Chapter 2

Motivation for Spread Spectrum

2.1 Introduction

The digital communications techniques discussed in the previous chapter were designed for the AWGN channel. When communication must occur in less benign environments, more sophisticated techniques are warranted. Two such environments that are often encountered in communications are the jamming environment and the fading environment. One communications technique that improves performance in both of these channel types is spread spectrum. Spread spectrum can be defined as any modulation technique which (a) uses bandwidth which is well beyond what is necessary for the data rate being transmitted and (b) uses a pseudo-random signal to obtain the increased bandwidth. The latter factor distinguishes spread spectrum techniques from standard communication techniques such as FM and high order orthogonal signaling which may also require high bandwidth as compared to the information rate. The random spreading is accomplished by adding a spreading block after the modulator as shown in Figure 2.1. Additionally, the pseudo-random nature of the spreading signal means that the receiver recovers the signal by correlating the received signal with some version of the spreading waveform (also called despreading) as also shown in Figure 2.1.

2.2 Reasons for Spreading

There are several reasons for spreading a signal to a bandwidth well beyond the information rate. These include resistance to interference (intentional or otherwise), the potential for low probability of intercept (LPI), resistance to multipath fading, improved multiple access capability, and ranging. In the following sections we will discuss each motivation for spread spectrum.

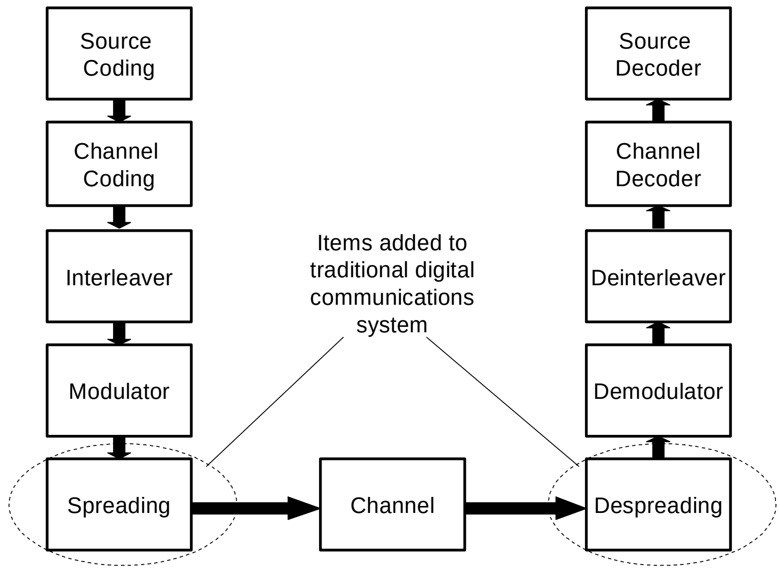

Figure 2.1: Block Diagram of a Digital Communications System with Spreading/Despreading

2.2.1 Anti-jamming

Jamming is a term typically used when an RF signal source of some type is used to disrupt the communications of an enemy link. This is done in various ways including sending a continuous, strong narrowband interferer, wideband noise or a pulsed jamming signal. Each has its advantages and disadvantages. Let us consider a pulsed signal with duty cycle ρ where continuous jamming is a special case of pulsed jamming with $\rho = 1$. Now, without jamming (and assuming an AWGN channel), the performance of BPSK modulation is

$$P_b = \mathrm{Q}\left(\sqrt{\frac{2E_b}{N_o}}\right) \tag{2.1}$$

Let us assume that a wideband jammer transmits a noise-like signal which is received with power J in bandwidth W. While the jammer is transmitting the performance of a receiver using BPSK modulation is

$$P_b = \mathrm{Q}\left(\sqrt{\frac{2E_b}{N_o + \frac{N_j}{\rho}}}\right) \tag{2.2}$$

where $N_j = \frac{J}{W}$ is the one-sided power spectral density of the jammer. Note that the jammer increases its impact while transmitting by transmitting less frequently (i.e., decreasing ρ). This concentrates the jammer's power in time. However, by decreasing ρ the jammer impacts performance less frequently. Thus, there is a trade-off by changing ρ. The average performance of a BPSK link is then

$$P_b = (1-\rho)\,\mathrm{Q}\left(\sqrt{\frac{2E_b}{N_o}}\right) + \rho\mathrm{Q}\left(\sqrt{\frac{2E_b}{N_o + \frac{N_j}{\rho}}}\right) \tag{2.3}$$

In a typical jamming situation, $\frac{E_b}{N_o} >> \frac{E_b}{N_j}$, thus

$$P_b \approx \rho\mathrm{Q}\left(\sqrt{\frac{2\rho E_b}{N_j}}\right) \tag{2.4}$$

The optimal choice of ρ (in terms of maximizing P_b) can be found by using an upper bound for the Q-function. Namely, it can be shown that

$$\mathrm{Q}\,(x) \leq \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} \tag{2.5}$$

Using this upper bound, we can bound the probability of error as

$$P_b \leq \frac{\rho}{\sqrt{4\pi\rho E_b/N_j}} e^{-\rho E_b/N_j} \tag{2.6}$$

The optimal choice of ρ can be shown to be (see Chapter 8)

$$\rho = \frac{N_j}{2E_b} \tag{2.7}$$

The resulting probability of error bound is then

$$P_b^{\max} \leq \frac{1}{\sqrt{2\pi e}} \frac{N_j}{2E_b} \tag{2.8}$$

In the case of continual jamming the performance from (2.3) is

$$P_b = \mathrm{Q}\left(\sqrt{\frac{2E_b}{N_j}}\right) \tag{2.9}$$

As shown in Figure 2.2 optimal pulsing severely degrades the performance of the victim receiver as compared to continuous jamming. For example, at a target error rate of $P_b = 10^{-4}$ optimal jamming degrades performance by over 22dB. It should be noted that optimal jamming is difficult to accomplish in practice since it requires the jammer to know the operating point of the receiver, but it does demonstrate the radical change that jamming can cause to receiver performance. While continuous wideband jamming is similar to AWGN, pulsed jamming results in performance where BER reduces linearly with E_b/N_j as opposed to exponential improvement. Thus, traditional modulation and coding techniques which are designed for AWGN channels are no longer appropriate. As we will show in Chapter 8, spread spectrum techniques can return the performance to the exponential improvement versus E_b/N_j.

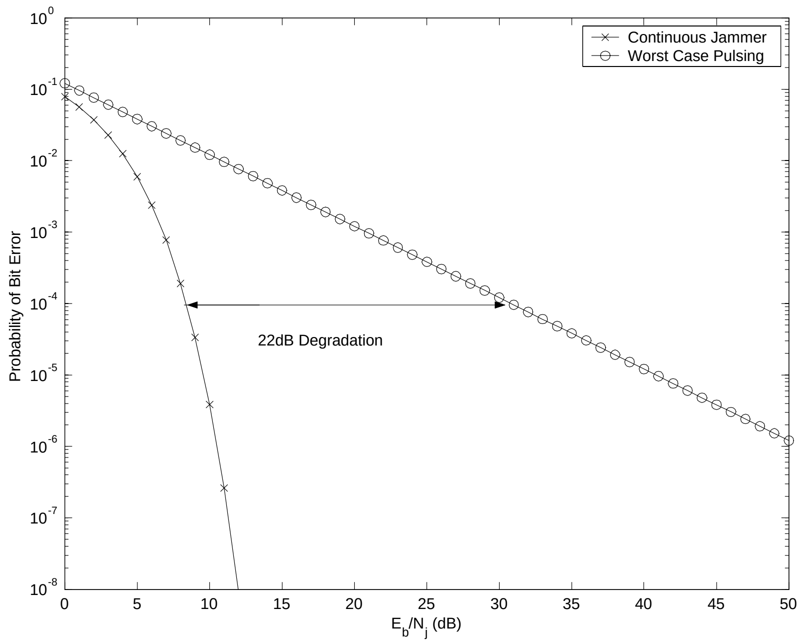

Figure 2.2: Impact of Pulsing on the Jammer's Effectiveness

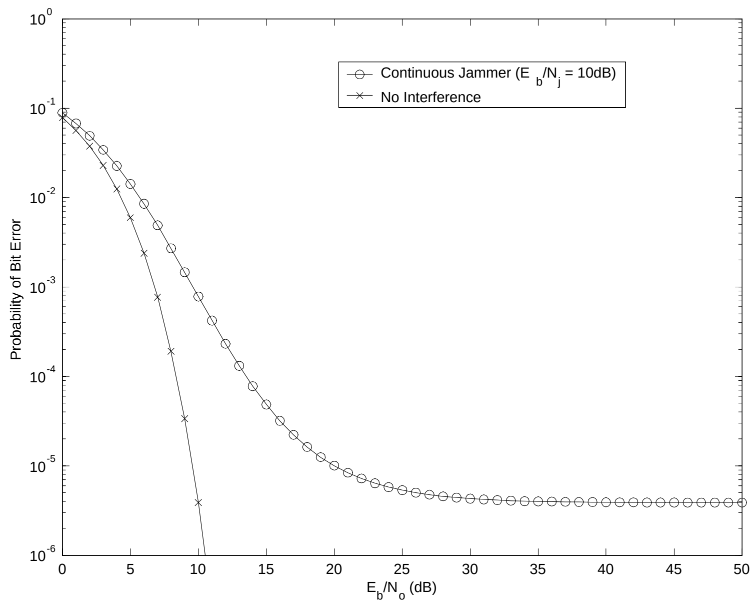

Figure 2.3: Error Floor Due to Continuous Jamming (An illustration of Interference Limited Scenario)

We should note that even without optimal pulsing, we can observe the impact of jamming. In Figure 2.3 the performance of a victim receiver using BPSK is plotted versus E_b/N_o with and without continuous jamming. In this case, the jammer is continuous with a resulting $\frac{E_b}{N_j} = 10dB$. The figure demonstrates that as $\frac{E_b}{N_o}$ goes to infinity, performance is interference limited due to constant $\frac{E_b}{N_j}$. We will show later that spread spectrum can improve this scenario as well.

As a simple example of the possible performance improvement with spread spectrum, recall that $E_b = ST_b$ where S is the received signal power of the desired signal and T_b is the bit duration and $N_j = \frac{J}{W}$. Using these definitions we can re-write equation (2.8) as

$$P_b^{\max} = \frac{1}{\sqrt{2\pi e}} \frac{J}{2S} \frac{R_b}{W} \tag{2.10}$$

where $R_b = \frac{1}{T_b}$ is the bit rate and the ratio $\frac{W}{R_b}$ can be termed the spreading factor since it describes the amount of excess bandwidth used. In other words, in traditional BPSK, $W = kR_b$ where $1 \leq k \leq 2$ depends on the pulse shape. However, in spread spectrum $W >> R_b$ and thus, the spreading factor is much greater than one. From equation (2.10) we can see that increasing the spreading factor directly reduces the impact of the jammer as shown in Figure 2.4. Note that the pulsing jammer is still able to impact performance such that the

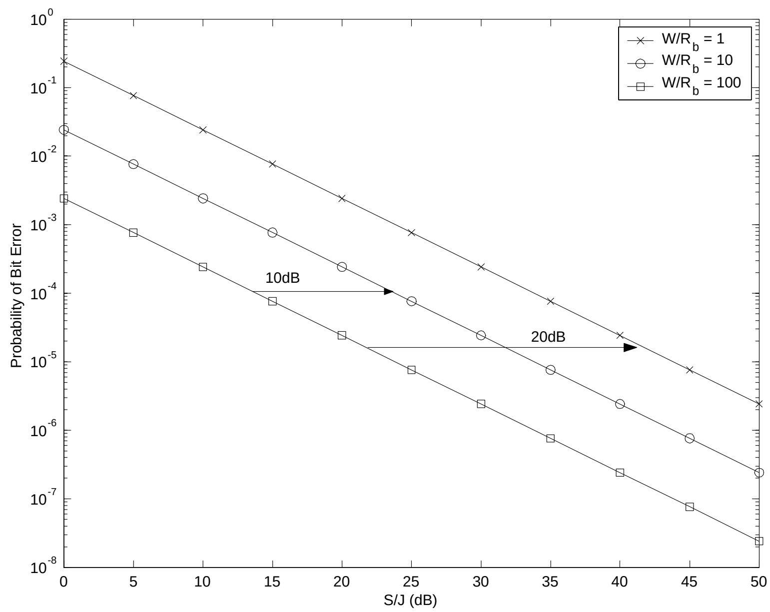

Figure 2.4: Illustration of Spreading Gain on Worst Case Jamming Performance

BER reduction is only linear with E_b/N_j, but its effectiveness is reduced in direct proportion to the spreading factor. We will show later that even better improvement can be obtained by including error correction coding along with increased bandwidth.

2.2.2 LPI/LPD

A second major application of spread spectrum is Low Probability of Intercept or Low Probability of Detection. In many military applications it is desirable (or perhaps necessary) that communications are carried out without knowledge of a third party. The ability of the third party to know that communications is taking place is defined by the probability of detection. We can demonstrate the benefit of spread spectrum to this scenario by examining the performance of a radiometer. A radiometer is a device used to detect RF energy and determine whether or not a signal is present. A typical radiometer block diagram is presented in Figure 2.6. The incoming signal is first filtered to the bandwidth of the signal of interest W and then amplified using a low power amplifier. The resulting signal is squared or rectified to eliminate phase modulation and consequently integrated over duration T_i. The output of the integration is then compared to a threshold to determine the existence of a signal. The threshold is an important parameter that detect the trade-off between the Probability of Detection (i.e., the probability of detecting that a signal is present when it

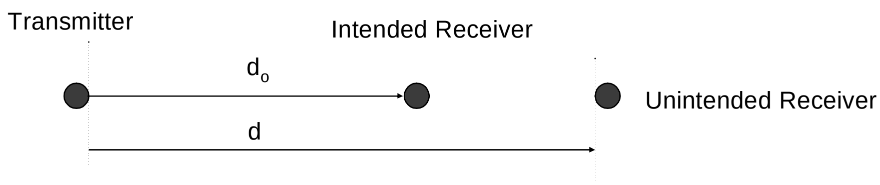

Figure 2.5: Assumed Interceptor Scenario

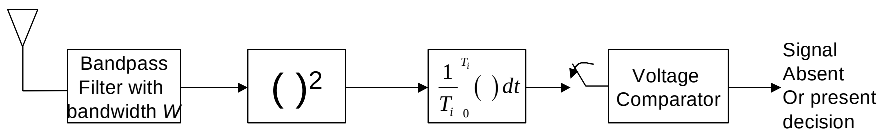

Figure 2.6: Block Diagram of a Simple Radiometer

is in fact present) and the Probability of False Alarm (i.e., the probability of erroneously determining a signal is present when it is not). Lower threshold values lead to higher probability of detection P_d, but also higher probabilities of false alarm P_{fa}. Conversely, increasing the threshold decreases the probability of false alarm, but also decreases the probability of detection. By choosing a threshold to obtain a desired P_{fa} the probability of detection in an AWGN channel can be shown to be

$$P_d = \Phi \left\{ \frac{S}{N_o} \sqrt{\frac{T_i}{W}} - \Phi^{-1}\left(1 - P_{fa}\right) \right\} \tag{2.11}$$

where $\Phi(x) = \int_{-\infty}^{x} exp\left(-\frac{1}{2}y^2\right) dy$. We can see that the probability of detection is directly related to the received signal power S and integration time T_i while it is inversely related to the bandwidth W. Consider the case depicted in Figure 2.5. Figure 2.7 plots the probability of detection versus the distance d between the transmitter and eavesdropper relative to the distance d_o between the transmitter and the desired receiver. In this example, $\frac{E_b}{N_o} = 7.25dB$, $R_b = 10kbps$, $T_i = 10ms$, $P_{fa} = 2\%$ and we assumed that the path loss exponent is 4 (i.e., the received power decays with d^4. As the figure shows, by increasing the signal bandwidth the eavesdropper must be closer to the transmitter to obtain the same probability of detection. For example assuming a data rate of 10kbps and a bandwidth of approximately 10kHz. To obtain a probability of detection of 80% the interceptor need only get within $1.5d_o$. However, if we increase the bandwidth to 10MHz to obtain the same probability of detection the interceptor must get within $0.6d_o$.

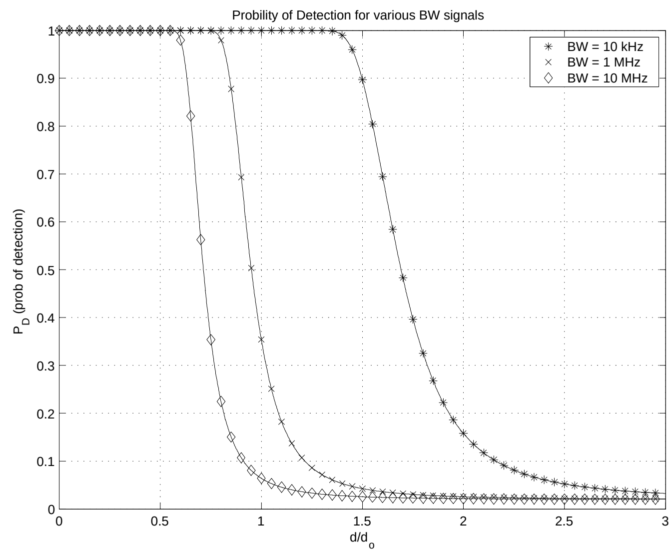

Figure 2.7: Impact of Spreading on Radiometer Performance

2.2.3 Resistance to Multipath

As we have stated previously, traditional modulation and coding schemes were designed for the relatively benign AWGN channel. A more harsh channel which is common in mobile wireless communications is the multipath fading channel. Due to the existence of a large number of scatterers in the environment, the receiver sees many reflected and delayed versions of the signal. If the relative delays of the signals are small as compared to the symbol rate, the receiver will observe a single waveform which varies in amplitude due to the random phase combinations which change with the mobile's movement (see Chapter 9). A common model for such fading signals is what is termed the flat Rayleigh fading model, since the amplitude of the signal follows a Rayleigh distribution. The fading is termed 'flat' fading since the coherence bandwidth of the channel is greater than the signal bandwidth and thus the channel transfer function is flat over the bandwidth of the signal. The probability of error is significantly worse than AWGN in this scenario. For example the probability of bit error for BPSK with coherent reception is

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{\frac{E_b}{N_o}}{1 + \frac{E_b}{N_o}}}\right) \tag{2.12}$$

The performance is plotted in Figure 2.8 along with the AWGN case. We can see that impact of fading is severe. For example, at a bit error probability of 10^{-3}, Rayleigh fading results in a 17dB degradation in performance.

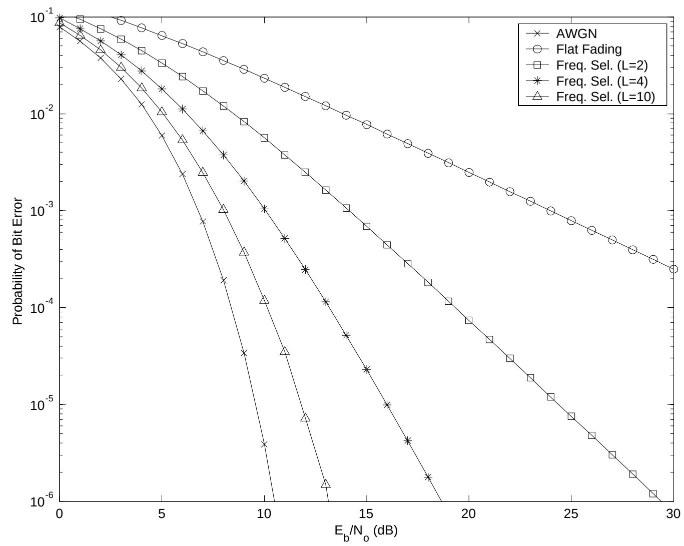

Figure 2.8: Impact of Frequency Selective Rayleigh Fading on the Performance of a Rake Receiver

Again, spread spectrum can be beneficial in such a case. By increasing the symbol rate beyond the channel coherence bandwidth, the severe fading can be mitigated. In other words, by decreasing the symbol duration so that it is smaller than the multipath delays, the multipath signals can be resolved and used as a form of diversity. This type of fading is termed 'frequency selective' fading since the channel coherence bandwidth is less than the signal bandwidth and the channel fades differently over different parts of the signal spectrum. For example, by increasing the symbol rate such that the bandwidth was approximately twice the coherence bandwidth, a diversity of order two can be obtained provided the proper spreading and receiver are used. As shown in Figure 2.8, if the second order diversity (L=2) can be harnessed by the receiver an improvement of nearly 10dB can be obtained at an error rate of 10^{-3}. By increasing the bandwidth by greater amounts results in greater diversity and thus better performance. With L=4 order diversity, performance is improved by approximately 14dB and is only 3dB from the AWGN case. Fading is considered in detail in Chapter 12.

2.2.4 Improved Multiple Access

The LPI and anti-jamming capabilities of spread spectrum have traditionally been used in military systems. However, these same characteristics can also be used in commercial systems. These characteristics allow spread spectrum to be used in overlay systems and for multiple access. In overlay systems, spread spectrum is used in spectrum where existing legacy systems already occupy the spectrum. The anti-jamming capabilities allow spread spectrum to operate in the existence of narrowband interference from the incumbent systems. The LPI capabilities mean that spread spectrum signals can be generated that cause minimal interference to the existing systems.

These same qualities can applied to spread spectrum signals operating in the presence of other spread spectrum signals from the same system. This is referred to as multiple access. In this section we compare the capacity, in terms of the number of simultaneous channels for TDMA, FDMA, and CDMA. A more detailed discussion of multiple access will given in Chapters 11-13. Let us first consider an FDMA system. Let us further assume that each user transmits at a rate of R_b b/s. Assuming optimal pulse shaping[1] and PSK, ASK or QAM modulation, the bandwidth per channel is $W = R_s = \frac{R_b}{k}$ where k is the number of bits per symbol. If there is a total system bandwidth of B_T, the number of channels supported by FDMA assuming that $k = 1$ is

$$K_{FDMA} = \frac{B_T}{R_b} \tag{2.13}$$

where we have ignored guard bands. If TDMA is used, all users transmit at a constant symbol rate $R_s = B_T$. If the desired bit rate is R_b, the number of channels that can be supported by the system is

$$\begin{aligned} K_{TDMA} &= \frac{R_s}{R_b} \\ &= \frac{B_T}{R_b} \end{aligned} \tag{2.14}$$

which is the same as FDMA. Now, let us consider a CDMA system. In general, CDMA (unlike TDMA and FDMA) is not an orthogonal multiple access scheme. That is, with proper filtering and timing, TDMA and FDMA channels are orthogonal and signals due not experience interference from other channels. With CDMA, however, this is difficult to maintain, esp. for the uplink. Thus, typically a CDMA system is designed to have a controlled amount of interference between channels. If we assume that each of the K_{CDMA} channels in the system causes interference to the other channels, the total interference power seen by any one signal is $(K_{CDMA} - 1)S$ where S is the received signal power for each signal (assumed to be the same for simplicity). Assuming that the mutual interference has the same properties as white noise (an assumption we will examine later), interference power spectral density is then

$$I_o = \frac{(N-1)S}{B_T} \tag{2.15}$$

[1] Note that here we are assuming the use of optimal pulse shaping as opposed to square pulses which were assumed in Chapter 1. The null-to-null bandwidth with square pulses is twice that shown here for optimal pulses.

where each signal uses the entire bandwidth B_T. The resulting E_b/I_o for each signal is then

$$\begin{aligned}\frac{E_b}{I_o} &= \frac{ST_b}{\frac{(K_{CDMA}-1)S}{B_T}} \\ &= \frac{B_T}{(K_{CDMA}-1)R_b} \qquad (2.16)\end{aligned}$$

Solving for the number of channels K_{CDMA} results in

$$K_{CDMA} \approx \frac{B_T}{R_b}\frac{I_o}{E_b} \qquad (2.17)$$

Now, since the required value of $\frac{E_b}{I_o}$ is certainly more than one, CDMA will support fewer users in a *single cell environment*. In fact, a typical value of $\frac{E_b}{I_o}$ is 6dB, meaning that CDMA will support 4 times fewer users than FDMA and TDMA. However, the value of CDMA comes in a cellular environment. Additionally, there are other factors that we haven't taken into account. First, for a cellular environment, the entire system bandwidth must be divided up in a frequency reuse pattern to minimize co-channel interference. If Q is the number of cells with distinct frequency bands, the capacity of a single cell in a TDMA or FDMA cell becomes

$$K_{TDMA} = K_{FDMA} = \frac{B_T}{QR_b} \qquad (2.18)$$

However, CDMA uses universal frequency reuse (Q=1). Although this increases the interference seen by an individual signal, the net result is a capacity improvement for CDMA as compared to FDMA and TDMA. Specifically, $\frac{E_b}{I_o}$ is now calculated as

$$\frac{E_b}{I_o} = \frac{ST_b}{\frac{(K_{CDMA}-1)S}{B_T}(1+f)} \qquad (2.19)$$

where f is the interference increase due to out-of-cell interference as fraction of in-cell interference and is typically in the range of 0.6. The number of channels supported is then

$$K_{CDMA} \approx \frac{B_T}{(1+f)R_b}\frac{I_o}{E_b} \qquad (2.20)$$

Another important factor to consider with CDMA is voice activity. Up to this point, the value of $\frac{E_b}{I_o}$ assumes that all signals are present at all times. However, it is a well established fact that the typical speaker is only speaking $\nu = 3/8$ of the time. Thus, if a transmitter suppresses transmissions during periods of inactivity, the amount of interference is reduced by ν and the number of channels is increased by $\frac{1}{\nu}$. That is

$$K_{CDMA} \approx \frac{B_T}{\nu(1+f)R_b}\frac{I_o}{E_b} \qquad (2.21)$$

Finally, we need to consider the impact of sectored antennas. In an FDMA or TDMA system, sectored antennas allow the reuse factor to decrease. Thus, for single sector systems $Q = 12$ is typical. However, for sectored systems $Q = 7$ is possible resulting in an increase in capacity of nearly 2. However, with sectored antennas, CDMA observes an interference reduction which directly improves $\frac{E_b}{I_o}$ and thus directly increases the number of supportable channels. That is

$$K_{CDMA} \approx \frac{GB_T}{\nu(1+f)R_b}\frac{I_o}{E_b} \tag{2.22}$$

where G is the antenna gain of the sector antenna in the azimuthal plane. A typical value for G is 4dB.

Now let us assume the use of three-sector antennas at the base station. A standard reuse pattern for three sector systems is $Q = 7$. The capacity of TDMA and FDMA is then

$$K_{TDMA} = K_{FDMA} = \frac{B_T}{7R_b} \tag{2.23}$$

Now for CDMA, let us assume that $\frac{E_b}{I_o} = 6\text{dB}$, $\nu = 3/8$, $G = 4\text{dB}$, and $f = 0.6$,

$$\begin{aligned} K_{CDMA} &\approx \frac{2.5B_T}{\frac{3}{8}(1.6)R_b}\frac{1}{4} \\ &\approx \frac{B_T}{R_b} \end{aligned} \tag{2.24}$$

which is an improvement of nearly an order of magnitude. Thus, due mainly to universal frequency reuse and the statistical multiplexing capability stemming from voice activity, CDMA can provide substantially better multiple access capability than TDMA or FDMA. A more detailed analysis will be provided in Chapter 11.

2.2.5 Ranging

Finally, the last application mentioned here that is useful for spread spectrum is ranging. Ranging is the process of attempting to determine the distance an object is from the transmitter by measuring the time difference between a reflection and the original transmit signal. However, the accuracy of this type of measurement is directly related to the symbol duration, with accuracy improving as the symbol duration decreases (or bandwidth increases). Thus, more accurate ranging capabilities are possible with very short symbol duration or very wide bandwidths.

2.3 Types of spread spectrum

There are two main types of spread spectrum that will be discussed in this book. The first is called direct-sequence spread spectrum (DS/SS). It involves increasing the signal bandwidth by multiplying the data signal by a high rate spreading waveform. In addition, the symbols of the spreading waveform are typically pseudo-random and are generated using a pseudo-random noise (PN) sequence. The ratio of the symbol rate of the spreading waveform to the symbol rate of the data signal is called the spreading gain and is what provides the benefits of spread spectrum. We will discuss DS/SS in more detail in Chapter 3.

The second main type of spread spectrum is called frequency hopping. In this form of spread spectrum the signal is moved in frequency from one band to another in a pseudo-random fashion. Again, this is due to the use of a pseudorandom noise sequence that is used to change the center frequency of the carrier to one of N different values. The number of different hopping frequencies N is also called the spreading gain.

We will examine each of these schemes in significantly more detail in later chapters.