More PDF properties

ii. $P[a \le X \le b] = \int_{a^-}^{b^+} f_X(x)\,dx$

iii. $F_X(x) = \int_{-\infty}^{x^+} f_X(t)\,dt$

the PDF completely specifies the behavior of continuous r.v.'s

iv. $\int_{-\infty}^{\infty} f_X(t)\,dt = F_X(\infty) = 1$ unit probability mass

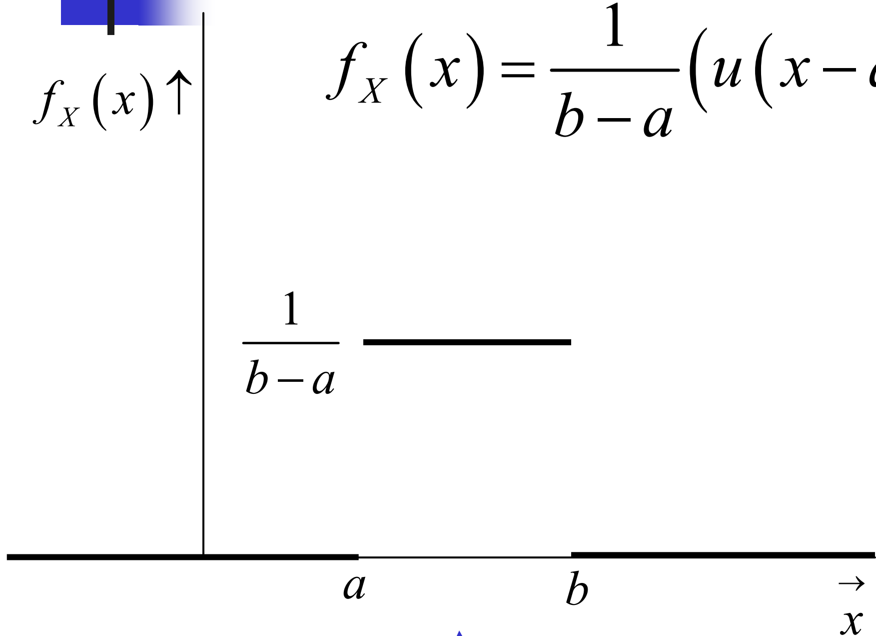
PDF is like "mass" density

a valid PDF can be formed from any nonnegative, piecewise continuous, integrable function

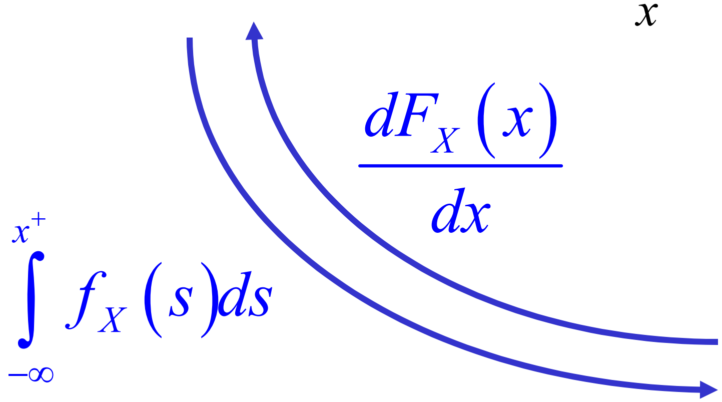
PDF – like CDF – is defined over $(-\infty,\infty)$

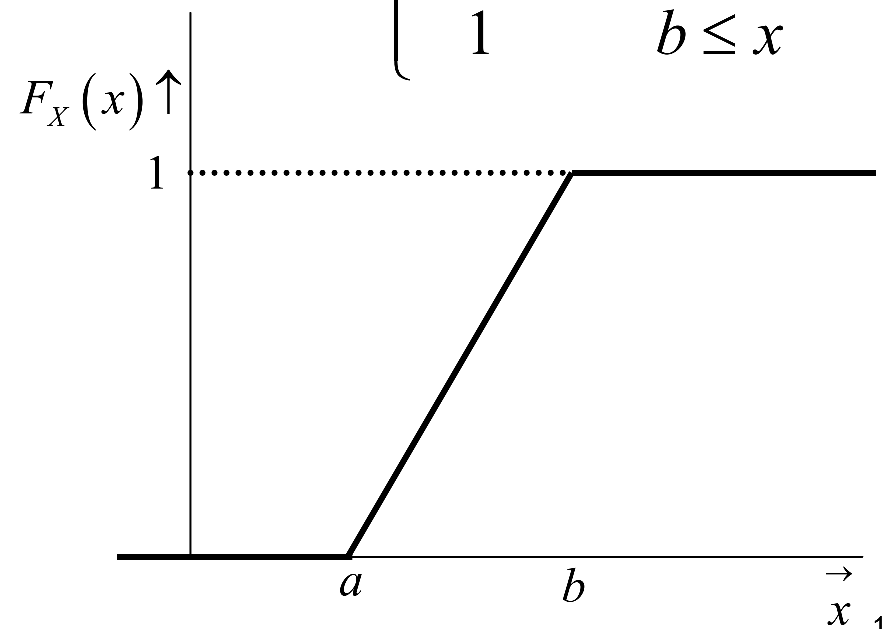
Ex 3.7 Uniform random variable

$$f_X(x) = \frac{1}{b-a}\left(u(x-a) - u(x-b)\right)$$



$$F_X(x) = \begin{cases} 0 & x < a \\ \dfrac{x-a}{b-a} & a \le x < b \\ 1 & b \le x \end{cases}$$





not magnitude

Ex 3.8 normalization

- PDF of sample values of speech waveforms is found to decay exponentially - at a rate α - for positive and negative values

$$f_X(x) = ce^{-\alpha|x|} \qquad -\infty < x < \infty$$

normalization:

$$\int_{-\infty}^{\infty} ce^{-\alpha|x|}dx = 2\int_{0}^{\infty} ce^{-\alpha|x|}dx = 2c\frac{e^{-\alpha\infty} - e^{-\alpha 0}}{-\alpha} = 1$$

$$\Downarrow$$

$$c = \frac{\alpha}{2}$$

$$P\left[|X| < \upsilon\right] = \frac{\alpha}{2}\int_{-\upsilon}^{\upsilon} e^{-\alpha|x|}dx = \alpha\int_{0}^{\upsilon} e^{-\alpha x}dx = \alpha\frac{e^{-\alpha\upsilon} - e^{-\alpha 0}}{-\alpha} = 1 - e^{-\alpha\upsilon}$$

Bernoulli r.v.

- *A* is an event related to the outcomes of a random experiment

indicator function for *A*: $I_A(\zeta) \triangleq \begin{cases} 0 & \zeta \notin A \\ 1 & \zeta \in A \end{cases}$

assigns # to outcome ⇓ $I_A(\zeta)$ is a r.v.

$$S_X = \{0,1\}$$

pmf: $p_I(0) = 1 - p\,; \quad p_I(1) = p = P[A]$

$I_A(\zeta) = 1 \sim \text{"success"} \rightarrow$ Bernoulli r.v.

"tossing of a biased coin": Bernoulli r.v. is a model for this fundamental mechanism for generating randomness"

Binomial r.v.

- Random experiment repeated *n* independent times. Let *X* be the number of times event *A* occurs in these *n* trials.

$$X = I_1 + I_2 + \cdots + I_n$$

sum of Bernoulli r.v.'s
(indicator functions for *A* in trial *j*)

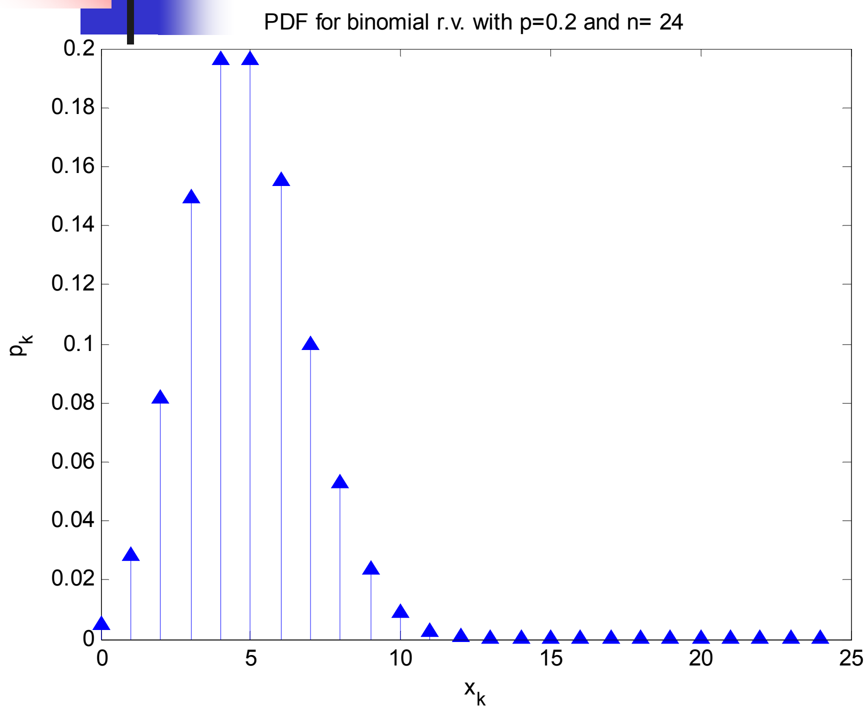
$$S_X = \{0, 1, \cdots, n\}$$

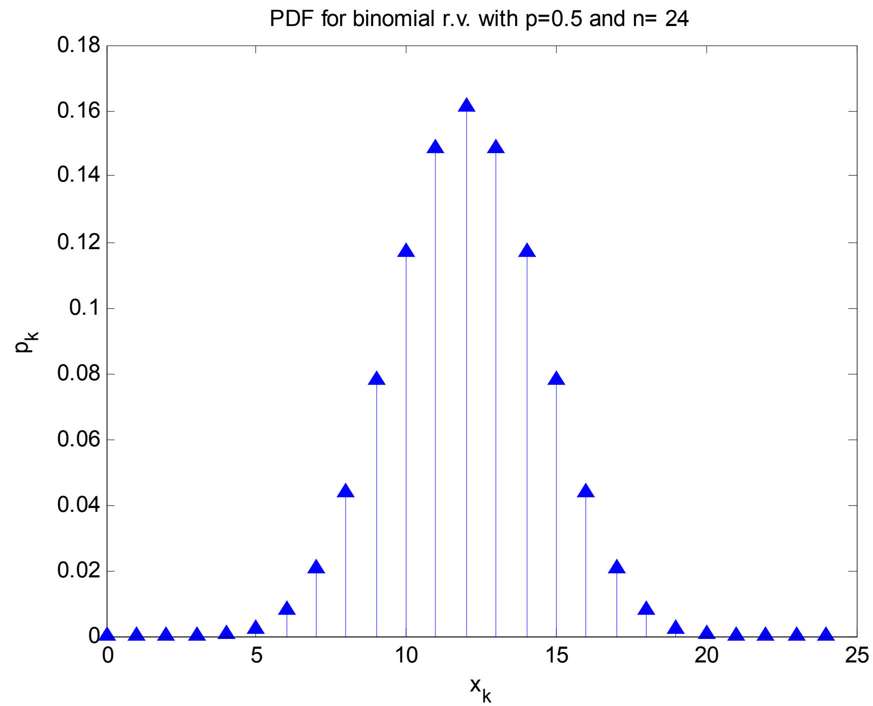
$$P[X = k] = \binom{n}{k} p^k (1-p)^{n-k} \quad \text{for } k = 0, 1, \cdots, n$$

Binomial PDF

$$k_{\max} = \arg\max_k P[X = k] = \lfloor (n+1)p \rfloor$$

$$\text{if } \lfloor (n+1)p \rfloor == \lceil (n+1)p \rceil \text{ then also max at } k_{\max -1}$$





Arises in applications where there are two types of object (heads/tails, good/defective, correct/in-error, active/silent), and we're interested in the number of type 1 objects in a randomly selected batch of size *n*, and the type of each object is independent of the types of the other objects in the batch

Geometric r.v.

- # of independent Bernoulli trials until first occurrence of "success"

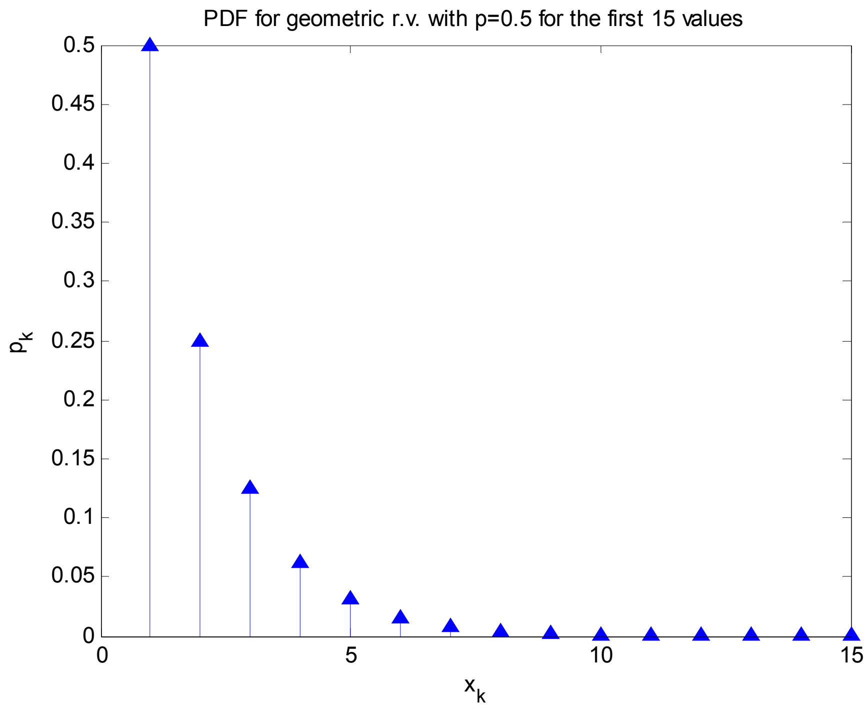
$$S_X = \{1, 2, \cdots\}$$

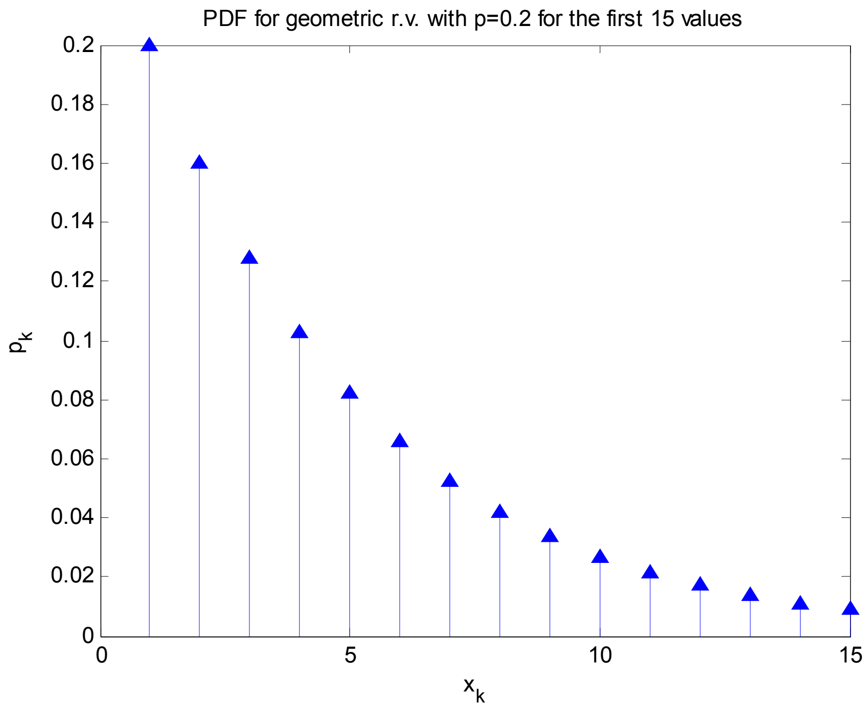
$$p = P[A] = P["success"] \text{ in each Bernoulli trial}$$

$$P[M = k] = (1-p)^{k-1} p \quad \text{for } k = 1, 2, \cdots$$

geometric decay

Geometric PDF





decay like 0.5^k and 0.8^k respectively

Exponential r.v.

- Arises in modeling of the time between occurrence of events, and in modeling lifetime of devices and systems; λ is the rate at which events occur

$$f_X(x) = \lambda e^{-\lambda x} u(x)$$

$$F_X(x) = \left(1 - e^{-\lambda x}\right) u(x)$$

shown earlier