Drawbacks of OFDM Systems

- High sensitivity to carrier frequency offsets (CFO)
- High peak-to-average power ratios (PAPR) problem
- Limited Frequency Diversity

PAPR Problem

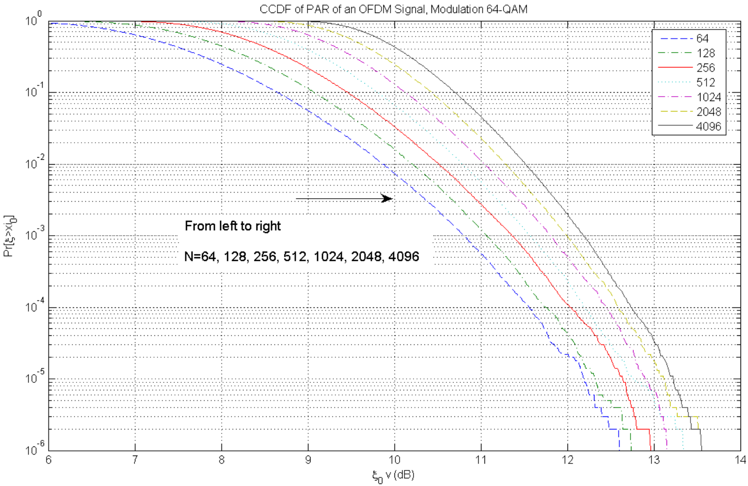
- OFDM suffers from high PAPR defined as,

$$\xi = \frac{\max_{t\in[0,T]} |x(t)|^2}{P_{av}} = \frac{\max_{t\in[0,T]} |x(t)|^2}{E\{|x(t)|^2\}} \quad (7)$$

- Large PAPR implies larger dynamic range of signals and causes problems in power amplifier, ADC and so on.
- PAPR can be characterized by CCDF.

CCDF of PAPR

Complementary cumulative distribution function (CCDF): Probability that the PAPR of an OFDM symbol exceeds a given threshold



Synchronization Issues in OFDM Systems

- Time synchronizations
  - Packet detection
  - Frame synchronization
- Frequency synchronizations
  - Carrier frequency synchronization
  - Sampling frequency synchronization

- Cause of synchronization error:
  - Asynchronous transmission => unknown transmit times
  - Circuit elements are never ideal
    - Local oscillators are never ideal
      - Frequency offset
      - Phase noise
    - Clocks are never ideal
      - Frequency offset
      - jitter

Synchronization Issues in OFDM Systems

Aalborg University

- What are their impacts of synchronization error?
- Time domain effect
  - *Incorrect packet start* => packet detection error, packet loss
  - *Incorrect symbol window* => Inter-symbol Interference.
  - *Carrier freq. offset* => phase rotation on symbol
  - *Sampling freq. offset* => incorrect sampling instant.
- Frequency domain effect
  - *Incorrect packet start* => rotation of constellation
  - *Incorrect symbol window* => rotation of constellation
  - *Carrier freq. offset* => Inter-Carrier-Interference
  - *Sampling freq. offset* => Inter-Carrier-Interference

Frequency Synchronization

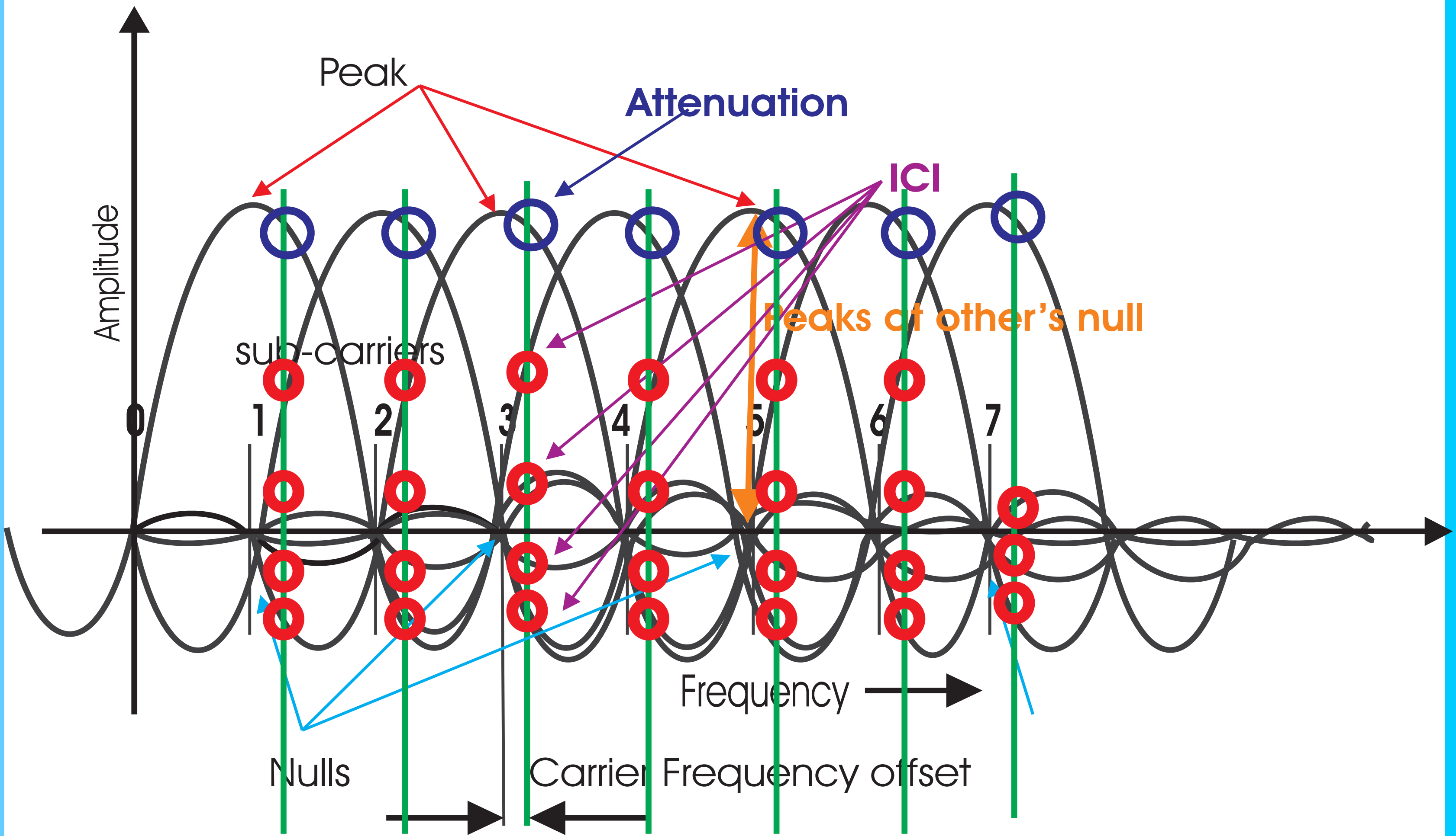
1. Carrier Frequency synchronization
2. Sampling frequency synchronization

- What is the cause for Carrier Frequency Offset Error ?
  - Mismatch in local oscillator frequency between transmitter and Receiver
  - Phase noise of the local oscillator at Transmitter and receiver

- How does it effect the system ?
  - Loss in orthogonally between sub-carriers
  - Inter carrier interference
  - Can be partly compensated; Partly irreducible noise floor

- What are the mitigation strategies ?
  - Using Training sequence
  - Using Pilots

Carrier Frequency offset



CFO Sensitivity

- Received signal with a frequency offset f_o

$$r_k = \sum_{m=0}^{N-1} H_m X_m e^{j2\pi k(m+\epsilon)/N} + n_k, k \in [0, N-1] \qquad (5)$$

where $\epsilon = f_o/\Delta f$ is the normalized CFO.

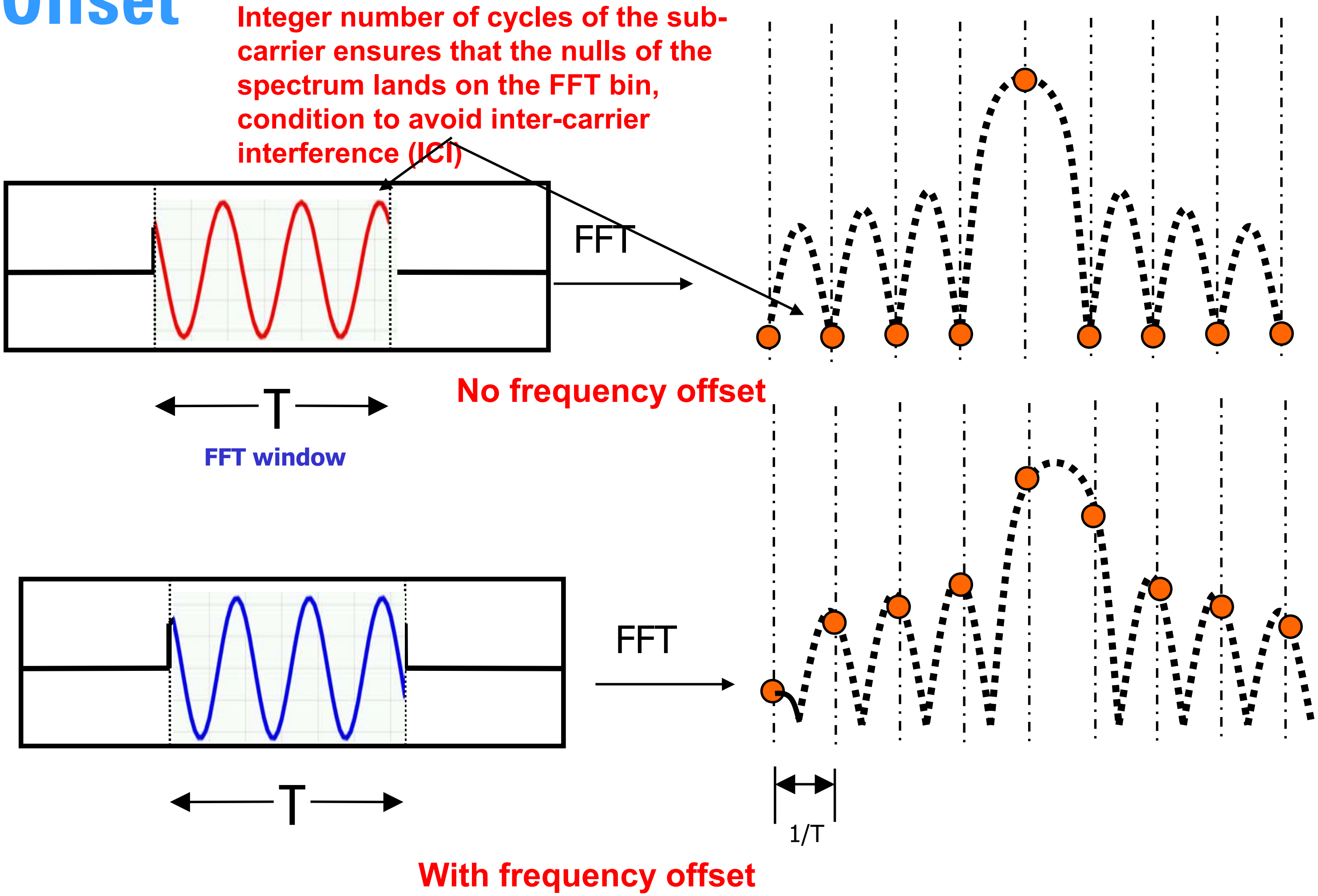
- After transferring to frequency domain

$$R_\ell = \sum_{k=0}^{N-1} r_k e^{-j2\pi \ell k/N} = X_\ell H_\ell \frac{\sin(\pi\epsilon)}{N\sin(\pi\epsilon/N)} e^{j\pi\epsilon(N-1)/N} + I_\ell + Z_\ell$$

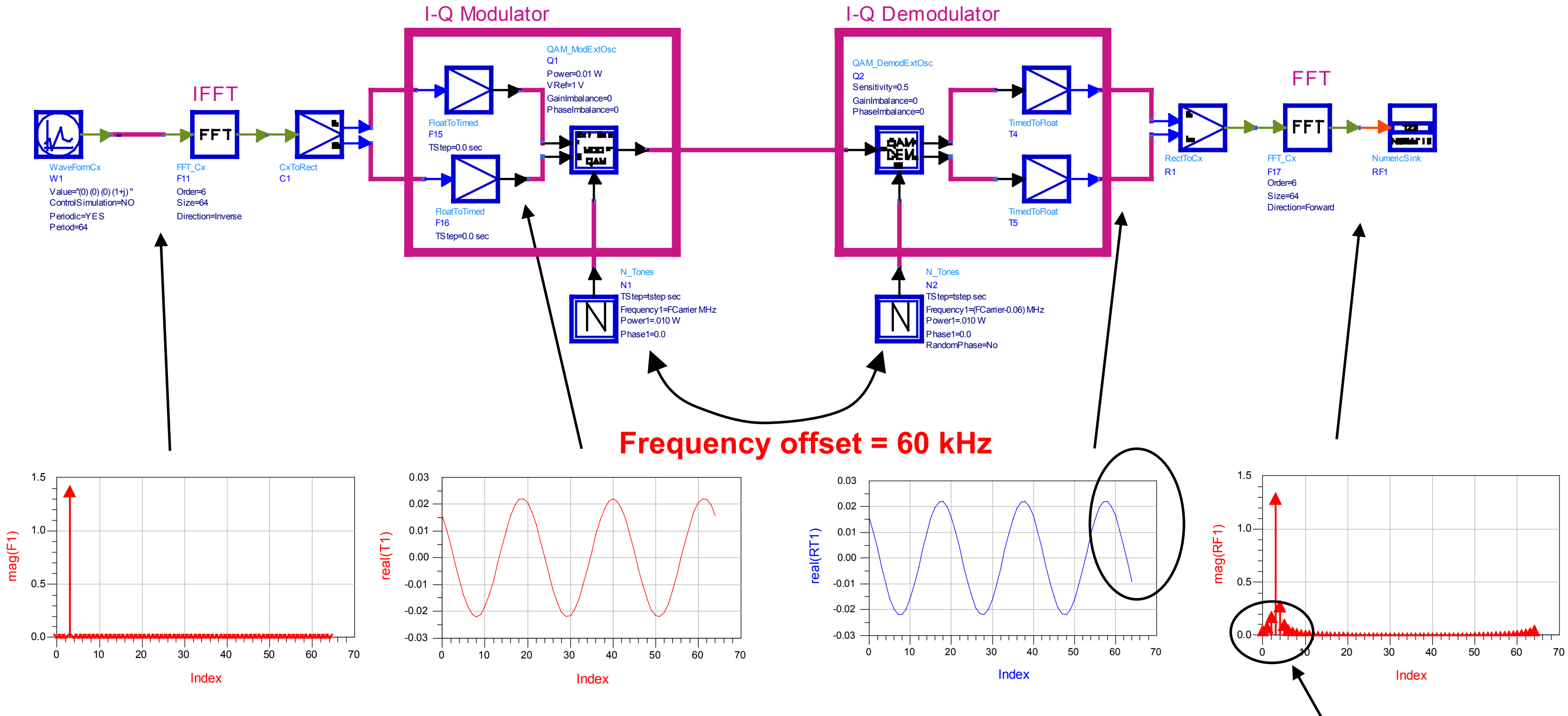
where $I_\ell = \sum_{l=0, l\neq\ell}^{N-1} X_l H_l \frac{\sin(\pi\epsilon)}{N\sin(\pi(l-\ell+\epsilon/N))} e^{j\pi\epsilon(N-1)/N} e^{j\pi(l-\ell)/N}$ is the ICI term.

- Different to single carrier systems, CFO causes ICI in OFDM systems.

Inter-Carrier Interference(ICI) Due to Frequency Offset



OFDM Operation (ICI problem)



Symbol to be transmitted (Magnitude spectrum)

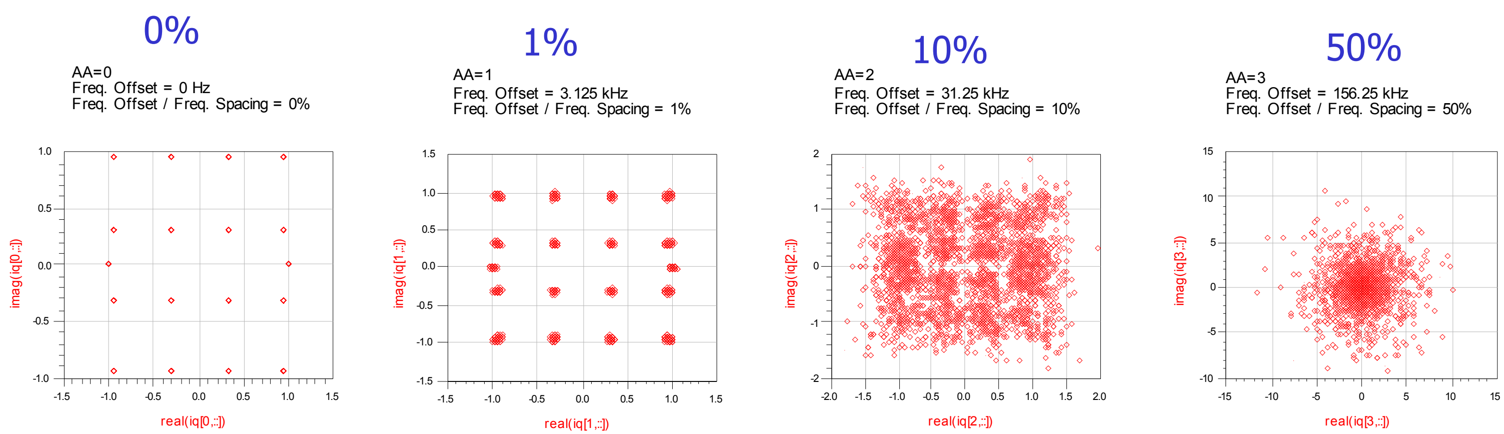
I-channel signal (after IFFT)

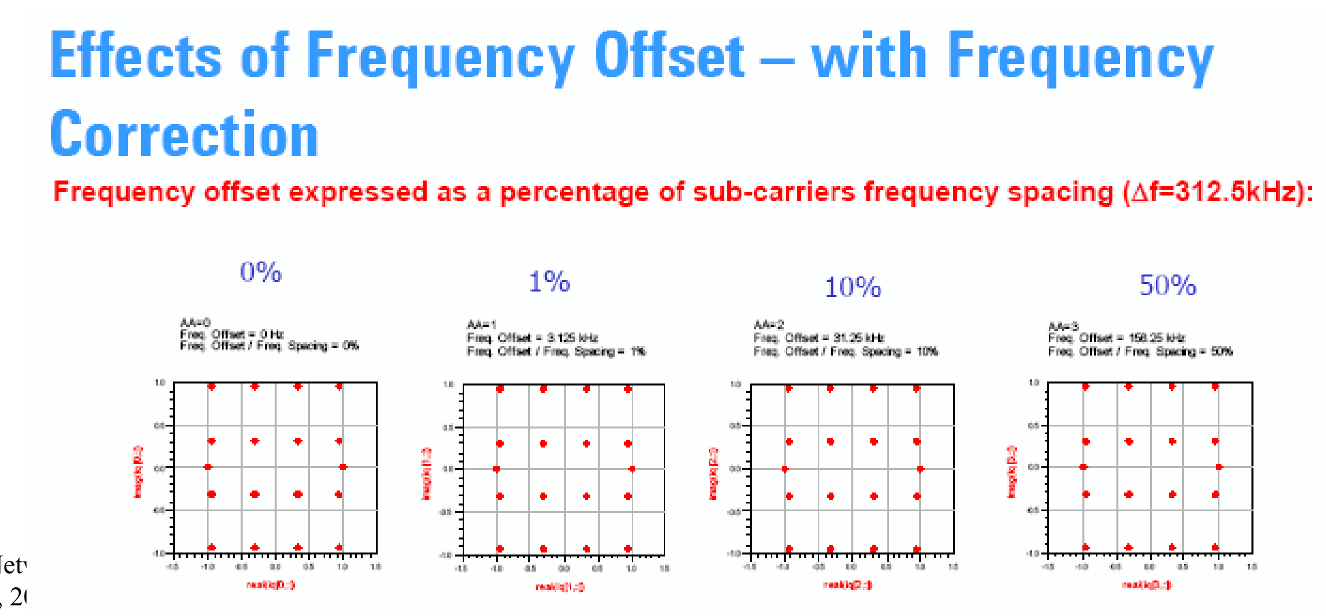
Demodulated I-channel signal

Recovered symbol contains ICI

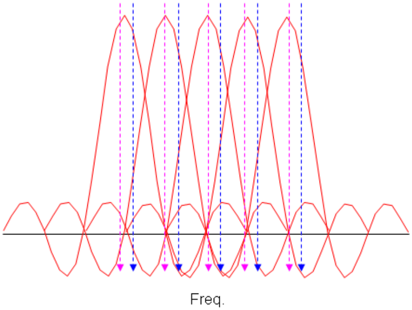
Effects of Frequency Offset – Without Frequency Correction

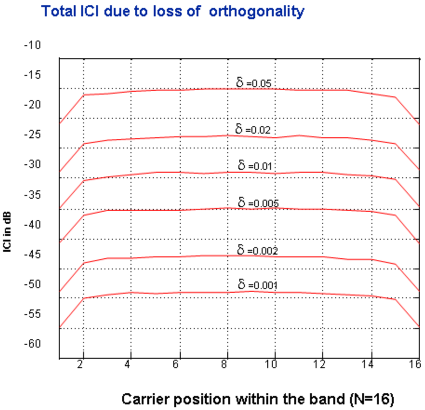
Frequency offset expressed as a percentage of sub-carriers frequency spacing (Δf=312.5kHz):





CFO Sensitivity (cont.)



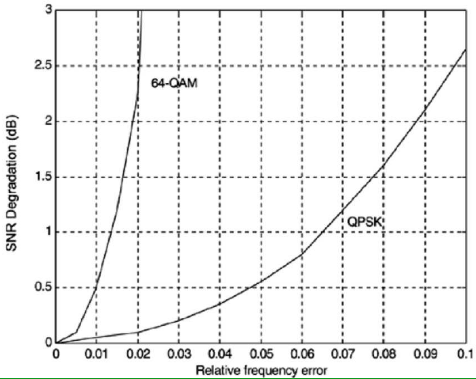


δ: normalized CFO

CFO Sensitivity - SNR Degradation

For relatively small frequency errors, the degradation in dB can be approximated by

$$\mathrm{SNR}_{\mathrm{loss}} = \frac{10}{3\ln 10}(\pi T_s \epsilon)^2 E_s/N_0 dB \quad (6)$$



Loss of orthogonality (by frequency offset)

Transmission pulses $\psi_k(t) = \exp(jk2\pi t/T)$ y $\psi_{k+m}(t) = \exp\left(j2\pi(k+m)t/T\right)$

Reception pulse with offset δ $\psi^{\delta}_{k+m}(t) = \exp\left(j2\pi(k+m+\delta)/T\right)$ *con* $|\delta| \le 1/2$

Interference between channels k and k+m

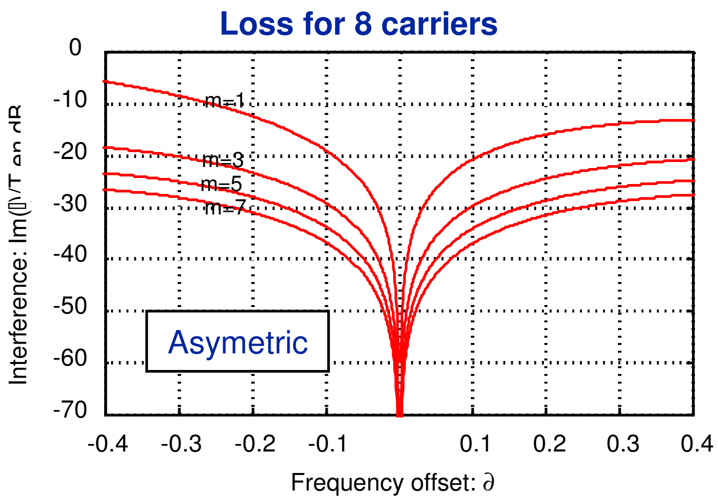
$$I_m(\delta) = \int_0^T \exp\left(jk2\pi t/T\right)\exp\left(-j(k+m+\delta)2\pi/T\right)dt = \frac{T(1-\exp(-j2\pi\delta))}{j2\pi(m+\delta)}$$

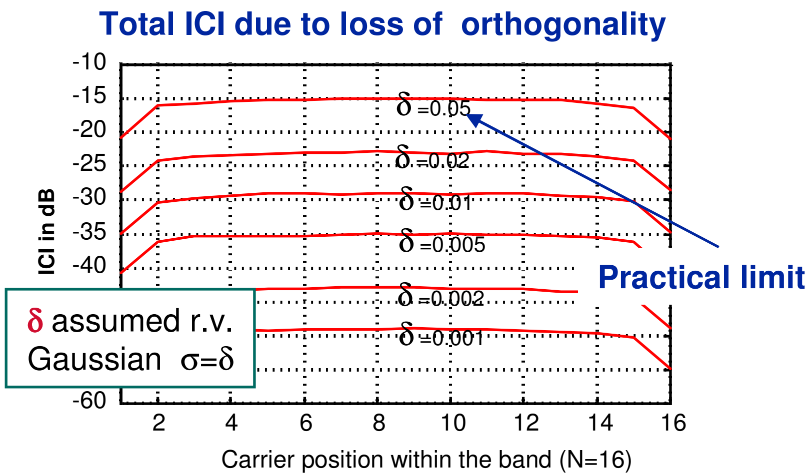
$$|I_m(\delta)| = \frac{T|\sin \pi\delta|}{\pi|m+\delta|}$$

Summing up ∀ m

$$\sum_m I_m^2(\delta) \approx (T\delta)^2 \sum_{m=1}^{N-1} \frac{1}{m^2} \approx (T\delta)^2 \frac{23}{14}$$

for $N >> 1$ ($N > 5$ *Is enough*)





Loss of orthogonality (time)

Let us assume a misadjustment τ

$$X_i = c_0 \int_{-T/2}^{-T/2+\tau} \psi_k(t)\psi_l^*(t-\tau)dt + c_1 \int_{-T/2+\tau}^{T/2} \psi_k(t)\psi_l^*(t-\tau)dt$$

2 consecutive symbols

Then if m=k-l

$$|X_i| = \begin{cases} 2T\left|\dfrac{\text{sen}\, m\pi \frac{\tau}{T}}{m\pi}\right|, & c_0 \neq c_1 \\ 0, & c_0 = c_1 \end{cases}$$

Or approximately, when $\tau \ll T$

$$\frac{|X_i|}{T} \approx \frac{2m\pi \frac{\tau}{T}}{m\pi} = 2\frac{\tau}{T}$$

independent on m

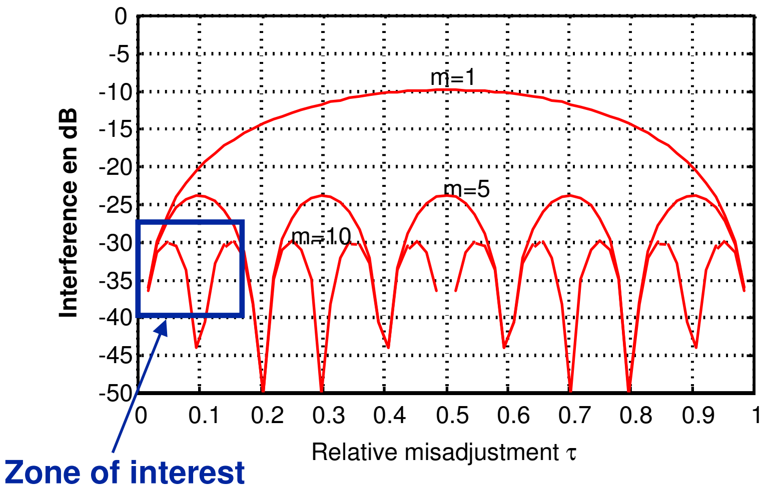
In average, the interfering power in any carrier is

$$E\left[\frac{|X_i|^2}{T^2}\right] = 4\left(\frac{\tau}{T}\right)^2 \frac{1}{2} + 0\frac{1}{2} = 2\left(\frac{\tau}{T}\right)^2$$

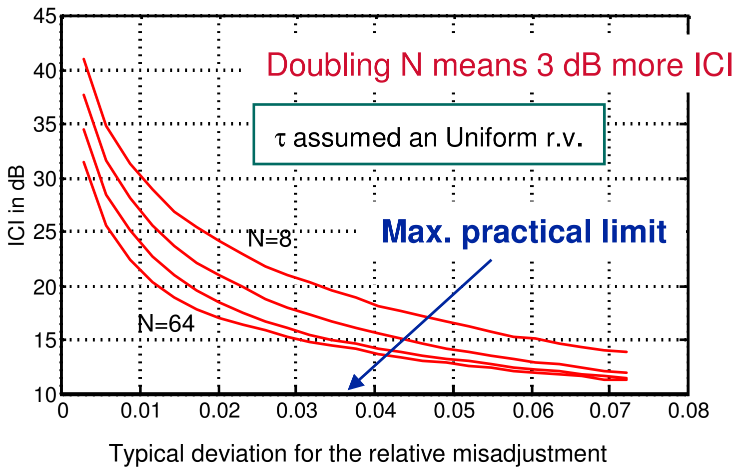
$$ICI \approx 20\log\left(\sqrt{2}\frac{\tau}{T}\right), \quad \tau \ll T$$

Per carrier

Loss for 16 carriers

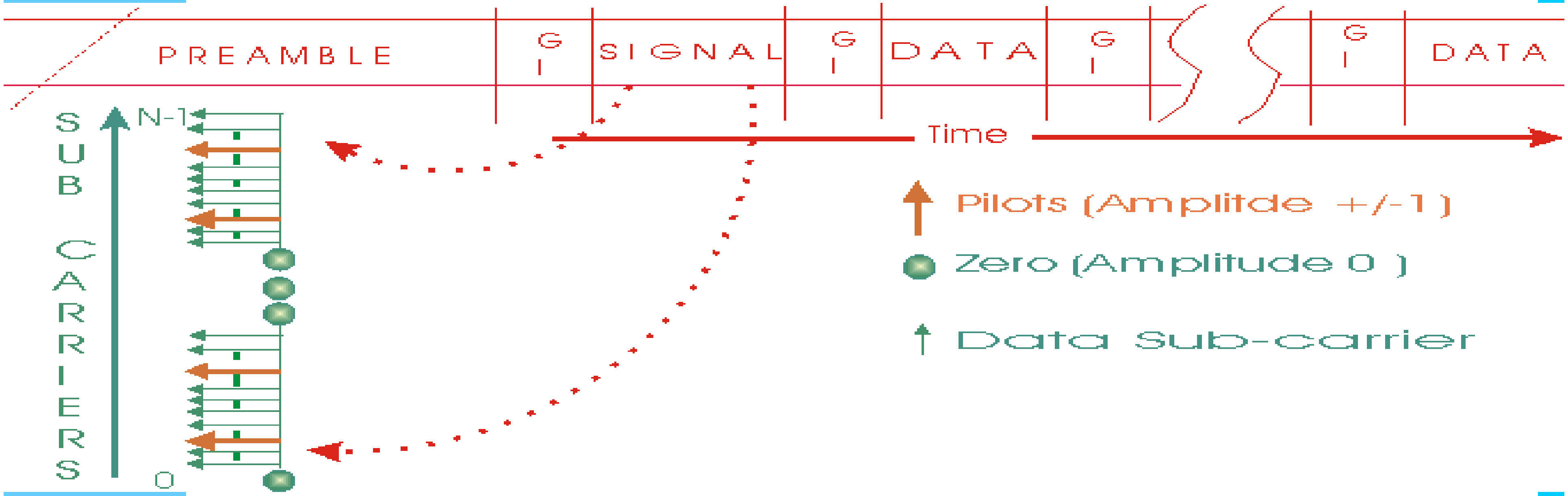


ICI due to loss of orthogonaliy



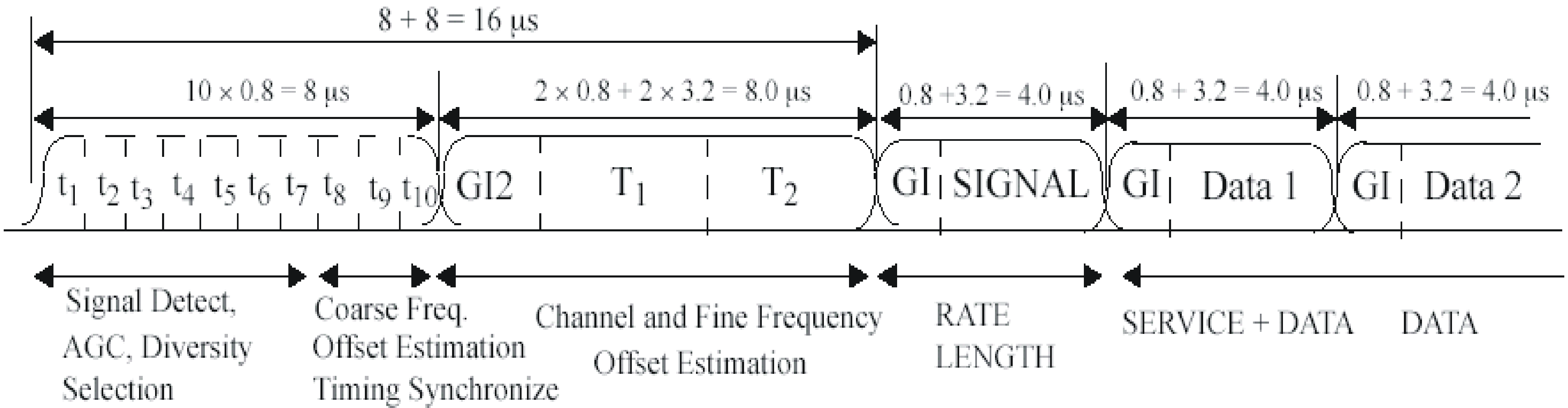
Example transmission format

IEEE 802.11a Frame format



Example transmission format

IEEE 802.11a Preamble (source IEEE 802.11a standard)



- $t_1, t_2 \ldots t_{10}$ are identical
  - $t_1, t_2, \ldots t_{10}$ are of duration ¼ times the FFT duration
  - => the sub—carrier spacing is 4 times more than in the Data part
- T_1 and T_2 are also identical
  - They are of same duration as the normal OFDM symbol

Packet Detection and AGC

Aalborg University

First training tasks of a digital receiver:

Packet detection

- Detect start of a signal, based on detecting energy jump or by a correlator exceeding some threshold

Automatic Gain Control

- Adjust RF gain such that A/D converter gets appropriate signal input level with best possible Signal-to-Quantization+Clipping Noise Ratio

Key Performance parameters:

1) 1) Probability of missed detect = missing a valid packet
2) Probability of false alarm = detecting a non-valid packet

   1) gives packet errors, 2) gives higher power consumption by spending processing power on non-valid packets, and it can also lead to missed detects as a valid packet comes in while the receiver thinks it is already decoding a packet.

Packet Detection - Signal Energy Detection

- Received Signal Energy Detection: Compare the decision variable m_n with a predefined threshold where m_n is the received signal energy accumulated over some window of length M

$$m_n = \sum_{k=0}^{M-1} |r_{n-k}|^2 \qquad (1)$$

  - Calculation of m_n can be simplified by noting that it is a moving sum of the received signal energy (Sliding window);
  - Sometimes implemented in analog domain to mitigate the impact of RF circuit including AGC;
  - A fixed threshold does not work well.

Double Sliding Window Packet Detection

- Double Sliding Window Packet Detection: Let m_n be the ratio of the received energy within two consecutive sliding windows.

$$m_n = \frac{\sum_{k=0}^{M_1-1} |r_{n+k}|^2}{\sum_{\ell=0}^{M_2-1} |r_{n-\ell}|^2} \qquad (2)$$

  - The value of m_n is more stable;
  - The peak point of m_n is approximately equal to the received SNR (SNR+1).

Packet Detection - Delay and Correlate Algorithm

- Exploiting the periodicity of the short training symbols in the preamble
- Algorithm similar to the approach presented in Schmidl and Cox [1]

$$m_n = \frac{\left| \sum_{k=0}^{M-1} r_{n+k} r_{n+k+D}^{*} \right|^2}{\left(\sum_{k=0}^{M-1} |r_{n+k+D}|^2 \right)^2} \quad (3)$$

where D is the period of the short training symbols, and generally $M \geq D$.

Packet Detection cont...

Autocorrelation based packet detection with IEEE 802.11a preamble

We define the decision variable as the normalized auto-correlation coefficient as:

$$\Phi(n) = \frac{\sum_{n=-N+1}^{0} x^*(n)x(n+N)}{\sum_{n=-N+1}^{0} x^*(n)x(n)} \quad (7)$$

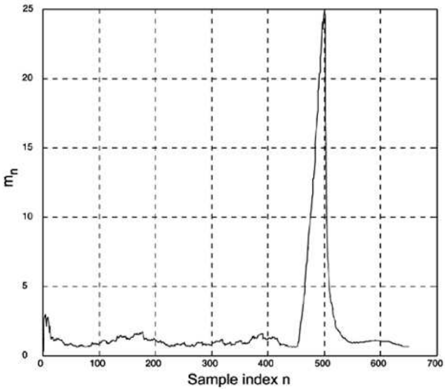
We consider a packet to be detected if for P consecutive samples

$$\Phi(n) > \zeta \quad (8)$$

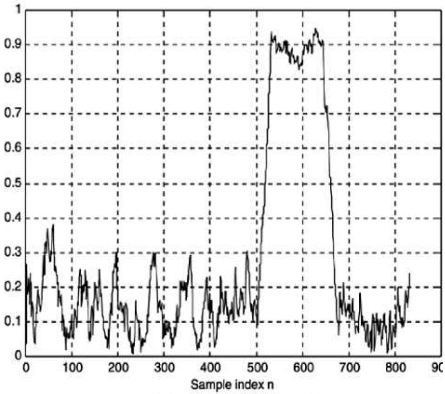
Where ζ is the threshold in this case; and N is the period of the short training sequence, in this case 16 samples (0.8 μs).

Packet Detection - Performance Comparison

The decision statistic m_n for IEEE802.11a preamble in 10dB SNR

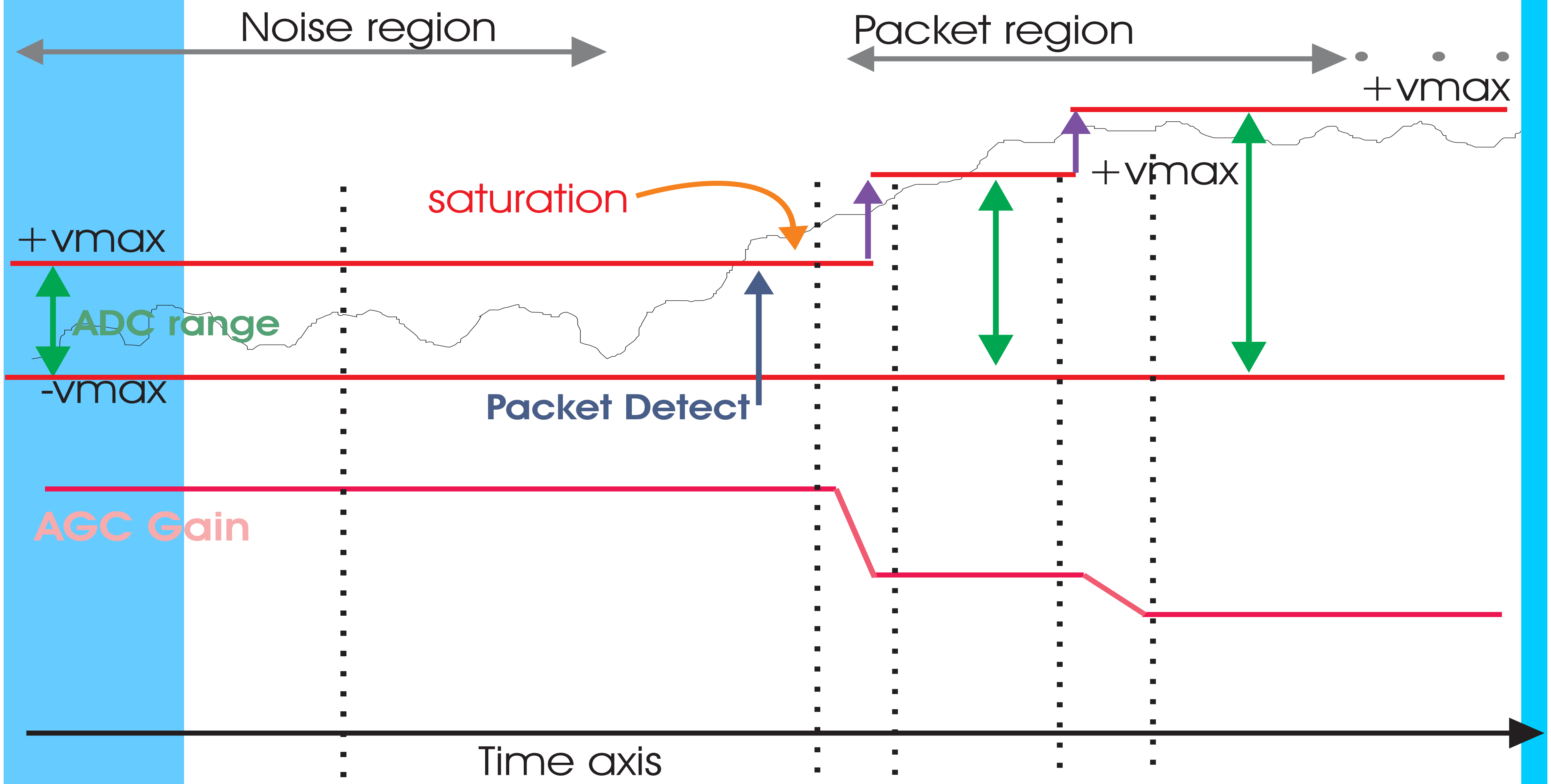


Double Sliding Window



Delay and Correlate

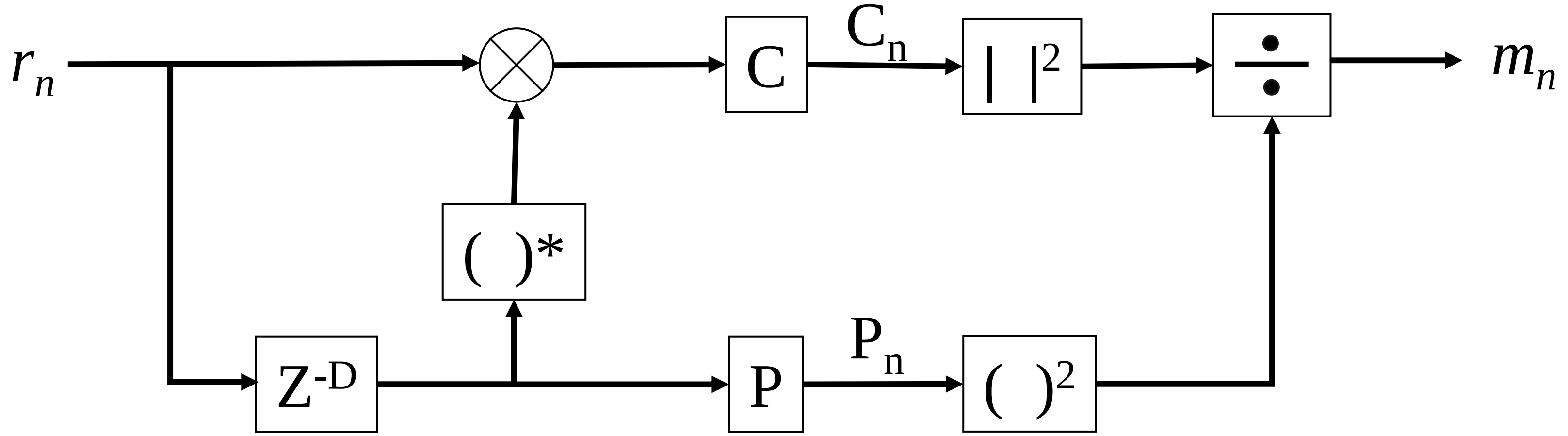
Implementing a simple packet detection algorithm



AGC Control during packet detection

timing synchronization (I)

- Using correlator:



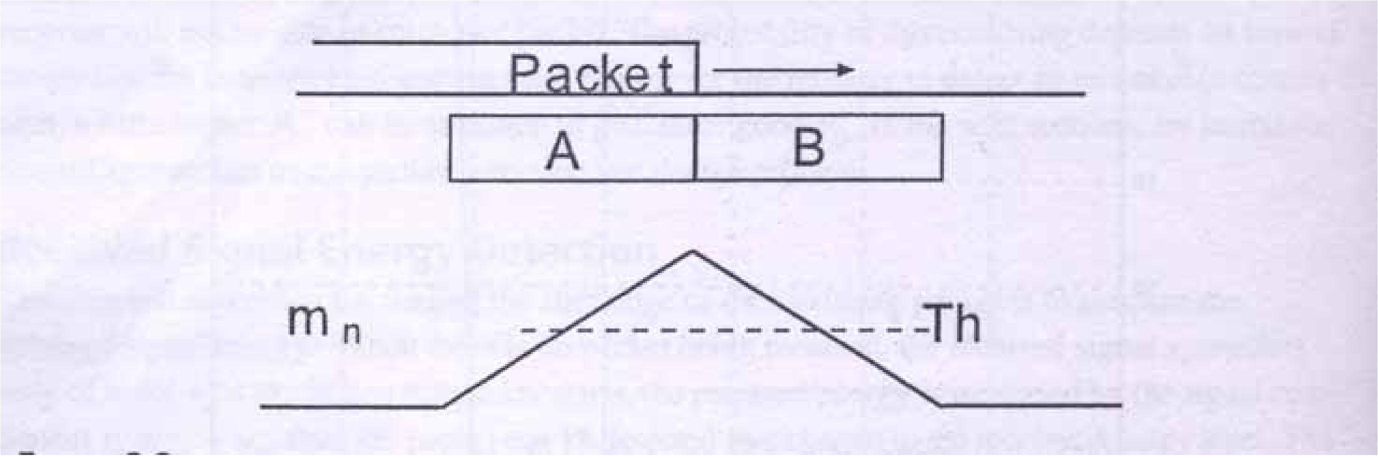
- From the delay correlate structure, the decision is calculate as

$$c_n = \sum_{k=0}^{L-1} r_{n+k} r_{n+k+D}^* \qquad p_n = \sum_{k=0}^{L-1} r_{n+k+D} r_{n+k+D}^* = \sum_{k=0}^{L-1} \left| r_{n+k+D} \right|^2$$

$$m_n = \frac{\left| c_n \right|^2}{\left(p_n \right)^2} \qquad \text{where } D = 16, L = 16$$

timing synchronization (II): Double sliding window

- The double sliding window packet detection algorithm calculates two consecutive sliding windows of the received energy
- The basic principle is to form the decision variable m_n as a ratio of total energy contained inside the two windows as follows



The effect of CFO

- In OFDM system, if there is any mismatch between the frequency and phase of Tx and Rx, it will result CFO
- There are two destructive effects caused by CFO
  - One is the reduction of signal amplitude
  - It will result in ICI which is caused by the loss of the subcarriers orthogonality
- The FFT output for each subcarrier will contain interference term from other subcarrier

WiTS Lab Inst. of Comm. Engr.

Math analysis of ICI

- An OFDM transmission symbol is given by the N point complex modulation sequence

$$x_n = \frac{1}{N} \sum_{k=-k}^{k} X_k e^{\frac{j2\pi nk}{N}}$$

- After passing through channel, the received sequence can be expressed as

$$y_n = \frac{1}{N} \left[\sum_{k=-k}^{k} X_k H_k e^{\frac{j2\pi n(k+\varepsilon)}{N}} \right] + w_n$$

- The output of the FFT for k th subcarrier consisting of three components

$$Y_k = \sum_{n=0}^{N-1} y_n e^{\frac{-j2\pi kn}{N}} = S_k + I_k + W_k$$

Cont'd

$$I_k = \sum_{\substack{l=-k \\ l\neq k}}^{k} \left(X_l H_l\right) \left\{ \frac{\sin \pi\varepsilon}{N \sin\left(\frac{\pi(l-k+\varepsilon)}{N}\right)} \right\} e^{\frac{j\pi\varepsilon(N-1)}{N}} e^{\frac{-j\pi(l-k)}{N}}$$

- Then ,the variance of interference signal

$$E\left(\left|I_k\right|^2\right) \leq 0.5947 \left|X\right|^2 \left|H\right|^2 \left(\sin \pi\varepsilon\right)^2$$

- Generally, the interference power is proportional to the frequency offset

SNR degradation

- The degradation D is given by

$$D \approx \frac{10}{\ln 10}\frac{1}{3}\left(\pi N \frac{\Delta f}{R}\right)^2 \frac{E_s}{N_0} \quad \text{OFDM}$$

$$D \approx \frac{10}{\ln 10}\frac{1}{3}\left(\pi \frac{\Delta f}{R}\right)^2 \quad \text{Single carrier}$$

where $R = N/T$ for OFDM , $R = 1/T$ for singel carrier

CFO estimation

- Using the correlator that takes maximum likelihood estimation (MLE) to estimate the CFO
- The received signal is

$$r_n = s_n e^{j2\pi f_{tx} n T_s} e^{-j2\pi f_{rx} n T_s}$$
$$= s_n e^{j2\pi (f_{tx} - f_{rx}) n T_s}$$
$$= s_n e^{j2\pi f_\Delta n T_s}$$

- The correlator output is

$$z = \sum_{k=0}^{L} r_k r_{k+D}^*$$
$$= e^{-j2\pi f_\Delta D T_s} \sum_{k=0}^{L} \left| s_n \right|^2$$

Cont'd

- Finally, the frequency error estimator is formed as

$$\hat{f}_{\Delta} = -\frac{1}{2\pi D T_s} \arg(z)$$

- The algorithm is simple and can use the same hardware of the delay and correlate algorithm

Algorithm of CFO

- The CFO algorithm is based on packet detection algorithm when packet is detected over the threshold
- The algorithm is described as

$$M(n) = \frac{C(n)}{P(n)} = \frac{\sum_{k=0}^{L-1} r_{n+k} r_{n+k+D}^{*}}{\sum_{k=0}^{L-1} \left| r_{n+k+D} \right|^2}$$

- Then, the coarse CFO is

$$\Delta \hat{f}_{coarse} = \frac{1}{2\pi D T_s} \arg\left(C(n)\right)\Big|_{M(n)>TH}$$

Algorithm of the fine CFO

- During short preamble, we get the coarse CFO , in this algorithm the correlator can be used again
- The algorithm is described as

$$r'_{long}(k) = r_{long}(k)\exp\left(-j2\pi k\Delta \hat{f}_{coarse}\right)$$

$$= r_{long}(k)\exp\left(-jk\cdot\arg\left(C(m)\right)/DT_s\right)$$

- The fine estimation of CFO is

$$\Delta \hat{f}_{fine} = \frac{1}{2\pi N_L T_s}\arg\left(\sum_{l=N_L}^{2N_L-1} r'_l\left(r'_{l-N_L}\right)^*\right) \qquad N_L = 64$$

WiTS Lab Inst. of Comm. Engr.

Cont'd

- After finishing the acquisition of CFO, both coarse and fine estimation is available

- Therefore, the received signal is described as

$$\hat{r}_k = r_k \exp\left(-j2\pi\left(\Delta \hat{f}_{coarse} + \Delta \hat{f}_{fine}\right)k\right)$$

Frame or Symbol Synchronization

Goal:

- To align the symbol window to reduce Inter symbol interference. i.e. To identify and locate the FFT window.

Why do we need it

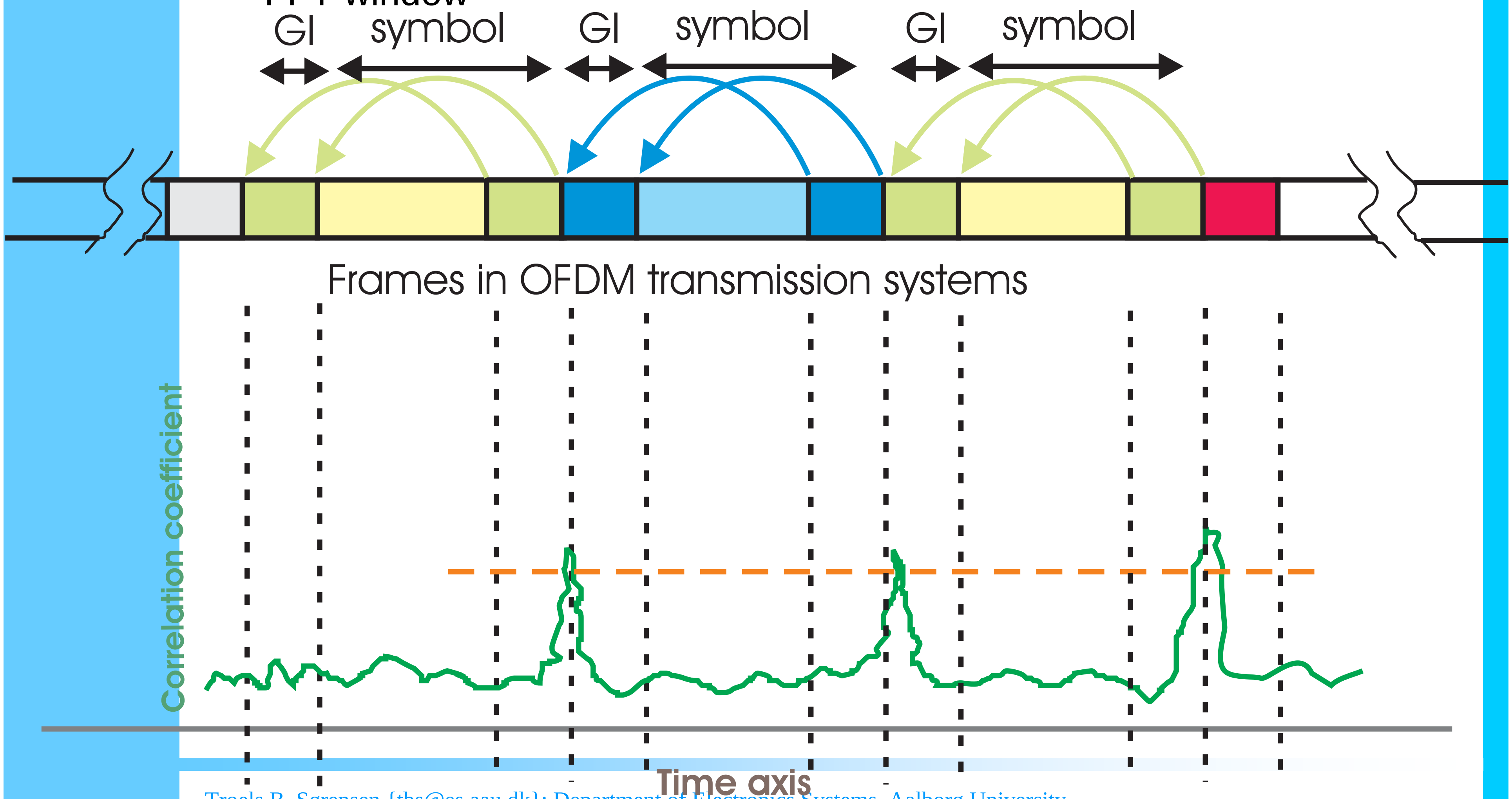
- Packet detection gives the approximate start of the frame, we need to find the exact start of the FFT window
- Otherwise, there will be ISI and irreducible error floor.

General method is using cross correlation in time domain; Can be done based on

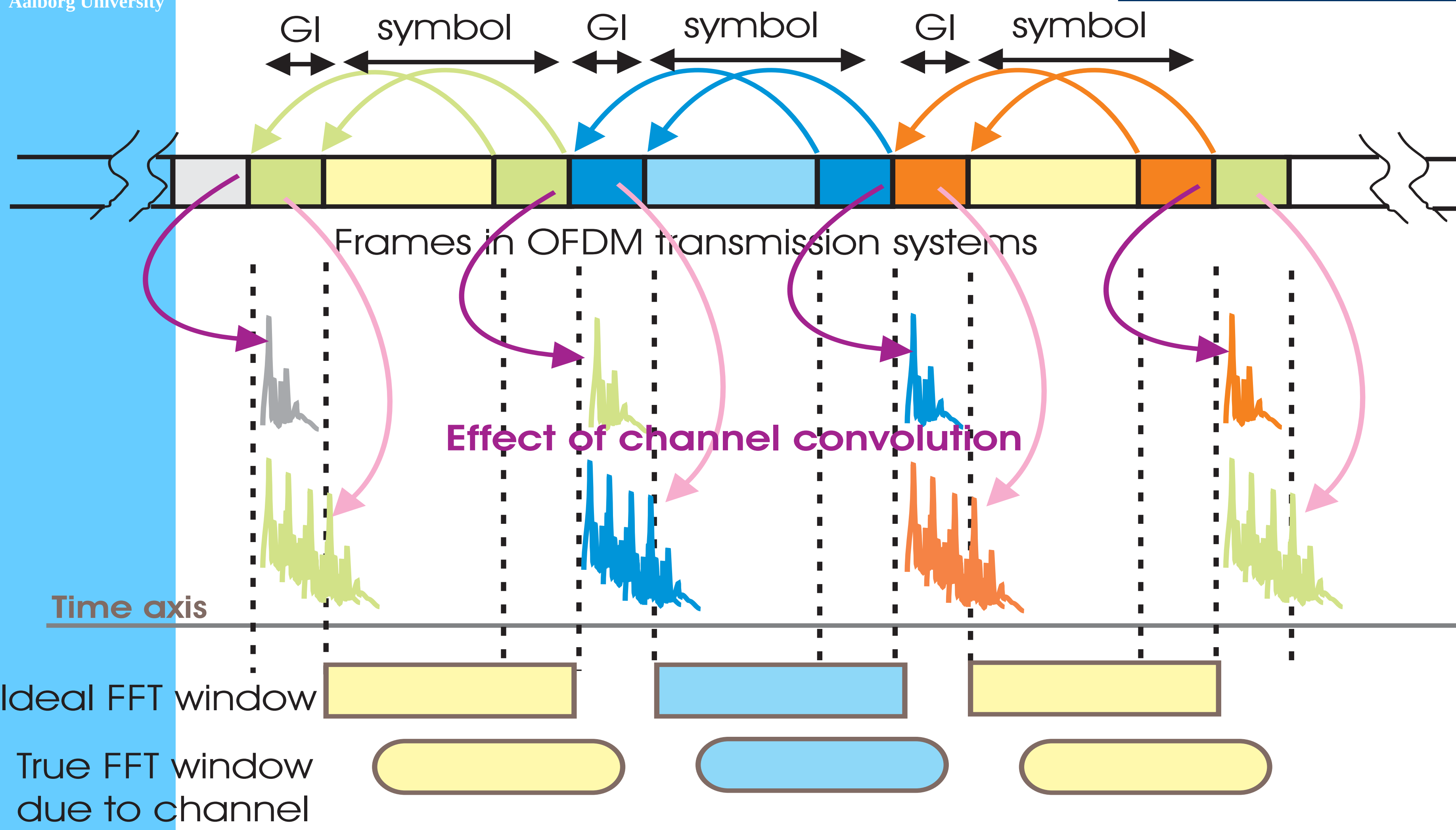
- Training sequence
- Cyclic prefix

Cyclic Prefix based Frame Synchronization

- Cross Correlate two portions of the received signal with a separation of “N” samples, where “N” is the number of samples in a FFT.
- Find the peaks of the correlation coefficients
- The location of the peaks gives the starting point of the frame or the FFT window



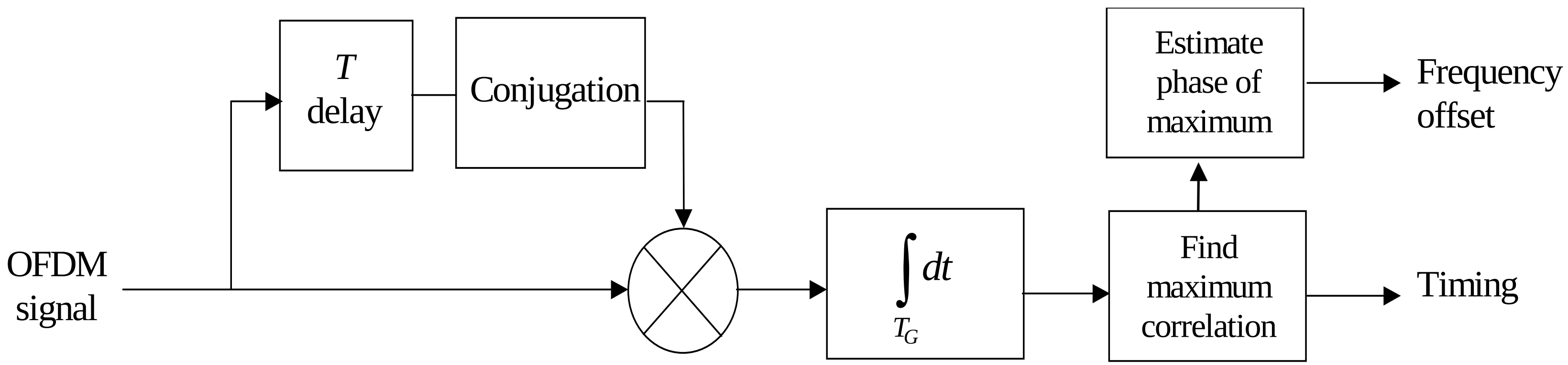
Frame or symbol timing



Need for Frame synchronization

Aalborg University

Synchronization Using Cyclic Prefix



Perform autocorrelation over guard interval to find both timing and frequency offset

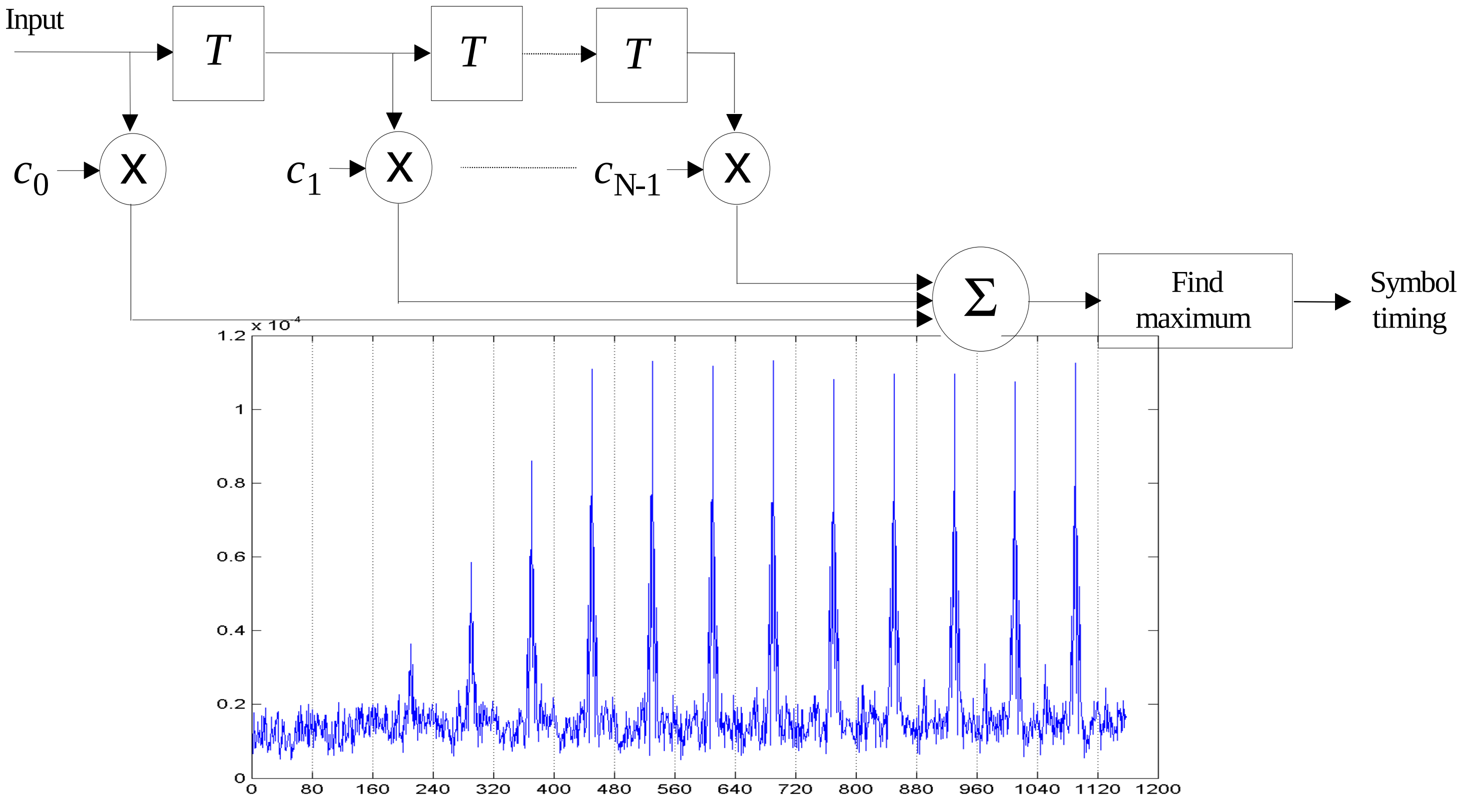
Average over several OFDM symbols to reduce undesired correlation sidelobes of random data

Cyclic Prefix based Frame Synchronization

- Cyclic prefix based synchronization is prone to errors because of channel convolution in the Guard Interval (Cyclic prefix region)
- Algorithm can be improved following similar steps as frame synchronization using training sequence
- Implementation can be optimized ……….needs detailed analysis of the system and the algorithm

- We generally use a two stage algorithm
  - Acquisition using Training sequence
  - Tracking using cyclic prefix

Synchronization with Special Training Symbols

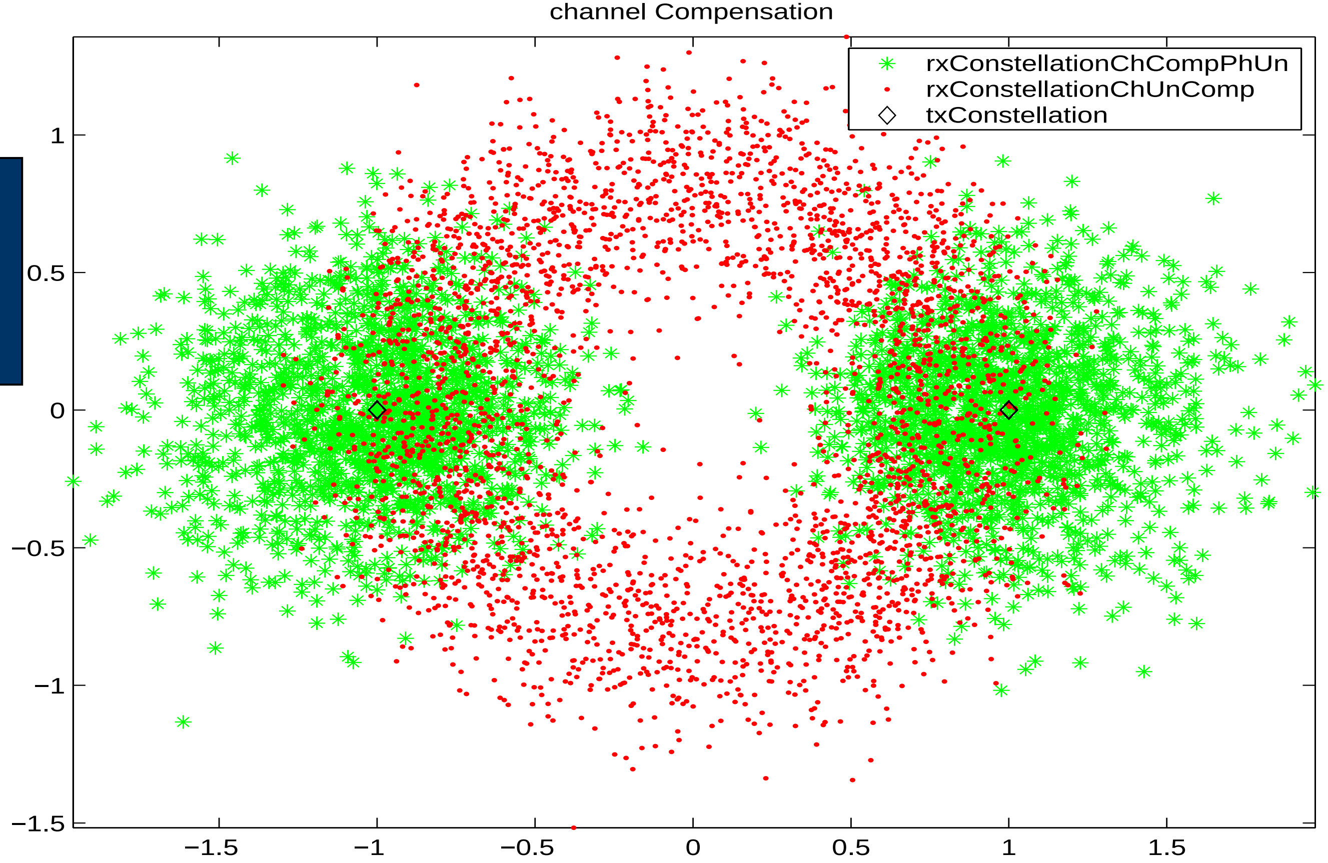


- ❑ Use matched filter matched to special training symbol
- ❑ Choose training symbol such that
  - ❑ Peak-to-average power ratio is minimal
  - ❑ Multipliers can be as simple as possible

Effects of Frame Synchronization Errors

- Constellation rotation
  - Correctable by channel equalization
  - ISI error floor
- Performance measure of algorithm in terms of SNR loss

constellation rotation due to frame synch error



Troels B. Sørensen {tbs@es.aau.dk}; Department of Electronics Systems, Aalborg University

Aalborg University

Carrier Frequency Synchronization (1)

- Carrier Frequency offset estimation using training sequence;

- Two step procedure
  - Estimation
  - Compensation

- Two stage Estimation in time domain
  - Coarse frequency acquisition using **short training sequence**
    - Short Training sequences are obtained by using only ¼ th of the number of FFT sub-carriers
    - In case of IEEE 802.11a it is 16 and hence sub-carrier spacing of 4 times that of the normal OFDM symbol
  - Fine frequency synchronization using **long Training Sequence**
    - Long Training sequence has as many sub-carriers as the normal OFDM symbol
    - In case of IEEE 802.11a the spacing is 312.5 kHz

Aalborg University

Carrier Frequency Synchronization (2)

- Compensation
  - Time domain de-rotation of the phase of the incoming samples
    - First coarse correction is done,
      - Coarse offset corrected signal are used for fine frequency correction
    - Combined Coarse + Fine frequency estimate is compensated together

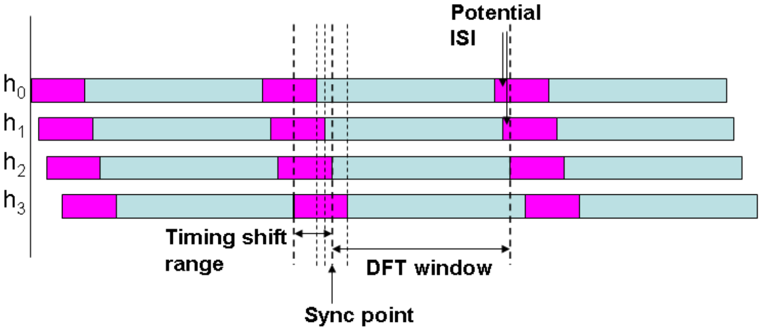
Symbol Timing

Refers to the task of finding the precise moment of when individual OFDM symbols start and end.

- OFDM is relatively robust to timing errors thanks to the guarding interval
- As long as the timing error is smaller than the guarding interval and does not cause multipath signals spread out of the guarding interval, timing error only causes a phase shift which can be absorbed by the channel coefficients in the stage of channel estimation.

Symbol Timing - Timing Shift

In practical systems using the correlator timing algorithm, the sync point is usually obtained by left shifting the estimated timing point by several samples.



Symbol Timing - Auto-correlation based algorithm

When two consecutive identical training symbols are available, the delay and correlator method proposed by Schmidl and Cox can be applied.

$$m_n = \frac{|P(n)|^2}{(R(n))^2} \quad (5)$$

where $P(n) = \sum_{k=0}^{M-1} r^*_{n+k} r_{n+k+M}$ and $R(n) = \sum_{k=0}^{M-1} |r_{n+k+M}|^2$.

- This algorithm can efficiently collect all the multipath energy when a training sequence with constant modulas in frequency domain is chosen.(Proof)
- The output of $P(n)$ can also be used to calculate fractional CFO.
- Increased noise due to autocorrelation

Carrier Frequency Offset (CFO) Estimation

Two types of CFO can be estimated separately

- Fractional CFO estimation
- Integral CFO estimation

Channel effects on the estimation

- General autocorrelation estimator
- Joint MLE estimator

General Fractional CFO Estimator[2, 1]

Time domain data-aided estimator: operating over received time domain training signal consisted of at least two repeated symbols. Down-sampled signal with CFO f_o in the receiver:

$$r(t) = y(t)e^{j2\pi f_o t},$$
$$r_k = r(t)_{t=kT_s} = y_k e^{j2\pi \epsilon k/N}, \qquad (6)$$

where $y(t) = x(t) * h(t)$. If we let

$$z = \sum_{k=0}^{M-1} r_k r_{k+D}^* = \sum_{k=0}^{M-1} y_k y_{k+D}^* e^{-j2\pi\epsilon D/N} \qquad (7)$$

It is easy to arrange training symbols to yield $y_k = y_{k+D}$, and we get

$$\epsilon = \frac{-N}{2\pi D}\angle z. \qquad (8)$$

The idea is also applicable in frequency domain.

General Fractional CFO Estimator (con.)

- Estimation range: $\epsilon < N/(D)$ - Inversely proportional to D
- Question: What's the relationship between the accuracy of estimates and D?

Consider the SINR of the following signal

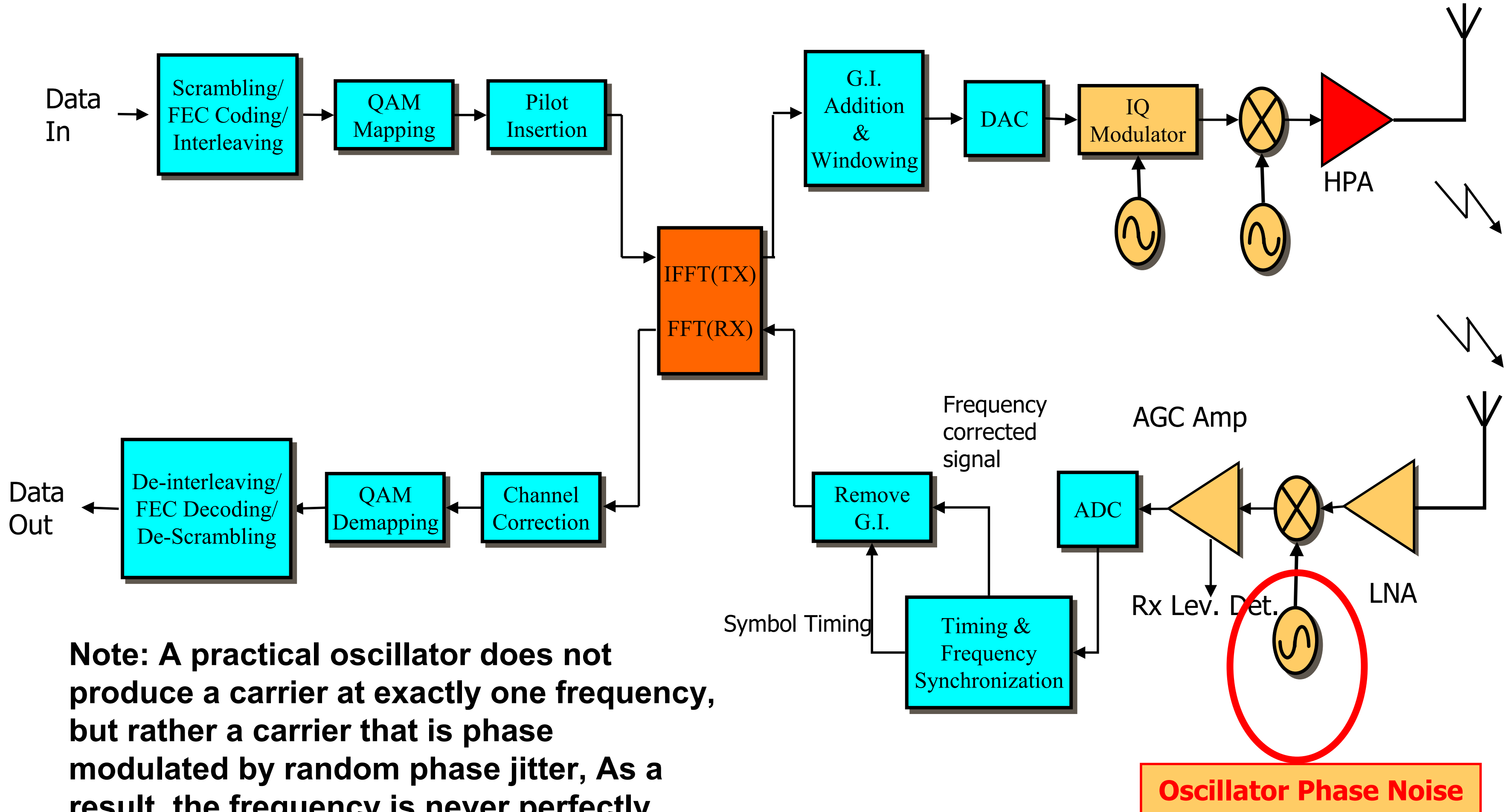
$$z = \sum_{k=0}^{M-1} r_k r_{k+D}^* = e^{-j2\pi\epsilon D/N} \sum_{k=0}^{M-1} |y_k|^2 + \sum_{k=0}^{M-1} y_k e^{j2\pi\epsilon k/N} n_{k+D}^* + \sum_{k=0}^{M-1} y_k^* e^{-j2\pi\epsilon(k+D)/N} n_k + \sum_{k=0}^{M-1} n_k n_{k+D}^*$$

$$\text{SINR} = \gamma_z = \frac{\sum_{k=0}^{M-1} |y_k|^2}{\sigma_n^2 \left(2 + \frac{M\sigma_n^2}{\sum_{k=0}^{M-1} |y_k|^2}\right)}$$

Integer CFO Estimation

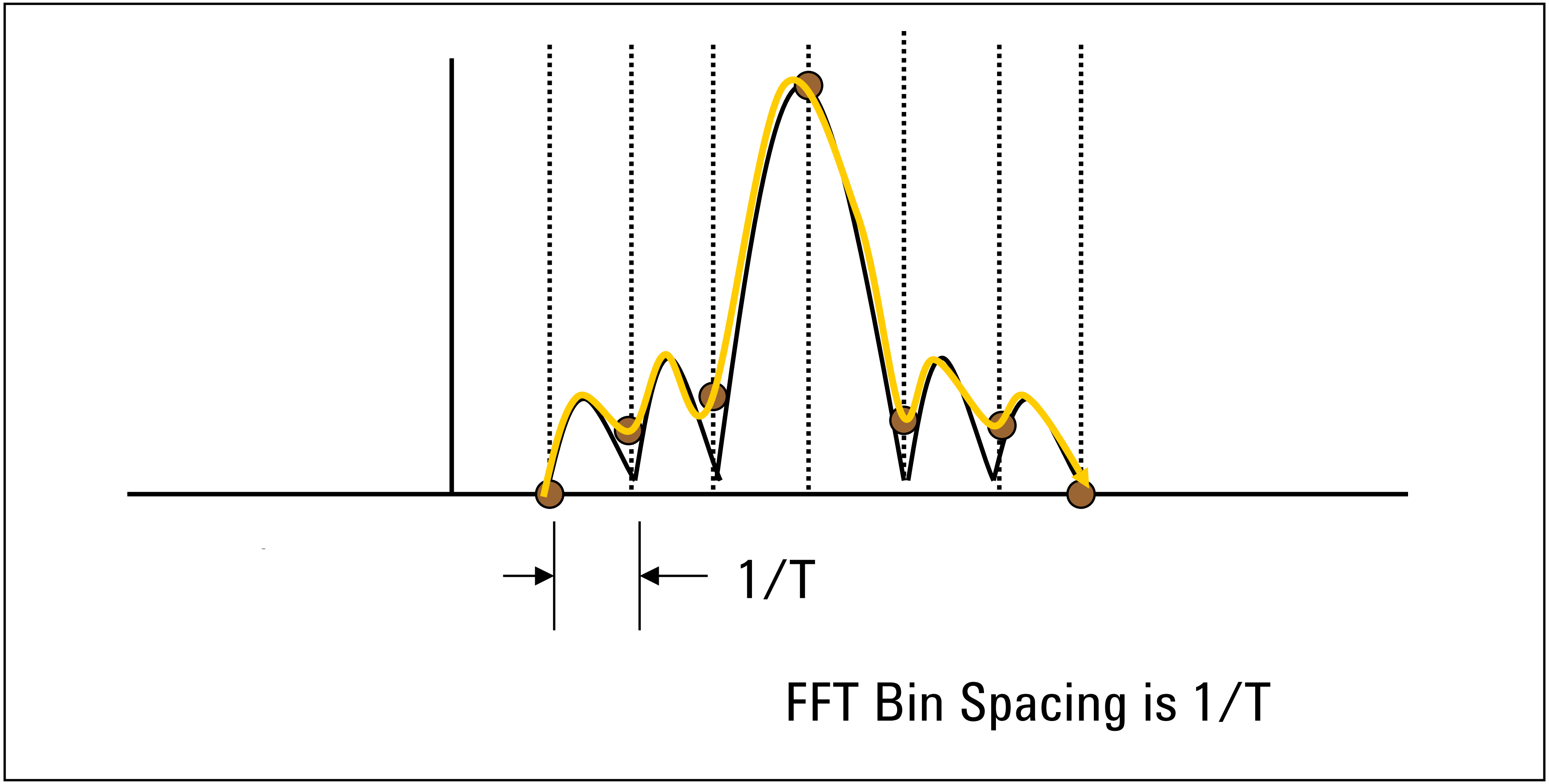
- Integer CFO causes symbol shifting at subcarriers.
- This property can be exploited to estimate the integer CFO according to the autocorrelation of symbols.
- [1] provides such an algorithm by requiring training sequences with good autocorrelation properties.

Effects of Oscillator Phase Noise

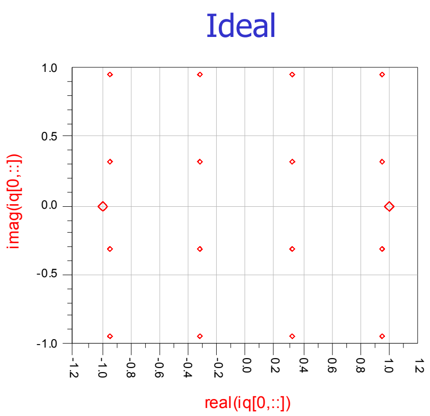


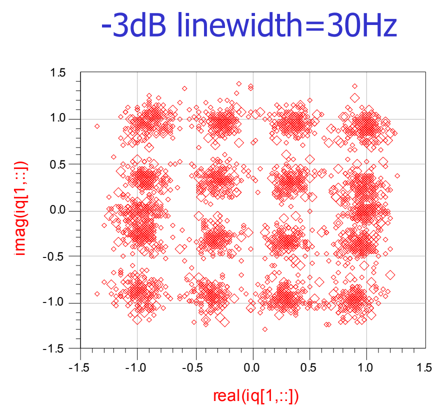
Note: A practical oscillator does not produce a carrier at exactly one frequency, but rather a carrier that is phase modulated by random phase jitter, As a result, the frequency is never perfectly constant, thereby causing ICI.

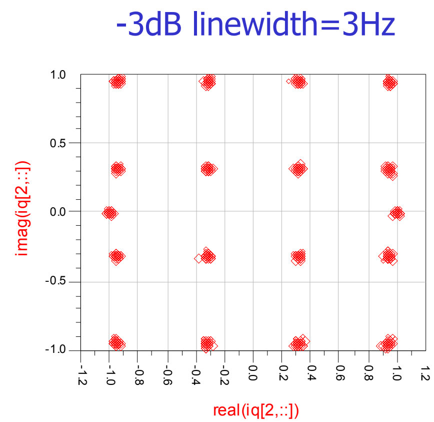
Effects of Oscillator Phase Noise

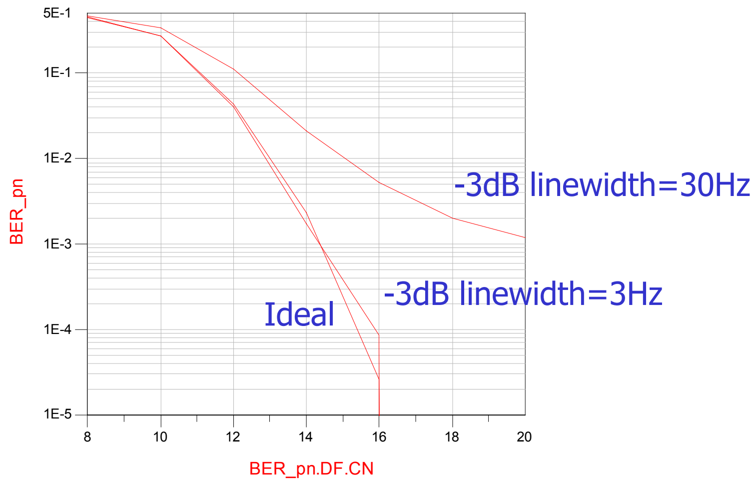


Effects of Oscillator Phase Noise (continued)





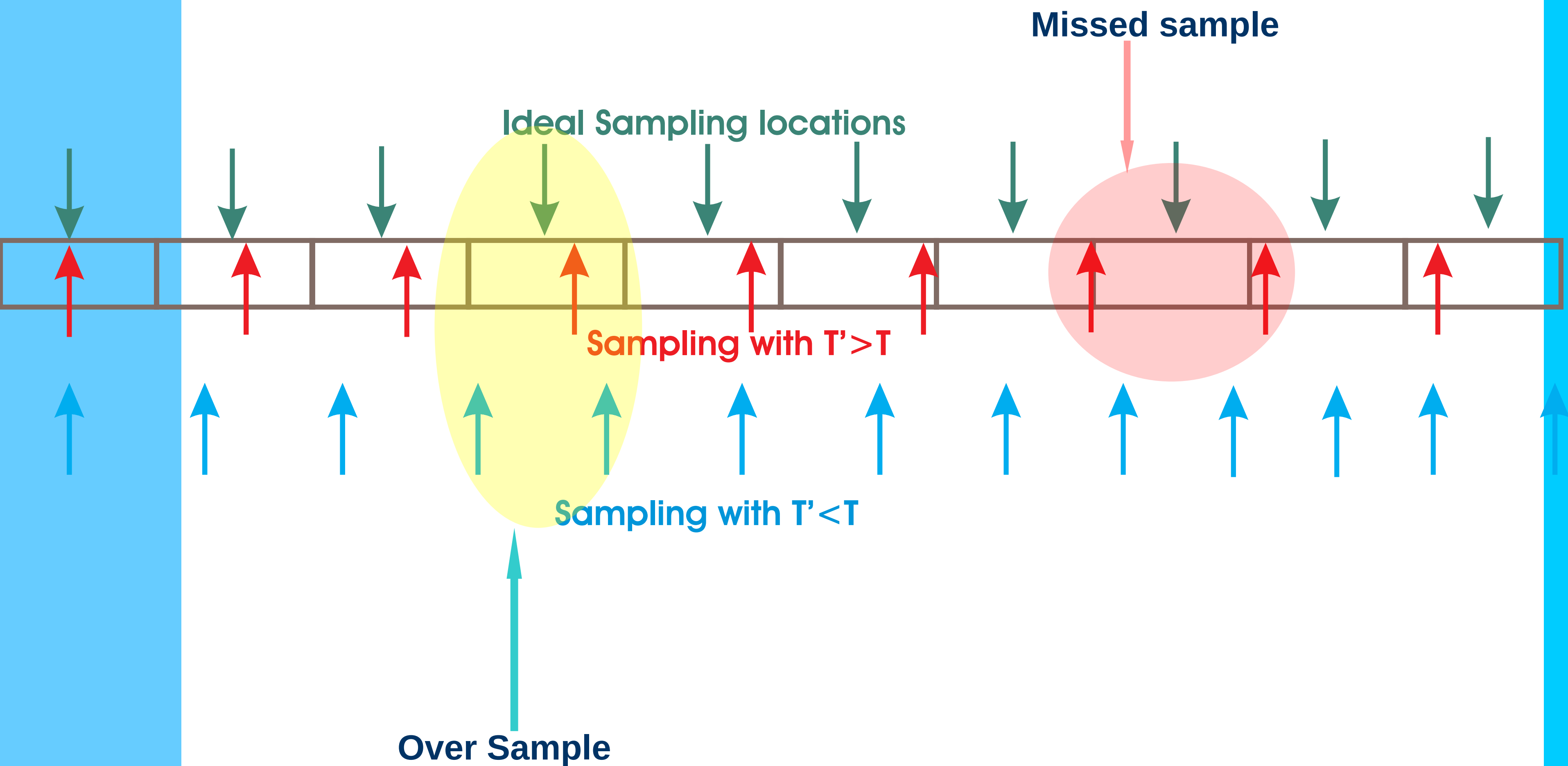




Aalborg University

Sampling Frequency Synchronization

Time Domain view of sampling frequency error



Basic Symbol Timing Algorithm - cross-correlation

- Basic timing algorithms are similar to single carrier systems, e.g., a correlator can be applied with local input identical to the transmitted signal, the output of the correlator is then used as a reference to determine the sync point.

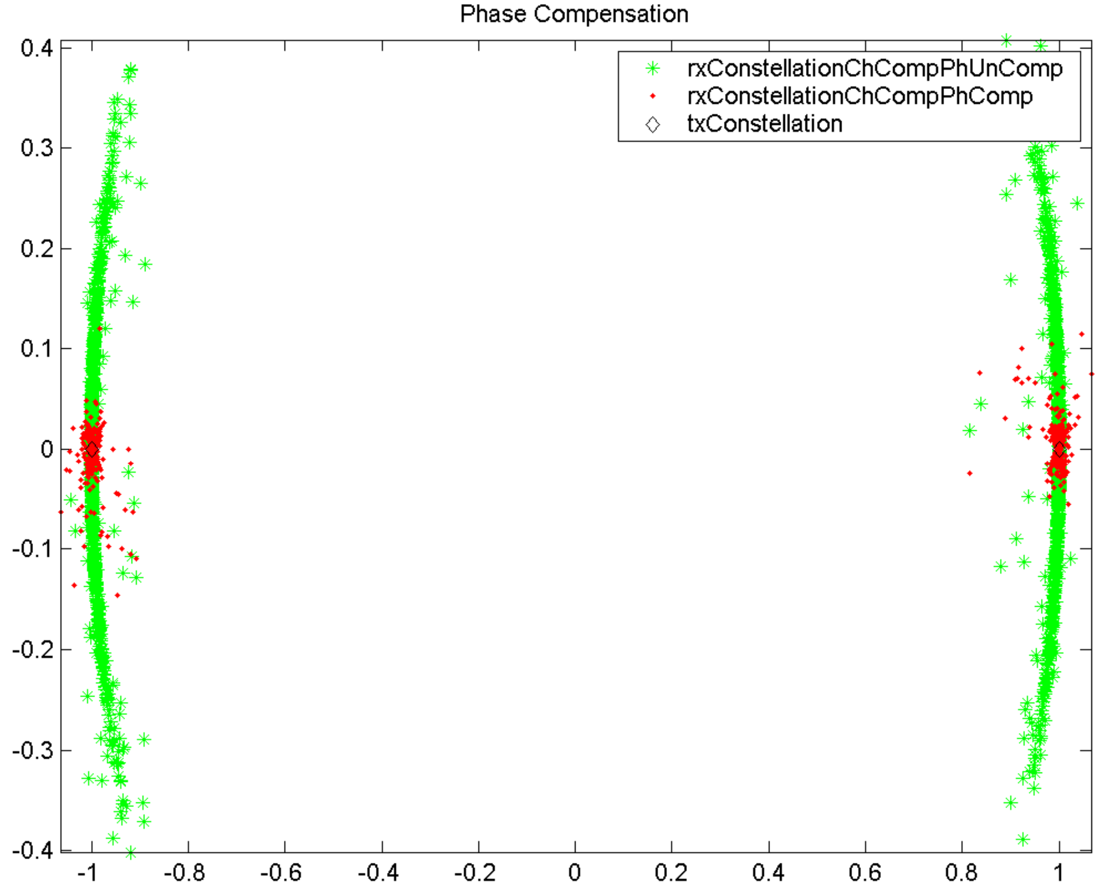
$$y_n = \arg\max_n \left| \sum_{k=0}^{M-1} r_{n+k} s_k \right| \quad (4)$$

where M is the length of the correlating window.

- The correlator-based timing algorithm will pick up the strongest multipath, which is not necessary the first multipath. ISI may be caused in this case.
- The ideal sync point should correspond to the first multipath channel when $T_g \geq T_d$.

Sampling Frequency Offset Compensation

Constellation rotation due to sampling frequency offset



Aalborg University

Conclusion of Synchronization Issues

- We have discussed
  - Synchronization error source
  - Types of synchronization
    - Time
      - Packet detection
      - Frame synchronization
    - Frequency
      - Carrier Frequency synchronization
      - Sampling Frequency synchronization
  - Examined how they effect the system
  - Seen as example some of estimation and compensation Algorithms