Lecture 17

Design of Highpass, Bandpass and Bandstop IIR filters

Design of IIR Highpass, Bandpass, and Bandstop Digital Filters

- First Approach -

D/A (1) Prewarp digital frequency specifications of desired digital filter $G_D(z)$ to arrive at frequency specifications of analog filter $H_D(s)$ of same type

Analog (2) Convert frequency specifications of $H_D(s)$ into that of prototype analog lowpass filter $H_{LP}(s)$ (As in freq. trans. Analog review lecture)

Analog (3) Design analog lowpass filter $H_{LP}(s)$

Analog (4) Convert $H_{LP}(s)$ into $H_D(s)$ using inverse frequency transformation used in Step 2

A/D (5) Design desired digital filter $G_D(z)$ by applying bilinear transformation to $H_D(s)$

- Second Approach -

Same(1) analog — (1) Prewarp digital frequency specifications of desired digital filter $G_D(z)$ to arrive at frequency specifications of analog filter $H_D(s)$ of same type

Same(1) analog — (2) Convert frequency specifications of $H_D(s)$ into that of prototype analog lowpass filter $H_{LP}(s)$

Same(1) analog — (3) Design analog lowpass filter $H_{LP}(s)$

Different digital — (4) Convert $H_{LP}(s)$ into an IIR digital transfer function $G_{LP}(z)$ using bilinear transformation

Spectral Transform (Next lecture) — (5) Transform $G_{LP}(z)$ into the desired digital transfer function $G_D(z)$

We illustrate the first approach

IIR Highpass Digital Filter Design

- Design of a Type 1 Chebyshev IIR digital highpass filter
- Specifications: $F_p = 700$ Hz, $F_s = 500$ Hz, $\alpha_p = 1$ dB, $\alpha_s = 32$ dB, $F_T = 2$ kHz Sampling freq.
- Normalized angular bandedge frequencies

$$\omega_p = \frac{2\pi F_p}{F_T} = \frac{2\pi \times 700}{2000} = 0.7\pi$$

$$\omega_s = \frac{2\pi F_s}{F_T} = \frac{2\pi \times 500}{2000} = 0.5\pi$$

- Prewarping these frequencies we get

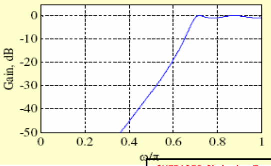
$$\hat{\Omega}_p = \tan(\omega_p/2) = 1.9626105$$

$$\hat{\Omega}_s = \tan(\omega_s/2) = 1.0$$

- For the prototype analog lowpass filter choose $\Omega_p = 1$
- Using $\boxed{\Omega = -\frac{\cancel{\Omega_p}\hat{\Omega}_p}{\hat{\Omega}}}$ we get $\Omega_s = 1.962105$
- Analog lowpass filter specifications: $\Omega_p = 1$, $\Omega_s = 1.926105$, $\alpha_p = 1$dB, $\alpha_s = 32$ dB

- MATLAB code fragments used for the design

```
[N, Wn] = cheb1ord(1, 1.9626105, 1, 32, 's')
[B, A] = cheby1(N, 1, Wn, 's');              Note: analog LP
[BT, AT] = lp2hp(B, A, 1.9626105);  Analog transform
[num, den] = bilinear(BT, AT, 0.5);
```



CHEBY1(N,R,Wn,'s'),CHEBY1(N,R,Wn,'high','s') and CHEBY1(N,R,Wn,'stop','s')

design analog Chebyshev Type I filters. In this case, Wn is in [rad/s] and it can be greater than 1.0.

P2HP Lowpass to highpass analog filter transformation.
[NUMT,DENT] = LP2HP(NUM,DEN,Wo) transforms the lowpass filter

prototype NUM(s)/DEN(s) with unity cutoff frequency to a highpass filter with cutoff frequency Wo.

[AT,BT,CT,DT] = LP2HP(A,B,C,D,Wo) does the same when the filter is described in state-space form.

CHEB1ORD Chebyshev Type I filter order selection.
[N, Wn] = CHEB1ORD(Wp, Ws, Rp, Rs) returns the order N of the lowest order digital Chebyshev Type I filter that loses no more than Rp dB in the passband and has at least Rs dB of attenuation in the stopband. Wp and Ws are the passband and stopband edge frequencies, normalized from 0 to 1 (where 1 corresponds to pi radians/sample).

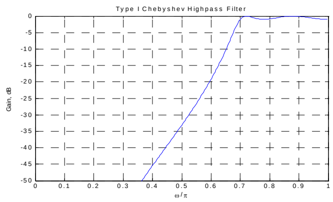
CHEB1ORD also returns Wn, the Chebyshev natural frequency to use with CHEBY1 to achieve the specifications.

[N, Wn] = CHEB1ORD(Wp, Ws, Rp, Rs, 's') does the computation for an analog filter, in which case Wp and Ws are in radians/second.

The above IIR digital highpass filter can be directly designed in the z domain using the M file: Cheb1ord and cheby1 as illustrated in 9.15

```
% Program 9_2
% Type 1 Chebyshev IIR Highpass Filter Design
Wp = input('Normalized passband edge = ');
Ws = input('Normalized stopband edge = ');
Rp = input('Passband ripple in dB = ');
Rs = input('Minimum stopband attenuation in dB = ');
[N,Wn] = cheb1ord(Wp,Ws,Rp,Rs);
[b,a] = cheby1(N,Rp,Wn,'high');
[h,omega] = freqz(b,a,256);
plot (omega/pi,20*log10(abs(h)));grid;
xlabel('\omega/\pi'); ylabel('Gain, dB');
title('Type I Chebyshev Highpass Filter');
```

```
>> Program_9_2
Normalized passband edge = 0.7
Normalized stopband edge = 0.5
Passband ripple in dB = 1
Minimum stopband attenuation in dB = 32
```



Before starting here
Review BP analog design

IIR Bandpass Digital Filter Design

- Design of a Butterworth IIR digital bandpass filter
- Specifications: $\omega_{p1} = 0.45\pi$, $\omega_{p2} = 0.65\pi$, $\omega_{s1} = 0.3\pi$, $\omega_{s2} = 0.75\pi$, $\alpha_p = 1$ dB, $\alpha_s = 40$dB
- Prewarping we get

$$\boxed{\begin{aligned} \hat{\Omega}_{p1} &= \tan(\omega_{p1}/2) = 0.8540807 \\ \hat{\Omega}_{p2} &= \tan(\omega_{p2}/2) = 1.6318517 \\ \hat{\Omega}_{s1} &= \tan(\omega_{s1}/2) = 0.5095254 \\ \hat{\Omega}_{s2} &= \tan(\omega_{s2}/2) = 2.41421356 \end{aligned}}$$

- Width of passband $B_w = \hat{\Omega}_{p2} - \hat{\Omega}_{p1} = 0.77777$

$$\hat{\Omega}_o^2 = \hat{\Omega}_{p1}\hat{\Omega}_{p2} = 1.393733$$

$$\hat{\Omega}_{s1}\hat{\Omega}_{s2} = 1.23010325 \neq \hat{\Omega}_o^2$$

- We therefore modify $\hat{\Omega}_{s1}$ so that $\hat{\Omega}_{s1}$ and $\hat{\Omega}_{s2}$ exhibit geometric symmetry with respect to $\hat{\Omega}_o^2$
- We set $\hat{\Omega}_{s1} = 0.5773031$
- For the prototype analog lowpass filter we choose $\Omega_p = 1$

- Using $\boxed{\Omega = -\Omega_p \dfrac{\hat{\Omega}_o^2 - \hat{\Omega}^2}{\hat{\Omega} B_w}}$ we get

$$\Omega_s = \frac{1.393733 - 0.3332788}{0.5773031 \times 0.777771} = 2.3617627$$

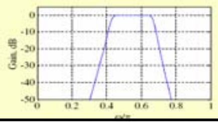
- Specifications of prototype analog Butterworth lowpass filter:

$$\Omega_p = 1,\ \Omega_s = 2.3617627,\ \alpha_p = 1 \text{ dB},\ \alpha_s = 40 \text{ dB}$$

- MATLAB code fragments used for the design

```
[N, Wn] = buttord(1, 2.3617627, 1, 40, 's')
[B, A] = butter(N, Wn, 's');
[BT, AT] = lp2bp(B, A, 1.1805647, 0.777771);
[num, den] = bilinear(BT, AT, 0.5);
```

Note: the above IRR digital band pass Filter can be directly designed in the z domain using only buttord and butter as ex 9.16



```
% Program 9_3
% Design of IIR Butterworth Bandpass Filter
Wp = input('Passband edge frequencies = ');
Ws = input('Stopband edge frequencies = ');
Rp = input('Passband ripple in dB = ');
Rs = input('Minimum stopband attenuation = ');
[N,Wn] = buttord(Wp, Ws, Rp, Rs);
[b,a] = butter(N,Wn);
[h,omega] = freqz(b,a,256);
gain = 20*log10(abs(h));
plot (omega/pi,gain);grid;
xlabel('\omega/\pi'); ylabel('Gain, dB');
title('IIR Butterworth Bandpass Filter');
```

BUTTORD Butterworth filter order selection.
[N, Wn] = BUTTORD(Wp, Ws, Rp, Rs) returns the order N of the lowest order digital Butterworth filter that loses no more than Rp dB in the passband and has at least Rs dB of attenuation in the stopband. Wp and Ws are the passband and stopband edge frequencies, normalized from 0 to 1 (where 1 corresponds to pi radians/sample). For example,

```
    Lowpass:    Wp = .1,      Ws = .2
    Highpass:   Wp = .2,      Ws = .1
    Bandpass:   Wp = [.2 .7], Ws = [.1 .8]
    Bandstop:   Wp = [.1 .8], Ws = [.2 .7]
```

BUTTORD also returns Wn, the Butterworth natural frequency (or, the "3 dB frequency") to use with BUTTER to achieve the specifications.

[N, Wn] = BUTTORD(Wp, Ws, Rp, Rs, 's') does the computation for an analog filter, in which case Wp and Ws are in radians/second.

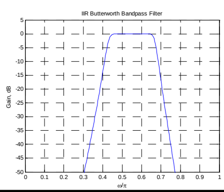
When Rp is chosen as 3 dB, the Wn in BUTTER is equal to Wp in BUTTORD.

FREQZ Digital filter frequency response.
[H,W] = FREQZ(B,A,N) returns the N-point complex frequency response vector H and the N-point frequency vector W in radians/sample of the filter:

```
          jw               -jw              -jmw
     jw  B(e)    b(1) + b(2)e + .... + b(m+1)e
  H(e) = ---- = ------------------------------------
          jw               -jw              -jnw
        A(e)    a(1) + a(2)e + .... + a(n+1)e
```

given numerator and denominator coefficients in vectors B and A. The frequency response is evaluated at N points equally spaced around the upper half of the unit circle. If N isn't specified, it defaults to 512.

```
>> Program_9_3
Passband edge frequencies = 0.25
Stopband edge frequencies = 0.55
Passband ripple in dB = -0.5
Minimum stopband attenuation = -15
```



- Design of an elliptic IIR digital bandstop filter
- Specifications: $\omega_{s1} = 0.45\pi$, $\omega_{s2} = 0.65\pi$, $\omega_{p1} = 0.3\pi$, $\omega_{p2} = 0.75\pi$, $\alpha_p = 1$ dB, $\alpha_s = 40$dB
- Prewarping we get

$$\hat{\Omega}_{s1} = 0.8540806, \quad \hat{\Omega}_{s2} = 1.6318517,$$
$$\hat{\Omega}_{p1} = 0.5095254, \quad \hat{\Omega}_{p2} = 2.4142136$$

- Width of stopband $B_w = \hat{\Omega}_{s2} - \hat{\Omega}_{s1} = 0.777771$

$$\hat{\Omega}_o^2 = \hat{\Omega}_{s2}\hat{\Omega}_{s1} = 1.393733$$
$$\hat{\Omega}_{p2}\hat{\Omega}_{p1} = 1.230103 \neq \hat{\Omega}_o^2$$

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- We therefore modify $\hat{\Omega}_{p1}$ so that $\hat{\Omega}_{p1}$ and $\hat{\Omega}_{p2}$ exhibit geometric symmetry with respect to $\hat{\Omega}_o^2$
- We set $\hat{\Omega}_{p1} = 0.577303$
- For the prototype analog lowpass filter we choose $\Omega_s = 1$
- Using $\Omega = \Omega_s \dfrac{\hat{\Omega} B_w}{\hat{\Omega}_o^2 - \hat{\Omega}^2}$ we get

$$\Omega_p = \frac{0.5095254 \times 0.777771}{1.393733 - 0.3332787} = 0.4234126$$

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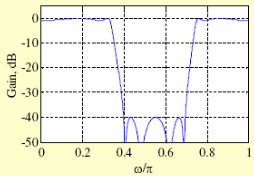
IIR Bandstop Digital Filter Design

- MATLAB code fragments used for the desig

```
[N, Wn] = ellipord(0.4234126, 1, 1, 40, 's');
[B, A] = ellip(N, 1, 40, Wn, 's');
[BT, AT] = lp2bs(B, A, 1.1805647, 0.777771);
[num, den] = bilinear(BT, AT, 0.5);
```



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The above band stop filter can be designed directly in the z domain using only the ellipord and ellip

IIR Digital Filter Design Using MATLAB

- Order Estimation -
- For IIR filter design using bilinear transformation, MATLAB statements to determine the order and bandedge are:

```
[N, Wn] = buttord(Wp, Ws, Rp, Rs);
[N, Wn] = cheb1ord(Wp, Ws, Rp, Rs);
[N, Wn] = cheb2ord(Wp, Ws, Rp, Rs);
[N, Wn] = ellipord(Wp, Ws, Rp, Rs);
```

- Example - Determine the minimum order of a Type 2 Chebyshev digital highpass filter with the following specifications:
  $F_p = 1$ kHz, $F_s = 0.6$ kHz, $F_T = 4$ kHz,
  $\alpha_p = 1$ dB, $\alpha_s = 40$ dB
- Here, $Wp = 2 \times 1/4 = 0.5$, $Ws = 2 \times 0.6/4 = 0.3$
- Using the statement
  [N, Wn] = cheb2ord(0.5, 0.3, 1, 40);
  we get N = 5 and Wn = 0.3224

- Filter Design -
- For IIR filter design using bilinear transformation, MATLAB statements to use are:
  [b, a] = butter(N, Wn)
  [b, a] = cheby1(N, Rp, Wn)
  [b, a] = cheby2(N, Rs, Wn)
  [b, a] = ellip(N, Rp, Rs, Wn)

- The form of transfer function obtained is

$$G(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \cdots + b(N+1)z^{-N}}{1 + a(2)z^{-1} + \cdots + a(N+1)z^{-N}}$$

- The frequency response can be computed using the M-file freqz(b, a, w) where w is a set of specified angular frequencies
- It generates a set of complex frequency response samples from which magnitude and/or phase response samples cn be computed

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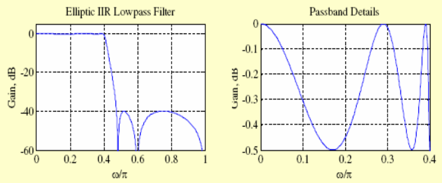
- Example - Design an elliptic IIR lowpass filter with the specifications: $F_p = 0.8$ kHz, $F_s = 1$ kHz, $F_T = 4$ kHz, $\alpha_p = 0.5$dB, $\alpha_s = 40$ dB
- Here, $\omega_p = 2\pi F_p/F_T = 0.4\pi$, $\omega_s = 2\pi F_s/F_T = 0.5\pi$
- Code fragments used are:
  [N,Wn] = ellipord(0.4, 0.5, 0.5, 40);
  [b, a] = ellip(N, 0.5, 40, Wn);

- Gain response plot is shown below:



Spectral Transformations of IRR filters

Spectral Transformations of IIR Digital Filters

- Objective - Transform a given lowpass digital transfer function $G_L(z)$ to another digital transfer function $G_D(\hat{z})$ that could be a lowpass, highpass, bandpass or bandstop filter
- z^{-1} has been used to denote the unit delay in the prototype lowpass filter $G_L(z)$ and $\hat{z}^{-1}$ to denote the unit delay in the transformed filter $G_D(\hat{z})$ to avoid confusion

- Unit circles in z- and $\hat{z}$-planes defined by

$$z = e^{j\omega}, \quad \hat{z} = e^{j\hat{\omega}}$$

- Transformation from z-domain to $\hat{z}$-domain given by

$$z = F(\hat{z})$$

- Then

$$G_D(\hat{z}) = G_L\{F(\hat{z})\}$$

- From $z = F(\hat{z})$, thus $|z| = |F(\hat{z})|$, hence

Points on the unit circle in z plane, must be mapped to points on the unit circle of the z^ plane

$$|F(\hat{z})| \begin{cases} >1, & \text{if } |z|>1 \\ =1, & \text{if } |z|=1 \\ <1, & \text{if } |z|<1 \end{cases}$$

To guarantee the stabilit stability

- Recall that a stable allpass function $A(z)$ satisfies the condition

$$|A(z)| \begin{cases} <1, & \text{if } |z|>1 \\ =1, & \text{if } |z|=1 \\ >1, & \text{if } |z|<1 \end{cases}$$

- Therefore $1/F(\hat{z})$ must be a stable allpass function whose general form is

$$\boxed{\frac{1}{F(\hat{z})} = \pm \prod_{\ell=1}^{L} \left(\frac{1-\alpha_\ell^* \hat{z}}{\hat{z}-\alpha_\ell} \right), \quad |\alpha_\ell| < 1}$$

Lowpass-to-Lowpass Spectral Transformation

- To transform a lowpass filter $G_L(z)$ with a cutoff frequency ω_c to another lowpass filter $G_D(\hat{z})$ with a cutoff frequency $\hat{\omega}_c$, the transformation is

$$\boxed{z^{-1} = \frac{1}{F(\hat{z})} = \frac{1-\alpha\hat{z}}{\hat{z}-\alpha}}$$

where α is a function of the two specified cutoff frequencies

---

- On the unit circle we have

$$e^{-j\omega} = \frac{e^{-j\hat{\omega}} - \alpha}{1-\alpha e^{-j\hat{\omega}}}$$

- From the above we get

$$\boxed{e^{-j\omega} \mp 1} = \boxed{\frac{e^{-j\hat{\omega}} - \alpha}{1-\alpha e^{-j\hat{\omega}}} \mp 1} = \boxed{(1 \pm \alpha)\cdot\frac{e^{-j\hat{\omega}} \mp 1}{1-\alpha e^{-j\hat{\omega}}}}$$

- Taking the ratios of the above two expressions

$$\tan(\omega/2) = \left(\frac{1+\alpha}{1-\alpha}\right)\tan(\hat{\omega}/2)$$

- Solving we get

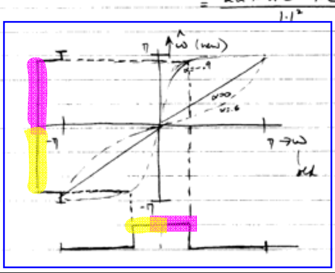
$$\alpha = \frac{\sin((\omega_c - \hat{\omega}_c)/2)}{\sin((\omega_c + \hat{\omega}_c)/2)}$$

$$\alpha = \frac{\sin\left(\frac{\omega_c - \hat{\omega}_c}{2}\right)}{\sin\left(\frac{\omega_c + \hat{\omega}_c}{2}\right)}$$

relates old & new 'cutoff' freqs

$$e^{-j\hat{\omega}} = \frac{\alpha + e^{-j\omega}}{1+\alpha e^{-j\omega}} = \frac{(1+\alpha e^{j\omega})(\alpha + e^{-j\omega})}{|\cdot|^2}$$

$$= \frac{2\alpha + \alpha^2 e^{j\omega} + e^{-j\omega}}{|\cdot|^2} = \frac{2\alpha + \alpha^2\cos\omega + \cos\omega + j(\alpha^2\sin\omega - \sin\omega)}{|\cdot|^2}$$



$$-\hat{\omega} = \operatorname{atan}\left(\frac{(\alpha^2-1)\sin\omega}{2\alpha + (1+\alpha^2)\cos\omega}\right)$$

a frequency warp

(from A.A.Beex lecture notes)

- <u>Example</u> - Consider the lowpass digital filter

$$G_L(z) = \frac{0.0662(1+z^{-1})^3}{(1-0.2593z^{-1})(1-0.6763z^{-1}+0.3917z^{-2})}$$

which has a passband from dc to 0.25π with a 0.5 dB ripple

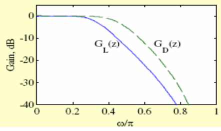
- Redesign the above filter to move the passband edge to 0.35π

- Here

$$\alpha = -\frac{\sin(0.05\pi)}{\sin(0.3\pi)} = -0.1934$$

- Hence, the desired lowpass transfer function is

$$\boxed{G_D(\hat{z}) = G_L(z)\Big|_{z^{-1}=\frac{\hat{z}^{-1}+0.1934}{1+0.1934\hat{z}^{-1}}}}$$



Check that using matlab

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- The lowpass-to-lowpass transformation

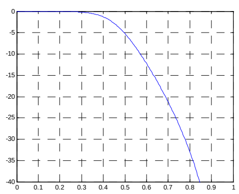
$$\boxed{z^{-1} = \frac{1}{F(\hat{z})} = \frac{1-\alpha\hat{z}}{\hat{z}-\alpha}}$$

can also be used as highpass-to-highpass, bandpass-to-bandpass and bandstop-to-bandstop transformations

```
>> poly([-1 -1 -1])

ans =

     1     3     3     1

>> b=0.13402309*ans

b =

    0.1340    0.4021    0.4021    0.1340

>> b=0.13402309*poly([-1 -1 -1])

b =

    0.1340    0.4021    0.4021    0.1340

>> z1=roots([1 -0.0694472])

z1 =

    0.0694

>> z2=roots([1 -0.1848053 0.337566])

z2 =

   0.0924 + 0.5736i
   0.0924 - 0.5736i

>> z=[z1;z2]

z =

   0.0694
   0.0924 + 0.5736i
   0.0924 - 0.5736i

>> a=poly(z)
```

or a=conv([ ],[ ])

```
a =

    1.0000   -0.2543    0.3504   -0.0234

>> [h,omega] = freqz(b,a,256);
>> plot (omega/pi,20*log10(abs(h)));grid;
>> axis([0 1 -0.5 0])
>> axis([0 1 -40 0])
```



Generation of Allpass Function Using MATLAB

- The allpass function needed for the spectral transformation from a specified lowpass transfer function to a desired highpass or bandpass or bandstop transfer function can be generated using MATLAB

- **Lowpass-to- Lowpass Transformation**
- Basic form:

```
[AllpassNum,AllpassDen] =
allpasslp2hp(wold,wnew)
```

where `wold` is the specified angular bandedge frequency of the original lowpass filter, and `wnew` is the desired angular bandedge frequency of the highpass filter

Lowpass-to-Highpass Spectral Transformation

- Desired transformation

$$\boxed{z^{-1} = -\frac{\hat{z}^{-1}+\alpha}{1+\alpha\,\hat{z}^{-1}}}$$

- The transformation parameter α is given by

$$\alpha = -\frac{\cos((\omega_c+\hat{\omega}_c)/2)}{\cos((\omega_c-\hat{\omega}_c)/2)}$$

where ω_c is the cutoff frequency of the lowpass filter and $\hat{\omega}_c$ is the cutoff frequency of the desired highpass filter

LP → HP

$$z^{-1} = -\frac{\hat{z}^{-1}+\alpha}{1+\alpha\hat{z}^{-1}}$$

$$e^{-j\omega} = -\frac{e^{-j\hat{\omega}}+\alpha}{1+\alpha e^{-j\hat{\omega}}}$$

$$e^{-j\omega}+\alpha e^{-j\hat{\omega}}e^{-j\omega} = -e^{-j\hat{\omega}}-\alpha \rightarrow \alpha = \frac{-e^{-j\hat{\omega}}-e^{-j\omega}}{1+e^{-j\hat{\omega}}e^{-j\omega}} = \frac{-e^{-j\frac{\hat{\omega}}{2}}e^{-j\frac{\omega}{2}}\left(e^{-j\frac{\hat{\omega}}{2}}e^{j\frac{\omega}{2}}+e^{j\frac{\hat{\omega}}{2}}e^{-j\frac{\omega}{2}}\right)}{e^{-j\frac{\hat{\omega}}{2}}e^{-j\frac{\omega}{2}}\left(e^{j\frac{\hat{\omega}}{2}}e^{j\frac{\omega}{2}}+e^{-j\frac{\hat{\omega}}{2}}e^{-j\frac{\omega}{2}}\right)}$$

$$= -\frac{\cos\left(\frac{\omega-\hat{\omega}}{2}\right)}{\cos\left(\frac{\omega+\hat{\omega}}{2}\right)}$$

relate cutoff freqs

$$e^{-j\hat{\omega}} = \frac{-\alpha-e^{-j\omega}}{1+\alpha e^{-j\omega}}$$

$$= \frac{-(\alpha+e^{-j\omega})(1+\alpha e^{j\omega})}{|\cdot|^2}$$

$$= \frac{-2\alpha-\cos\omega-\alpha^2\cos\omega+j(\sin\omega-\alpha^2\sin\omega)}{|\cdot|^2}$$

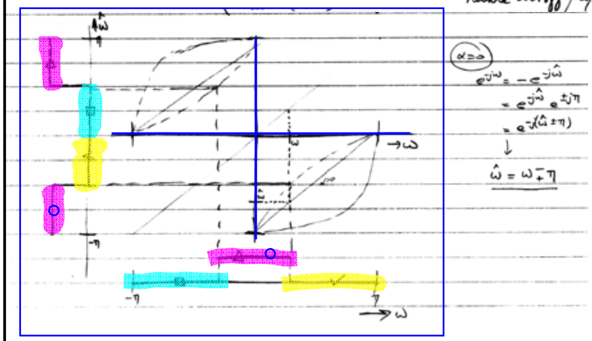
$$\Downarrow$$

$$-\hat{\omega} = \operatorname{atan}\left(\frac{(1-\alpha^2)\sin\omega}{-2\alpha-(1+\alpha^2)\cos\omega}\right)$$

(from A.A.Beex lecture notes)

$$-\hat{\omega} = \operatorname{atan}\left(\frac{(1-\alpha^2)\sin\omega}{-2\alpha-(1+\alpha^2)\cos\omega}\right) \qquad \alpha = -\frac{\cos\left(\frac{\omega-\hat{\omega}}{2}\right)}{\cos\left(\frac{\omega+\hat{\omega}}{2}\right)}$$

rotate cutoff freqs



$\alpha = 0$

$$e^{-j\hat{\omega}} = -e^{-j\hat{\omega}} = e^{-j\hat{\omega}}e^{\pm j\pi} = e^{-j(\hat{\omega}\pm\pi)}$$

$$\hat{\omega} = \omega \mp \pi$$

(from A.A.Beex lecture notes)

- Example - Transform the lowpass filter

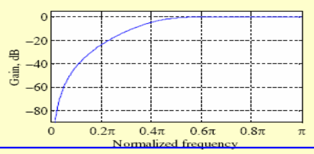
$$G_L(z) = \frac{0.0662(1+z^{-1})^3}{(1-0.2593z^{-1})(1-0.6763z^{-1}+0.3917z^{-2})}$$

- with a passband edge at 0.25π to a highpass filter with a passband edge at 0.55π
- Here $\alpha = -\cos(0.4\pi)/\cos(0.15\pi) = -0.3468$
- The desired transformation is

$$z^{-1} = -\frac{\hat{z}^{-1} - 0.3468}{1 - 0.3468\hat{z}^{-1}}$$

- The desired highpass filter is

$$G_D(\hat{z}) = G(z)\Big|_{z^{-1} = -\frac{\hat{z}^{-1}-0.3468}{1-0.3468\hat{z}^{-1}}}$$



- The lowpass-to-highpass transformation can also be used to transform a highpass filter with a cutoff at ω_c to a lowpass filter with a cutoff at $\hat{\omega}_c$

  and transform a bandpass filter with a center frequency at ω_o to a bandstop filter with a center frequency at $\hat{\omega}_o$

- **Lowpass-to-Highpass Transformation**
- Basic form:

```
[AllpassNum,AllpassDen] =
allpass1p2hp(wold,wnew)
```

where wold is the specified angular bandedge frequency of the original lowpass filter, and wnew is the desired angular bandedge frequency of the highpass filter

Generation of Allpass Function Using MATLAB

- **Lowpass-to-Highpass Example** – wold = 0.25π, wnew = 0.55π
- The MATLAB statement

```
[APnum, APden]
= allpass1p2hp(0.25, 0.55)
```

yields the mapping

$$z^{-1} \rightarrow \frac{-z^{-1} + 0.3468}{-0.3468z^{-1} + 1}$$

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Spectral Transformation Using MATLAB

- The pertinent M-files are iirlp2lp, iirlp2hp, iirlp2bp, and iirlp2bs
- **Lowpass-to-Highpass Example** –

$$G_{LP}(z) = \frac{0.066(1+z^{-1})^3}{1-0.9353z^{-1}+0.5669z^{-2}-0.1015z^{-3}}$$

Passband edge wold = 0.25π

Desired passband edge of highpass filter wnew = 0.55π

- The MATLAB code fragments used are

```
b = 0.066*[1 3 3 1];
a = [1.00 -0.9353 0.5669 -0.1015];
[num,den,APnum,APden]
= iirlp2hp(b,a,0.25,0.55);
```

- The desired highpass filter obtained is

$$G_{HP}(z) = \frac{0.218(1-z^{-1})^3}{1-0.3521z^{-1}+0.3661z^{-2}-0.0329z^{-3}}$$

Lowpass-to-Bandpass Spectral Transformation

- Desired transformation

$$\boxed{z^{-1} = -\frac{\hat{z}^{-2} - \frac{2\alpha\beta}{\beta+1}\hat{z}^{-1} + \frac{\beta-1}{\beta+1}}{\frac{\beta-1}{\beta+1}\hat{z}^{-2} - \frac{2\alpha\beta}{\beta+1}\hat{z}^{-1} + 1}}$$

- The parameters α and β are given by

$$\alpha = \frac{\cos((\hat{\omega}_{c2}+\hat{\omega}_{c1})/2)}{\cos((\hat{\omega}_{c2}-\hat{\omega}_{c1})/2)}$$

$$\beta = \cot((\hat{\omega}_{c2}-\hat{\omega}_{c1})/2)\tan(\omega_c/2)$$

where ω_c is the cutoff frequency of the lowpass filter, and $\hat{\omega}_{c1}$ and $\hat{\omega}_{c2}$ are the desired upper and lower cutoff frequencies of the bandpass filter

Important note:
The above transformations such as lp2lp, lp2hp, lp2bp and lp2bs. Can be used only to map one frequency point ω_c in the magnitude response of the lowpass prototype filter into a new position $\hat{\omega}_c$ with the same mag. Response value for the transformed lp and hp filters; or into two new positions $\hat{\omega}_{c1}$ and $\hat{\omega}_{c2}$ with the same mag. Response values for the transformed bp and bs filters.
Hence; it is possible only to map either the pasband edge or the stopband edge of the lp prototype filter onto the desired position(s) but not both

- Special Case - The transformation can be simplified if $\omega_c = \hat{\omega}_{c2} - \hat{\omega}_{c1}$
- Then the transformation reduces to

$$z^{-1} = -\hat{z}^{-1}\frac{\hat{z}^{-1} - \alpha}{1 - \alpha\hat{z}^{-1}}$$

where $\alpha = \cos\hat{\omega}_o$ with $\hat{\omega}_o$ denoting the desired center frequency of the bandpass filter

Lowpass-to-Bandstop Spectral Transformation

- Desired transformation

$$z^{-1} = \frac{\hat{z}^{-2} - \frac{2\alpha\beta}{1+\beta}\hat{z}^{-1} + \frac{1-\beta}{1+\beta}}{\frac{1-\beta}{1+\beta}\hat{z}^{-2} - \frac{2\alpha\beta}{1+\beta}\hat{z}^{-1} + 1}$$

- The parameters α and β are given by

$$\alpha = \frac{\cos((\hat{\omega}_{c2} + \hat{\omega}_{c1})/2)}{\cos((\hat{\omega}_{c2} - \hat{\omega}_{c1})/2)}$$

$$\beta = \tan((\hat{\omega}_{c2} - \hat{\omega}_{c1})/2)\tan(\omega_c/2)$$

where ω_c is the cutoff frequency of the lowpass filter, and $\hat{\omega}_{c1}$ and $\hat{\omega}_{c2}$ are the desired upper and lower cutoff frequencies of the bandstop filter

- **Lowpass-to-Bandpass Transformation**
- Basic form:

```
[AllpassNum,AllpassDen] =
allpasslp2bp(wold,wnew)
```

where wold is the specified angular bandedge frequency of the original lowpass filter, and wnew is the desired angular bandedge frequency of the bandpass filter

- **Lowpass-to-Bandstop Transformation**
- Basic form:

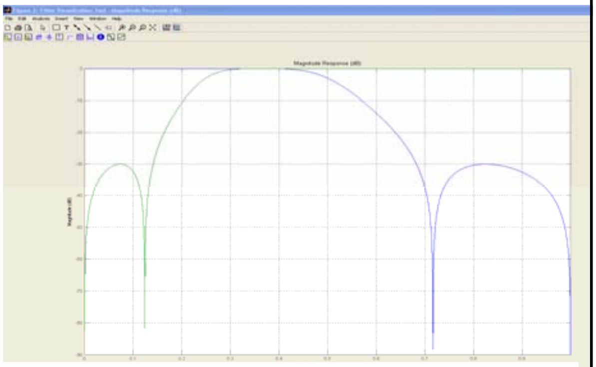
```
[AllpassNum,AllpassDen] =
allpasslp2bs(wold,wnew)
```

where wold is the specified angular bandedge frequency of the original lowpass filter, and wnew is the desired angular bandedge frequency of the bandstop filter

Spectral Transformation Using MATLAB

- The pertinent M-files are iirlp2lp, iirlp2hp, iirlp2bp, and iirlp2bs example

Filter visualization tool (check it)




```
[b, a]= ellip(3, 0.1, 30, 0.409); % IIR halfband filter
[num, den] = iirlp2hp(b, a, 0.5, 0.25);
fvtool(b, a, num, den);
```