Home work (5)

Linear Block codes

1. The generator matrix for a linear binary code is

$$G = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

a) Express **G** in systematic [**I**|**P**] form.
b) Determine the parity check matrix **H** for the code.
c) Construct the table of syndromes for the code.
d) Determine the minimum distance of the code.
e) Demonstrate that the code word corresponding to the information sequence 101 is orthogonal to **H**.

2. Consider a (7, 4) code whose generator matrix is

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Find all the codewords of the code.
(b) Find **H**, the parity-check matrix of the code.
(c) Compute the syndrome for the received vector 1 1 0 1 1 0 1. Is this a valid code vector?
(d) What is the error-correcting capability of the code?
(e) What is the error-detecting capability of the code?

3. Consider a systematic block code whose parity-check equations are

$$p_1 = m_1 + m_2 + m_4$$
$$p_2 = m_1 + m_3 + m_4$$
$$p_3 = m_1 + m_2 + m_3$$
$$p_4 = m_2 + m_3 + m_4$$

where m_i are message digits and p_i are check digits.
(a) Find the generator matrix and the parity-check matrix for this code.
(b) How many errors can the code correct?
(c) Is the vector 10101010 a codeword?
(d) Is the vector 01011100 a codeword?