Midterm Exam Fall 2009-2010
Wireless communications
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Submit no later than 1 February 2010

Exam is take-home exam
Please solve individually. No grouping is allowed

1. A convolutional code is described by:

$$\overline{g_0} = [100], \qquad \overline{g_1} = [101], \qquad \overline{g_2} = [111]$$

   (a) Draw the encoder corresponding to this code.
   (b) Describe the encoder, that is, determine the constraint length K and the coding rate r.
   (c) Draw the state-transition diagram for this code.
   (d) Draw the trellis diagram until depth 3 for this code.
   (e) Assume that there are $L = 6$ data bits entering the encoder. How many zeros should follow the 6 data bits?

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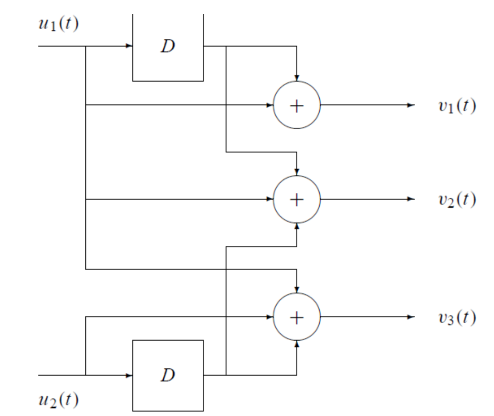
2. The block diagram of a convolutional code of rate $\frac{2}{3}$ is shown in Fig



   (a) Draw the state diagram of the code.
   (b) Draw the trellis diagram for this code.
   (c) Suppose that the received sequence is (111, 111, 101, 111, 011, 111). Use the Viterbi decoding algorithm to find the most likely data sequence.

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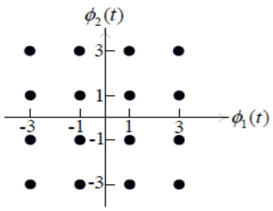
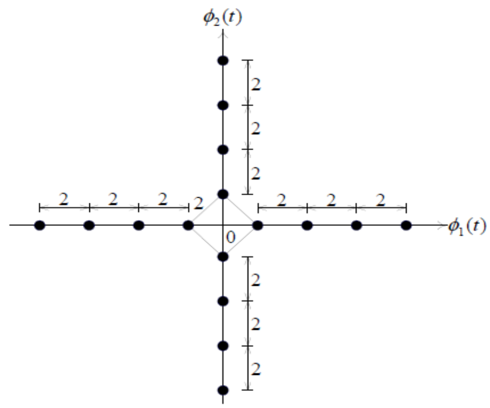
3. An OFDM communication system should operate at f_c=2 GHz and support velocities up to 250 Km/h with coverage range of up to 2Km in diameter.
   a. Give an approximate value for the delay spread
   b. Give an approximate value for Doppler shift
   c. Give an approximate value for the coherent time and coherent bandwidth
   d. State how to find the subcarrier spacing of your system

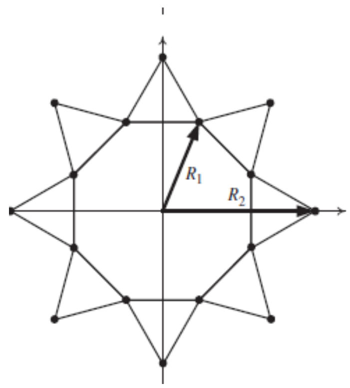
4.



(a)



(b)



(c)

Find the decision boundary and regions and roughly find the probability of error for the three shown constellation- make any assumption you find needed.

5-

In the so-called 8k-mode, DVB-T uses 6817 consecutive carriers out of $N = 8192$ (FFT size). The lowest and the highest used carrier are 5705357 Hz apart. The TV program is broadcast in the band 790–798 MHz (*i.e.*, the RF modulator runs at frequency $F_c = 794$ MHz).

(a) Determine the carrier spacing Δf, the OFDM-symbol duration T_{MC} (excluding the cyclic prefix) and the sampling period T_{S} of the discrete-time baseband transmit multiplex.

(b) Assume a maximum delay spread of 50 μs. The length T_{CP} of the cyclic prefix for DVB-T can be $T_{\text{MC}}/4$, $T_{\text{MC}}/8$, $T_{\text{MC}}/16$, or $T_{\text{MC}}/32$. Choose T_{CP} such that the bit rate is maximised and there is neither ISI nor ICI. Compute the bit rate R if all carriers use 16-QAM and a channel code of rate 2/3.

(c) Assume that the system specification requires a coherence time (assume that the propagation velocity of the radio wave is $c = 3 \cdot 10^8$ m/s) that is at least 20 times larger than the multicarrier symbol duration ($T_{\text{MC}} + T_{\text{CP}}$). What is the maximum velocity at which we can still watch TV?

6-

X and Y are zero mean jointly Gaussian random variables with correlation coefficient ρ and unit variances. The joint PDF is

$$f_{XY}(x,y) = \frac{1}{2\pi}\sqrt{\frac{1}{1-\rho^2}}\, e^{\frac{-x^2-y^2+2xy\rho}{2(1-\rho^2)}}$$

(a) Suppose $\rho = 0$ and $W = aX$ where $a \in \Re$. What is the PDF of W?

(b) Suppose $\rho = 0$ and $Z = aX + bY$ where $a, b \in \Re$. PROVE that Z is Gaussian. What is the expected value of Z? What is its variance?

(c) For general ρ, PROVE that the PDF for $Z = aX + bY$ is Gaussian where $a, b \in \Re$.

(d) For general ρ, suppose $W = cX + dY$ and $Z = aX + bY$. PROVE that Z and W are JOINTLY Gaussian for $a, b, c, d \in \Re$.

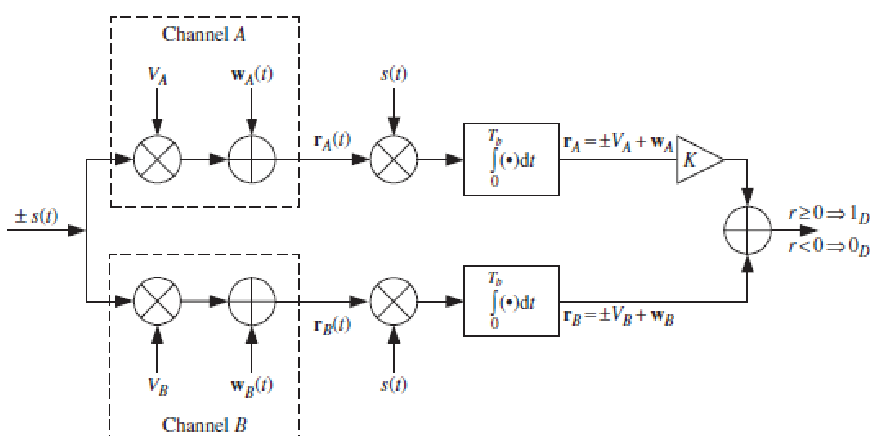
(e) Assume the result of the previous part. Can you use this result to show that

$$Z_j = \sum_{i=1}^{N} a_{ij} X_i$$

are jointly Gaussian random variables if the X_i are jointly Gaussian and the $a_i \in \Re$?

(f) For $W = cX + dY$ and $Z = aX + bY$, assume that W and Z are jointly Gaussian and find a relationship between a, b, d and d which makes W and Z INDEPENDENT.

7- Consider the antipodal signaling system in the following Figure, where the signals $s(t)$ and $-s(t)$ are used to transmit the information bits "1" and "0" respectively. The bit duration is Tb and $s(t)$ is assumed to have unit energy. The signal is sent via two different channels, denoted "A" and "B," to the same destination. Each channel is described by a gain factor (VA or VB) and AWGN ($\mathbf{w}A(t)$ or $\mathbf{w}B(t)$). The noises $\mathbf{w}A(t)$ and $\mathbf{w}B(t)$ both have zero means and PSDs of $\sigma 2$ A and $\sigma 2$ B respectively. Furthermore, they are independent noise sources



The receiver for such a system consists of two correlators, one for each channel, as shown in Figure. The output of one correlator, say the one corresponding to channel

A, is passed through an amplifier with an adjustable voltage gain K. The signals are then added before being compared with a threshold of zero to make the decision.
(a) For a fixed K, find the probability of error of this system. The noise samples $\mathbf{w}A$ and $\mathbf{w}B$ are independent, zero-mean, Gaussian random variables with variances $\sigma 2\ A$ and $\sigma 2\ B$ respectively. Furthermore, the sum of two independent Gaussian random variables is a Gaussian random variable whose variance equals the sum of the individual variances.
(b) Find the value of K that minimizes the probability of error.
(c) What is the probability of error when the optimum value of K is used?
(d) What is the probability of error when K is simply set to 1?
(e) if V_A and V_B were considered Rayleigh distributed random variables. Find the averge probability of error for such a system assuming K=1.

8-

(*Amount of fading*) For coherent demodulation in fading, it is assumed that the random phase can be perfectly estimated and accounted for at the receiver. This means that only the random amplitude affects the quality of the demodulation. Let $f_\alpha(\alpha)$ be the pdf of the random amplitude α. Then, the severity of fading due to random amplitude can be quantified through a single parameter, called the "amount of fading" or fading figure, defined as

$$\mathrm{AF} = \frac{\mathrm{var}\{\alpha^2\}}{(E\{\alpha^2\})^2}. \tag{P10.3}$$

In general, the smaller the AF, the less severe the fading is.

(a) What is the value of AF when the amplitude is a constant.
(b) Show that, for a Rayleigh amplitude, AF = 1. Recall that the Rayleigh distribution is

$$f_\alpha(\alpha) = \frac{2\alpha}{\sigma_F^2}\mathrm{e}^{-\alpha^2/\sigma_F^2}u(\alpha). \tag{P10.4}$$

(c) Consider the following Nakagami-m distribution:

$$f_\alpha(\alpha) = \frac{2m^m\alpha^{2m-1}}{\Gamma(m)\sigma^{2m}}\mathrm{e}^{-m\alpha^2/\sigma^2}u(\alpha), \quad m \geq 0.5. \tag{P10.5}$$

Show that $\mathrm{AF} = 1/m$.