}$

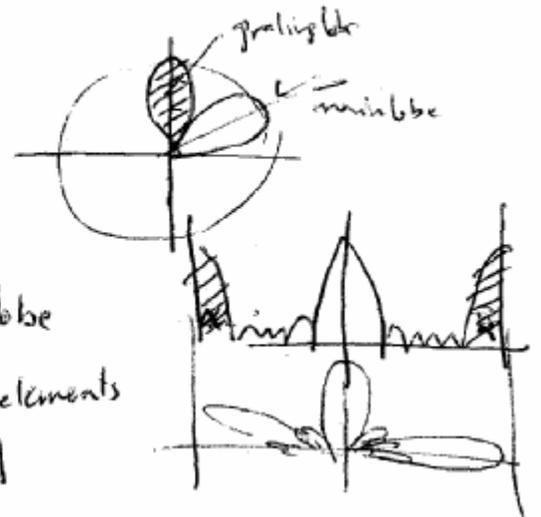
$\theta = \cos^{-1} \left[ \frac{\alpha}{\beta d} \right]$



## ② grating lobe :

\* has the same mag. as the main lobe  
but in an undesired direction.

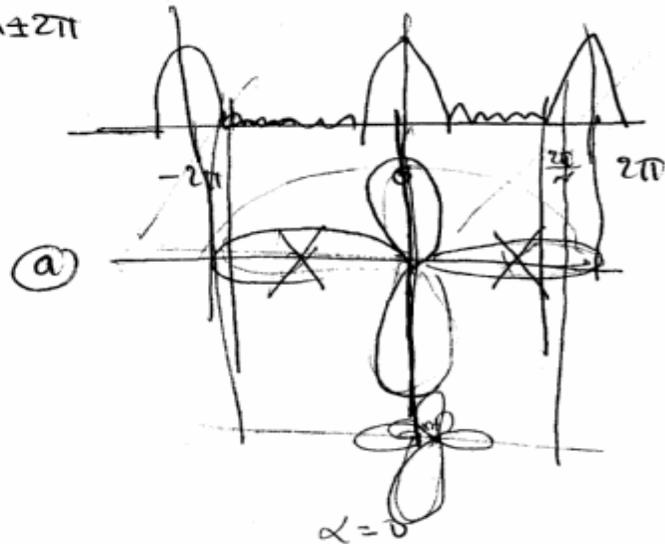
\* ~~to get rid of the grating lobe~~ to get rid of the grating lobe  
the distance between elements  
should be decreased



## ② Grating lobe

(2)

- ① if  $d > \lambda/2$  G.L. appears
- ②  $AF(\Psi)$  is repeated each  $\pm 2\pi$   
thus to eliminate the grating lobes  
the special cases



### ① broadside array

① No grating lobes

$$|\Psi| \leq 2\pi - \frac{2\pi}{N}$$

inlet one side

$$\therefore \frac{2\pi d}{\lambda} \leq 2\pi \left(1 - \frac{1}{N}\right)$$

eliminate grating lobe

$$d \leq \left(1 - \frac{1}{N}\right) \lambda$$

broadside array with

for  $N \gg 1$   $d \leq \lambda$

② Sometimes, we can allow a portion of the grating lobe, usually half of its width in  $\Psi$  units,

$$|\Psi| \leq 2\pi - \frac{\pi}{N}$$

eliminate half grating lobe

$$d \leq \left(1 - \frac{1}{2N}\right) \lambda$$

Max. allowed portion of GL

lobe

(b) endfire array  $\Rightarrow$  fan  $d = \frac{\lambda}{2}$   $N = 5$   
 $\alpha = \frac{\pi}{2}$   $\beta d = \pi$

(1) no grating lobe (single beam):

just one main beam

$$\alpha + \beta d = 2\pi - \frac{2\pi}{N} = 1.6\pi$$

$$\alpha - \beta d = 0 \Rightarrow \alpha = \beta d$$

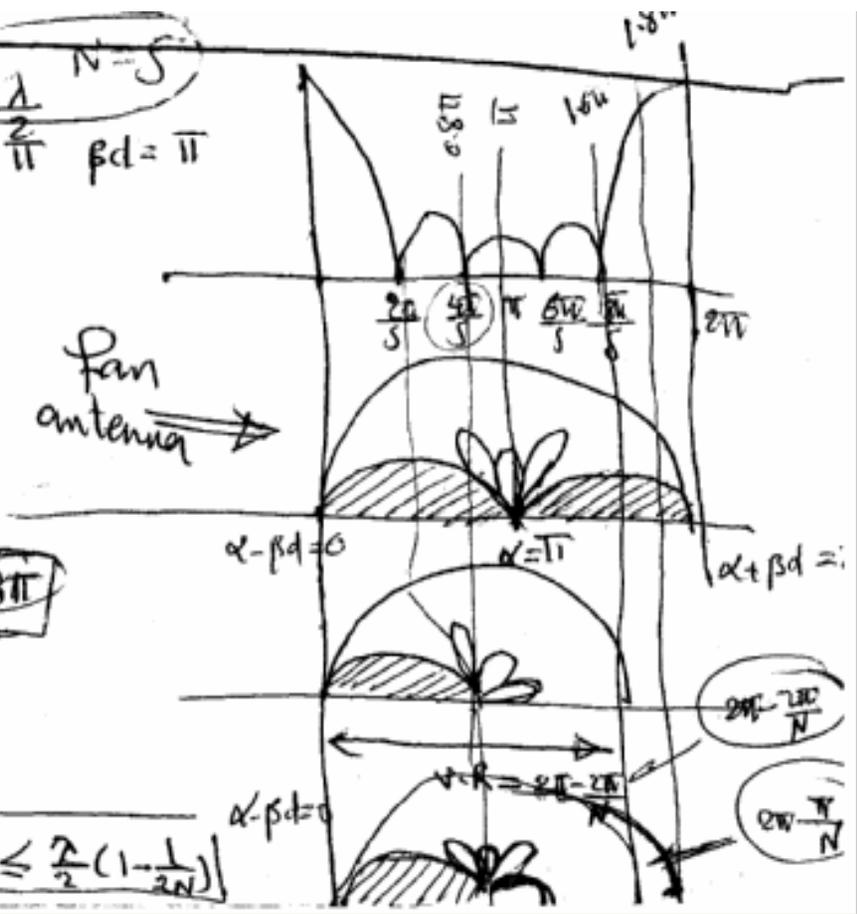
v.r  $\approx 2\beta d = 0.8\pi$   $\Rightarrow \alpha = \beta d = 0.8\pi$

in general

$$2\beta d \leq 2\pi - \frac{2\pi}{N}$$

$$\Rightarrow d \leq \frac{\lambda}{2} \left(1 - \frac{1}{N}\right)$$

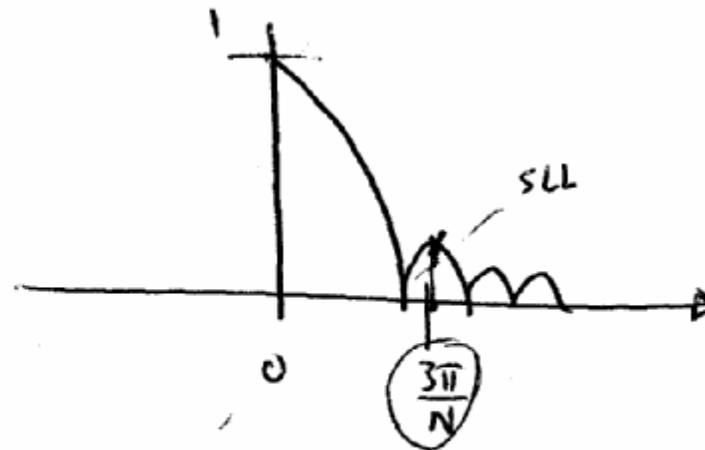
(2) half grating lobe:  $d \leq \frac{\lambda}{2} \left(1 - \frac{1}{2N}\right)$



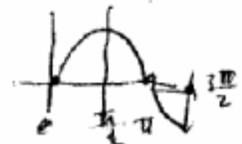
③ Side Lobe level

$$PSLL \Big|_{\gamma = \frac{3\pi}{N}} = \left| \frac{\sin \left( N \cdot \frac{3\pi}{N} / 2 \right)}{N \sin \left( \frac{3\pi}{N} / 2 \right)} \right|$$

$$PSLL \Big|_{\gamma = \frac{3\pi}{2}} = \frac{1}{N \sin \left( \frac{3\pi}{2N} \right)}$$



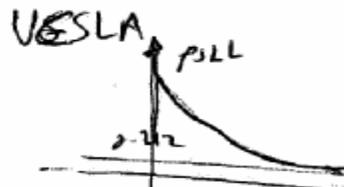
$$\sin \frac{\pi}{2} = 1$$



Minimum Limit of PSL ( $N \rightarrow \infty$ )

$$\lim_{N \rightarrow \infty} \frac{1}{N \sin\left(\frac{3\pi}{2N}\right)} = \frac{1}{N \frac{3\pi}{2N}} = \frac{2}{3\pi} = \boxed{0.212}$$

$$= 20 \log 0.212 = \boxed{-13.5 \text{ dB}}$$



Example

$N=3$	0.333
$N=4$	0.272
$N=5$	0.247

Side lobe level doesn't exceed a certain level

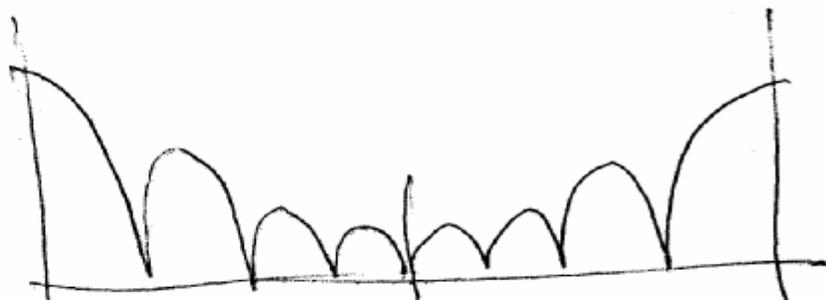
<del>0.282</del>	0.282
$N=4$	

or

-11 dB

$$10^{-11/20} = 0.282$$

From table  $N=4$

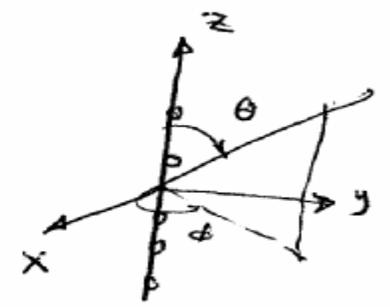


# (4) Pattern multiplication

Arrays Along z axis

$$\Psi = \alpha + \beta d \cos \theta$$

$$AF(\theta, \phi) = AF(\theta)$$

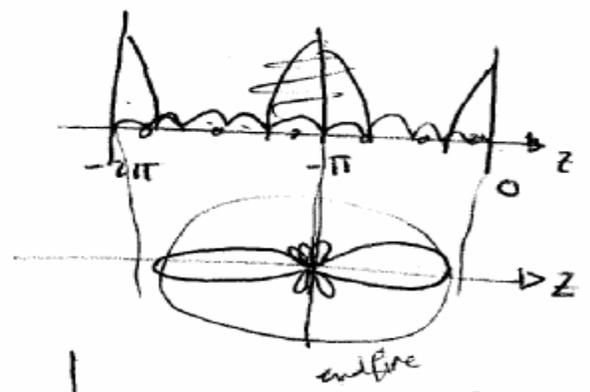
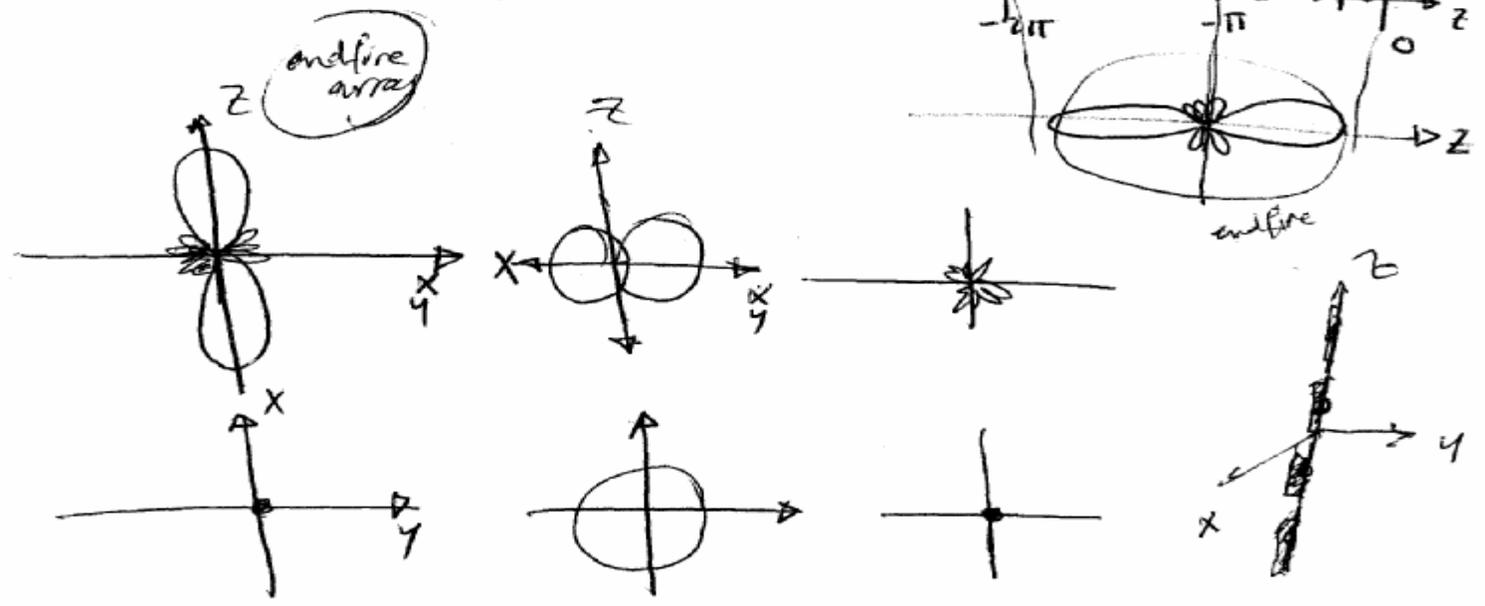


Example (1)  
ordinary Endfire

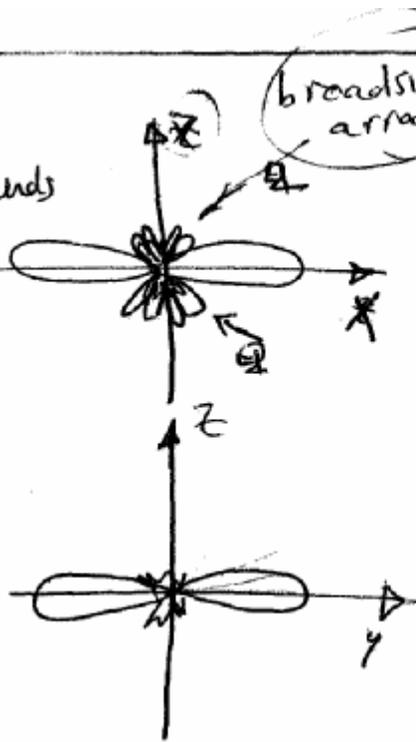
$$N=10 \quad d=\lambda/2 \quad \alpha = -\pi$$

Array Line z axis  
short dipole oriented to z axis

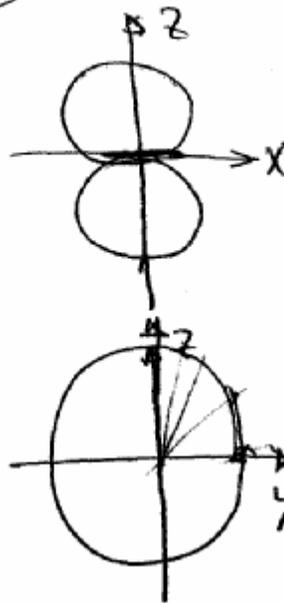
draw the total pattern



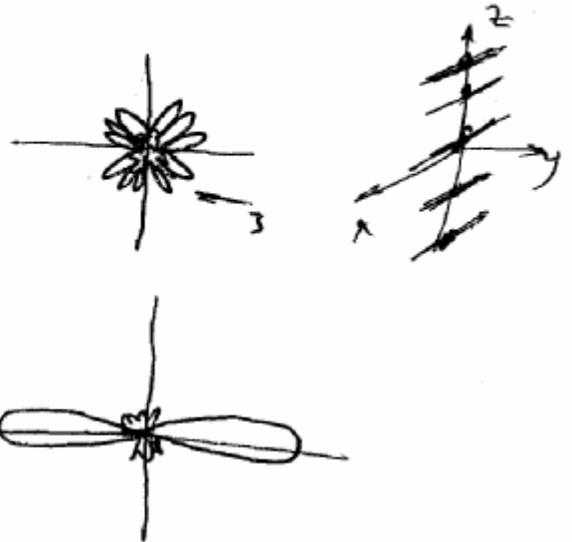
broad side  
short dipole towards  
X direction



broadside array



short dipole towards X



Design ~~array~~ array with  $\theta_0 = 45^\circ$  No GL  $PSL \leq 0.24$  N  
 &  $2\lambda$  dipoles oriented along  $y$  axis & arrayline in  $z$  direction

$$\alpha = -\beta d \cos \theta_0 = \frac{-\beta d}{\sqrt{2}} \Rightarrow \checkmark \quad \text{not given } d??$$

$$PSLL = \frac{1}{N \sin \frac{\pi}{2N}} \leq 0.24$$

N	PSL
5	0.247
6	0.235

$$\therefore N = 6$$

~~array~~



$$\left| \frac{V_{\max}}{V_{\min}} \right| = \left| 2\pi - \frac{2\pi}{6} \right| = 1.666\pi = \frac{5\pi}{3}$$

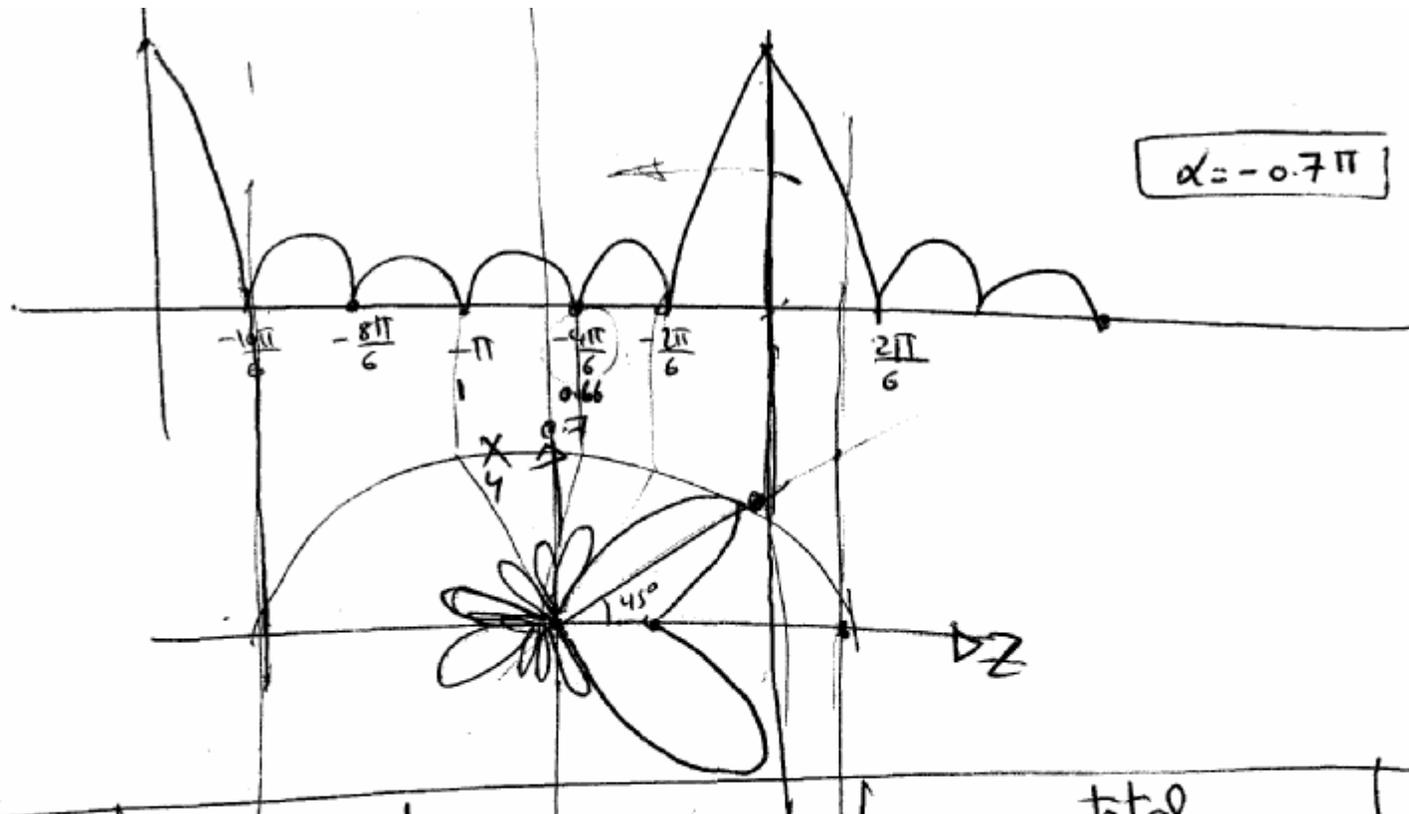
$$\alpha + \beta d = 0$$

$$\alpha - \beta d =$$

$$-\beta d \left( 1 + \frac{1}{\sqrt{2}} \right) = \frac{5\pi}{3}$$

$$\therefore \beta d = \frac{0.98\pi}{2\pi} \lambda$$

$$\therefore d = \lambda (0.49)$$



	$\epsilon_1$	AF	total
X-Y			
X-Z			