Chapter 3

Direct Sequence Spread Spectrum

In Chapter 2 we presented a motivation for the use of a communication signal whose bandwidth is much larger than the data rate being sent. In this chapter we begin our discussion of specific techniques to accomplish this. The first spread spectrum technique described is called Direct Sequence Spread Spectrum or DS/SS.

3.1 Definition/Description

Direct sequence spread spectrum is perhaps the most common form of spread spectrum in use today. DS/SS accomplishes bandwidth spreading through the use of a high rate symbol sequence which directly multiplies the information symbol stream. Since the symbol sequence has a rate much higher than the data rate, the bandwidth is increased. The simplest form of DS/SS uses BPSK modulation with BPSK spreading and is illustrated in Figure 3.1. Note that this is equivalent to a standard BPSK system with a matched filter receiver with the addition of the spreading and despreading process. Note that the receiver is equivalent to a matched filter provided that square pulses are used. If pulse shaping is employed, the simple integrator should be replaced by a filter that is matched to the pulse shape used. The transmit signal can be represented by

$$\begin{aligned} s(t) &= \sqrt{2P}a(t)\cos\left(2\pi f_c t + \theta_d(t)\right) \\ &= \sqrt{2P}a(t)b(t)\cos\left(2\pi f_c t\right) \qquad (3.1) \end{aligned}$$

where $\theta_d(t)$ is the binary phase shift due to the information sequence, $b(t) = \sum_{i=-\infty}^{\infty} b_i p_b(t - iT_b)$ is the information signal where $b_i \in \{+1, -1\}$ represent the information bits, each bit has duration T_b, $p_b(t)$ is the pulse shape used for the information waveform (assumed to be rectangular), $a(t)$ is the spreading signal where each symbol (usually called a 'chip') has duration $T_c = \frac{T_b}{N}$, f_c is the

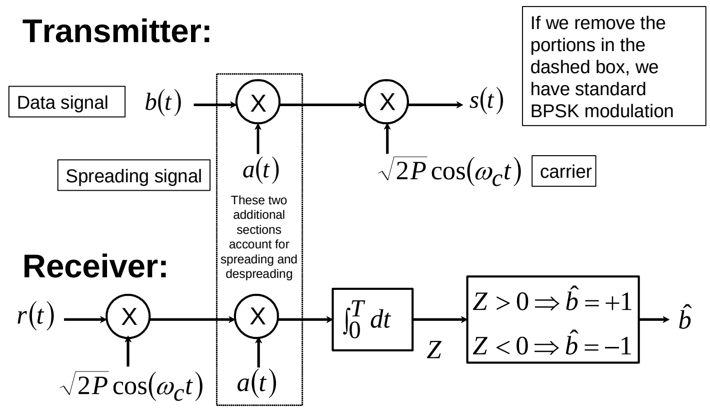

Figure 3.1: Transmitter and Receiver Block Diagram for BPSK Spreading and BPSK Modulation

center frequency of the transmit signal, P is the power of the signal and N is the bandwidth expansion factor also sometimes called the *spreading gain*. Example waveforms for the case of rectangular pulses are given in Figure 3.2. It can be seen that the chip rate is N times that of the bit rate, resulting in a signal whose bandwidth is much larger than necessary for transmission of the information. Specifically, as we will show shortly, the bandwidth is commensurate with the chip rate or N times what a traditional BPSK signal would be. We will discuss the performance of DS/SS in more detail in Chapter 8.

At the receiver, the opposite operations are performed. Specifically, the signal is first down-converted to baseband[1]. After down-conversion, the signal is despread and passed to a standard BPSK detector. This process can be envisioned in two ways. First, we can view the spreading/despreading operations as transparent additions to a standard BPSK transmit/receiver pair. The spreading is applied after BPSK symbol creation and despreading occurs before the BPSK detector. Secondly, we can view DS/SS as a BPSK modulation scheme where the 'pulse' is the spreading waveform. Thus, at the receiver the despreading operation can be viewed as part of a correlator version of a matched filter receiver.

At the receiver, the received signal can be modeled as

$$\begin{aligned} r(t) &= s(t) + n(t) \\ &= \sqrt{2P}a(t)b(t)\cos(2\pi f_c t) + n(t) \end{aligned} \tag{3.2}$$

[1]Despreading can also be done at IF, although baseband is currently more common.

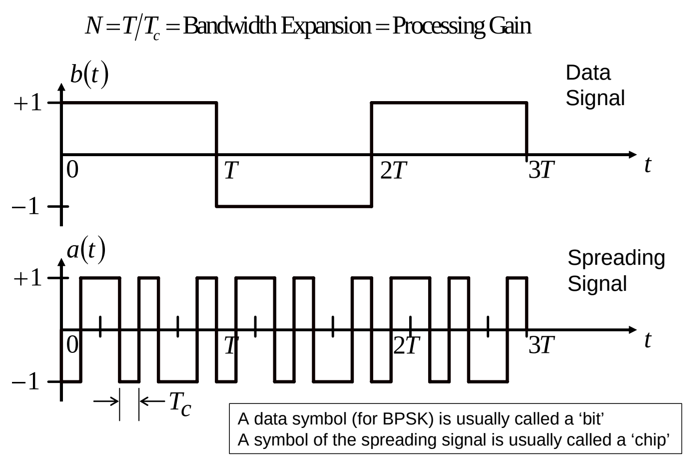

Figure 3.2: Example Data and Chip Sequences for DS/SS with BPSK information and BPSK spreading

where $n(t)$ is bandpass Additive White Gaussian Noise (AWGN). The maximum likelihood receiver then calculates the decision statistic as

$$\begin{aligned}
Z &= \int_0^{T_b} r(t)a(t)\cos(2\pi f_c t)\,dt \qquad (3.3)\\
&= \int_0^{T_b} \left(\sqrt{2P}a(t)b(t)\cos(2\pi f_c t)\right) a(t)\cos(2\pi f_c t)\,dt... \\
&\quad + \int_0^{T_b} n(t)\,a(t)\cos(2\pi f_c t)\,dt \qquad (3.4)\\
&= \int_0^{T_b} \left(\sqrt{2P}a^2(t)b(t)\cos^2(2\pi f_c t)\right) dt + N
\end{aligned}$$

where we have assumed perfect phase coherence, bit timing and chip timing and N is the noise at the output of the matched filter. Now, in BPSK spreading, the spreading signal $a(t)$ can be modeled as

$$a(t) = \sum_{i=-\infty}^{\infty} a_i p_c(t - iT_c) \qquad (3.5)$$

where $a_i \in \{+1, -1\}$ is the spreading sequence and $p_c(t)$ is the chip pulse shape, assumed to be rectangular for this discussion. We will discuss the properties of the spreading sequence in Chapter 5. It can be readily discerned that $a^2(t) = 1$. Further, ignoring the double frequency term in (3.3), the decision statistic becomes

$$Z = \frac{\sqrt{2P}T_b}{2} b_0 + N$$

where we have assumed that $p_b(t)$ is a rectangular pulse of duration T_b and N is due to AWGN and will be analyzed later. Thus, we can see that we obtain a decision variable that is comprised of the original bit along with a noise term, just as in standard BPSK. We will analyze the performance of this scheme shortly.

3.2 Quadrature spreading

Until this point we have considered only the simplest form of DS/SS, BPSK spreading with BPSK modulation. In general many modulation formats are theoretically possible, including PSK and PAM. However, PAM formats are not generally used because (a) an unmodulated carrier makes it easier to detect (i.e., by an interceptor) and (b) it is less energy efficient than PSK. General PSK formats are common including BPSK, QPSK, and MSK. Additionally, QPSK spreading can be used in addition to general PSK data modulation. The transmitter for such a case is illustrated in Figure 3.3. The output of the phase modulator is passed through a quadrature hybrid circuit which produces the original phase modulated signal and a version 90^o out of phase (*i.e.,* delayed by $\pi/2$. The top branch is then spread by a binary spreading waveform (termed the in-phase portion of the spreading waveform) and the bottom branch is independently spread by a negated version of a second binary spreading waveform (the quadrature portion of the spreading waveform). Using separate spreading codes in this manner improves the phase characteristics of the signal making it more difficult to detect in military applications and reducing its impact to other signals in commercial applications [4].

The receiver for QPSK spreading is given in Figure 3.4. Despreading is accomplished by removing both in-phase and quadrature portions of the spreading waveform as shown. Other versions of the receiver are possible, with either IF despreading or baseband despreading. That latter is depicted in Figure 3.4. The motivation for the receiver structure will become more obvious after we discuss the complex baseband notation for DS/SS in the next section.

3.3 Complex Baseband Representation

The model assumed in the previous section can at times be cumbersome since we must maintain the sinusoidal carrier. Thus, it is more convenient to use complex baseband (or complex envelope) notation. The complex baseband notation is derived from the following observation of bandpass signals. Any bandpass signal $v(t)$ can be written as

$$v(t) = \Re\left\{\tilde{v}(t)e^{j2\pi f_c t}\right\} \tag{3.6}$$

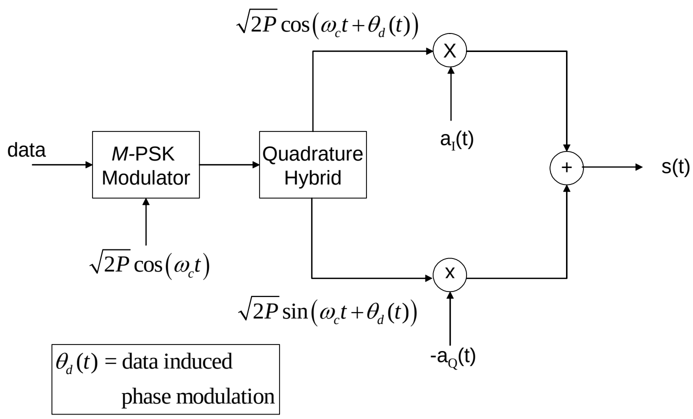

Figure 3.3: QPSK Spread Transmitter

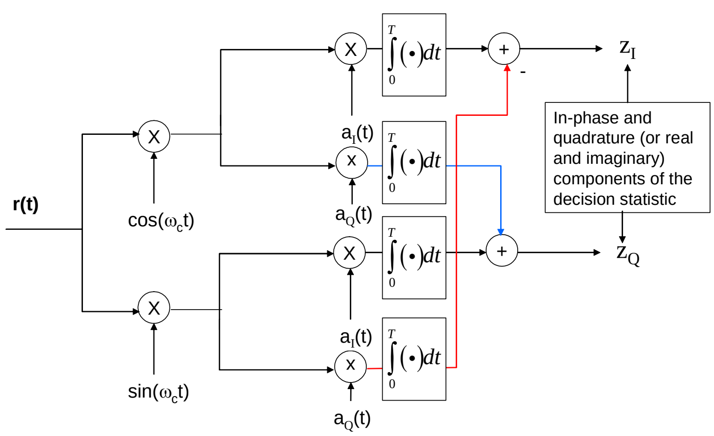

Figure 3.4: QPSK Spreading Receiver

where $\tilde{v}(t)$ is a complex baseband signal. $\tilde{v}(t)$ is thus referred to as the complex baseband equivalent of $v(t)$. Whenever possible, we will use complex baseband notation in this text. Thus, the complex baseband version of the transmit signal for DS/SS with BPSK spreading is

$$\tilde{s}(t) = \sqrt{P}a(t)b(t) \tag{3.7}$$

Note that we will use $\tilde{}$ to represent complex baseband quantities and that the power of a bandpass signal is $\frac{1}{2}$ the power in the complex baseband. Whenever $\tilde{x}(t)$ appears, context will determine whether $x(t)$ is a baseband or bandpass signal. If the latter, it will be understood that we are dealing with a complex baseband equivalent. In the former case, we will be referring to a complex version of a baseband signal. The received signal can be represented in complex baseband as

$$\begin{aligned} \tilde{r}(t) &= \tilde{s}(t) + \tilde{n}\,(t) \\ &= \sqrt{P}a(t)b(t) + \tilde{n}(t) \end{aligned} \tag{3.8}$$

where $\tilde{n}(t)$ is a complex Gaussian random process which represents the thermal noise.

Complex baseband notation can be particularly useful for QPSK spreading formats. For example, consider the transmit waveform described in Figure 3.3. Specifically,

$$s(t) = \sqrt{2P}a_I(t)\cos(\omega_c t + \theta_d(t)) - \sqrt{2P}a_Q(t)\sin(\omega_c t + \theta_d(t)) \tag{3.9}$$

where $a_I(t)$ is the in-phase portion of the spreading waveform, $a_Q(t)$ is the quadrature portion of the spreading waveform, and $\theta_d(t)$ is the data-induced phase modulation. Now using basic trigonometric identities we can rewrite (3.9) as

$$\begin{aligned} s(t) &= \sqrt{2P}a_I(t)\left[\cos(\omega_c t)\cos(\theta_d(t)) - \sin(\omega_c t)\sin(\theta_d(t))\right] \\ &\quad -\sqrt{2P}a_Q(t)\left[\sin(\omega_c t)\cos(\theta_d(t)) + \cos(\omega_c t)\sin(\theta_d(t))\right] \\ &= \sqrt{2P}\left[a_I(t)\cos(\theta_d(t)) - a_Q(t)\sin(\theta_d(t))\right]\cos(\omega_c t) \\ &\quad -\sqrt{2P}\left[a_I(t)\sin(\theta_d(t)) + a_Q(t)\cos(\theta_d(t))\right]\sin(\omega_c t) \end{aligned} \tag{3.10}$$

Thus, the complex baseband version of the transmit signal is

$$\begin{aligned} \tilde{s}(t) &= \sqrt{P}\left[a_I(t)\cos(\theta_d(t)) - a_Q(t)\sin(\theta_d(t))\right]... \\ &\quad +j\sqrt{P}\left[a_I(t)\sin(\theta_d(t)) + a_Q(t)\cos(\theta_d(t))\right] && (3.11) \\ &= \sqrt{P}\left(a_I(t) + ja_Q(t)\right)\left[\cos(\theta_d(t)) + j\sin(\theta_d(t))\right] \\ &= \sqrt{P}\tilde{a}(t)\tilde{d}(t) && (3.12) \end{aligned}$$

where $\tilde{a}(t)$ is the complex spreading code and $\tilde{d}(t)$ is the complex phase modulation. Thus, we can view spreading in baseband as a simple complex multiply which can be done prior to modulation. Despreading is thus accomplished as

$$\begin{aligned}
\tilde{y}(t) &= \tilde{s}(t)\tilde{a}^*(t) \\
&= \sqrt{P}\left(a_I^2(t) + a_Q^2(t)\right)\left[\cos\left(\theta_d(t)\right) + j\sin\left(\theta_d(t)\right)\right] \\
&= \sqrt{P}\left[\cos\left(\theta_d(t)\right) + j\sin\left(\theta_d(t)\right)\right] \qquad (3.13)
\end{aligned}$$

Note that $a_I^2(t) + a_Q^2(t) = 1$ since typically $\tilde{a}(t)$ is defined such that $|\tilde{a}(t)|^2 = 1$. Throughout this text we will use the complex baseband notation whenever possible.

3.4 Power Spectral Density of DS/SS

The power spectral density (PSD) of DS/SS depends on the modulation scheme used as well as the pulse shape used. To this point we have assumed the use of square pulses for convenience. Based on the Wiener-Khintchine Theorem the PSD [1] of a random process is the Fourier Transform of the autocorrelation function of that process. For a PAM signal of the form

$$x(t) = \sum_{i=-\infty}^{\infty} a_i p(t - iT_s) \qquad (3.14)$$

where a_i are arbitrary pulse amplitudes and $p(t)$ is the pulse shape, the power spectral density can be shown to be [3]

$$S_x(f) = \frac{|P(f)|^2}{T_s} \sum_{k=-\infty}^{\infty} R(k) e^{-j2\pi f k T_s}$$

where $P(f)$ is the Fourier Transform of the pulse shape, $R(k) = \overline{a_i a_{i+k}}$ is the autocorrelation function of the data sequence, and T_s is the symbol duration. Now, if the data is uncorrelated[2]

$$\begin{aligned}
R(k) &= \begin{cases} \overline{a_i^2} & k = 0 \\ \overline{a_i a_{i+k}} & k \neq 0 \end{cases} \\
&= \begin{cases} \sigma_a^2 + m_a^2 & k = 0 \\ m_a^2 & k \neq 0 \end{cases} \qquad (3.15)
\end{aligned}$$

[2] Note that this is an approximation for the DS/SS spreading waveform since the spreading code is pseudo-random and periodic. For extremely long spreading codes, however, this approximation is very good.

where m_a and σ_a^2 are the mean and variance of the data amplitude sequence respectively. Returning to the power spectral density we have

$$\begin{aligned} S_x(f) &= \frac{|P(f)|^2}{T_s} \sum_{k=-\infty}^{\infty} R(k) e^{-j2\pi f k T_s} \\ &= \frac{|P(f)|^2}{T_s} \left(\sigma_a^2 + m_a^2 \sum_{k=-\infty}^{\infty} e^{-j2\pi f k T_s} \right) \\ &= \frac{|P(f)|^2}{T_s} \left(\sigma_a^2 + m_a^2 \sum_{k=-\infty}^{\infty} \delta \left(f - \frac{k}{T_s} \right) \right) \\ &= \frac{\sigma_a^2}{T_s} |P(f)|^2 + \frac{m_a^2}{T_s} \sum_{k=-\infty}^{\infty} \left| P\left(\frac{k}{T_s} \right) \right|^2 \delta \left(f - \frac{k}{T_s} \right) \end{aligned} \tag{3.16}$$

Now for phase modulation $m_a = 0$ and $\sigma_a^2 = 1$. Further, if square pulses are assumed, $P(f) = T_s \mathrm{sinc}\,(T_s f)$. Thus,

$$S_x(f) = T_s \mathrm{sinc}^2\,(T_s f) \tag{3.17}$$

Since both the spreading waveform and the data waveform have the same format, we have the power spectral density of both. Now it remains to find the PSD of the transmitted waveform.

The transmitted signal $\tilde{s}(t)$ is an ergodic random process and the power spectral density can be found from the Fourier Transform of the autocorrelation function. Since the data and the spreading sequence are independent, the autocorrelation function of the transmit signal is the multiplication of the autocorrelation functions of the two signals. That is

$$\tilde{s}(t) = \sqrt{P} a(t) b(t) \tag{3.18}$$

Now,

$$\begin{aligned} R_s(\tau) &= \mathrm{E}\left\{ \tilde{s}(t) \tilde{s}^*(t+\tau) \right\} \\ &= \mathrm{E}\left\{ a(t) b(t) a(t+\tau) b(t+\tau) \right\} \\ &= \mathrm{E}\left\{ a(t) a(t+\tau) \right\} \mathrm{E}\left\{ b(t) b(t+\tau) \right\} \\ &= R_a(\tau) R_b(\tau) \end{aligned} \tag{3.19}$$

The power spectral density is then the Fourier Transform of the autocorrelation function:

$$\begin{aligned} S(f) &= \int_{-\infty}^{\infty} S_b(\phi) S_a(f-\phi) d\phi \\ &= \int_{-\infty}^{\infty} T_b \mathrm{sinc}^2\,(\phi T_b)\, T_c \mathrm{sinc}^2\,([f-\phi] T_c)\, d\phi \\ &= \int_{-\infty}^{\infty} T_b \mathrm{sinc}^2\,(\phi T_b)\, \frac{T_b}{N} \mathrm{sinc}^2 \left([f-\phi] \frac{T_b}{N} \right) d\phi \end{aligned} \tag{3.20}$$

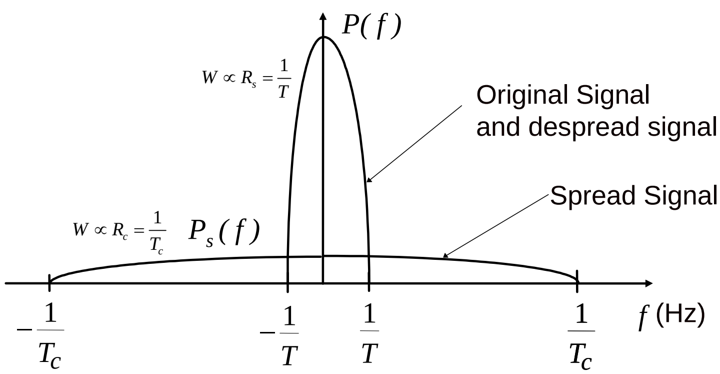

Figure 3.5: Illustration of Spectrum in Direct Sequence

Now, examining the last line in (3.20) we can see that if $N >> 1$, the second term will be approximately constant over the significant values of the first term. Thus,

$$\begin{aligned} S_x(f) &\approx \frac{T_b}{N}\text{sinc}^2\left(\frac{fT_b}{N}\right)\int_{-\infty}^{\infty} T_b\text{sinc}^2\left(\phi T_b\right)d\phi \\ &\approx \frac{T_b}{N}\text{sinc}^2\left(\frac{fT_b}{N}\right) \end{aligned} \tag{3.21}$$

An illustrative sketch of the spectra (main lobe only) for the original information signal and the signal after spreading is plotted in Figure 3.5. A more concrete example is plotted in Figure 3.6 for N=128. From the perspective of the spread signal the information signal $s_b(t)$ appears to be a strong narrowband tone. From the perspective of the narrowband signal (see inset) the spread signal appears to be white noise. Further, we can see that the bandwidth of the spread signal is N times that of the original information signal. Thus, we call N the bandwidth expansion factor which is closely related to the processing gain.

The PSD of the bandpass signal can be shown to be

$$S_s(f) \approx \frac{A^2T_c}{4}\left[\left(\frac{\sin\pi(f-f_c)T_c}{\pi(f-f_c)T_c}\right)^2 + \left(\frac{\sin\pi(f+f_c)T_c}{\pi(f+f_c)T_c}\right)^2\right] \tag{3.22}$$

where A is the signal amplitude.

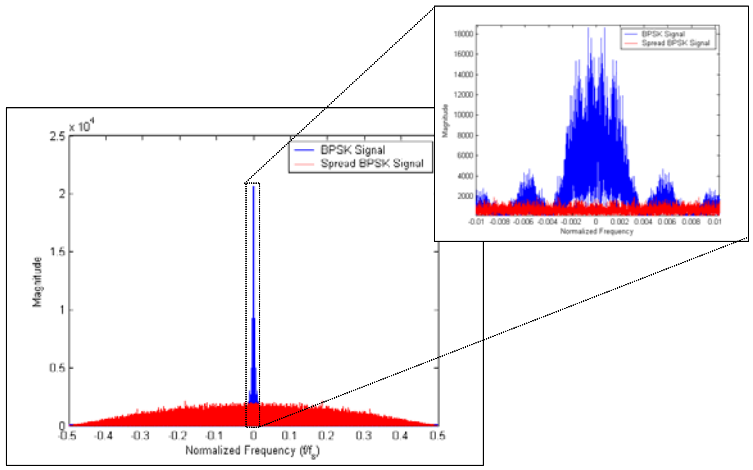

Figure 3.6: Example Spectrum for BPSK Signal

3.5 Performance in AWGN

The decision statistic of a BPSK signal with BPSK spreading in an AWGN channel was shown to be

$$Z = \frac{\sqrt{2P}T_b}{2}b + N \tag{3.23}$$

where N is a zero mean Gaussian random variable with variance $\sigma^2 = \frac{N_o T_b}{4}$. The performance of the maximum likelihood receiver in AWGN can be determined using

$$P_b = \mathrm{Q}\left(\sqrt{\frac{\mathrm{E}\left\{Z\right\}^2}{\mathrm{var}\left\{Z\right\}}}\right) \tag{3.24}$$

Now, from equation (3.23) we know that

$$\mathrm{E}\left\{Z\right\} = T_b\frac{\sqrt{2P}}{2}b \tag{3.25}$$

and

$$\mathrm{var}\left\{Z\right\} = \frac{N_o T_b}{4} \tag{3.26}$$

Thus,

$$\begin{aligned} P_b &= \mathrm{Q}\left(\sqrt{\frac{T_b^2 \frac{2P}{4}}{\frac{N_o T_b}{4}}}\right) \\ &= \mathrm{Q}\left(\sqrt{\frac{2PT_b}{N_o}}\right) \\ &= \mathrm{Q}\left(\sqrt{\frac{2E_b}{N_o}}\right) \qquad (3.27) \end{aligned}$$

which is equivalent to BPSK without spreading. Thus, spreading provides no performance benefit in AWGN.

3.6 Resistance to Jamming

As we saw in the last section, DS spread spectrum has no benefit in an AWGN channel. The performance benefit is found in other channels such as fading and jamming. We will study both in detail later in this text. Now we will briefly demonstrate the benefit in one type of jamming scenario, the tone jammer. Consider a BPSK DS/SS signal

$$s(t) = \sqrt{2P}a(t)b(t)\cos\omega_c t \qquad (3.28)$$

and a jamming signal defined as

$$j(t) = \sqrt{2J}\cos\omega_c t \qquad (3.29)$$

For a received signal $r(t) = s(t) + j(t)$ we have an input signal to interference ratio of

$$\frac{S}{I} = \frac{E\left\{s^2(t)\right\}}{E\left\{j^2(t)\right\}} = \frac{P}{J}. \qquad (3.30)$$

The power spectral density of the received signal, assuming that $s(t)$ and $j(t)$ are uncorrelated is

$$S_r(f) = \frac{A^2 T_c}{4}\left[\mathrm{sinc}^2\left((f-f_c)T_c\right) + \mathrm{sinc}^2\left((f+f_c)T_c\right)\right] + \frac{J}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] \qquad (3.31)$$

If we assume that a simple matched filter receiver is used, the decision statistic is

$$Z = \int_0^{T_b} r(t)a(t)\cos\omega_c t dt \qquad (3.32)$$

If we define $y(t) = r(t)a(t)$ we can find the resulting signal-to-noise ratio after despreading. The power spectral density of $y(t)$ is [2]

$$\begin{aligned} Y(f) &= \frac{PT_b}{2}\left[\mathrm{sinc}^2\left((f-f_c)T_b\right) + \mathrm{sinc}^2\left((f+f_c)T_b\right)\right] \\ &\quad + \frac{JT_c}{2}\left[\mathrm{sinc}^2\left((f-f_c)T_c\right) + \mathrm{sinc}^2\left((f+f_c)T_c\right)\right] \qquad (3.33) \end{aligned}$$

The desired signal power after despreading was been shown previously to be P. Now the output jammer power J_{out} can be determined by examining the second term in equation (3.33). We can approximate the impact of the matched filter by using a filter $H(f)$ with an ideal transfer function and a noise equivalent bandwidth $R = 1/T$.

$$\begin{aligned}
J_{out} &= \int_{-\infty}^{\infty} S_j(f)\,|H(f)|^2\,df \\
&= \int_{-f_c-1/2T}^{-f_c+1/2T} S_j(f)df + \int_{f_c-1/2T}^{f_c+1/2T} S_j(f)df \\
&\approx \int_{-f_c-1/2T}^{-f_c+1/2T} \frac{JT_c}{2}\left[\mathrm{sinc}^2\left((f-f_c)T_c\right) + \mathrm{sinc}^2\left((f+f_c)T_c\right)\right] df \\
&\quad + \int_{f_c-1/2T}^{f_c+1/2T} \frac{JT_c}{2}\left[\mathrm{sinc}^2\left((f-f_c)T_c\right) + \mathrm{sinc}^2\left((f+f_c)T_c\right)\right] df \\
&\approx \frac{JT_c}{2}\frac{1}{T_b} + \frac{JT_c}{2}\frac{1}{T_b} \\
&= \frac{JT_c}{T} \\
&= \frac{J}{N}
\end{aligned} \tag{3.34}$$

Thus, the signal to interference ratio after despreading is

$$\frac{S}{I} = \frac{PN}{J} \tag{3.35}$$

and despreading has provided a gain of N to the signal-to-interference ratio. This gain is termed the *spreading gain* and can be seen in the spectra of an example. Figure 3.7 presents the baseband spectra of a spread signal (N=100) and a tone jammer (note that the frequency is not commensurate with the desired signal carrier in this case)with $\frac{P}{J} = 1$. Figure 3.8 presents the spectrum of the joint output signal after despreading with an incorrect alignment of the spreading waveform. Thus, proper despreading is not taking place. The spectrum of the jammer is spread over the same band as the desired signal (which has not been despread). Figure 3.9 presents the spectrum of the joint output signal after despreading with a correct alignment of the spreading waveform. Now we see that while the jammer remains spread over the band of the original desired signal, the desired signal spectrum has collapsed to the band of the information signal ($\frac{1}{N}$ times the original bandwidth). Subsequent filtering will remove most of the jammer's power.

3.7 Conclusion

In this chapter we have introduced the concept of Direct Sequence Spread Spectrum (DS/SS), described common implementations and demonstrated one situ-

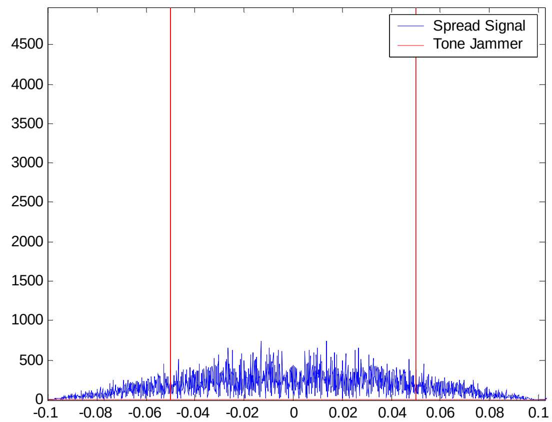

Figure 3.7: Spectrum of Received DS/SS Signal and Tone Jammer

ation where DS/SS provides performance benefits (viz. in the presence of a tone jammer). We will more fully investigate the performance of DS/SS in Chapter 8. In the following chapter we will introduce other forms of spread spectrum including Frequency Hopping and Ultra-Wideband.

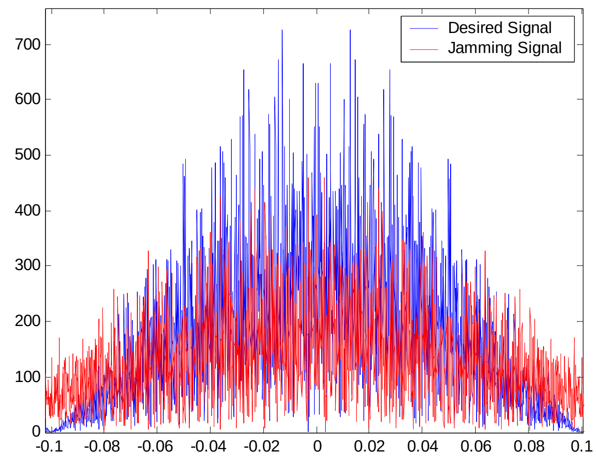

Figure 3.8: Spectrum of DS/SS and Tone Jammer after Incorrect Despreading

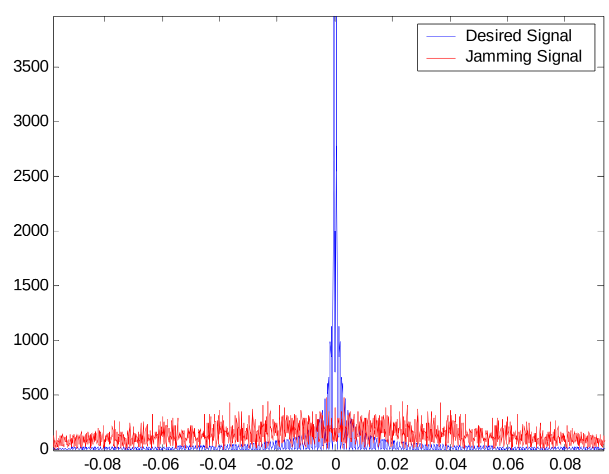

Figure 3.9: Spectrum of DS/SS and Tone Jammer after Correct Despreading

Bibliography

[1] A. Papoulis. *Probability, Random Variables, and Stochastic Processes*. McGraw-Hill, New York, NY, third edition, 1991.

[2] R.L. Peterson, R.E. Ziemer, and D.E. Borth. *Introduction to Spread Spectrum Communications*. Prentice-Hall, 1995.

[3] J.G. Proakis. *Digital Communications*. McGraw-Hill, New York, NY, fourth edition, 2001.

[4] A.J. Viterbi. *CDMA: Principles of Spread Spectrum Communication*. Addison-Wesley, 1995.