Digital Modulation for Wireless communications
(BPSK, QPSK)

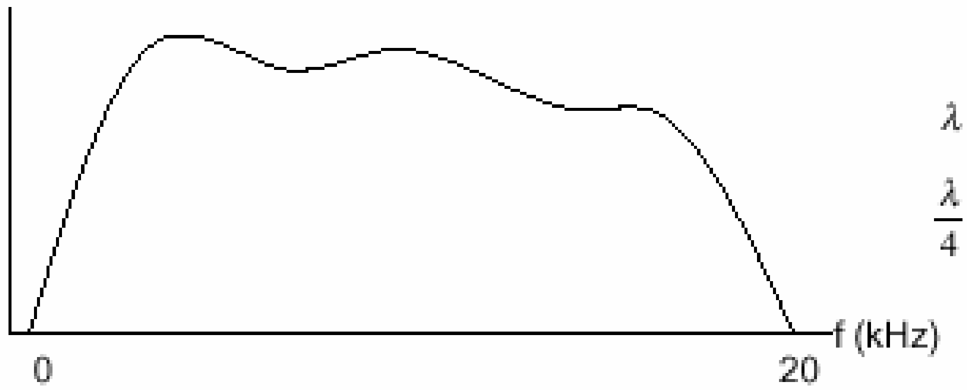
Signal Spectrum Examples

Power Spectrum of Speech

$$\lambda \approx \frac{3\times10^{8}\ \text{m/s}}{1800\ \text{Hz}} = 166{,}667\ \text{m}$$

$$\frac{\lambda}{4} = 41{,}667\ \text{meters}$$

Power Spectrum of CD quality music



$$\lambda \approx \frac{3\times10^{8}\ \text{m/s}}{10000\ \text{Hz}} = 30{,}000\ \text{m}$$

$$\frac{\lambda}{4} = 7{,}500\ \text{meters}$$

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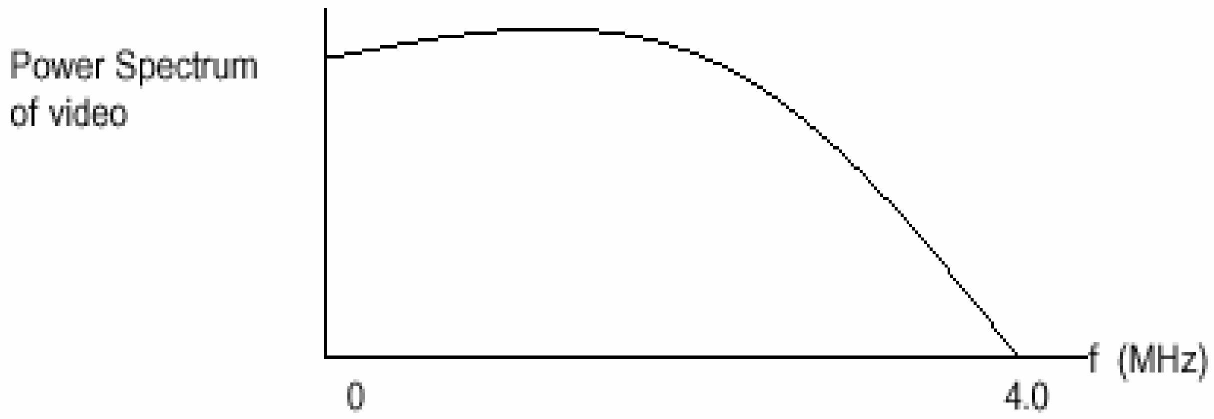
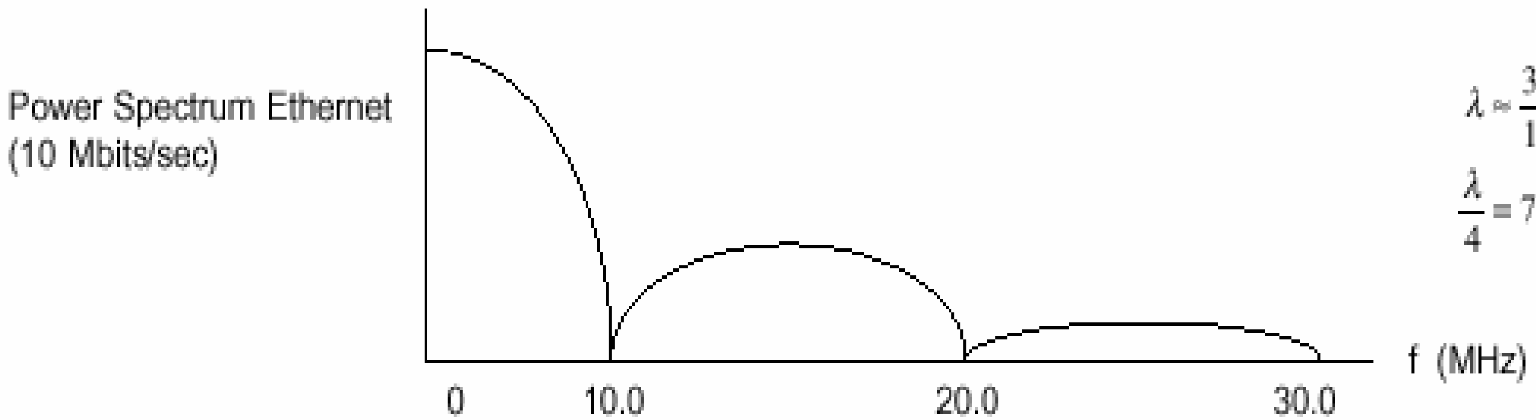
More examples



$$\lambda \approx \frac{3\times10^{8}\ \text{m/s}}{2\times10^{6}\ \text{Hz}} = 150\ \text{m}$$

$$\frac{\lambda}{4} = 37.5\ \text{meters}$$



$$\lambda \approx \frac{3\times10^{8}\ \text{m/s}}{10\times10^{6}\ \text{Hz}} = 30\ \text{m}$$

$$\frac{\lambda}{4} = 7.5\ \text{meters}$$

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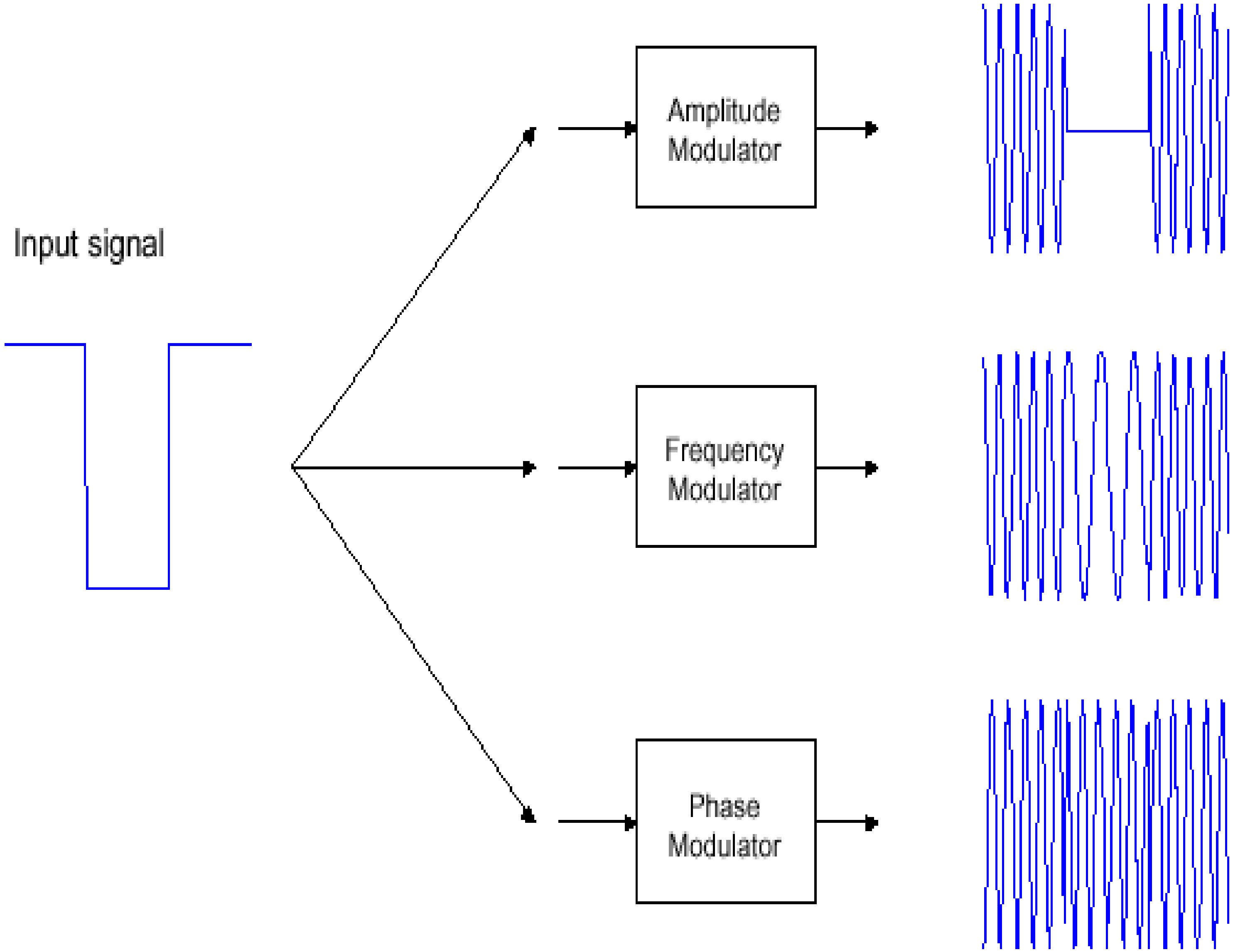

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Modulator

- Converts digital data to a continuous waveform suitable for transmission over channel - usually a sinusoidal wave
- Information is transmitted by varying one or more parameters of waveform:
  - Amplitude
  - Phase
  - Frequency
- Although we modulate a high frequency sinusoid, we will study the modulation in terms of complex baseband (using a signal space approach)

Examples of Modulation

- Amplitude Shift Keying (ASK) or On/Off Keying (OOK):

$$\boxed{\begin{aligned} &1 \Rightarrow A\cos(2\pi f_c t) \\ &0 \Rightarrow 0 \end{aligned}}$$

- Frequency Shift Keying (FSK):

$$\boxed{\begin{aligned} &1 \Rightarrow A\cos(2\pi f_1 t) \\ &0 \Rightarrow A\cos(2\pi f_0 t) \end{aligned}}$$

- Phase Shift Keying (PSK):

$$\boxed{\begin{aligned} &1 \Rightarrow A\cos(2\pi f_c t) \\ &0 \Rightarrow A\cos(2\pi f_c t + \pi) = -A\cos(2\pi f_c t) \end{aligned}}$$

Performance of Digital Modulation

- Let us first consider the case of binary signaling with *coherent* detection in an AWGN channel
- For equally likely symbols the minimum probability of error is obtained using a *maximum liklihood receiver*
- The probability of symbol (and equivalently bit) error is

$$P_b = Q\left(\sqrt{\frac{E_b}{N_o}(1-\rho)}\right)$$

P_b = bit err. prob.
P_s = sym. err. prob.

- Where $\boxed{Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du}$ is the standard *Q*-function
- E_b/N_o is the *average* energy per bit divided by the one sided power spectral density N_o
- and ρ is correlation coefficient between symbols:

$$\boxed{\rho = \frac{1}{\sqrt{E_1 E_2}} \int_0^T s_1(t) s_2(t) dt}$$

E_1 = energy in symbol 1
E_2 = energy in symbol 2
T = symbol duration

Performance of Binary Digital Modulation

$$P_b = Q\left(\sqrt{\frac{E_b}{N_o}(1-\rho)}\right)$$

- Specific cases:
  - $\rho = 0$
    - orthogonal modulation (e.g., BFSK, BASK)
  - $\rho = -1$
    - Antipodal signaling (e.g., BPSK)
    - Best performance for binary modulation
  - $\rho > 0$ results in worse performance than either case

3.6 Modulation

The digital bit stream has to be modulated onto an RF carrier in order for it to be transmitted. The modulated signal is then transmitted through space in the form of a propagating *electromagnetic* (EM) field.

One immediate question is why do we need to modulate the bit stream onto an RF carrier; why can't we just transmit the baseband signal through space to the desired designation. There are two answers to this question. First, the government regulatory agency (i.e., the FCC) specifies the frequency at which a particular service can transmit. Thus, not everyone can transmit at the baseband frequency. Second, in order to transmit at baseband, which is at a much lower frequency, the required antenna size would be enormous in order to allow an efficient coupling between the transmitter and free space. For example, in order to efficiently couple power to free space, the antenna size needs to be at least on the order of the wavelength. If one wishes to transmit a baseband signal at 9.6 kHz, the antenna size would be 31.25 km!

There is a difference between analog and digital modulation techniques. Readers may be familiar with analog modulation schemes such as *amplitude modulation* (AM) and *frequency modulation* (FM). In analog modulation, information is contained in the continuous-waveform shape of the signal. Digital modulation schemes, on the other hand, are used to transmit discrete units of information called symbols, and the information may be contained in the amplitude (e.g., on-off keying), the phase (e.g., phase-shift keying), or the amplitude and phase (e.g., quadrature-amplitude modulation) of the signal.

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3.6.1 Binary Phase-Shift Keying (BPSK)

3.6.1.1 Modulator

Let's first examine a *basic digital modulation scheme* called BPSK and its performance in a Gaussian noise environment. The concept is simple. Whenever the transmitter wants to send a +1, it will transmit a positive cosinusoid; whenever the transmitter wants to send a −1, it will transmit a negative cosinusoid. The analytic expression for BPSK is

$$+1: s_{+1}(t) = \sqrt{\frac{2E}{T}}\cos(2\pi ft) \qquad 0 < t < T \tag{3.10}$$

$$-1: s_{-1}(t) = \sqrt{\frac{2E}{T}}\cos(2\pi ft + \pi) = -\sqrt{\frac{2E}{T}}\cos(2\pi ft) \qquad 0 < t < T \tag{3.11}$$

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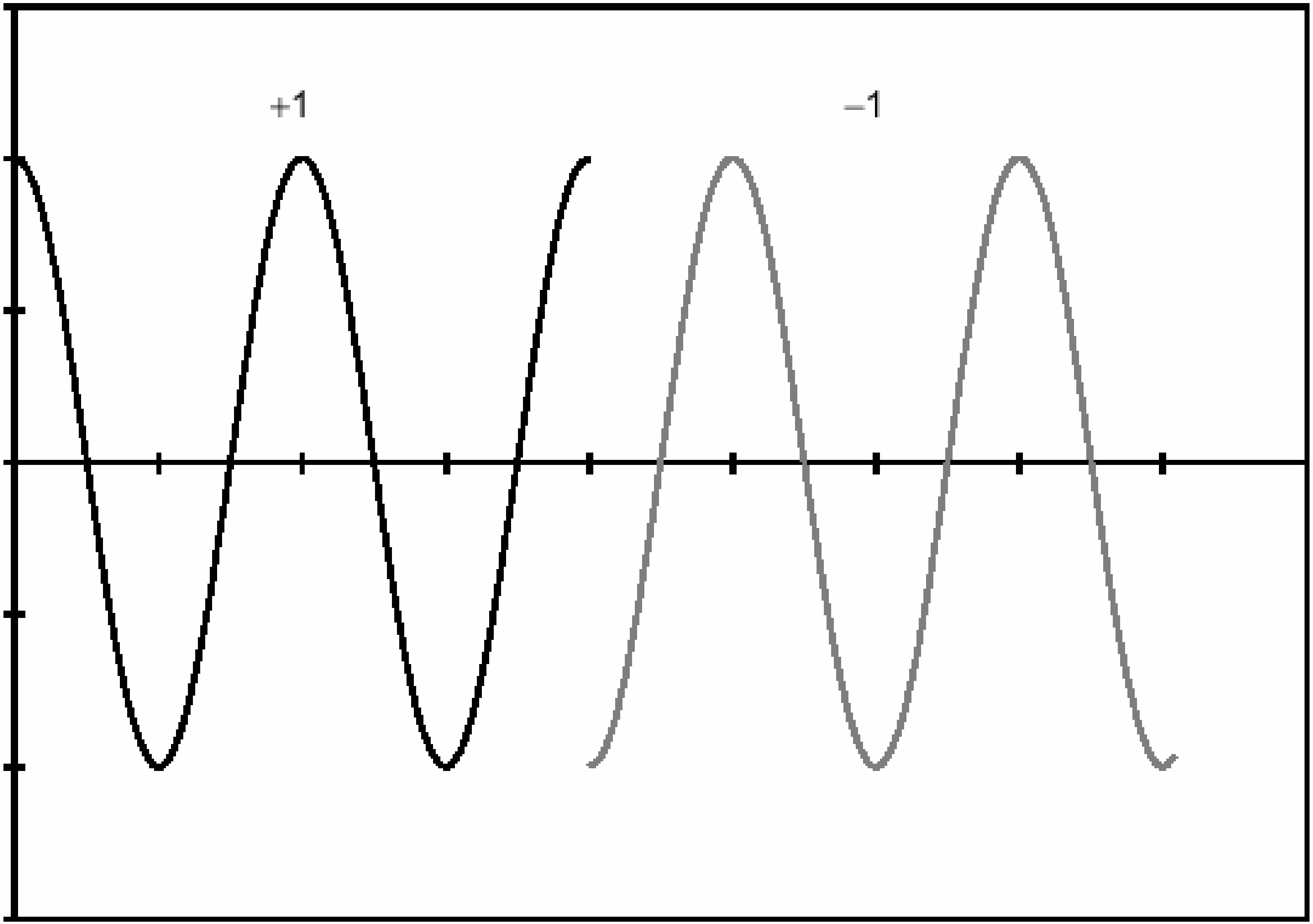

Figure 3.12 Modulated signal in the time domain using BPSK.

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where E is the energy per symbol, and T is the time duration of each symbol. From these expressions, we can see that the information is indeed stored in the phase of the modulated signals $s_{+1}(t)$ and $s_{-1}(t)$. If the transmitted information is 1, the modulated signal $s_{+1}(t)$ has a phase of 0. If the transmitted information is −1, then the modulated signal $s_{-1}(t)$ has a phase of π, or 180 degrees. Figure 3.12 shows what the modulated signals look like in the time domain.

The BPSK modulator is quite simple to implement. The modulator itself is no more than a multiplier. Figure 3.13 shows the block diagram of a BPSK modulator. The input to the modulator consists of the data symbols. The data can be either a +1 or a −1. The data is multiplied by the carrier $\cos(2\pi ft)$ scaled by the coefficient $\sqrt{2E/T}$. The output of the multiplier is the corresponding modulated signal.

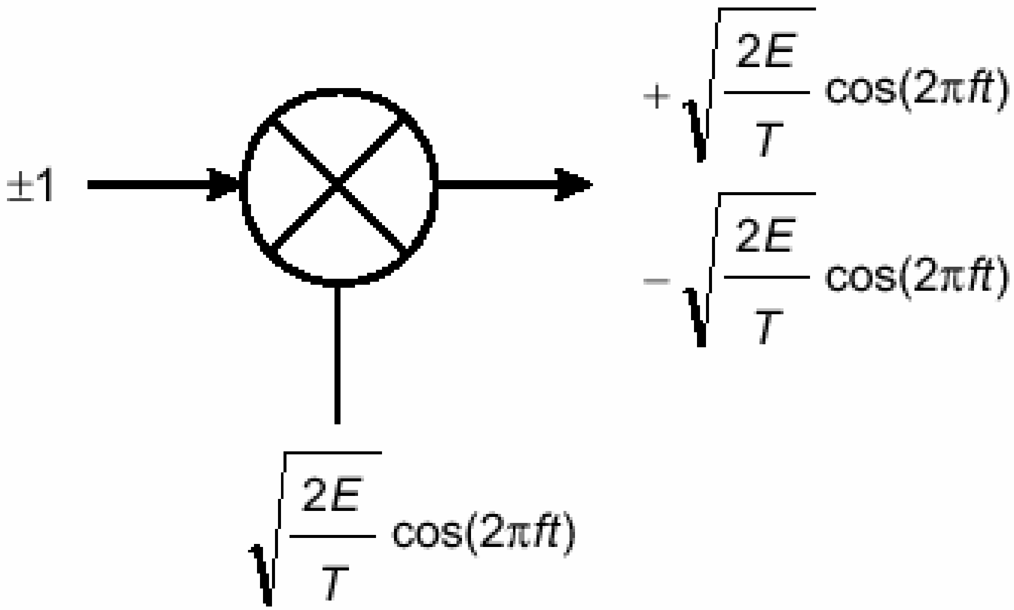

Figure 3.13 BPSK modulator.

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3.6.1.2 Demodulator

One implementation of the BPSK demodulator is the matched-filter approach. Figure 3.14 illustrates such an implementation. The received signal $r(t)$ has two components: the originally transmitted signal $s_i(t)$ where i could be either +1 or −1; and noise $n(t)$, which has been introduced by the channel. The received signal $r(t)$ is multiplied by the reference signals $s_{+1}(t)$. The multiplied result is then integrated over one bit interval T.

If the transmitter sent a +1 (i.e., $s_i(t) = s_{+1}(t)$), then the integrated result is

$$y = \frac{2E}{T}\int_0^T \cos^2(2\pi ft)\,dt + \sqrt{\frac{2E}{T}}\int_0^T \cos(2\pi ft)\,n(t)\,dt \qquad (3.12)$$

where the first term is the *signal* term that is utilized by the decision threshold to make the decision and the second term is the *noise* contribution. In the absence of noise, we see that the first term reduces to

$$\frac{2E}{T}\int_0^T \cos^2(2\pi ft)\,dt = \frac{2E}{T}\left(\frac{1}{2}\right) = +\frac{E}{T} \qquad (\text{for } +1)$$

If the transmitter sent a -1 (i.e., $s_i(t) = s_{-1}(t)$), then the integrated result is

$$y = -\frac{2E}{T}\int_0^T \cos^2(2\pi ft)\,dt - \sqrt{\frac{2E}{T}}\int_0^T \cos(2\pi ft)\,n(t)\,dt \qquad (3.13)$$

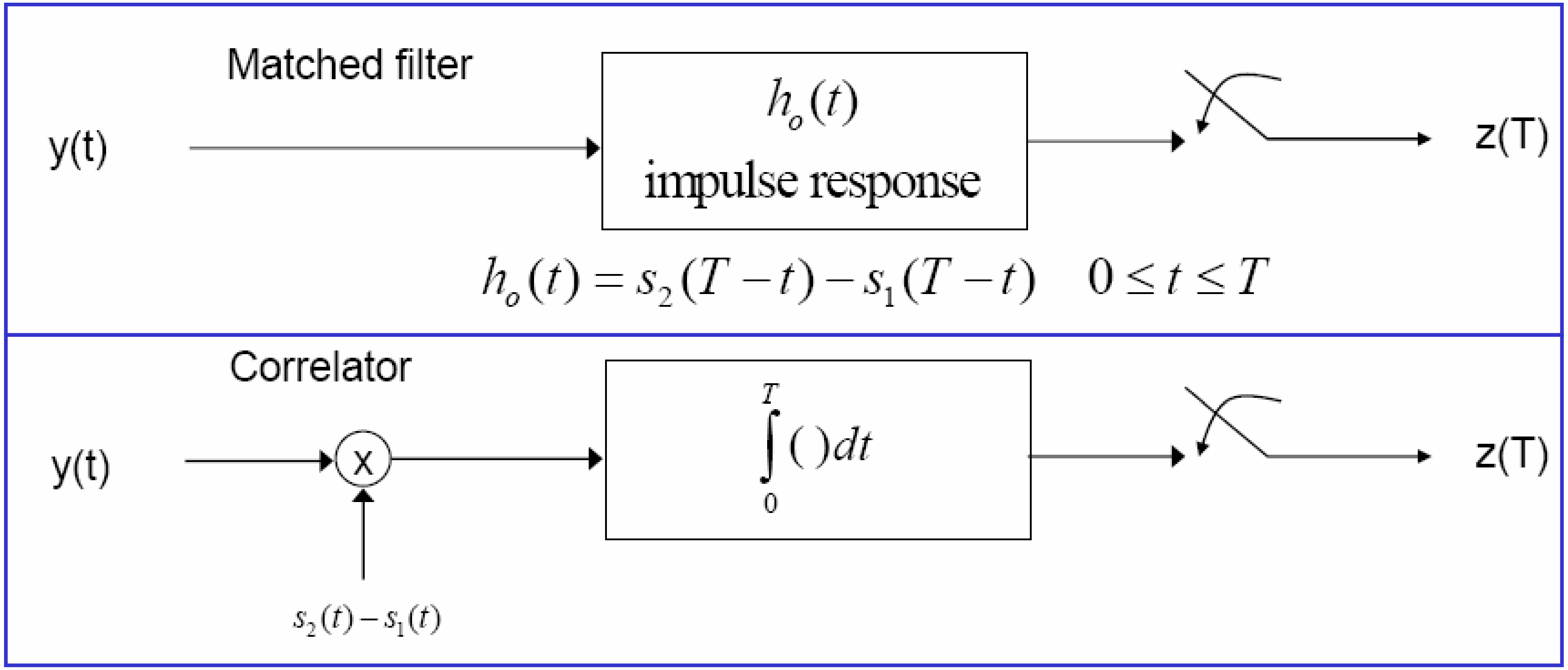
In the absence of noise, we see that the first term reduces to

$$-\frac{2E}{T}\int_0^T \cos^2(2\pi ft)\,dt = -\frac{2E}{T}\left(\frac{1}{2}\right) = \frac{E}{T} \qquad (\text{for } -1)$$

Therefore, the decision threshold decides that the transmitter sent a +1 if the integrated result is greater than 0 and −1 if the integrated result is less than 0. In this *maximum likelihood detector* implementation, we have assumed that the probability of sending a +1 is equal to the probability of sending a −1. Furthermore, it is assumed that the demodulator is coherent (i.e., the phase of its reference signal perfectly matches the phase of the transmitter).

Maximum Likelihood Receiver

- The ML receiver can be implemented as either a *correlator* or a *matched filter*



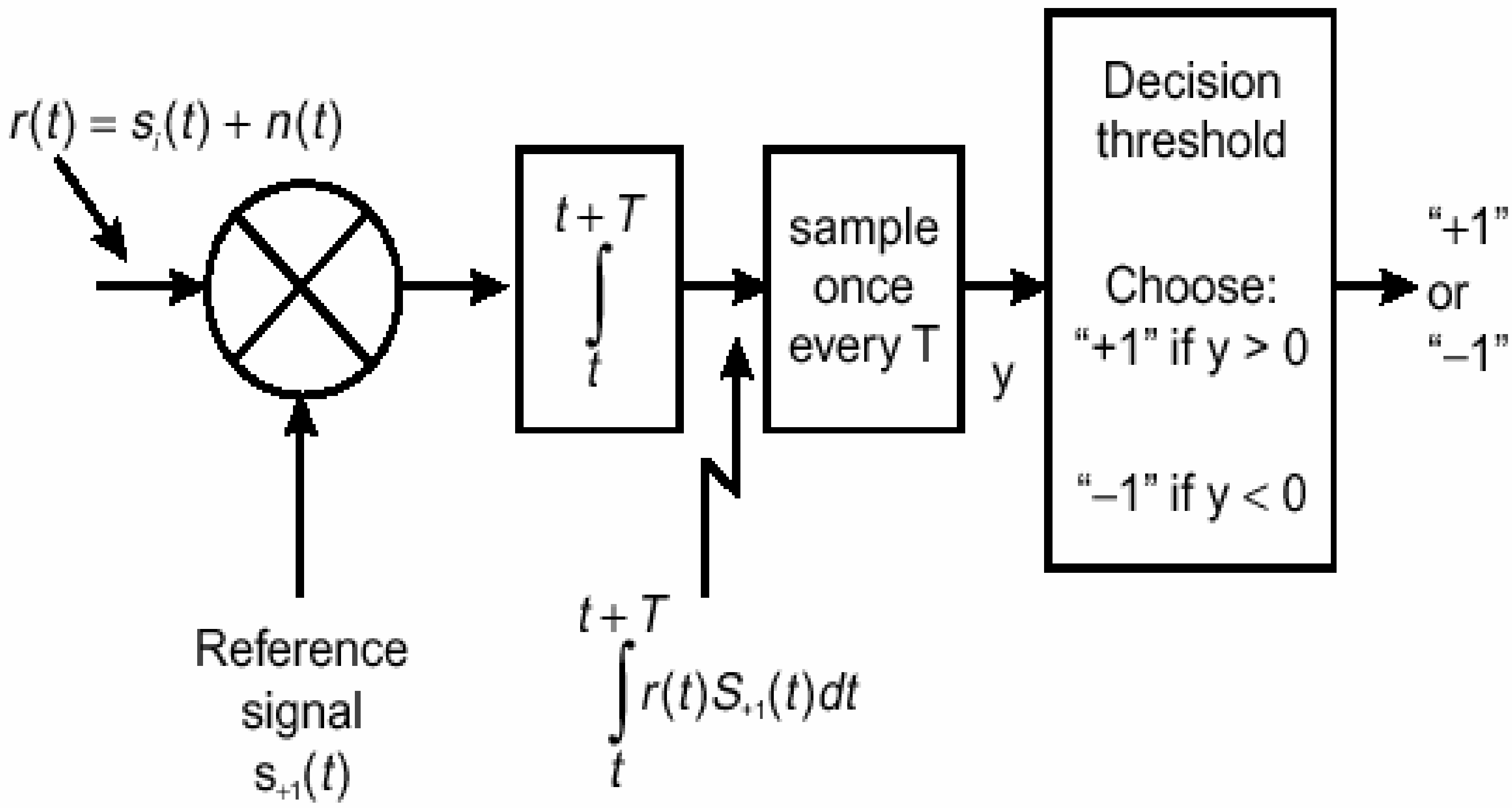


Figure 3.14 Correlator demodulator for BPSK.

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3.6.1.3 Error Performance of BPSK

Thus far, we have ignored the noise term at the output of the integrator. The noise terms (i.e., the second terms in (3.12) and (3.13)) are

$$\pm\sqrt{\frac{2E}{T}}\int_0^T \cos(2\pi f t)\,n(t)\,dt$$

The noise terms could be large enough to cause the decision threshold function to make a wrong decision. For example, suppose that the transmitter sent a +1. The output of the integrator is

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$$y = \frac{2Ew}{T}\int_0^T \cos^2(2\pi ft)dt + \sqrt{\frac{2E}{T}}\int_0^T \cos(2\pi ft)n(t)dt$$

$$= +\left(\frac{E}{T}\right) + \sqrt{\frac{2E}{T}}\int_0^T \cos(2\pi ft)n(t)dt$$

The noise $n(t)$ is frequently modeled as an *additive white Gaussian noise* (AWGN) process. If the noise power is large (i.e., if the variance of $n(t)$ is large), it could cause the second term in the above expression to become less than $-(E/T)$. If that is the case, y would be less than zero and the decision threshold would then decide −1. Since the transmitter in reality sent a +1, the demodulator is said to have made an error.

To characterize the error performance of a coherent BPSK system, we use the signal-space representation of the BPSK signals in (3.10) and (3.11). The signal-space representation is nothing more than another representation of the signals. The representation depicts a signal in its in-phase and quadrature components. Every real-valued signal can be de-composed into an I and a Q component, and the signal-space representation effectively draws the signal in a space defined by the I and the Q axes. Figure 3.15 shows the signal-space representation of the BPSK signals defined in equations (3.10) and (3.11)

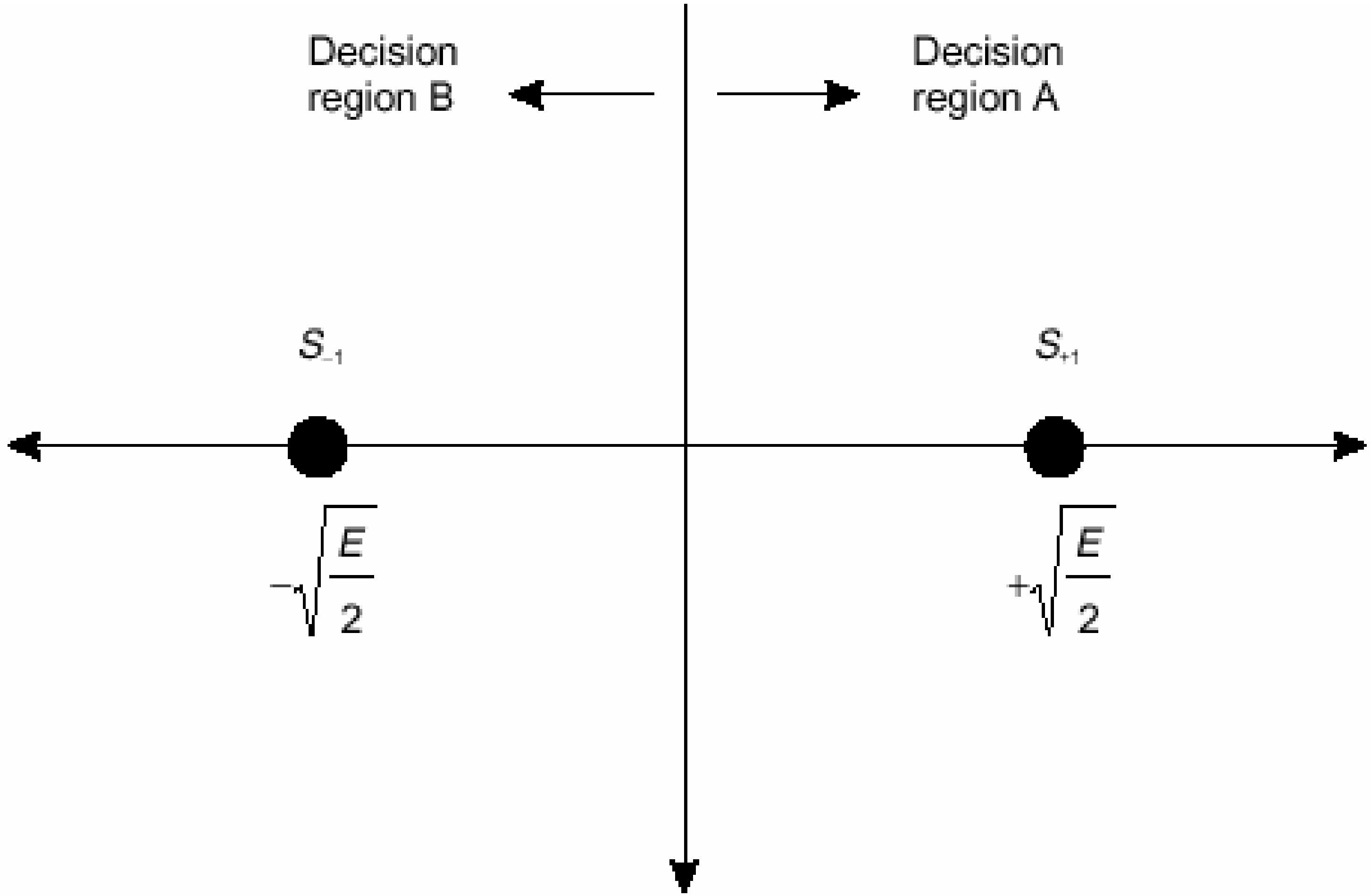

Figure 3.15 Signal-space representation of a BSPK signal set.

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To obtain the magnitude of a signal along the I axis, we extract the in-phase component of that particular signal. The in-phase component is calculated by multiplying that signal by a cosinusoid and then integrating over the bit period. For example, the in-phase component of $s_{+1}(t)$ is

$$\int_0^T \cos(2\pi ft)s_{+1}(t)dt = \sqrt{\frac{2E}{T}}\int_0^T \cos^2(2\pi ft)dt = \sqrt{\frac{2E}{T}}\left(\frac{1}{2}\right) = \sqrt{\frac{E}{2T}} = \sqrt{\frac{E}{2}}$$

For simplicity, we use $T = 1$.

The probability that an error has occurred happens when the transmitter sent a +1 but the receiver made a decision of −1, and vice versa. In other words,

$$P_e = P\left(\text{decide} - 1 \middle| \text{sent} + 1\right)P(\text{sent} + 1) + P\left(\text{decide} + 1 \middle| \text{sent} - 1\right)P(\text{sent} - 1)$$

If we assume that the probability of sending a +1 is equal to the probability of sending a −1, then

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$$P_e = \frac{1}{2} P(\text{decide} - 1 | \text{sent} + 1) + \frac{1}{2} P(\text{decide} + 1 | \text{sent} - 1) \tag{3.14}$$

The two probabilities in the above equation can be obtained by noting the fact that in its *IQ* form, the received signal y is given by $y = s + n$, where s is the signal and n is the noise. Note that s is a constant in the *IQ* representation, and n is effectively a Gaussian random variable; thus, y is also Gaussian distributed with a mean of s. Therefore, the probability of deciding −1 given that +1 is sent is the probability of y (given +1 is sent) falling in decision region B. This probability can be evaluated by integrating the Gaussian probability density function over the error area. See Figure 3.16.

Since the two conditional probability density functions are symmetrical, the two conditional probabilities are identical; that is,

$$P(\text{decide} - 1 | \text{sent} + 1) = P(\text{decide} + 1 | \text{sent} - 1)$$

and (3.14) reduces to

$$P_e = P(\text{decide} - 1 | \text{sent} + 1) \tag{3.15}$$

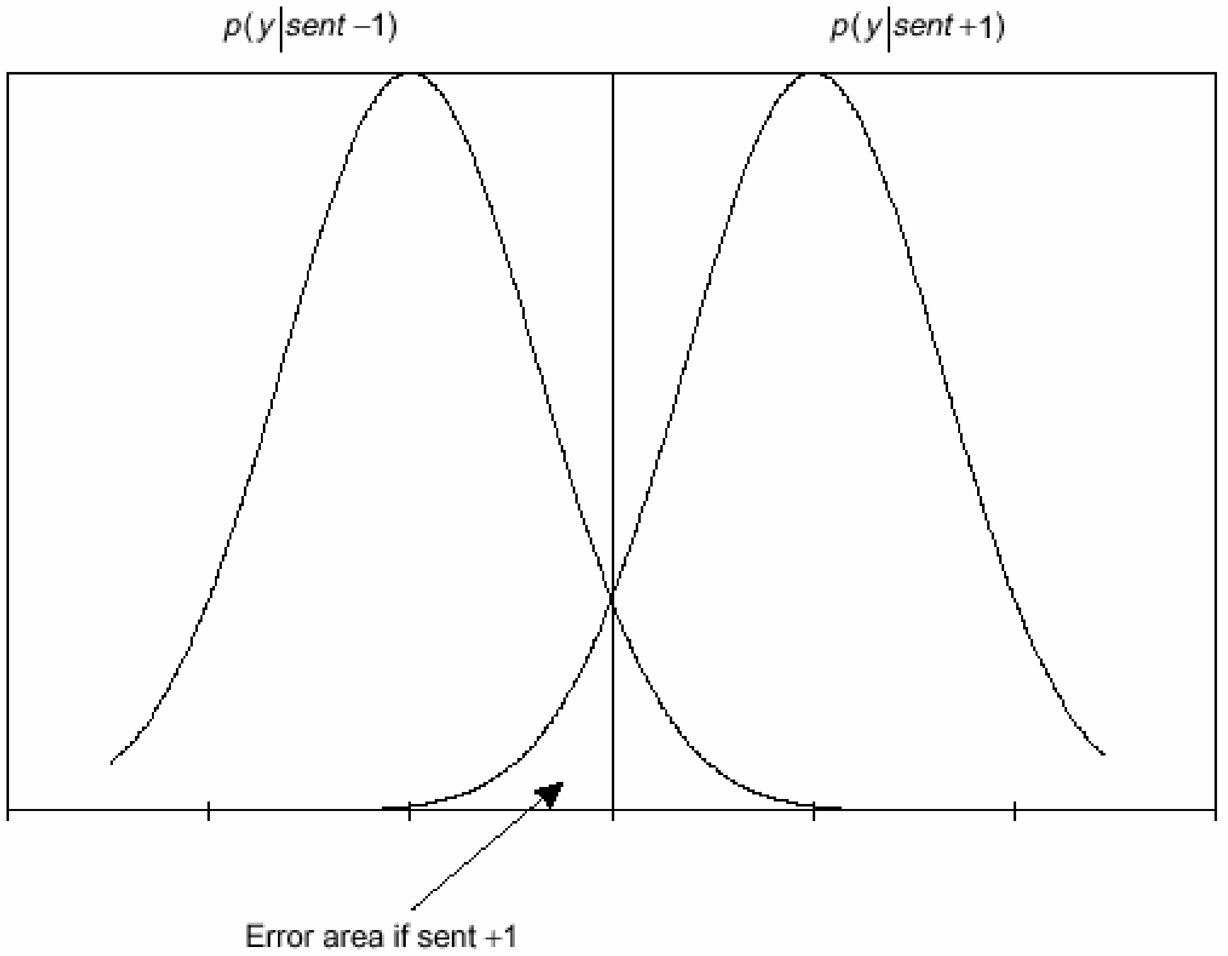

Figure 3.16 Conditional probability density function for the received signals.

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Equation (3.15) can be evaluated as

$$P_e = \int_{-\infty}^{0} p\left(y \middle| \text{sent} + 1\right) dy \tag{3.16}$$

Since $p(y|\text{sent} + 1)$ is a Gaussian probability density function, we can evaluate (3.16) by using the complementary error function $Q(x)$:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp\left(-\frac{v^2}{2}\right) dv \tag{3.17}$$

By substituting the appropriate variables, and by recognizing the fact that the noise variance σ^2 equates to half of the noise power density N_0 [1], we obtain

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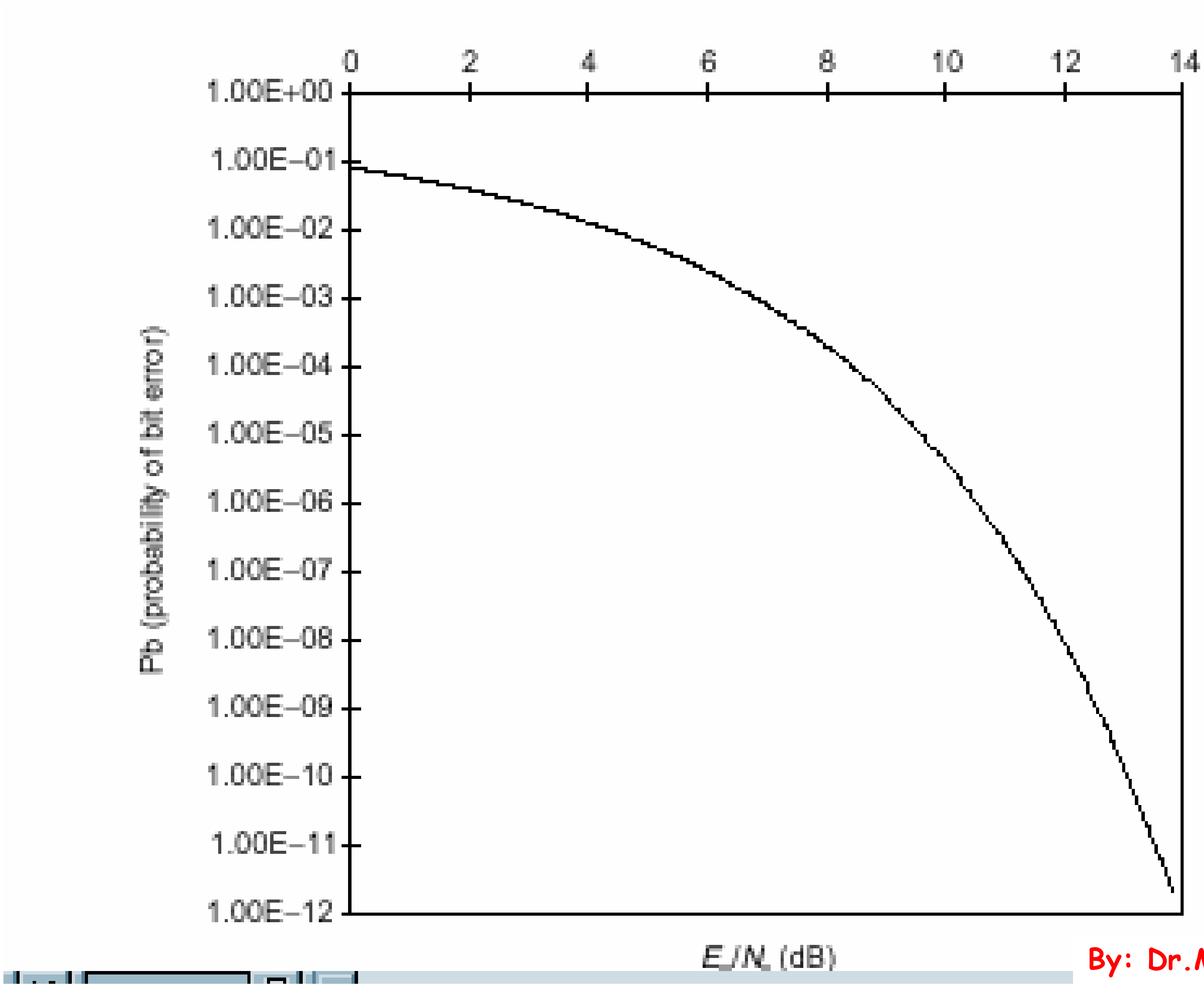
$$P_s = \frac{1}{2} Q\left(\sqrt{\frac{E}{N_0}}\right) \tag{3.18}$$

Since in BPSK each symbol is also an individual bit, the probability of symbol error shown in (3.18) is also equal to the probability of bit error. The energy per symbol E is also the energy per bit E_b:

$$P_b = \frac{1}{2} Q\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{3.19}$$

Figure 3.17 shows the curve of probability of bit error versus E_b/N_0.

Bit error performance of coherent BPSK system



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