

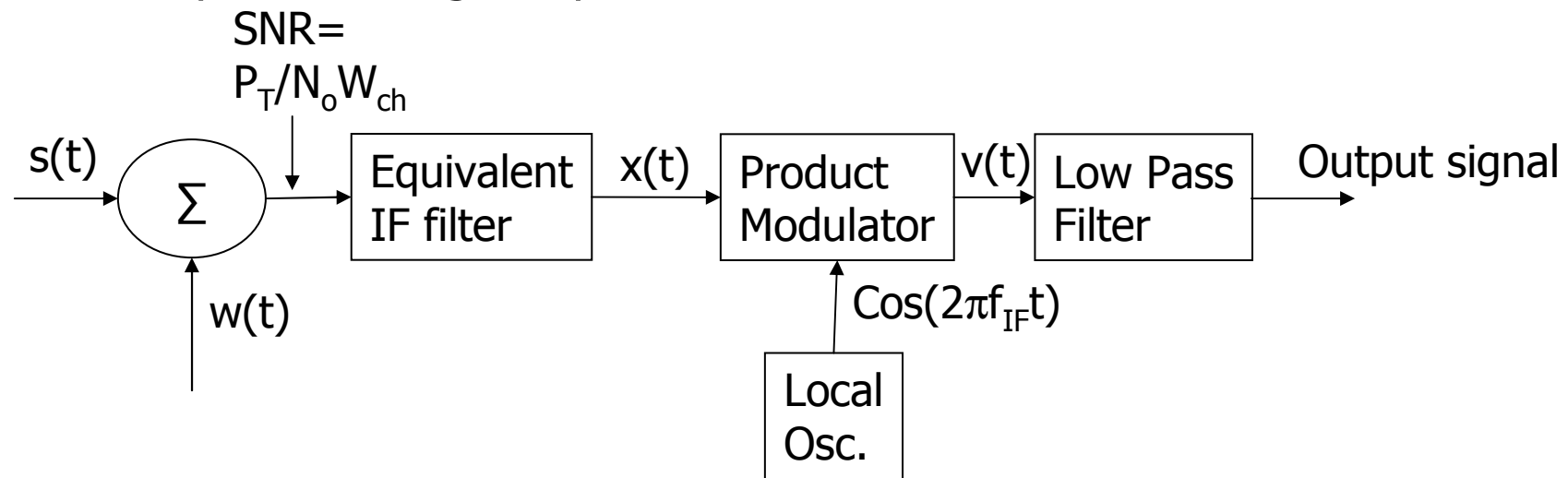


Communication Systems II

Noise in CW modulation
SSB, QAM, Envelope Detection

SNR for Coherent Reception of SSB Modulation

- The IF filter output is multiplied by a local carrier with frequency f_{IF} and then low pass filtering the product



- Consider SSB-LSB

$$s(t) = \frac{A_c}{2} m(t) \cos(2\pi f_c t) + \frac{A_c}{2} \hat{m}(t) \sin(2\pi f_c t)$$
 where $m(t)$ is considered a sample function of a stationary process of zero mean and PSD $S_M(f)$ limited to maximum frequency W



SNR for Coherent Reception of SSB Modulation

- $m(t)$ and $\hat{m}(t)$ are orthogonal to each other.
 - If $m(t)$ is zero mean $\rightarrow m(t)$ and $\hat{m}(t)$ are uncorrelated
 - Their PSD are additive \longrightarrow
 - $\hat{m}(t)$ is obtained by passing $m(t)$ on a LTIS with $H(f) = -j\text{sgn}(f) \rightarrow$ square magnitude is equal to one $\rightarrow \hat{m}(t)$ have the same PSD as $m(t)$

$$z = m(t) + \hat{m}(t)$$

$$R_z(t) = E[z(t)z(t + \tau)]$$

$$= E[m(t)m(t + \tau)] + E[\hat{m}(t)\hat{m}(t + \tau)] + \text{zeros}$$

$$R_z(\tau) = R_m(\tau) + R_{\hat{m}}(\tau)$$

$$S_z(f) = S_m(f) + S_{\hat{m}}(f)$$



SNR for Coherent Reception of SSB Modulation

- Inphase component $\frac{A_c}{2} m(t) \cos(2\pi f_c t)$
- Quadrature component $\frac{A_c}{2} \hat{m}(t) \sin(2\pi f_c t)$
- Average power in the inphase component = $\frac{A_c^2}{8} P_m$
average power in quadrature component = $\frac{A_c^2}{8} P_m$
- Average power in modulated signal $s(t) = \frac{A_c^2}{4} P_m$
 - Half that of DSB-SC
- Channel noise power in message bandwidth is $N_o W$
- $$SNR_c = \frac{A_c^2 P_m}{4 N_o W}$$

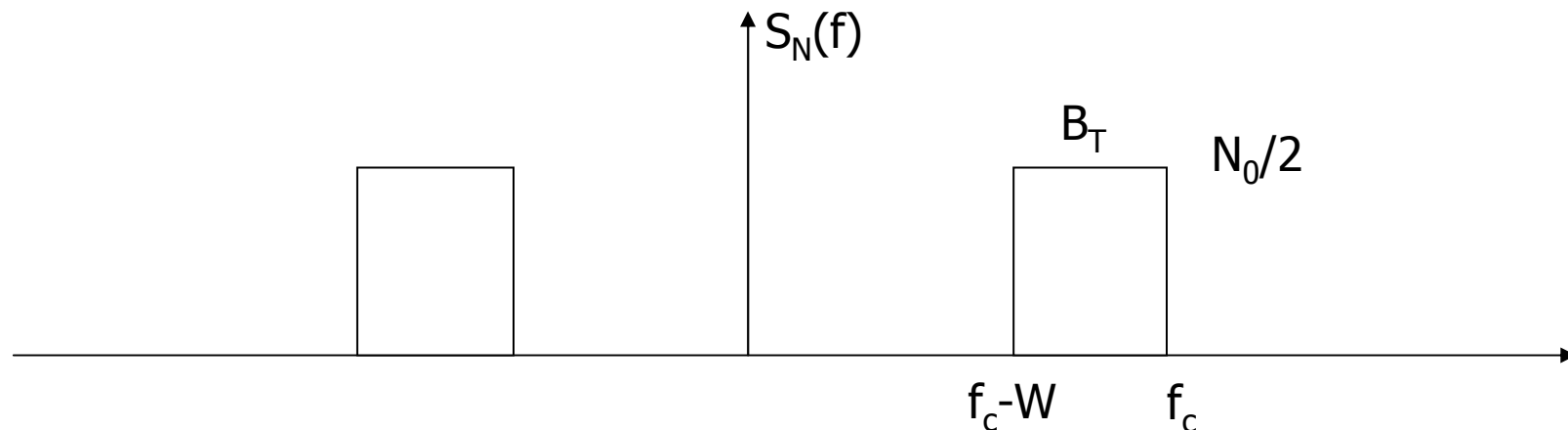
SNR for Coherent Reception of SSB Modulation

- The NBN can be represented as

$$n(t) = n_c(t) \cos(2\pi(f_c - \frac{w}{2})t) - n_s(t) \sin(2\pi(f_c - \frac{w}{2})t)$$

- The output of the coherent detector

$$y(t) = [s(t) + n(t)] \cos(2\pi f_c t)$$





SNR for Coherent Reception of SSB Modulation

- The output of the coherent detector

$$y(t) = \frac{A_c}{2} m(t) \cos^2(2\pi f_c t) + \frac{A_c}{2} \hat{m}(t) \sin(2\pi f_c t) \cos(2\pi f_c t)$$

+

$$n_c(t) \cos(2\pi(f_c - \frac{w}{2})t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi(f_c - \frac{w}{2})t) \cos(2\pi f_c t)$$

$$= \frac{A_c}{4} m(t) + \frac{A_c}{4} m(t) \cos(2\pi 2 f_c t) + \frac{A_c}{4} \hat{m}(t) \sin(2\pi 2 f_c t)$$

+

$$\frac{n_c(t)}{2} \cos(2\pi \frac{w}{2} t) + \frac{n_s(t)}{2} \sin(\frac{w}{2} t) + \text{higher frequency terms}$$

SNR for Coherent Reception of SSB Modulation

- The output of the after LPF

$$= \frac{A_c}{4} m(t) + \frac{n_c(t)}{2} \cos(2\pi \frac{w}{2} t) + \frac{n_s(t)}{2} \sin(\frac{2\pi w}{2} t)$$

- Average power in the detected message is $\frac{A_c^2}{16} P_m$
- Average power in the received noise is

$$SNR_o = \frac{A_c^2 P_m}{16N_o W / 4} = \frac{A_c^2 P_m}{4N_o W} = \frac{N_o W}{8} + \frac{N_o W}{8} = \frac{N_o W}{4}$$

- Figure of Merit=1

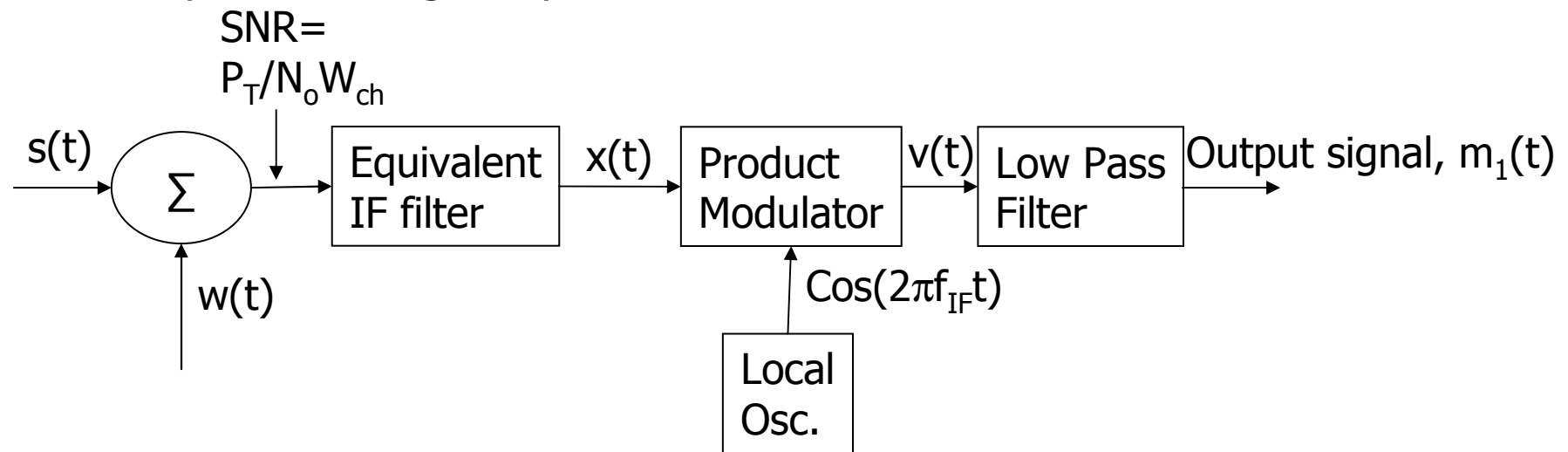


Comparison between SSB, DSB-SC

- HW assignment
- Compare between the SNR of the DSB-SC, and the SSB

SNR for Coherent Reception of QAM Modulation

- The IF filter output is multiplied by a local carrier with frequency f_{IF} and then low pass filtering the product



- Consider SSB-LSB $s(t) = A_c m_1(t) \cos(2\pi f_c t) + A_c m_2(t) \sin(2\pi f_c t)$
 where $m_1(t)$ and $m_2(t)$ are considered a sample functions of stationary processes of zero mean and PSD $S_M(f)$ limited to maximum frequency W

SNR for Coherent Reception of QAM Modulation

$$s(t) = A_c m_1(t) \cos(2\pi f_c t) + A_c m_2(t) \sin(2\pi f_c t)$$

$$R_s(\tau) = \frac{A_c^2}{2} R_{m_1}(\tau) \cos(2\pi f_c \tau) + \frac{A_c^2}{2} R_{m_2}(\tau) \cos(2\pi f_c \tau)$$

$$S_s(f) = \frac{A_c^2}{4} [S_{m_1}(f - f_c) + S_{m_1}(f + f_c) + S_{m_2}(f - f_c) + S_{m_2}(f + f_c)]$$

Received Signal

$$s(t) + w(t)$$

Received Signal after IF filter

$$X(t) = s(t) + n(t)$$

$$= A_c m_1(t) \cos(2\pi f_c t) + A_c m_2(t) \sin(2\pi f_c t) + n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)$$

$$= [A_c m_1(t) + n_c(t)] \cos(2\pi f_c t) + [A_c m_2(t) - n_s(t)] \sin(2\pi f_c t)$$

Received Signal after coherent detector

$$= [A_c m_1(t) + n_c(t)] \cos(2\pi f_c t) \cos(2\pi f_c t) + [A_c m_2(t) - n_s(t)] \sin(2\pi f_c t) \cos(2\pi f_c t)$$

Received Signal after LPF

$$[A_c m_1(t) + n_c(t)] / 2$$

$$SNR_o = \frac{A_c^2 P_{m_1}}{2 N_o W}$$

Noise in AM receivers using Envelope Detection

- The transmitted signal $s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$
- The average power in $s(t) = A_c^2 [1 + k_a^2 P_m] / 2$
- The average noise power in the message bandwidth = $N_o W$
- The $SNR_{c,AM} = \frac{A_c^2 [1 + k_a^2 P_m]}{2N_o W}$
- The received signal at the envelope detector $x(t)$ consists of the transmitted signal $s(t)$ and NBN $n(t)$
 - $x(t) = s(t) + n(t)$
 - $= A_c [1 + k_a m(t)] \cos(2\pi f_c t) + n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)$
 - $= [A_c + A_c k_a m(t) + n_c(t)] \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)$

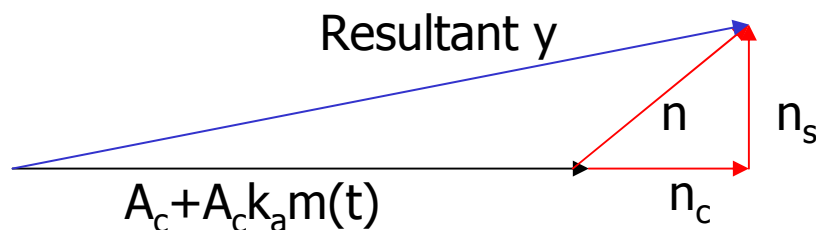
Phasor diagram for AM wave plus NBN

- From the phasor diagram, the received output $y(t)$ is obtained as the envelope of $x(t)$

$$y(t) = \sqrt{[A_c + A_c k_a m(t) + n_c(t)]^2 + n_s^2(t)}$$

- $Y(t)$ need to be approximated to a message term plus a noise and distortion term.
- If average carrier power is larger compared to noise, then the first term in $y(t)$ will be larger than the second term

$$y(t) \approx [A_c + A_c k_a m(t) + n_c(t)]$$





Phasor diagram for AM wave plus NBN

- DC component can be removed with DC block.
- The output signal to noise ratio

$$SNR_{o,AM} = \frac{A_c^2 k_a^2 P}{2WN_o}$$

- This expression is only valid IFF
 - The noise at the receiver input is small compared to the signal.
 - The K_a is adjusted for a percentage modulation less than or equal to 100%
- The Figure of merit is always less than unity

$$\frac{SNR_{o,AM}}{SNR_{c,AM}} = \frac{A_c^2 k_a^2 P}{2WN_o} \cdot \frac{2WN_o}{A_c^2 [1 + k_a^2 P]} = \frac{k_a^2 P}{[1 + k_a^2 P]}$$

The threshold effect

- When the signal to noise ratio is small, the noise component dominates and the performance of the envelope detector completely changes.

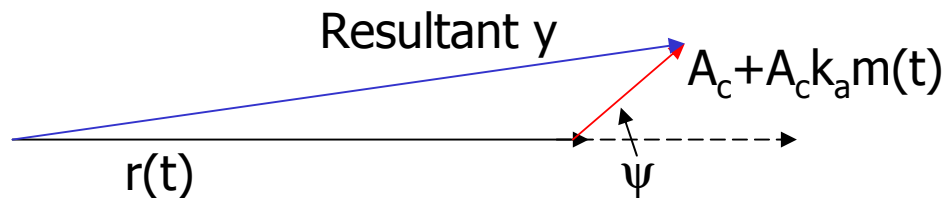
- NBN can be represented as shown

$$n(t) = r(t) \cos(2\pi f_c t + \psi(t))$$

Received signal at envelope detector input

$$x(t) = s(t) + n(t)$$

- We can neglect the quadrature component of the signal $y(t) \approx [r(t) + A_c \cos(\psi(t)) + A_c k_a m(t) \cos(\psi(t))]$





The threshold effect

- When the signal to noise ratio is small, The envelope detector output does not have a component proportional to the message $m(t)$, the noise angle is uniformly distributed from 0 to 2π
- The threshold effect is defined as the value of the carrier to noise ratio below which the noise performance of a detector deteriorates much more rapidly than proportionately to the carrier to noise ratio