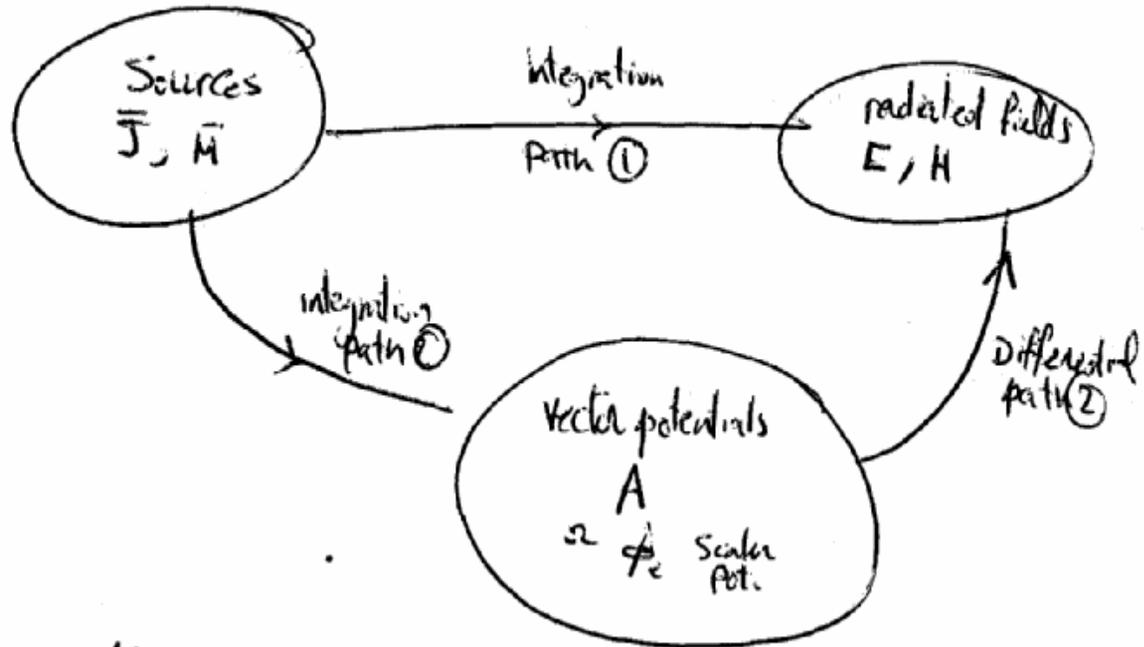


(10) 7 jobs

(1)

Radiation integrals & auxiliary potential functions

How to find
 E & H
fields radiated
from antennas



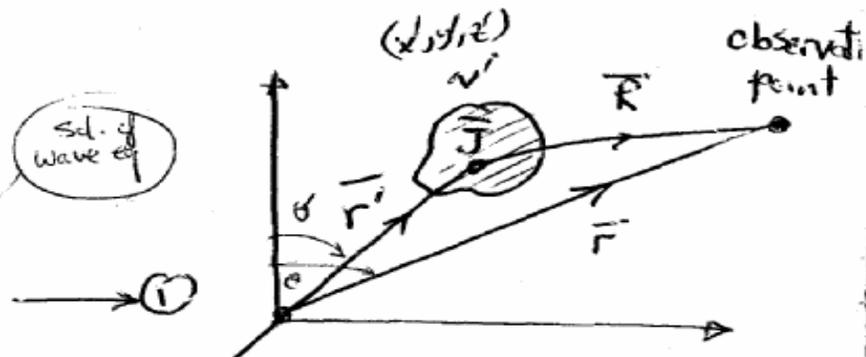
« Computing radiated field from electric & magnetic sources »

auxiliary potential functions

\vec{A} = magnetic vector potential fn.

ϕ_e = electric scalar ~ ~ ~

$$\vec{A}(\vec{r}, t) = \frac{\mu}{4\pi} \iiint_V \vec{J}(\vec{r}', t') \frac{e^{-jkR}}{R} dv'$$



$\nabla(\vec{A} + k^2 \vec{A} = -\vec{J})$ From the solution of inhomogeneous vector potential WE

$$\vec{R} = \vec{r} - \vec{r}'$$

where \vec{J} is the electric current density (A/m²)

R is the distance from any point in the source to the observation point.

$k = \omega\sqrt{\mu\epsilon}$ propagation constant (phase shift) = $\frac{\omega}{c}$

r' (origin to source)

r = (origin to observation)

$$\vec{H} = \frac{1}{\mu} \nabla \times \vec{A} \quad \text{--- (2)}$$

$$\vec{E} = -\nabla \phi_e - j\omega \vec{A}$$

Lorentz condition
 $\nabla \cdot \vec{A} = -j\omega\mu \phi_e$

$$\vec{E} = -j\omega \vec{A} - j \frac{1}{\omega\mu\epsilon} \nabla(\nabla \cdot \vec{A}) \quad \text{--- (3)}$$

$$\phi_e = -\frac{1}{j\omega\mu\epsilon} (\nabla \cdot \vec{A}) \quad \text{--- (3')}$$

$$\nabla \times \vec{E} = \nabla \times (-j\omega \vec{A}) = -j\omega \nabla \times \vec{A} = -j\omega \mu \vec{H}$$

once \vec{A} is known \vec{H} can be found from (2) & \vec{E} from (3)

($\vec{A} = ??$ How to find \vec{A} from \vec{J})

②

$$\bar{A}(r) = \frac{\mu}{4\pi} \iiint_V \bar{J}(r') \frac{e^{-jkR}}{R} dv'$$

$$\bar{A}(r) = \frac{\mu}{4\pi} \iint_S \bar{J}_s(r') \frac{e^{-jkR}}{R} ds'$$

(if \bar{J} represent linear densities)

A/m

$$\bar{A}(r) = \frac{\mu}{4\pi} \oint_C \boxed{\bar{I}_e(x', y', z')} \frac{e^{-jkR}}{R} dl'$$

$\bar{I}_e(r')$

(if \bar{I}_e electric current (A))

★ Far Field Radiation

could be neglected
the 2nd term in (3) $\propto \frac{1}{r^2}$

$$\left\{ \begin{array}{l} E_r \approx 0 \\ E_\theta = -j\omega A_\theta \\ E_\phi = -j\omega A_\phi \end{array} \right.$$

$$\Rightarrow \boxed{E_{ff} = -j\omega \bar{A}} \rightarrow (4)$$

(\perp trans. components only)

$$\boxed{\begin{array}{l} A_r = A_z \cos\theta \\ A_\theta = -A_z \sin\theta \end{array}}$$

$$\boxed{H_{ff} \approx \frac{\hat{r}}{\eta} \times E_{ff} = -j\frac{\omega}{\eta} \hat{r} \times \bar{A}} \rightarrow (5)$$

$$H_\phi = \frac{E_\theta}{\eta}$$

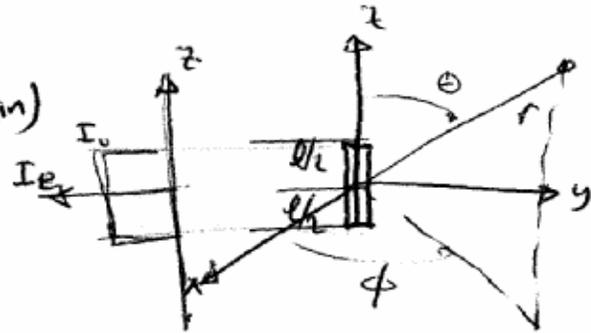
$$H_\theta = -\frac{E_\phi}{\eta}$$

for θ & ϕ components only
 $H_r \approx 0$

Radiation from an infinitesimal dipole

(3)

elementary
 ideal electric dipole ($l \ll \lambda$)
 Hertzian
 oriented along z-direction (center at origin)
 the current is assumed to be constant

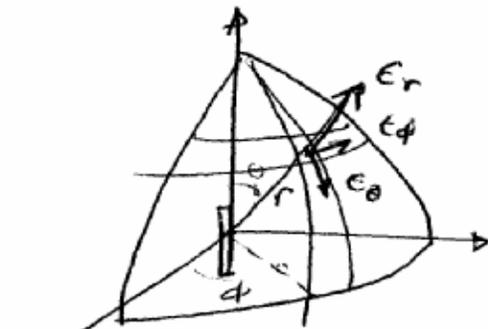


$$I(z') = I_0 \hat{z}$$

Constant

* radiated field (everywhere ~~except at the source~~ at the source)

- ① Find \vec{A} (from integral) $\rightarrow (\vec{J})$
- ② Find \vec{E} & \vec{H} (① & ②)
Parallel to ① & ②



$$A(x, y, z) = \frac{\mu_0}{4\pi} \int_C I_0(x', y', z') \frac{e^{-j\beta R}}{R} dz'$$

$$\int_C ds \vec{J} = \int_C du = \int_C dz$$



- where
- (x, y, z) : observation point coordinates
 - (x', y', z') : represents the coordinates of the source
 - R : distance from any point on the source to the observation pt.
 - path C : along the length of the source

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} = \sqrt{x^2 + y^2 + z^2}$$

$$\therefore x' = y' = z' = 0 \quad (\text{infinitesimal dipole})$$

$$R = r = \text{constant}$$

$$dl' = dz'$$

$$\therefore \bar{A}(x, y, z) = A_z \hat{z} = \hat{z} \frac{\mu I_0}{4\pi r} e^{jkr} \int_{-l/2}^{l/2} dz' = l$$

$$A_z = \frac{\mu I_0 l e^{-jkr}}{4\pi r}$$



$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta \cos\phi & \sin\theta \sin\phi & \cos\theta \\ \cos\theta \cos\phi & \cos\theta \sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Annotations: $A_x \rightarrow$ zero, $A_y \rightarrow$ zero, $A_z \rightarrow$ zero for this problem. (4)

(spherical to rectangular transformation)

$A_r = A_z \cos\theta$	$= \frac{\mu I l e^{-jkr}}{4\pi r} \cos\theta$	\rightarrow (6)
$A_\theta = -A_z \sin\theta$	$= -\frac{\mu I l e^{-jkr}}{4\pi r} \sin\theta$	\rightarrow (7)
$A_\phi = 0$	$= 0$	

Now \vec{H} & \vec{E} can be found from \vec{A}

from ①

$$\vec{H} = \frac{1}{\mu} \nabla \times \vec{A}$$

$$= \frac{1}{\mu} \cdot \frac{1}{r^2 \sin \theta} \begin{vmatrix} \hat{r} & r \hat{\theta} & r \sin \theta \hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin \theta A_\phi \end{vmatrix}$$

$$\therefore A_\phi = 0$$

$$\& \frac{\partial}{\partial \phi} = 0$$

no variation in the azimuth
(isotropic)

$$\vec{H} = \hat{\phi} \frac{1}{\mu r} \left[\frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right]$$

substituting

$$H_r = H_\theta = 0$$

$$H_\phi = j \frac{k I_0 \rho \sin \theta}{4\pi r} \left[1 + \frac{1}{jkr} \right] e^{-jkr}$$

⑨

Pr. (3)

$$\vec{E} = \frac{1}{j\omega\epsilon} \nabla \times \vec{H} = \frac{1}{j\omega\epsilon} \frac{1}{r^2 \sin\theta}$$

$$\begin{vmatrix} \hat{r} & \hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ H_r & H_\theta & r\sin\theta H_\phi \end{vmatrix}$$

$\frac{k}{\omega\epsilon} = \eta$
Wave impedance
 $\approx 120\pi$
 $\approx 377\Omega$

$$E_r = \frac{1}{j\omega\epsilon} \frac{1}{r\sin\theta} \frac{\partial}{\partial \theta} (\sin\theta H_\phi)$$

$$E_r = \eta \frac{I_0 l \cos\theta}{2\pi r^2} \left[1 + \frac{1}{jkr} \right] e^{-jkr}$$

→ (6)

(5)

$$E_{\theta} = \frac{-1}{j\omega\epsilon_0} \frac{1}{r} \frac{\partial}{\partial r} (r H_{\phi})$$

prove

$$\therefore E_{\theta} = j\eta \frac{K I d \sin\theta}{4\pi r} \left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2} \right] e^{-jkr} \quad (6')$$

$$E_{\phi} = 0$$

Far field approximation (Radiation far field)

$$\frac{2\pi}{\lambda} \leftarrow kr \gg 1 \text{ (very large)}$$

$$r \gg \lambda$$

then all terms having inverse power of $kr \sim (kr)^2$ are small compared to unity

$$\begin{aligned} \vec{E} &= j \frac{k I_0 R \sin\theta}{4\pi r} e^{-jkr} \hat{\theta} \\ \vec{H} &= j \frac{k I_0 R \sin\theta}{4\pi r} e^{-jkr} \hat{\phi} \end{aligned}$$

note
 $E_{\phi} = \frac{E_{\theta}}{\eta}$

behaves
as
plane wave

$$e^{j\omega t}$$

$$|E| \propto \frac{1}{r} e^{j(\omega t - kr)}$$

$$E = \frac{E_0}{r} e^{-jkr} \sin\theta$$

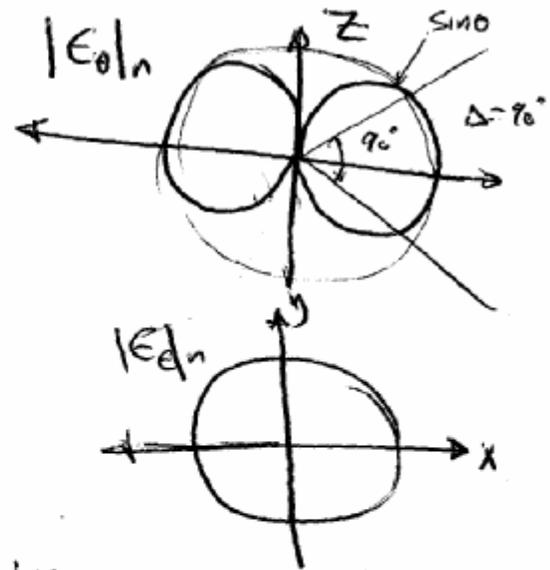
$$|E| = \frac{|E_0|}{r} \sin\theta$$

and the normalized value of $|E|$ at a given distance r is given by

$$E_n(\theta, \phi) = \sin\theta$$

$$S_{av} = \frac{1}{2} \text{Re}(E \times H^*) = \hat{r} \frac{1}{2\eta} |E_0|^2$$

$$S_{av} = (\hat{r} \eta/2 (E_0^2/4\pi)) \sin^2\theta/r^2$$



$$S_{av} = S_0 \sin^2\theta, \quad U_{av} = r^2 S_{av}$$

prove Beam width = 90°

Directivity = 3/2

prove that

$$w = r^2 S_{av}$$

$$mx = A_0$$

$$D_0 = 3/2$$

average radiated power from ideal dipole

①

$$P_{\text{rad}} = \oint_S \vec{S} \cdot \hat{n} da$$
$$= \int_0^{2\pi} \int_0^{\pi} S_r r^2 \sin\theta d\theta d\phi$$

$$S_r = \frac{1}{2} (\epsilon_0 H_\phi^* + \epsilon_\phi H_\theta^*) = \frac{1}{2} \epsilon_0 H_\phi^*$$

\swarrow
 ω

$$\therefore S_r(\hat{r}) = \frac{1}{2\eta} |\epsilon_0|^2 (\hat{r}) \quad \text{w/m}^2$$

$$S_r = \frac{1}{2} \left(\frac{I_0 l}{4\pi} \right)^2 \eta k^2 \frac{\sin^2\theta}{r^2}$$

$$\omega^2 \mu_0 \epsilon_0 = \omega/k$$

$$P_{\text{rad}} = \frac{I_0^2 l^2}{16\pi} \eta k^2 \left(\int_0^\pi \sin^3 \theta d\theta \right) \Rightarrow \left(\frac{4}{3} \right)$$

$$\therefore P_{\text{rad}} = \frac{I_0^2 l^2}{16\pi} \eta k^2$$

$$P_{\text{rad}} = 10 \cdot \underbrace{I_0^2 l^2}_{2I_{\text{rms}}^2} \cdot \underbrace{k^2}_{\left(\frac{2\pi}{\lambda}\right)^2} = \boxed{40 \pi^2 \frac{I_0^2 l^2}{\lambda^2}} \quad \downarrow \quad 2I_{\text{rms}}^2$$

$$P_{\text{rad}} = 80 \pi^2 I_{\text{rms}}^2 \left(\frac{l}{\lambda} \right)^2$$

Radiation Resistance

$$Z_{in} = R_{in} + jX_{in}$$



$$P_{rad} = I_{rms}^2 R_{rad} = \frac{1}{2} R_{rad} I_0^2$$

$$R_{rad} = 80 \pi^2 \frac{L^2}{\lambda^2} \Omega$$

ideal
Elementary
dipole

input impedance

the impedance presented by an antenna at its terminal
or the ratio of $\frac{V}{I}$ at the feed point

$$Z_{in} = R_{in} + jX_{in}$$

↓ ↓
resistance reactance

$$R_{in} = R_r + R_L$$

↓ ↘
radiation loss
resistance resistance

* Beam width = 90°

* Directivity = $3/2$

* $S_{av} = \int \sin^2 \theta$

$U_{av} = r^2 \int \sin^2 \theta$

Directivity

$S_{av} = \frac{1}{2} \text{Re} (\mathbf{E} \times \mathbf{H}^*) = \hat{r} \frac{1}{2\eta} |\mathbf{E}|^2$

average PFD

$S_{av} = \hat{r} \frac{\eta}{2} \left(\frac{k I_0 l}{4\pi} \right)^2 \frac{\sin^2 \theta}{r^2}$

radiation intensity

$U_{av} = r^2 S_{av} = \underbrace{\frac{\eta}{2} \left(\frac{k I_0 l}{4\pi} \right)^2}_{A_0} \sin^2 \theta$

$U_{av, \max} = A_0$

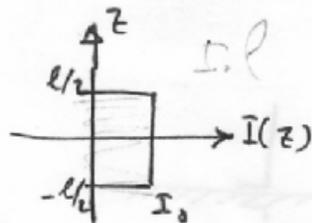
$\therefore D_0 = 3/2$

Linear dipoles & monopoles

Small (short) dipole

infinitesimal dipole $l \ll \lambda/50$:

- the current is constant (assumed)
- not realizable (mathematical quantity)



small dipole : $\lambda/50 \leq l \leq \lambda/10$

$$I_z(x', y', z') = \begin{cases} I_0 \left(1 - \frac{z}{l} z'\right) \hat{z} & 0 \leq z' \leq l/2 \\ I_0 \left(1 + \frac{z}{l} z'\right) \hat{z} & -l/2 \leq z' \leq 0 \end{cases} \quad \text{--- } \textcircled{1}$$

where $I_0 = \text{constant}$.

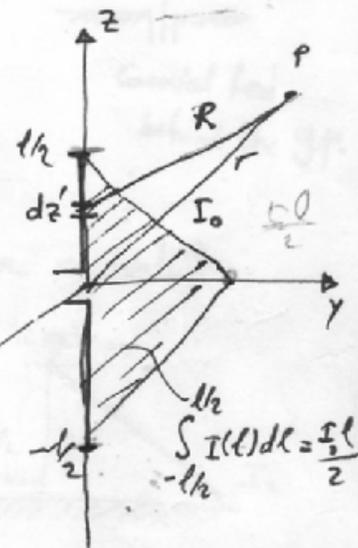
$$\vec{A}(x, y, z) = \frac{\mu}{4\pi} \left[\hat{z} \int_{-l/2}^0 I_0 \left(1 + \frac{z}{l} z'\right) e^{-jkR} \frac{dz'}{R} + \hat{z} \int_0^{l/2} I_0 \left(1 - \frac{z}{l} z'\right) e^{-jkR} \frac{dz'}{R} \right]$$

integral can be found directly from calculating the area under the curve.

$$\int = \frac{I_0 l}{2}$$

$\therefore l \leq \lambda/10$

$R \approx r$



$$\bar{A} = \hat{z} A_z = \hat{z} \left(\frac{1}{2} \right) \left[\frac{\mu I_0 l e^{-jkr}}{4\pi r} \right] \quad \rightarrow \textcircled{2}$$

$$A(r) = \frac{\mu}{4\pi r} \int I_0 e^{-jkr} d\tau$$

$\vec{H} = \nabla \times \vec{A}$ which is one half of that obtained for ideal dipole
 $\vec{E} = \nabla \times \vec{A} \times \hat{r}$ thus

E & H fields radiated by a small dipole as: (Far field $kr \gg 1$)

$$\vec{E} = -j\omega \vec{A}_T$$

$$H = \frac{E}{\eta}$$

$$E_\theta = j\omega \sin\theta A_z \hat{\theta}$$

$$A_\theta = -A_z \sin\theta$$

$$A_r = A_z \cos\theta$$

$$E_\theta = j\omega A_\theta$$

$$E_r = -j\omega A_r$$

$$\therefore E_\theta = j\eta \frac{k I_0 l e^{-jkr}}{8\pi r} \sin\theta$$

$$H_\phi = \frac{j k I_0 l e^{-jkr}}{8\pi r} \sin\theta$$

$$E_r = E_\phi = H_r = H_\theta \approx 0$$

$\rightarrow \textcircled{3}$

$$P_{rad} = \iint \vec{S}_r \cdot d\vec{a}$$

$$= \frac{1}{4} P_{rad}(\text{ideal dipole}) = \frac{1}{4} \left[40\pi^2 \left(\frac{l}{\lambda}\right)^2 I_0^2 \right]$$

small

$$P_{rad} = \frac{20\pi^2 \left(\frac{l}{\lambda}\right)^2 I_0^2}{10\pi^2}$$

$$R_r = \frac{2P_{rad}}{|I_0|^2} = \frac{20\pi^2 \left(\frac{l}{\lambda}\right)^2}{20} \quad \text{radiation resistance}$$

Monopoles (small dr monopole)

A monopole antenna is a dipole that has been divided in half at its feed point and fed against a ground plane.
(Center)

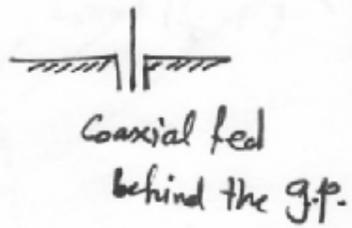
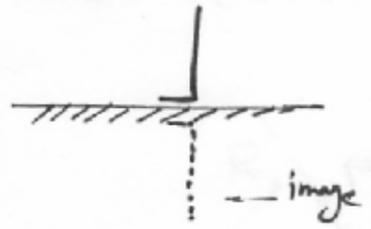
$$\text{input impedance} = Z_{in} = \frac{V_{in, \text{mono}}}{I_{in, \text{mono}}} = \frac{1/2 V_A}{I_A}$$

① $P_{rad, \text{mono}} = \frac{1}{2} P_{rad, \text{dip}}$

② $Z_{in, \text{mono}} = \frac{1}{2} Z_{in, \text{dipole}}$

Radiation resistance of monopole $R_{r, \text{mono}} = \frac{P_{\text{power}}}{\frac{1}{2} (I_A, \text{mono})^2}$

③ $R_{r, \text{mono}} = \frac{\frac{1}{2} P_{rad, \text{dipole}}}{\frac{1}{2} |I_A, \text{dip}|^2} = \frac{1}{2} R_{r, \text{dip}}$

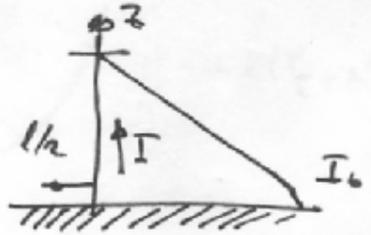
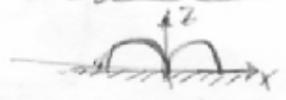


For short monopole with $l/2$, the P_{rad} is half of that for equivalent dipole as it radiates from the upper half only.

$$P_{rad} = \frac{30 \pi^2 (l/A)^2 I_0^2}{10}$$

$$R_r = \frac{10 \pi^2 (l/A)^2}{20}$$

$D_{\text{mono}} = \frac{U_{in}}{\frac{1}{2} P_{in}} = \frac{1}{2} D_{\text{dipole}}$

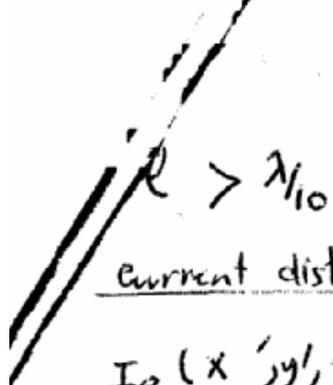


the effective length:

$$L_{\text{eff}} = \frac{1}{I_0} \int_{-l/2}^{l/2} I(z) dz$$

$$\begin{aligned} \therefore \text{short dipole} &= l/2 \\ \sim \text{monopole} &= l/4 \end{aligned}$$

Finite length dipole



$l > \lambda/10$

"(special attention to $\lambda/2$ dipole) important case study"

current distribution

$$I_e(x', y', z') = \begin{cases} \hat{z} I_0 \sin(k(\frac{l}{2} - z')) & 0 \leq z' \leq \frac{l}{2} \\ \hat{z} I_0 \sin(k(\frac{l}{2} + z')) & -\frac{l}{2} \leq z' \leq 0 \end{cases}$$

Linear approximation of short dipole is not

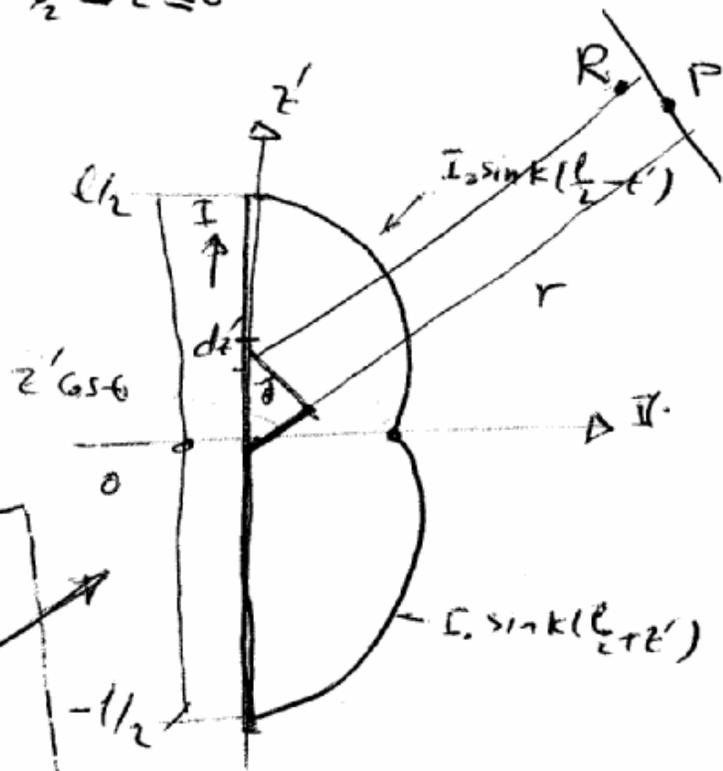
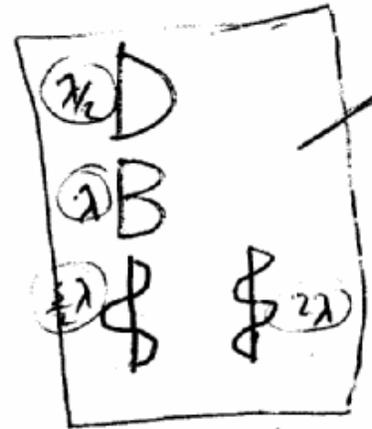
$\vec{A}(x, y, z) = A_z \hat{z}$

$$A_z(x, y, z) = \int_{-l/2}^0 \frac{\mu}{4\pi R} I_0 \frac{e^{-jkR}}{k} \sin(k(\frac{l}{2} + z')) dz' + \frac{\mu}{4\pi} \int_0^{l/2} I_0 \frac{e^{-jkR}}{R} \sin(k(\frac{l}{2} - z')) dz'$$

using the far field approximation

$R \approx r - z' \cos \theta$

$\frac{1}{R} \approx \frac{1}{r}$



$$\therefore A_z = \frac{4}{4\pi} I_0 \frac{e^{-jkr}}{r} \int_0^l \sin k \left(\frac{l}{2} + z' \right) e^{jkz' \cos \theta} dz'$$

$$+ \frac{4}{4\pi} I_0 \frac{e^{-jkr}}{r} \int_0^l \sin k \left(\frac{l}{2} - z' \right) e^{jkz' \cos \theta} dz'$$

Can be solved to give

$$A_z = \frac{4 I_0 e^{-jkr}}{2\pi r k} \left[\frac{\cos \left(\frac{kl}{2} \cos \theta \right) - \cos \left(\frac{kl}{2} \right)}{\sin^2 \theta} \right]$$

For field

$$\therefore E_\theta = j\omega \sin \theta A_z$$

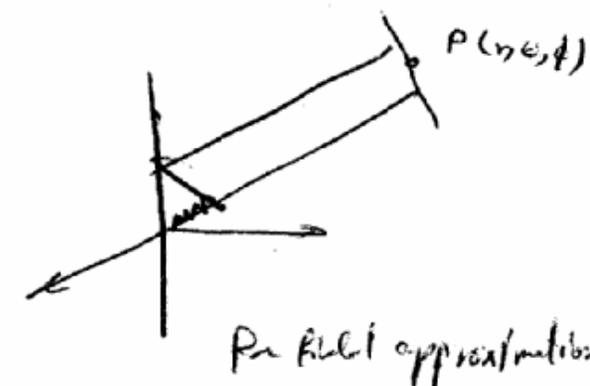
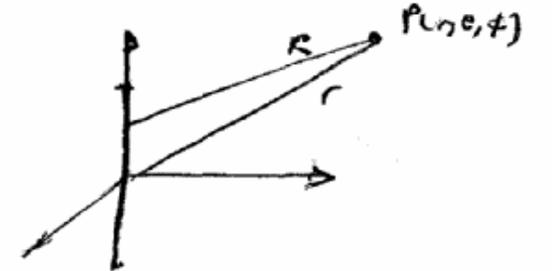
$$E_\theta = \frac{j\omega I_0 e^{-jkr}}{2\pi r} \left[\frac{\cos \left(\frac{kl}{2} \cos \theta \right) - \cos \left(\frac{kl}{2} \right)}{\sin^2 \theta} \right]$$

$$H_\phi = \frac{E_\theta}{\eta}$$

$$E_r = H_\theta = E_\phi = 0$$

$$S_{av} = \frac{1}{2} \text{Re} [E \times H^*] = \hat{r} \frac{1}{2\eta} |E_\theta|^2 \Rightarrow U = S_{av} \cdot r^2$$

$$\frac{30 I_0^2}{2\pi} \left[\frac{4 I_0^2}{8\pi^2} \left[\frac{\cos \left(\frac{kl}{2} \cos \theta \right) - \cos \left(\frac{kl}{2} \right)}{\sin^2 \theta} \right]^2 \right]$$



P.P. (21)
Figure 4.5
repeat on
the Computer

$l \ll r$
 $\rho = \lambda/4$
 $l = \lambda/2$
 $l = 5\lambda/11$

(C.S.N)

Half length dipole & $\lambda/4$ monopoles

(4)

For $l = \lambda/2$ $k l = \frac{\pi}{\lambda} \cdot \frac{\lambda}{2} = \frac{\pi}{2}$ $\cos \frac{k l}{2} = 0$

$$E_{\theta} = j \frac{60}{\lambda} I_0 \frac{e^{-jkr}}{r} \left[\frac{\cos\left(\frac{\pi}{2} \cos\theta\right)}{\sin\theta} \right]$$

$$H_{\phi} = \frac{E_{\theta}}{\eta}$$

$$S_{av} = \frac{\eta I_0^2}{8\pi^2 r^2} \left[\frac{\cos^2\left(\frac{\pi}{2} \cos\theta\right)}{\sin^2\theta} \right] \quad \text{W/m}^2 \quad \xi_w = \frac{E_{\theta}^2}{2\eta}$$

$$P_{rad} = \oint S_{av} \cdot d\Omega = \frac{\eta I_0^2}{8\pi^2 r^2} \int_0^{2\pi} \int_0^{\pi} \left[\frac{\cos^2\left(\frac{\pi}{2} \cos\theta\right)}{\sin^2\theta} \right] r^2 \sin\theta d\theta d\phi$$

1.2186

Numerically

$$P_{rad} = \frac{30 I_0^2}{2\pi} \cdot 2\pi \cdot 1.2186 = \frac{1}{2} I_0^2 (73)$$

$$\therefore \left[R_{rad} = 73 \Omega \text{ dipole } \lambda/2 \right]$$

$$\therefore \boxed{R_{rad} = 73 \Omega \quad \text{dipole } \lambda/2}$$

$$R_{rad} = 36.5 \Omega \quad \text{monopole } \lambda/2$$

note $Z_{in} = R_{in} + jX_{in} = \frac{V_{in}}{I_{in}}$
 lossless $\lambda/2$ dipole $R_{in} = R_r = 73 \Omega$ } could be matched to 75Ω coaxial cable easily

Directivity of $\lambda/2$ dipole & Gain

$$D = \frac{U_{max} \cdot 4\pi}{P_{rad}} = \frac{4\pi U_{max}}{I_0^2 R_{rad}/2} = \frac{30 I_0^2 \sin^2 \theta}{13 \cdot 73 \cdot \frac{1}{2}}$$

$$\therefore \boxed{D = 1.64 \neq 1} \\ = 2.15 \text{ dB}$$

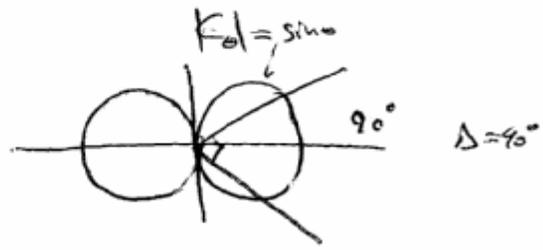
for $\lambda/4$ monopole

$$R_{rad} = 36.5$$

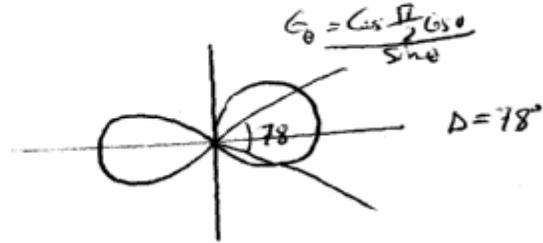
$$D_{max} = 2 D_{dipole} = \boxed{3.28} \\ = 5.15 \text{ dB}$$

Gain $\Rightarrow D$ (lossless)

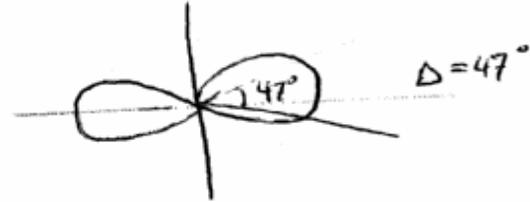
$$L = \frac{\lambda}{10}$$



$$L = \frac{\lambda}{12}$$



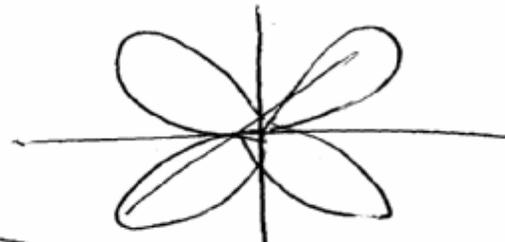
$$L = \lambda$$



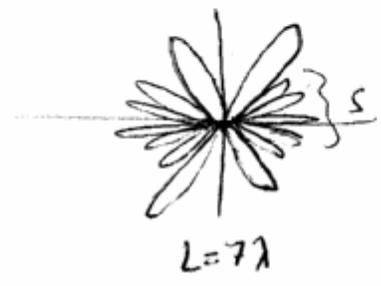
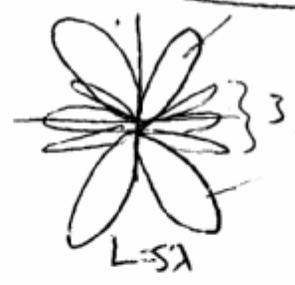
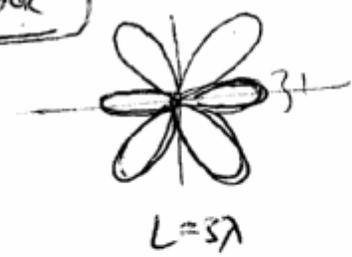
$$L = \frac{3}{2}$$



$$L = 2\lambda$$



longer dipole



2 main Lobes
in the backward direction

L ↑ number
of "