Chapter 4

Frequency Hopped Spread Spectrum

In the previous chapter we introduced the concept of direct sequence spread spectrum. While this is the most common form of spread spectrum, it is by no means the only important technique. In this chapter we describe a second common form, namely frequency hopped spread spectrum. Additionally, we will also briefly describe other techniques including chirp modulation and time-hopping.

4.1 Definition/Description

The goal of spread spectrum systems is to increase the dimensionality of the signal. By increasing the dimensionality, we make eavesdropping and/or jamming more difficult since there are more dimensions of the signal to consider. In commercial applications, this means that the increased dimensionality provides robustness in the presence of other systems and less interference caused to those same systems. The main method of increasing the dimensionality of the signal is to increase the signal's spectral occupancy. Last chapter we discussed in detail one method of accomplishing this, direct sequence spread spectrum. In DS/SS the bandwidth is increased by directly multiplying the data signal by a high rate pseudo-random spreading sequence. A second method of accomplishing this bandwidth expansion is through frequency hopping. In frequency hopped spread spectrum (FH/SS) the carrier frequency of the data modulated sinusoidal carrier is periodically changed over some predetermined bandwidth. By "hopping" the center frequency to one of N contiguous but non-overlapping frequency bands, the overall spectrum occupancy is increased by the factor N. This hopping is typically done in a pseudo-random manner. In military applications this makes interception and jamming more difficult. In commercial applications, it reduces the impact of a particular co-channel interferer as well as the impact to another system since it will only be present in a particular

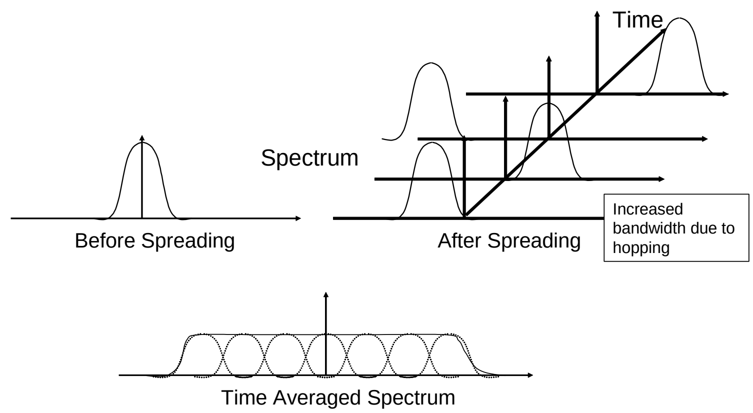

Figure 4.1: Illustration of Spectrum Spreading through Frequency Hopping

band on average $1/N$ of the time.

The hopping signal can be represented as

$$h(t) = \sum_{i=-\infty}^{\infty} p(t - iT_c) \cos(2\pi f_i t + \phi_i) \tag{4.1}$$

where $p(t)$ is the pulse shape used for the hopping waveform (typically assumed to be a square pulse), $f_i \in \{f_1, f_2, \ldots f_N\}$ are the N hop frequencies, T_c is the hop period also called the chip period, and ϕ_i are the phases of each oscillator. The resulting frequency hopped transmit signal is then

$$\begin{aligned} s(t) &= [s_d(t)h(t)]_{BPF} \\ &= \left[s_d(t) \sum_{i=-\infty}^{\infty} p(t - nT_c) \cos 2\pi f_i t + \phi_i \right]_{BPF} \end{aligned} \tag{4.2}$$

where $s_d(t)$ is the bandpass data signal which depends on the modulation scheme employed and the bandpass filter (applied to the quantity within $[\cdot]_{BPF}$) is designed to transmit the sum frequencies only. The concept of frequency hopping is illustrated in Figure 4.1. As time advances the signal occupies a separate frequency band as determined by the hopping sequence. On average the power spectral density is spread over the entire band as shown. Provided that each frequency band is used $\frac{1}{N}$ of the time, the spectrum will be similar to that seen in DS/SS systems when averaged over a sufficiently long time period.

The transmitter and receiver for a typical implementation are shown in Figures 4.2 and 4.3 respectively. As shown in the figures, any modulation scheme (with either coherent or non-coherent demodulation) can theoretically be used.

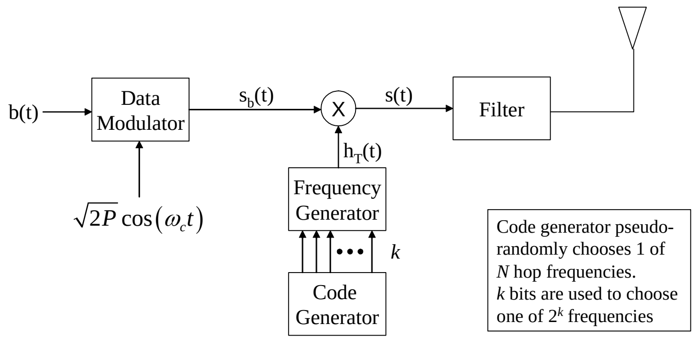

Figure 4.2: Typical Frequency-Hopping Transmitter Architecture

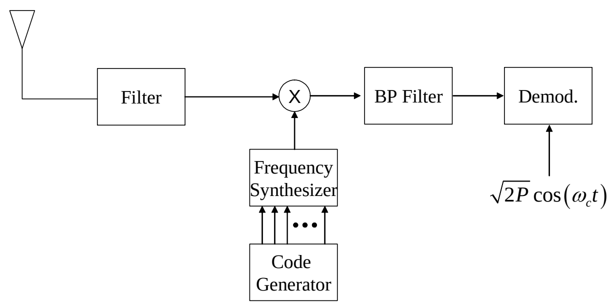

Figure 4.3: Typical Frequency Hopping Receiver Architecture

As in DS/SS the frequency hopping is ideally transparent to the data demodulator. The data modulated carrier is hopped to one of N carrier frequencies every "chip" period T_c which may be greater than the data symbol period T_s. At the receiver the same hopping pattern is generated such that the received signal is ideally mixed back down to the original carrier frequency. Data demodulation is then accomplished as in standard digital communications. Note that the bandwidth expansion factor is equal to N the number of hop frequencies. Unlike in DS/SS, the bandwidth expansion is not dependent on the chip period T_c. In fact, as mentioned, the chip period can be greater than the symbol period. In other words, the hopping may be slower than the symbol rate. We will discuss the consequences of this relationship later.

Although, any modulation format can be used with FH/SS, coherent demodulation techniques require that the frequency hopping maintain frequency coherence each hop. This can be difficult to maintain and thus non-coherent demodulation techniques are more commonly used with FH/SS. Specifically, M-FSK is commonly used in conjunction with FH/SS.

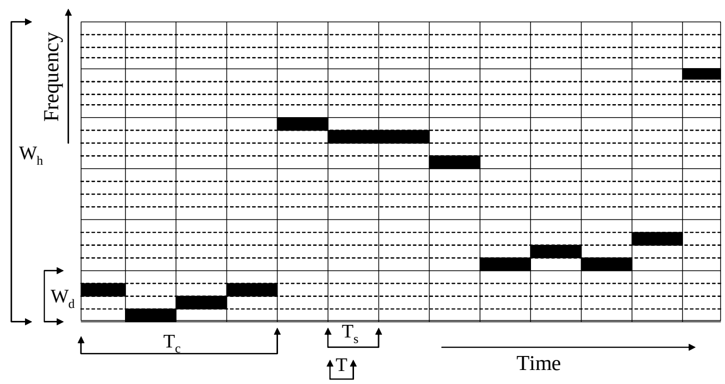

Figure 4.4: Example of a Time-Frequency Plot for Slow Hopping (T_c=Hop period, T_s = symbol period, T_b = bit period, W_h = spread bandwidth, W_d = symbol bandwidth)

4.1.1 Slow versus Fast Hopping

As mentioned earlier, the hop period (also called the chip period T_c) may be greater or less than the symbol duration. The bandwidth expansion factor is related only to the number of hop frequencies N, not the hop period. Thus, we are free to choose the hop period based on other considerations. Specifically, the hop frequency should be chosen based on implementation and performance considerations. First let us consider the case where $T_c > T_s$, or slow hopping. Additionally, let us assume that FSK modulation is used. Figure 4.4 plots the frequency occupancy versus time considering both the data modulation and frequency hopping. In this example $T_c = 4T_s$, or the frequency is hopped every four symbols, $N = 6$ and $M = 4$ ($T_b = T_s/2$). Further, in the figure we have defined W_d as the bandwidth of the MFSK signal and W_h as the spread bandwidth. As can be seen, every T_s seconds, the frequency is changed to one of 4 symbols based on the data. Additionally, every T_c seconds, the center frequency of these symbols is changed based on the frequency hopping pattern. At the receiver the pseudo-random hopping is removed, leaving only the data modulation as shown in Figure 4.5.

In contrast to slow hopping, with fast frequency hopping $T_c < T_s$. That is, frequency hopping occurs faster than the modulation. This is depicted in Figure 4.6 where $T_c = \frac{T_s}{2}$, N=6, and M=4. In this case coherent modulation is extremely difficult since it would require extremely fast carrier synchronization. Thus, non-coherent FSK is almost universally used with fast hopping. The despread or de-hopped signal is plotted in Figure 4.5 which shows that the despread data is the same as in slow hopping. Fast hopping, although more difficult to implement, offers some advantages over slow hopping. First, unlike slow hopping, fast hopping provides frequency diversity at the symbol level which provides substantial benefit in fading channels or versus narrowband jamming.

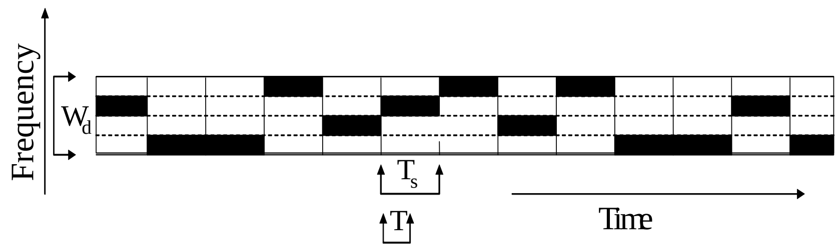

Figure 4.5: Example of Time-Frequency Plot after Despreading

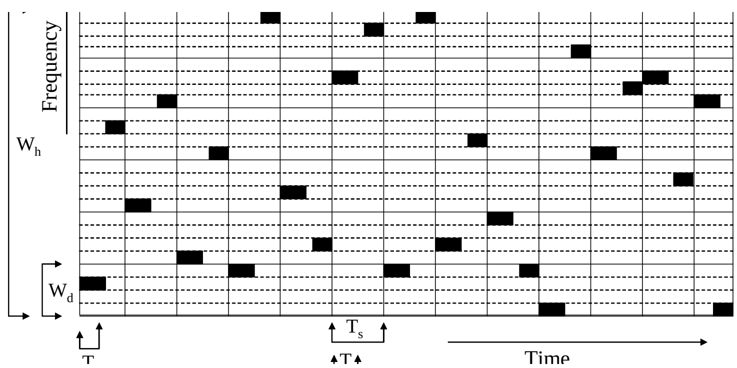

Figure 4.6: Example of Time-Frequency Plot for Fast Hopping (T_c=Hop period, T_s = symbol period, T_b = bit period, W_h = spread bandwidth, W_d = symbol bandwidth)

Slow hopping can obtain these same benefits through error correction coding as we will see later, but fast hopping offers this benefit before coding, which can provide better performance, especially when punctured codes are used.

The reception of FH/SS is accomplished as shown in Figure 4.3. The despread signal $y(t)$ is obtained by multiplying the incoming signal by the hopping signal and filtering out the images:

$$\begin{aligned} y(t) &= [r(t)h(t)]_{BPF} \\ &= \left[(s(t)+n(t))\sum_{i=-\infty}^{\infty} p(t-nT_c)\cos 2\pi f_i t + \phi_i\right]_{BPF} \\ &= s_d(t) + n'(t) \end{aligned} \tag{4.3}$$

where $n\prime(t)$ is the noise process after despreading and filtering.

4.2 Complex Baseband Representation

As with DS/SS, the complex envelope representation can be convenient for analysis and simulation. Thus, we would like to introduce the complex baseband for FH/SS systems. Specifcally, the hopping waveform can be represented in complex baseband as

$$\tilde{h}(t) = \sum_{i=-\infty}^{\infty} p(t - iT_c) e^{j(2\pi f_i t + \phi_i)} \tag{4.4}$$

where f_i, ϕ_i and $p(t)$ were defined earlier. For M-FSK modulation, the complex baseband version of the data signal is

$$\tilde{d}(t) \sum_{i=-\infty}^{\infty} p(t - iT_s) e^{j(2\pi f_i t + \phi_i)} \tag{4.5}$$

where $f_i \in \{f_1, f_2, \ldots f_M\}$ are the M symbol frequencies. The transmit signal is then

$$\tilde{s}(t) = \tilde{d}(t)\tilde{h}(t) \tag{4.6}$$

and dehopping (depsreading) is accomplished by $\tilde{y}(t) = \tilde{s}(t)\tilde{h}^*(t) = \tilde{h}(t)\tilde{d}(t)\tilde{h}^*(t) = \tilde{d}(t)$ since $\left|\tilde{h}\left(t\right)\right|^2 = 1$ assuming square pulses.

4.3 Power Spectral Density of FH/SS

The power spectral density of FH/SS can be found as

$$S(f) = S_d(f) * H(f) \tag{4.7}$$

where $S_d(f)$ is the power spectral density of the data modulated carrier before hopping and $H(f)$ is the power spectral density of the hopping waveform. If we define N as the number of hop frequencies it can be shown that the PSD of the hopping waveform is [Peterson *et al.*, 1995]

$$\begin{aligned} H(f) \;\; = \;\; & \frac{1}{T_c^2} \sum_{i=-\infty}^{\infty} \left| \sum_{k=1}^{N} p_k G_k\left(\frac{i}{T_c}\right)\right|^2 \delta\left(f - \frac{i}{T_c}\right) + \frac{1}{T_c}\sum_{k=1}^{N} p_k(1-p_k)\left|G_k(f)\right|^2 \\ & -\frac{1}{T_c}\sum_{k=1}^{N} \sum_{\substack{m=1 \\ m \neq k \\ m > k}}^{N} p_k p_m \Re\left\{G_k(f) G_m^*(f)\right\} \end{aligned} \tag{4.8}$$

where $G_m(f)$ is the Fourier Transform of the pulsed carrier $p(t)\cos\left(2\pi f_m t + \phi_m\right)$ defined over $0 \leq t \leq T_c$ and p_m is the probability of using the mth carrier. We can find $G_m(f)$ as (assuming $p(t)$ is a square pulse)

$$\begin{aligned} G_m(f) &= \mathrm{F}\{g_m(t)\} \\ &= \mathrm{F}\{p(t)\cos(2\pi f_m t + \phi_m)\} \\ &= T_c e^{-j[\pi(f-f_m)T_c - \phi_m]}\mathrm{sinc}((f-f_m)T_c) \\ &\quad + T_c e^{-j[\pi(f+f_m)T_c + \phi_m]}\mathrm{sinc}((f+f_m)T_c) \end{aligned} \tag{4.9}$$

Now, if the carrier spacing is such that the spectra of of $G_m(f)$ and $G_k(f)$ do not overlap for $m \neq k$ (i.e., if the hop rate $\frac{1}{T_c}$ is slow compared to the minimum carrier spacing), and we assume that all hop frequencies are equally likely we obtain

$$\begin{aligned} H(f) &\approx \frac{1}{T_c^2 N^2}\sum_{i=-\infty}^{\infty}\sum_{k=1}^{N}\left|G_k\left(\frac{i}{T_c}\right)\right|^2 \delta\left(f - \frac{i}{T_c}\right) \\ &\quad + \frac{1}{T_c}\frac{1}{N}\left(1 - \frac{1}{N}\right)\sum_{k=1}^{N}|G_k(f)|^2 \end{aligned} \tag{4.10}$$

Inserting for equation (4.9) for $G_m(f)$ into (4.10), the resulting PSD is

$$\begin{aligned} H(f) &\approx \frac{1}{N^2}\sum_{i=-\infty}^{\infty}\sum_{k=1}^{N}\left(\mathrm{sinc}^2(i - f_k T_c) + \mathrm{sinc}^2(i + f_m T_c)\right)\delta\left(f - \frac{i}{T_c}\right) \\ &\quad + \frac{T_c}{N}\left(1 - \frac{1}{N}\right)\sum_{k=1}^{N}\left[\mathrm{sinc}^2((f-f_k)T_c) + \mathrm{sinc}^2((f+f_k)T_c)\right] \end{aligned} \tag{4.11}$$

Now, if we choose the frequency spacing to be an integer multiple of the hop rate for illustration purposes, we sample the sinc function at integer values eliminating all terms except the first:

$$\begin{aligned} H(f) &\approx \frac{1}{N^2}\sum_{k=1}^{N}\left[\delta(f - f_k) + \delta(f + f_k)\right] \\ &\quad + \frac{T_c}{N}\left(1 - \frac{1}{M}\right)\sum_{k=1}^{N}\left[\mathrm{sinc}^2((f-f_k)T_c) + \mathrm{sinc}^2((f+f_k)T_c)\right] \end{aligned} \tag{4.12}$$

As an example, let us consider the power spectral density when BPSK with coherent frequency hopping is used. Now, from previous developments we know that the PSD of BPSK is

$$S_d(f) = \frac{A^2 T_b}{4}\left[\mathrm{sinc}^2((f-f_c)T_b) + \mathrm{sinc}^2((f+f_c)T_b)\right] \tag{4.13}$$

In order to find the PSD of transmit signal $S(f)$, we must convolve $H(f)$ with

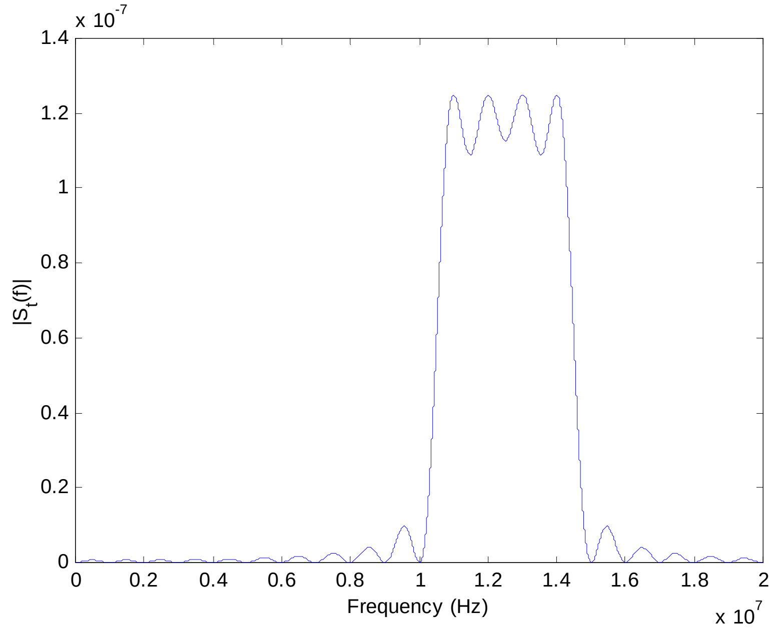

Figure 4.7: Example Power Spectrum of Frequency Hopped Signal with BPSK Modulation (R_b=1Mbps, f_h = 11MHz,12MHz, 13MHz, 14MHz)

$S_d(f)$ resulting in

$$\begin{aligned} S(f) &\approx \frac{PT_b}{2N^2}\sum_{k=1}^{N}\left[\mathrm{sinc}^2\left((f-f_k-f_c)T_b\right)+\mathrm{sinc}^2\left((f+f_k+f_c)T_b\right)\right]\ldots \\ &\quad+\left(1-\frac{1}{N}\right)\frac{PT_b}{2N}\sum_{k=1}^{N}\left[\mathrm{sinc}^2\left((f-f_k-f_c)T_b\right)+\mathrm{sinc}^2\left((f+f_k+f_c)T_b\right)\right] \\ &= \frac{PT_b}{2N}\sum_{k=1}^{N}\left[\mathrm{sinc}^2\left((f-f_k-f_c)T_b\right)+\mathrm{sinc}^2\left((f+f_k+f_c)T_b\right)\right] \end{aligned} \tag{4.14}$$

which is an intuitively satisfying result as it says that the PSD of the frequency-hopped signal is the sum of N replicas of the information signal PSD each centered at the hopping frequencies. An example is plotted in Figure 4.7 for R_b=1Mbps and hop frequencies of 11MHz, 12MHz, 13MHz, and 14MHz (i.e., 4 hop frequencies)

4.4 Performance of FH/SS

As with DS/SS, FH/SS results in no performance benefit in AWGN channels. This can be readily seen by examining the dehopped signal. If we use BPSK modulation, the received signal after despreading is

$$\begin{aligned}\tilde{y}(t) &= \tilde{r}(t)\tilde{h}^*(t) \\ &= \left(\tilde{h}(t)b(t) + n(t)\right)\tilde{h}^*(t) \\ &= b(t) + n(t)\tilde{h}^*(t)\end{aligned} \tag{4.15}$$

Now, if the hop sequence is completely random, dehopping will not impact the noise characteristics. Thus, the performance of coherent BPSK with coherent hopping results in a probability of error

$$P_b = \mathrm{Q}\left(\sqrt{\frac{2E_b}{N_o}}\right) \tag{4.16}$$

and other modulation schemes also achieve the same performance as without frequency hopping. Now consider a narrowband noise jammer with bandwith B. Assuming that the jammer knows the frequency band of the signal of interest, and $N_j = J/B$, the performance without frequency hopping is

$$P_b = \mathrm{Q}\left(\sqrt{\frac{1}{\frac{N_o}{2E_b} + \frac{N_j}{2E_b}}}\right) \tag{4.17}$$

which is plotted in Figure 4.8 (curve labeled "Narrowband Signal") for $\frac{E_b}{N_o} =$ 20dB and $\frac{E_b}{N_j} = 10$dB. We see that the presence of the jammer results in a probability of error of approximately 0.1%. Now, consider a frequency hopping signal which randomly hops to N different frequency bands. Since the frequency hopper will land in the band of the jammer only 1 out of N hops, the performance becomes

$$P_b = \frac{1}{N}\mathrm{Q}\left(\sqrt{\frac{1}{\frac{N_o}{2E_b} + \frac{N_j}{2E_b}}}\right) + \frac{N-1}{N}\mathrm{Q}\left(\sqrt{\frac{2E_b}{N_o}}\right) \tag{4.18}$$

which is also plotted in Figure 4.8 for the same parameters, but allowing N to vary. Obviously, as we let N increase, we are impacted by the jammer less frequently, improving performance. We can also plot the performance vs. $\frac{E_b}{N_j}$ as shown in Figure 4.9. The benefit of frequency hopping is evident, although this gain is clearly diminshed as $\frac{E_b}{N_j}$ increases. Note that the gains are different than DS/SS shown in the previous chapter. With DS/SS the received signal is always subject to intereference albeit at reduced levels after despreading. In FH/SS the effective power level of the jammer is not reduced, rather the frequency of the jammer's impact is reduced.

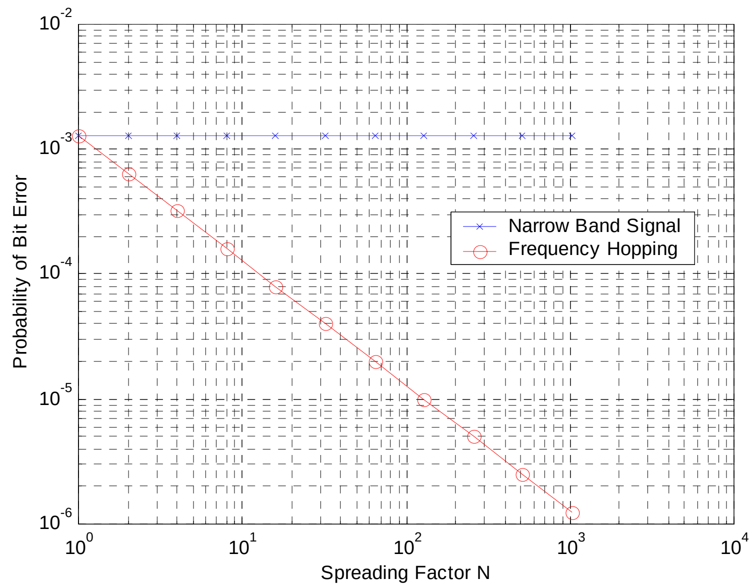

Figure 4.8: Performance of BPSK Narrowband Signal and BPSK Frequency-Hopped Signal in the Presence of Narrowband Interference (E_b/N_j=10dB, E_b/N_o=20dB)

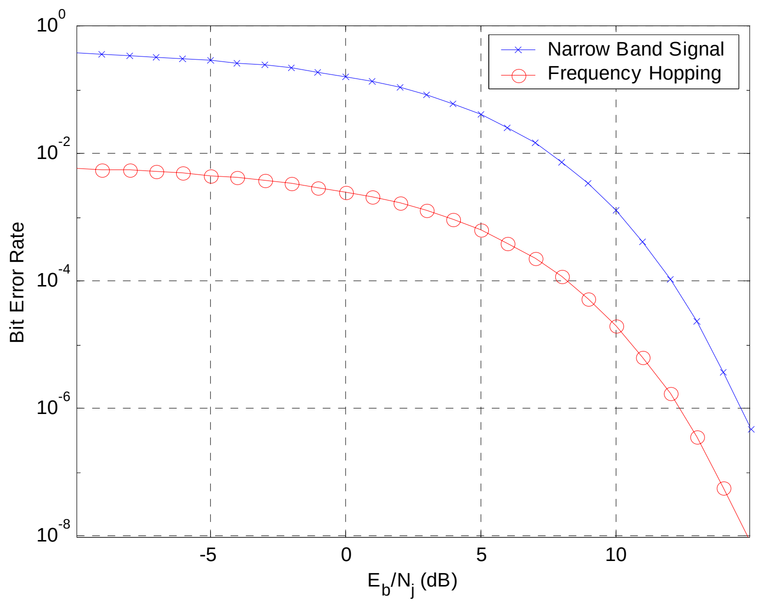

Figure 4.9: Performance of Narrowband BPSK Signal and Frequency-Hopped BPSK Signal in the Presesnce of a Narrowband Jammer $\left(\frac{E_b}{No} = 20dB, N = 64\right)$

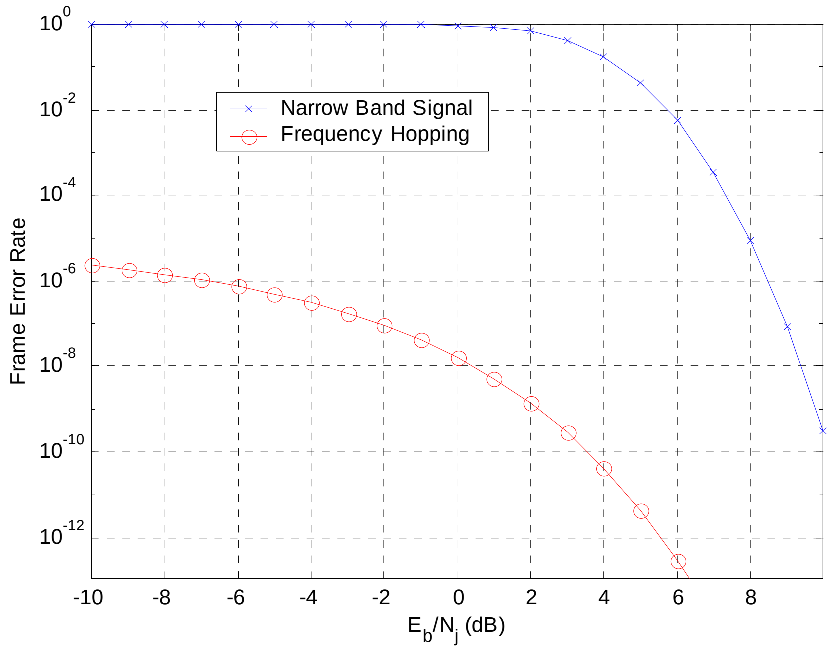

Figure 4.10: Performance of Narrowband BPSK and Frequency Hopped BPSK with Generic Block Coding

Now consider the use block codes which can correct up to e errors out of a block of B bits. The probability of codeword error can be written as

$$\begin{aligned} P_w &= 1 - P_c \\ &= 1 - \sum_{i=0}^{e} \left(\begin{array}{c} B \\ i \end{array} \right) (1 - P_b)^{31-i} (P_b)^i \end{aligned} \tag{4.19}$$

As an example, consider the previous example but with error correction coding on top of frequency hopping. Specifically, assume B=64 and e=5 and N=64. The resulting performance of standard coherent BPSK with block coding is plotted in Figure 4.10 along with the performance of frequency hopping with coding. We can see that the combination of frequency hopping and error correction coding provides a tremendous improvement in the presence of jamming. While coding improves narrowband performance, the combination of FH and coding is especially powerful. We will examine the performance FH/SS in more detail in Chapter 10.

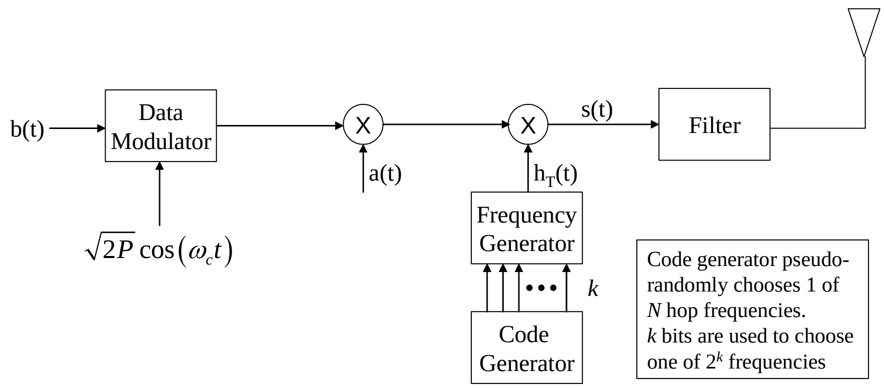

Figure 4.11: Transmitter for Hybrid DS/FH Spread Spectrum System

4.5 Other Techniques

In addition to DS/SS and FH/SS there are several other forms of spread spectrum. Three specific forms of spread spectrum that we will discuss in this section include

- hybrid DS/FH/SS
- chirp modulation
- time hopping

4.5.1 Hybrid DS/FH/SS

Another common form of spread spectrum is known as Hybrid DS/FH/SS which combines frequency hopping and direct sequence techniques. The technique is described in Figures 4.11 and 4.12. As described in Figure 4.11 the data signal is first spread using a direct sequence technique and then further spread by hopping the center frequency to one of N hop frequencies. This technique is useful for obtaining extremely high spreading factors since it can spread the bandwidth more than either DS/SS or FH/SS alone. Additionally, this technique increases the complexity needed for an interceptor and provides benefits of both DS and FH.

4.5.2 Chirp Modulation

Chirp modulation is another form of spread spectrum which linearly increases the transmit frequency over the symbol duration. That is the frequency is

$$f_1 + \mu t \quad 0 \leq t \leq T_s \tag{4.20}$$

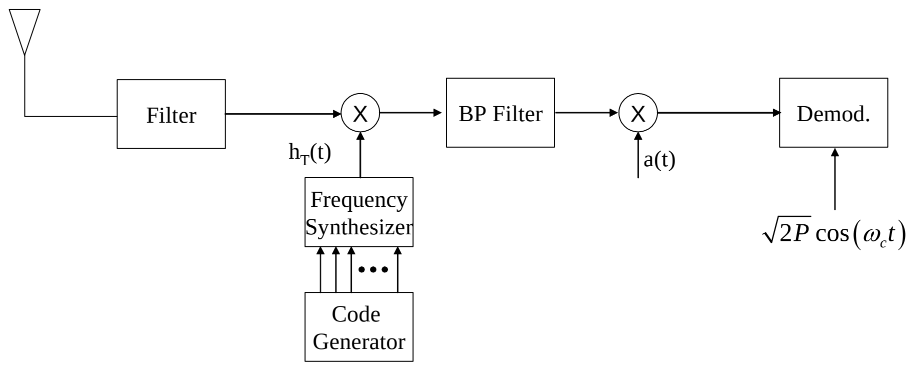

Figure 4.12: Receiver for Hybrid DS/FH Spread Spectrum System

where f_1 is the initial frequency and $\mu = \frac{df}{dt}$ is the rate of the chirp. The transmit signal is then

$$\cos\left(2\pi(f_1 + \mu t)t + \phi\right) \quad 0 \leq t \leq T_s \tag{4.21}$$

An example (f_1 = 10kHz, $\mu = 90MHz$, and T_s = 1ms) is plotted in Figure 4.13. Note that up-chirp signals and down-chirp signals are possible. In fact one method of modulation is the use of an up-chirp for '1' and down-chirp for '0'.

4.5.3 Time Hopping (Ultra Wideband)

The final spread spectrum technique that we will discuss is Time Hopped Spread Spectrum (TH/SS). Time hopping is typically used with pulse position modulation (PPM), thus we briefly describe PPM first. Binary PPM is described in Figure 4.14. With PPM, the position of the pulse is moved forward or backward from the nominal symbol time by δ seconds based on the data being sent. Provided that the pulses are non-overlapping, the modulation scheme is orthogonal and achieves the performance equivalent to any orthogonal modulation scheme. In other words

$$P_b = \mathrm{Q}\left(\sqrt{\frac{E_b}{N_o}}\right) \tag{4.22}$$

However, depending on the pulse shape used, slightly better performance can be obtained with non-orthogonal pulses.

To improve the probability of intercept, as well as to reduce the impact of jamming, time hopping can be added to PPM. In order to allow additional time modulation for time hopping, a frame time is defined as shown in Figure 4.16. A single pulse is transmitted each frame where the frame duration T_f is much larger than the pulse duration T_p. The bandwidth is determined by the

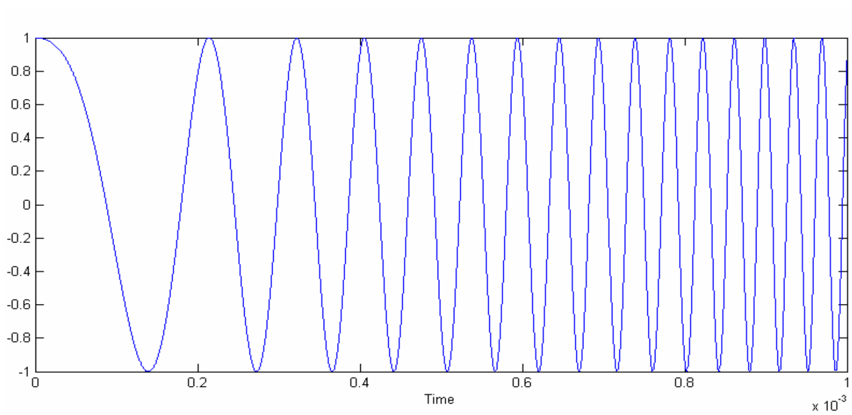

Figure 4.13: Example of an Up-chirp Time Waveform

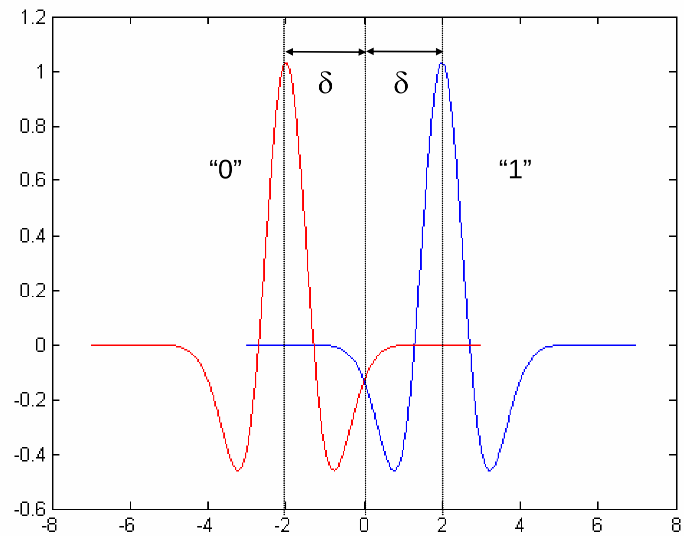

Figure 4.14: Pulse Position Modulation

pulse duration T_p while the data rate is proportional to the pulse repitition rate $R_f = 1/T_f$. Time hopping modulates the position of the pulse within the frame as shown in Figures 4.17. Further, this additional modulation is pseudo-random which makes unauthorized detection and jamming more difficult.

The frame size depends on the number of possible time hopping positions which have granularity T_c (Figure 4.17). Additionally, multiple (N_s)pulses can be transmitted per information bit, which decreases the data rate, increasing the spreading factor B/R_b. This is described in Figure 4.14. The final form of the transmit signal can be expressed as

$$s(t) = \sum_{i=-\infty}^{\infty} Ap\left(t - iT_f - c_i T_c - \delta d_{\lfloor j/N_s \rfloor}\right) \tag{4.23}$$

where c_j represents the pseudo-random time hopping code and d_j represents the data modulation. Demodulation is accomplished by first de-hopping the signal and then performing maximimum likelihood detection on the PPM signal.

4.6 Conclusions

In this chapter we have briefly introduced frequency-hopped spread spectrum and other less common forms of spread spectrum. We will discuss the performance of FH/SS in detail in Chapter 9. In the next chapter we will discuss the

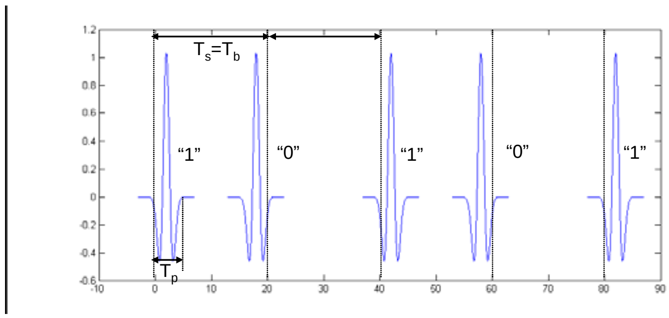

Figure 4.15:

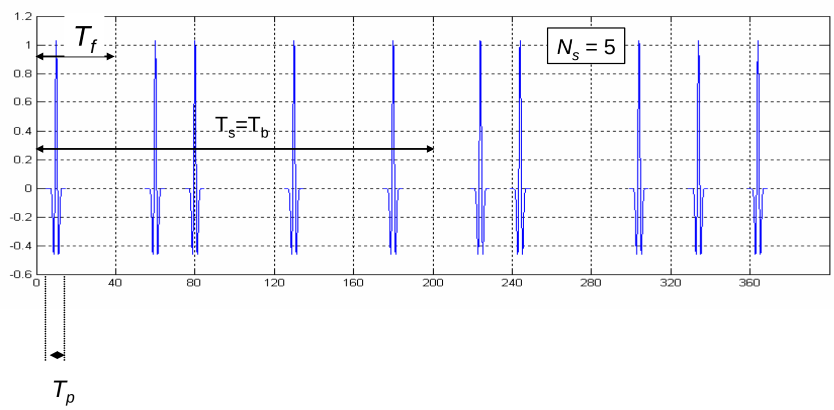

Figure 4.16: Example of Pulse Position Modulation

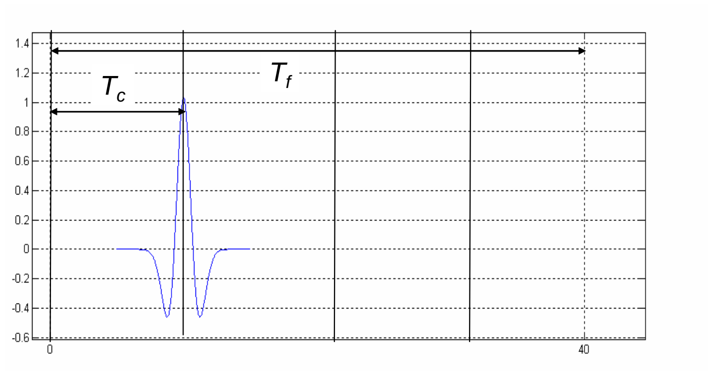

Figure 4.17: Definitions for Time-Hopping

properties of the spreading waveforms that are vital to the proper operation of spread spectrum.

Bibliography

[Peterson *et al.*, 1995] R.L. Peterson, R.E. Ziemer, and D.E. Borth. *Introduction to Spread Spectrum Communications*. Prentice-Hall, 1995.