EE359
Wireless Communications

Lecture 9

Digital Modulation

Performance over AWGN Channels
Performance over Fading Channels

Capacity of Fading Channels

- Capacity depends on degree of channel knowledge CDI only, CSI at receiver only, CSI at both receiver and transmitter
- Capacity with TX/RX knowledge: variable-rate variable-power transmission (water filling) optimal
  - Flat fading channels water filling in instantaneous SNR
  - Frequency selective fading water filling in frequency
- Channel inversion practical, but should truncate
- Water-filling provides significance improvement at low SNR

Review of Digital Modulation

- Design Issues (Often Conflicting)
  - Data Rate
  - Spectral Efficiency
  - Power Efficiency
  - Performance (Channel Impairments and Noise)
  - Cost

Binary message sequence is divided into words of length K Bits, sent every T seconds

M possible symbols $\{m_1,\ldots m_M\}$ with probabilities $\{p_1,\ldots p_M\}$

$M = 2^K$, $K = \log_2 M$, $R = K/T$ bits per second

Geometric Signal Representation

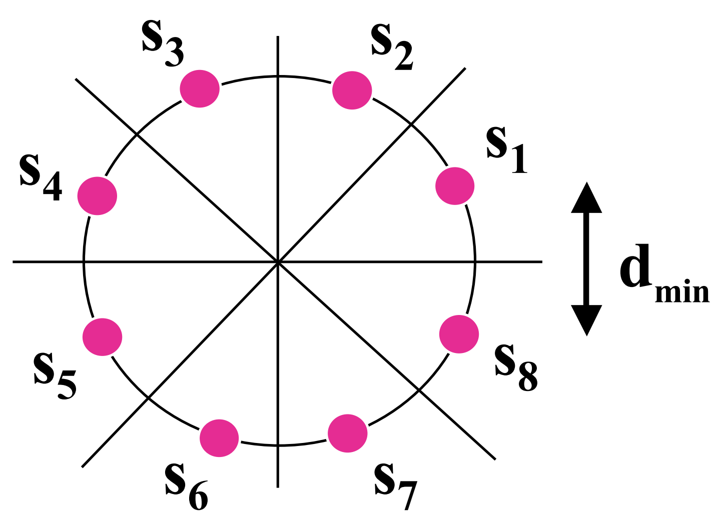
Want to minimize P_e=p(decode $m_j | m_i$ sent)

Vector space analysis

$s_i(t)$ is characterized by vector $\mathbf{s}_i$

$\mathbf{s}_i=(s_{i1}, s_{i2},\ldots,s_{iN})$

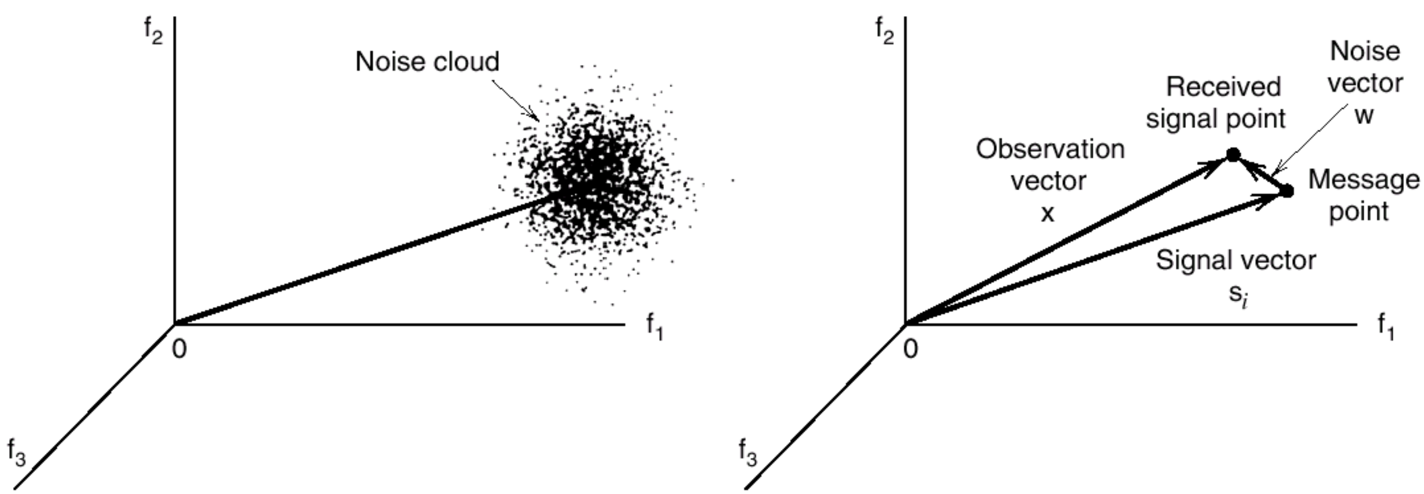
Signal Constellation:



Decision Regions

Optimum receiver (minimum probability of Error) is based on Maximum Likelihood (ML) estimation

ML receiver decodes s_i closest to x (observation vector)



Decision Regions - Error Probability

Assign decision regions:

$$Z_i = \{r : p(s_i \text{ sent} | r) > p(s_j \text{ sent} | r \text{ for all } j \neq i\}$$

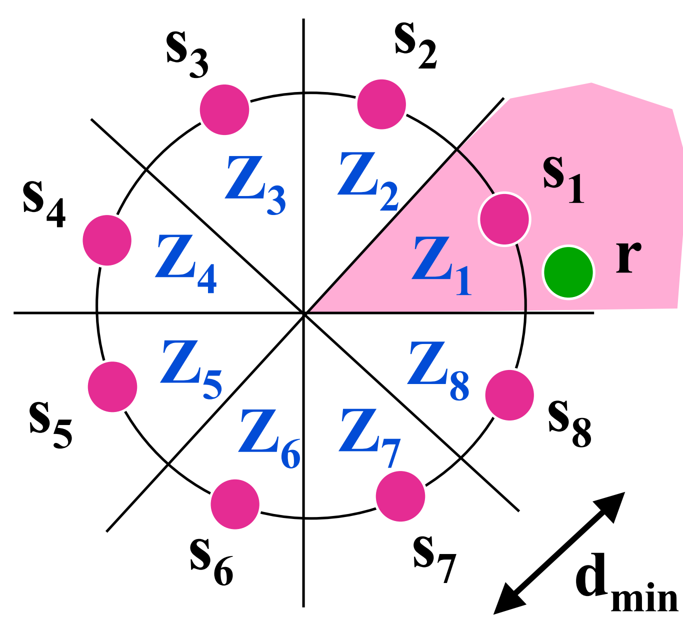
$$= \{r : |x - s_i| < |x - s_j| \text{ all } j \neq i\}$$

$$r \in Z_i \Rightarrow m = m_i$$

P_e is based on noise distribution

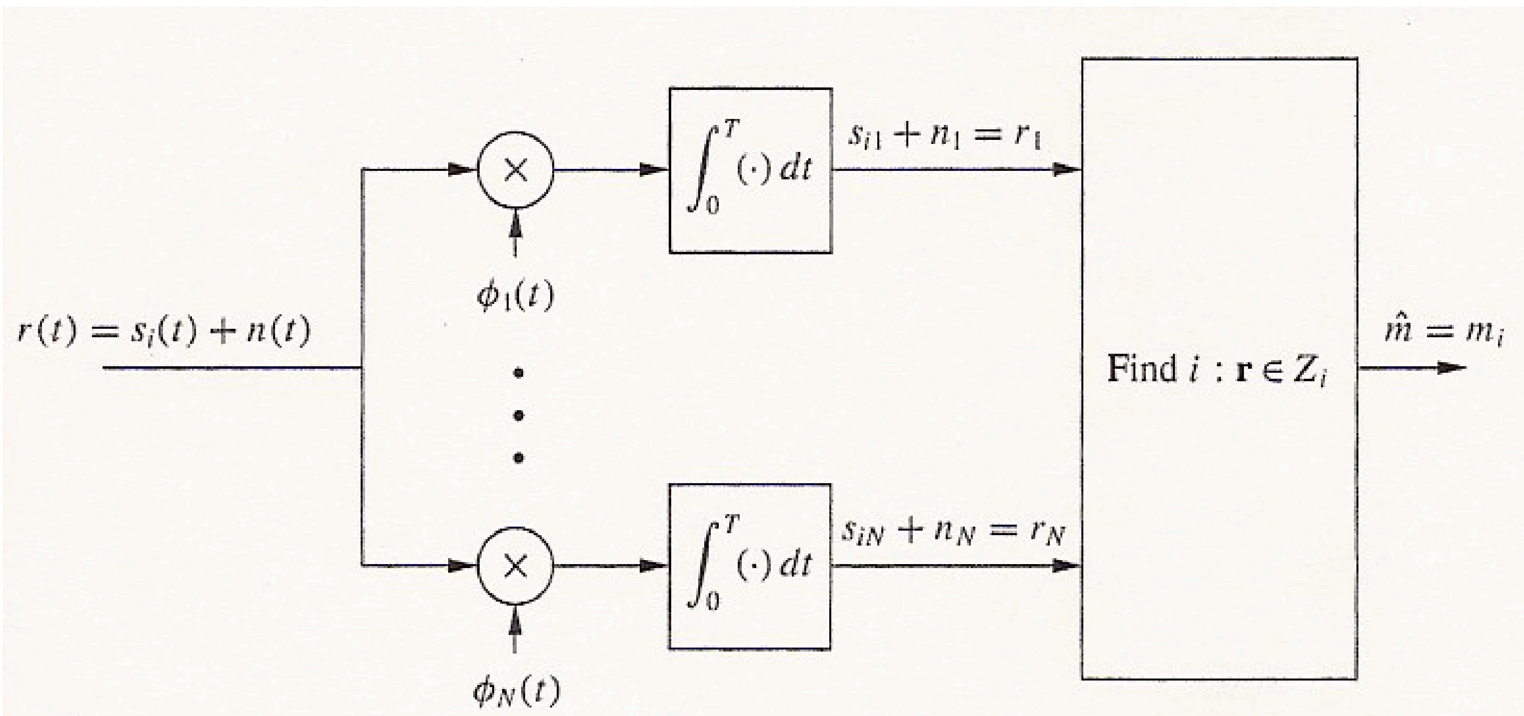
$$P_s \leq (M-1)Q\left(\sqrt{d_{\min}^2/(2N_0)}\right)$$

Signal Constellation



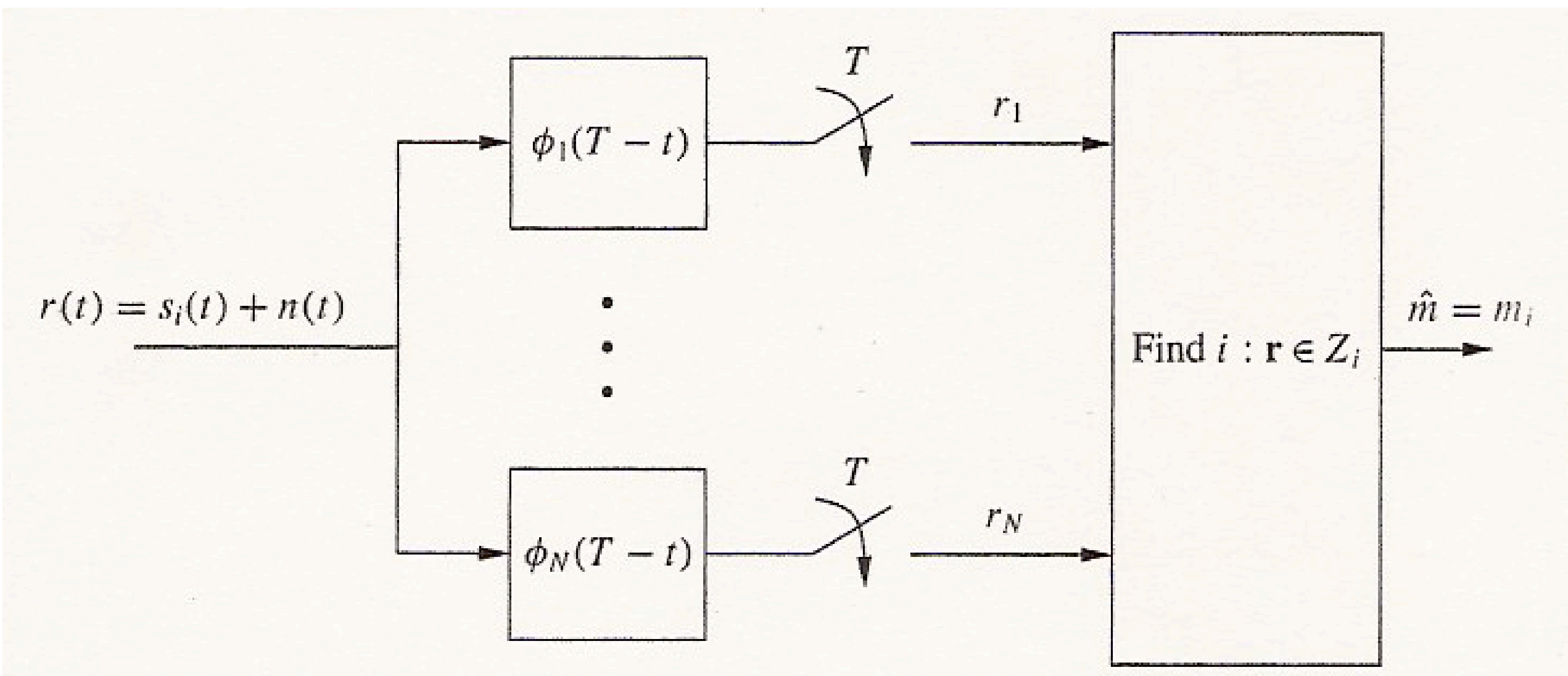
Optimum Coherent Detection Correlation Receiver

Received signal (transmitted signal plus Gaussian noise) is correlated with each basis function. Maximum is selected.



Optimum Coherent Detection
Matched Filter Receiver

Received signal (transmitted signal plus White Gaussian noise) is passed through a bank of filters, each matched to one basis function. Maximum is selected.



Linear Modulation

- Bits encoded in carrier amplitude or phase

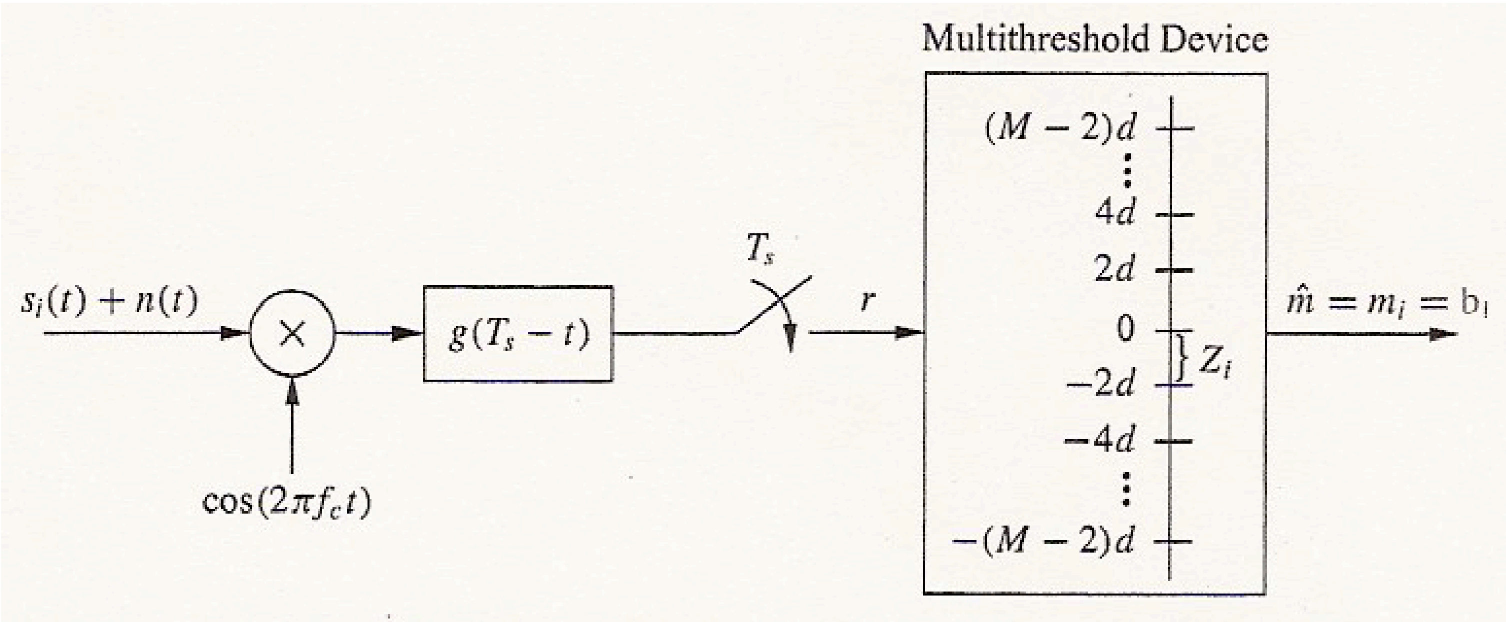
$$s(t) = \sum_n a_n g(t - nT_s)\cos(2\pi f_c t) - \sum_n b_n g(t - nT_s)\sin(2\pi f_c t)$$

  - Pulse shape g(t) typically Nyquist
  - Signal constellation defined by (a_n,b_n) pairs
  - Can be differentially encoded
  - M values for $(a_n,b_n) \Rightarrow \log_2 M$ bits per symbol

- P_s depends on
  - Minimum distance d_{min} *(depends on γ_s)*
  - # of nearest neighbors α_M
  - Approximate expression:

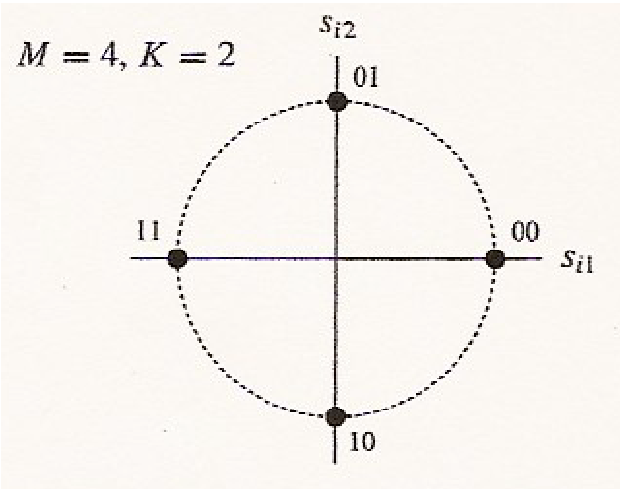
$$P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)$$

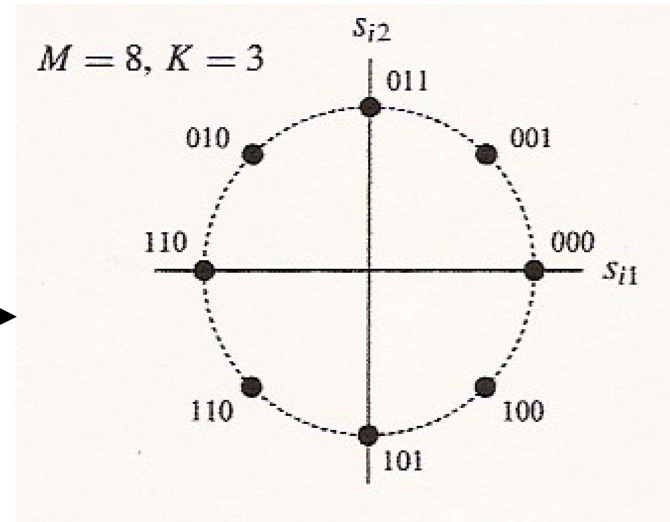
Coherent Demodulation - MPAM



$$z_i = \begin{cases} (-\infty, A_i + d) & i = 1 \\ (A_i - d, A_i + d) & 2 \le i \le M-1 \\ (A_i - d, \infty) & i = M \end{cases}$$

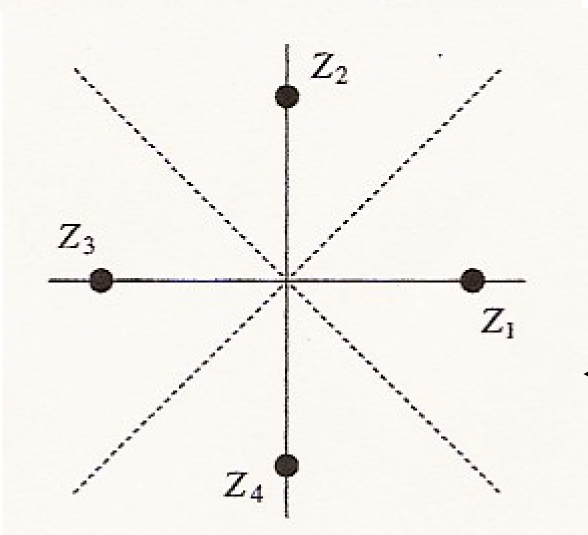
Signal Constellation and Decision Regions - MPSK

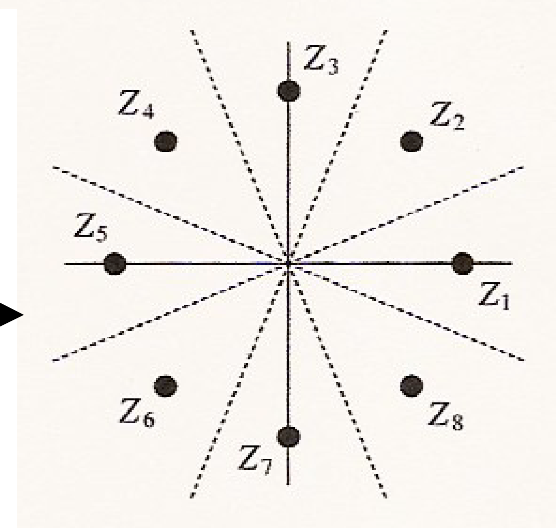




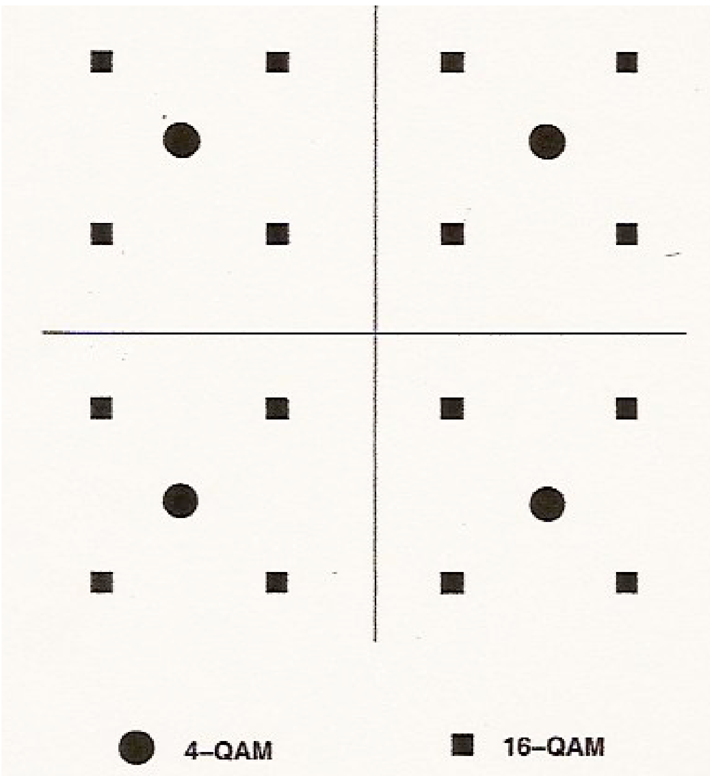
Bit Mapping by Gray Encoding

Decision Regions

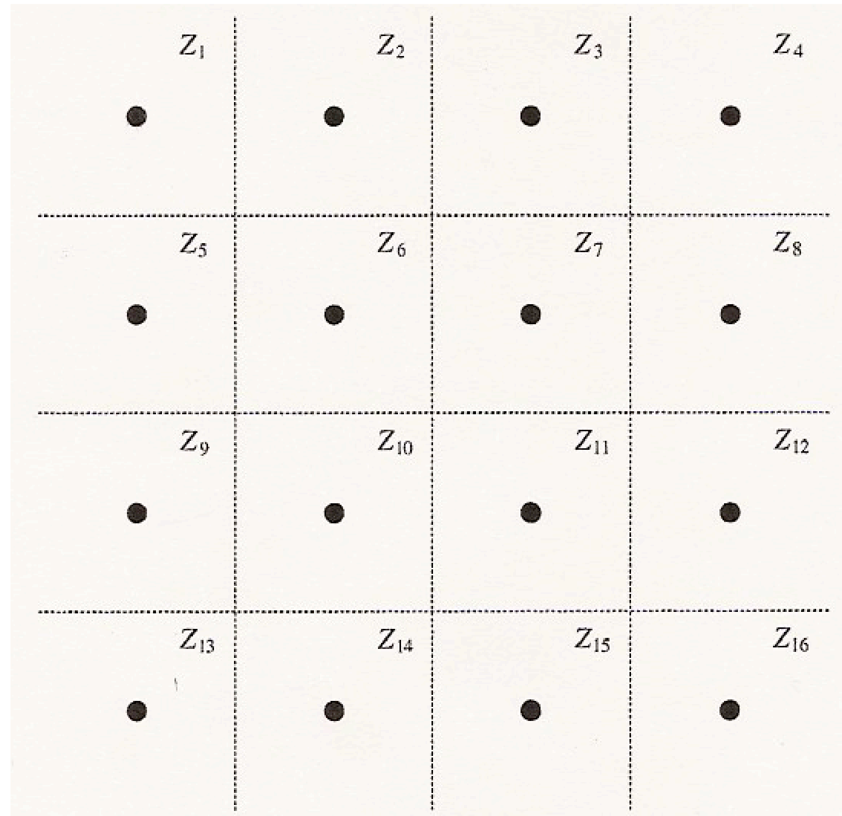




Signal Constellation and Decision Regions - MQAM



Signal Constellation



Decision Regions

Probability of Error Analysis AWGN - Non-Fading - Coherent

T_S = Symbol Time
T_b = Bit Time

E_S = Signal Energy per Symbol
E_b = Signal Energy per Bit

Probability of error is a function of SNR

$$SNR = \frac{P_r}{N_0 B} = \frac{E_S}{N_0 B T_S} = \frac{E_b}{N_0 B T_b}$$

For pulse shaping with T_S = 1/B:

$$\gamma_S = \frac{E_S}{N_0}$$ **For M-ary**

$$\gamma_b = \frac{E_b}{N_0}$$ **For Binary**

Using Gray Coding and assuming errors only between neighboring symbols leads to one bit error for each symbol error

Probability of Error Analysis
AWGN - Non-Fading - Coherent

M-ary versus Binary: $\gamma_b \approx \dfrac{\gamma_S}{\log_2 M}$ $P_b \approx \dfrac{P_S}{\log_2 M}$

Table 6.1: Approximate symbol and bit error probabilities for coherent modulations

Modulation	$P_s(\gamma_s)$	$P_b(\gamma_b)$
BFSK		$P_b = Q(\sqrt{\gamma_b})$
BPSK		$P_b = Q(\sqrt{2\gamma_b})$
QPSK, 4-QAM	$P_s \approx 2Q(\sqrt{\gamma_s})$	$P_b \approx Q(\sqrt{2\gamma_b})$
MPAM	$P_s = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6\bar{\gamma}_s}{M^2-1}}\right)$	$P_b \approx \frac{2(M-1)}{M\log_2 M} Q\left(\sqrt{\frac{6\bar{\gamma}_b \log_2 M}{M^2-1}}\right)$
MPSK	$P_s \approx 2Q\left(\sqrt{2\gamma_s}\sin\left(\frac{\pi}{M}\right)\right)$	$P_b \approx \frac{2}{\log_2 M} Q\left(\sqrt{2\gamma_b \log_2 M}\sin\left(\frac{\pi}{M}\right)\right)$
Rectangular MQAM	$P_s \approx 4Q\left(\sqrt{\frac{3\bar{\gamma}_s}{M-1}}\right)$	$P_b \approx \frac{4}{\log_2 M} Q\left(\sqrt{\frac{3\bar{\gamma}_b \log_2 M}{M-1}}\right)$
Nonrectangular MQAM	$P_s \approx 4Q\left(\sqrt{\frac{3\bar{\gamma}_s}{M-1}}\right)$	$P_b \approx \frac{4}{\log_2 M} Q\left(\sqrt{\frac{3\bar{\gamma}_b \log_2 M}{M-1}}\right)$

Error Probability Approximation Coherent Demodulation

For all coherent demodulation probability of error has a functional form:

$$P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)$$

where: α_m is the number of nearest neighbors in the constellation
β_m is a constant which depends on the specific modulation

$$Q(z) = \int_z^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$

An alternate but simpler Q-Function suitable for AWGN and fading channels is:

$$Q(z) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left[-\frac{z^2}{2\sin^2\phi}\right] d\phi \quad z > 0$$

Main Points

- **Major Linear Modulation Schemes are MPAM, MPSK, MQAM**
- **Linear modulation more spectrally efficient but less robust than nonlinear modulation**
- **Decision regions are based on Maximum Likelihood**
  - **Optimum coherent detection structure is based on Correlation Receivers and Matched Filter Receivers**
  - P_e **depends on constellation minimum distance**
  - P_e **in AWGN approximated by:** $P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)$
  - **Pulse-shaping improves spectral characteristics**
  - **An alternate Q-function is more suitable for error analysis**

Linear Modulation in Fading

- **For fading channels SNR γ_s and, therefore, probability of error P_s are random**

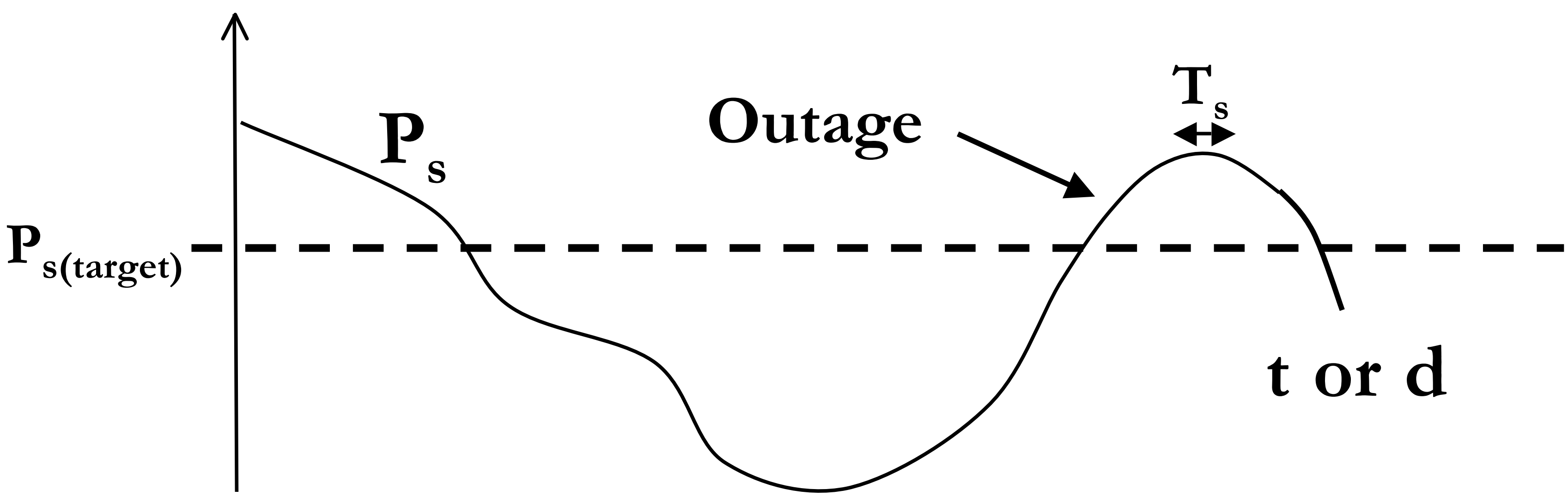
- **Performance metrics:**
  - **Outage probability: pr($P_s > P_{target}$) = pr($\gamma < \gamma_{target}$)**
  - **Average P_s , $\overline{P}_s$:**

$$\overline{P_s} = \int_0^\infty P_s(\gamma)\, p(\gamma)\, d\gamma$$

  - **Combined outage and average P_s**

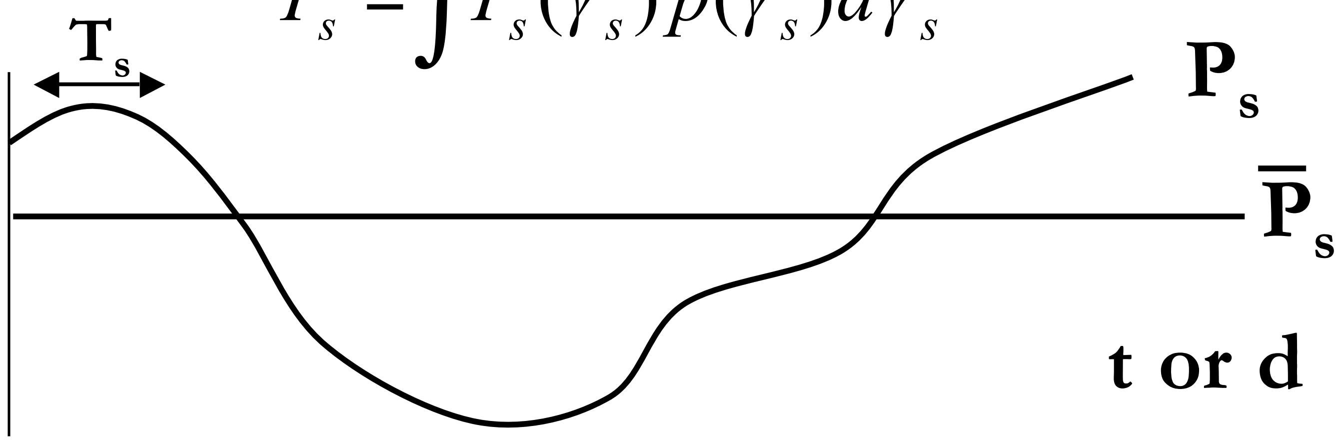
Outage Probability

- Probability that P_s is above target
- Equivalently, probability γ_s below target
- Used when $T_c >> T_s$



Average P_s

$$\overline{P}_s = \int P_s(\gamma_s)p(\gamma_s)d\gamma_s$$



- Expected value of random variable P_s
- Used when $T_c \sim T_s$
- Error probability much higher than in AWGN alone
- Alternate Q function approach greatly simplifies calculations

Average BER in AWGN and Rayleigh

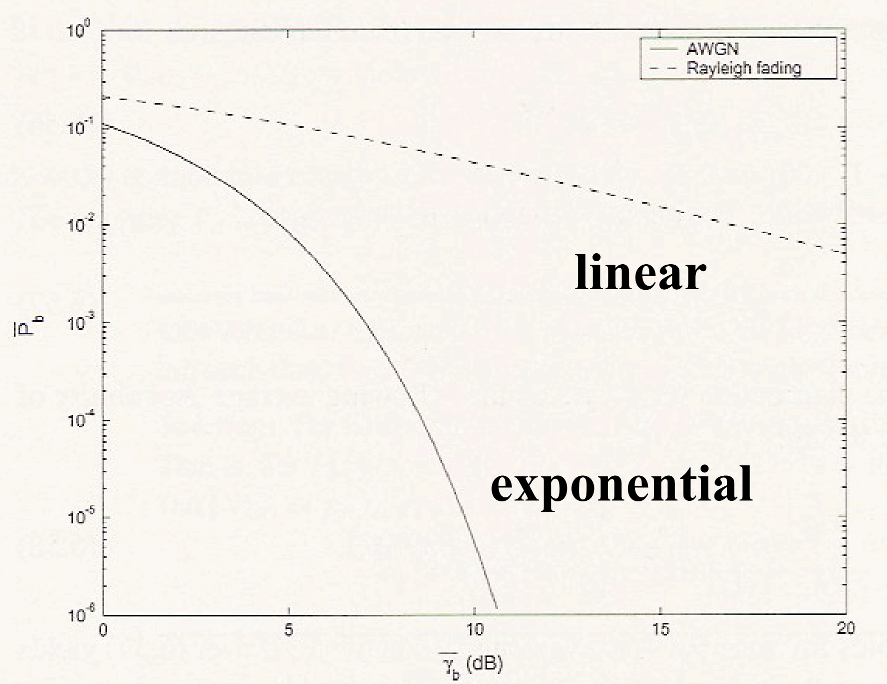


Figure 6.1: Average P_b for BPSK in Rayleigh fading and AWGN.

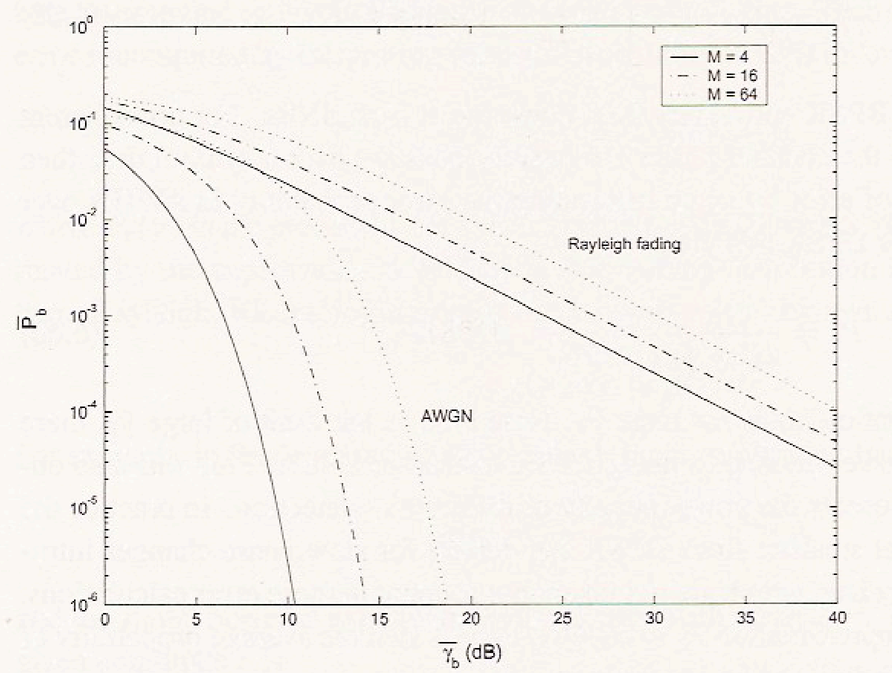
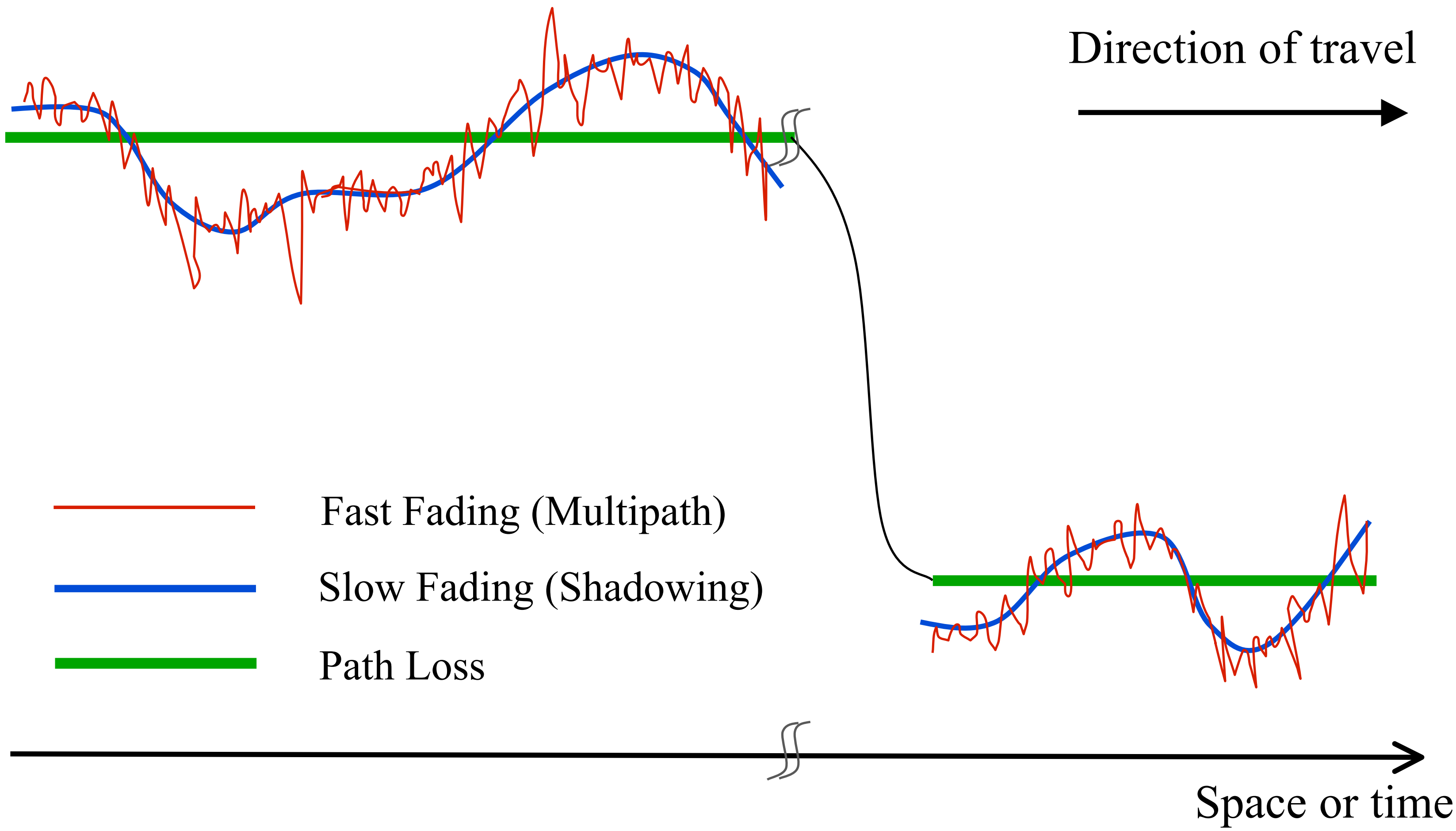


Figure 6.2: Average P_b for MQAM in Rayleigh fading and AWGN.

BER = 10^{-3} SNR = 8dB for AWGN

24dB in Rayleigh

Signal Variations



Combined Outage and Average P_s

γ_s = random SNR for fixed path loss and shadowing, but random fast fading.

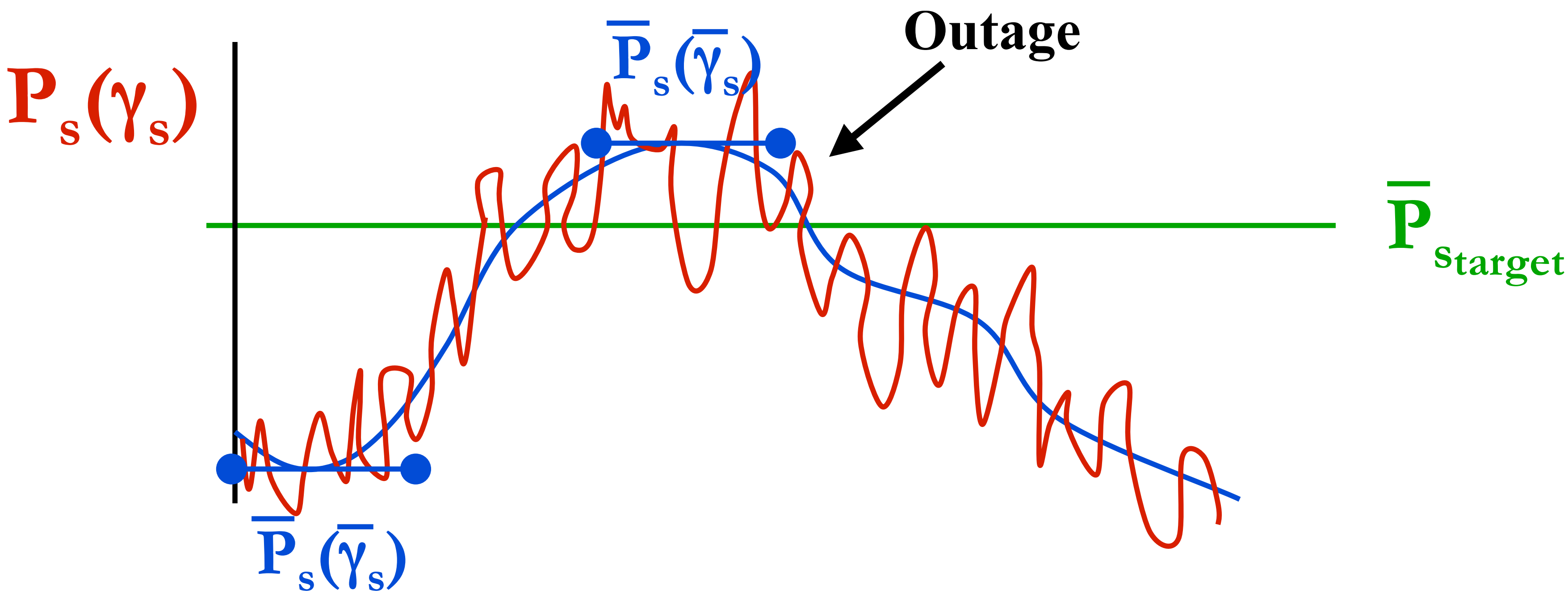
Probability of Error = $P_s(\gamma_s)$

$\overline{\gamma_s}$ = random SNR for fixed path loss and random shadowing, but averaged over fast fading. $\overline{\gamma_s} = E(\gamma_s)$.

Average Probability of Error = $\overline{P}_s(\overline{\gamma_s})$

$\overline{\overline{\gamma_s}}$ = average SNR for a fixed path loss. Averaging over fast fading and shadowing. $\overline{\overline{\gamma_s}} = E(\overline{\gamma_s}) = E\,[E(\gamma_s)]$.

Combined Outage and Average P_s



- Used in combined shadowing and flat-fading
- $\overline{P}_s$ varies slowly, locally determined by flat fading
- Declare outage when $\overline{P}_s$ above target value

Doppler Effects

- High Doppler causes channel phase to decorrelate between symbols
- Leads to an irreducible error floor for differential modulation schemes such as DPSK
  - Increasing power does not reduce error
- Error floor depends on $B_D T_s$ (equivalently, T_s/T_c)

DPSK - Rician Fading Channel

$$\overline{P}_b = \frac{1}{2}\left(\frac{1+K+\overline{\gamma}_b(1-\rho_c)}{1+K+\overline{\gamma}_b}\right)\exp\left[-\frac{K\overline{\gamma}_b}{1+K+\overline{\gamma}_b}\right]$$

ρ_c = channel correlation coefficient after a bit time T_b

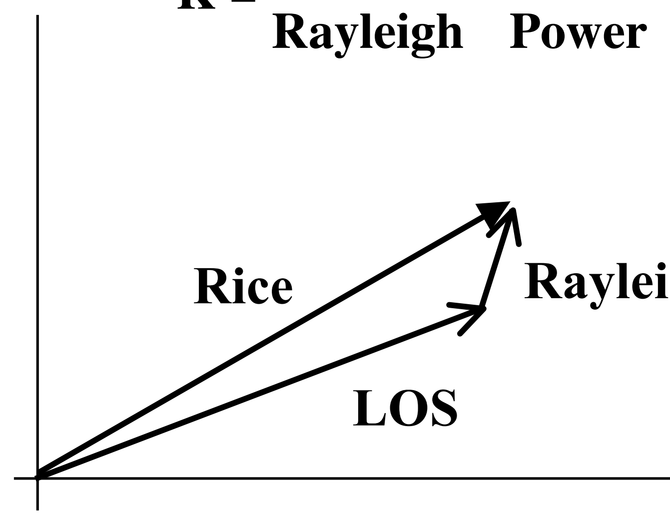
$$\overline{P}_{floor} = \left(\frac{(1-\rho_c)e^{-K}}{2}\right) \quad Letting \quad \overline{\gamma}_b \rightarrow \infty$$

$K = \frac{\text{LOS Power}}{\text{Rayleigh Power}}$

$$\overline{P}_b = \frac{1}{2}\left(\frac{1+\overline{\gamma}_b(1-\rho_c)}{1+\overline{\gamma}_b}\right)$$

For K = 0
(Rayleigh)

$$\overline{P}_b = \frac{1-\rho_c}{2} \quad \rightarrow \frac{1}{2} \quad when \quad \rho \rightarrow 0$$

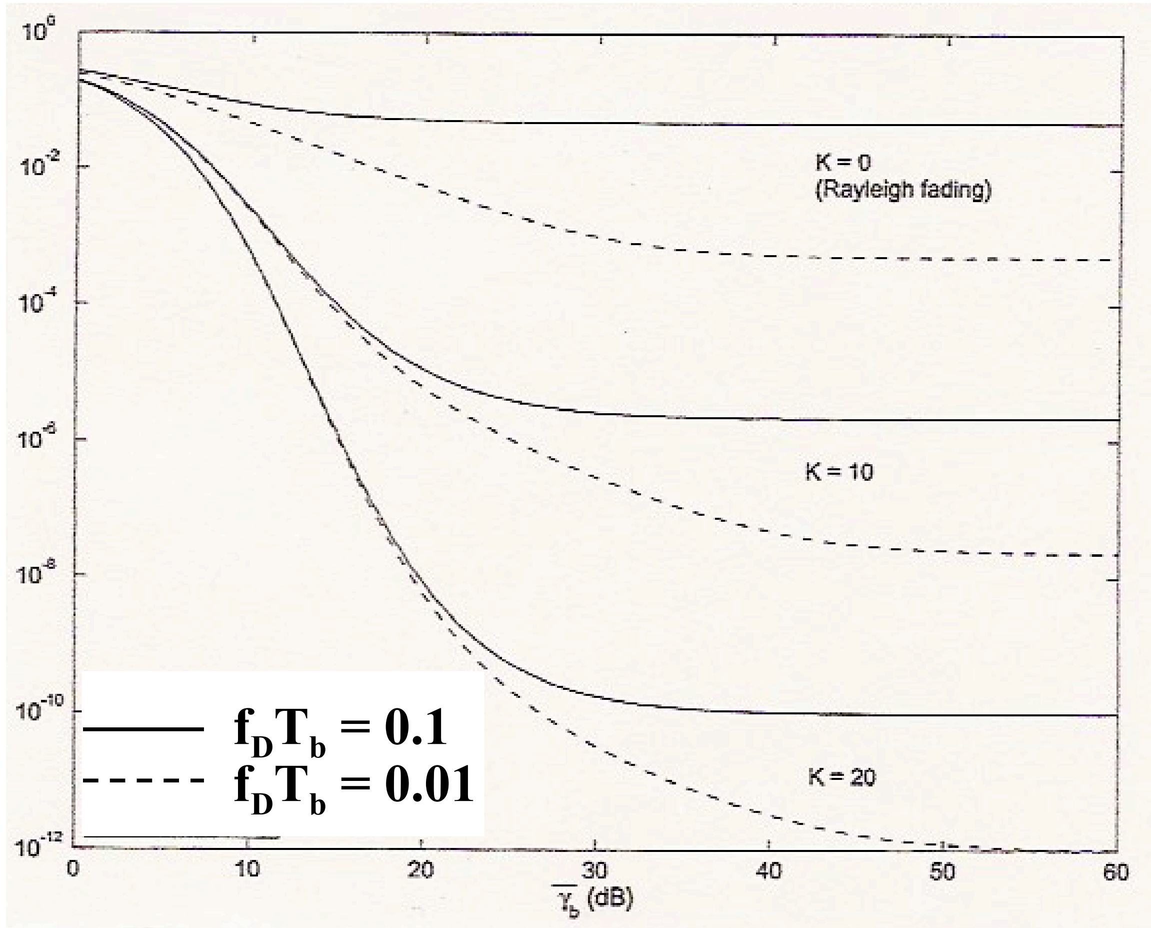


Irreducible BER due to Doppler

$f_D T_b \; \alpha \; T_b / T_c$

For fixed f_D decreasing T_b (increasing R_b) decreases error floor

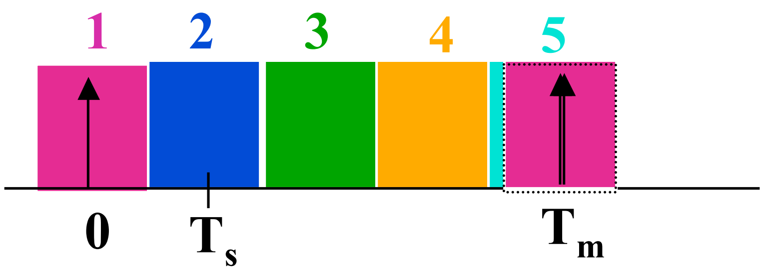
$\overline{P}_b$



DPSK in fast Rician fading $\overline{\gamma}_b$ **(dB)**

ISI Effects

- Delay spread exceeding a symbol time causes ISI (self interference).



- ISI leads to irreducible error floor
  - Increasing signal power increases ISI power
- ISI-free transmission requires that $T_s >> T_m$ ($R_s << B_c$)

ISI Effects

Irreducible error rate is difficult to analyze. It Depends on the specific channel model and choice of linear modulation. Closed form solution is often not possible. Simulation studies available.

$$\hat{\gamma}_s = \frac{P_r}{N_0 B + 1}$$

P_r = Power of the LOS component

I = Power associated with ISI

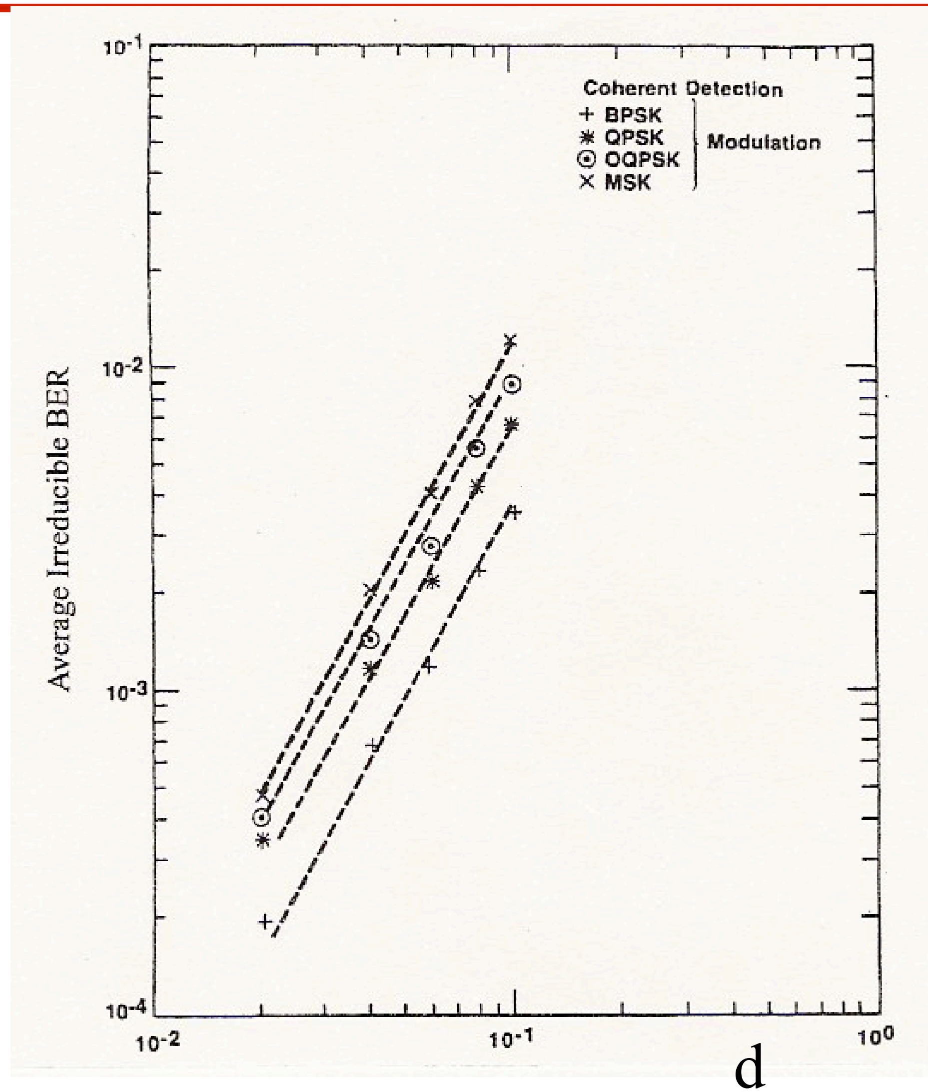
$$\overline{P}_s = \int P_s(\hat{\gamma}_s)\, p(\hat{\gamma}_s)\, d\hat{\gamma}_s$$

Irreducible BER due to ISI

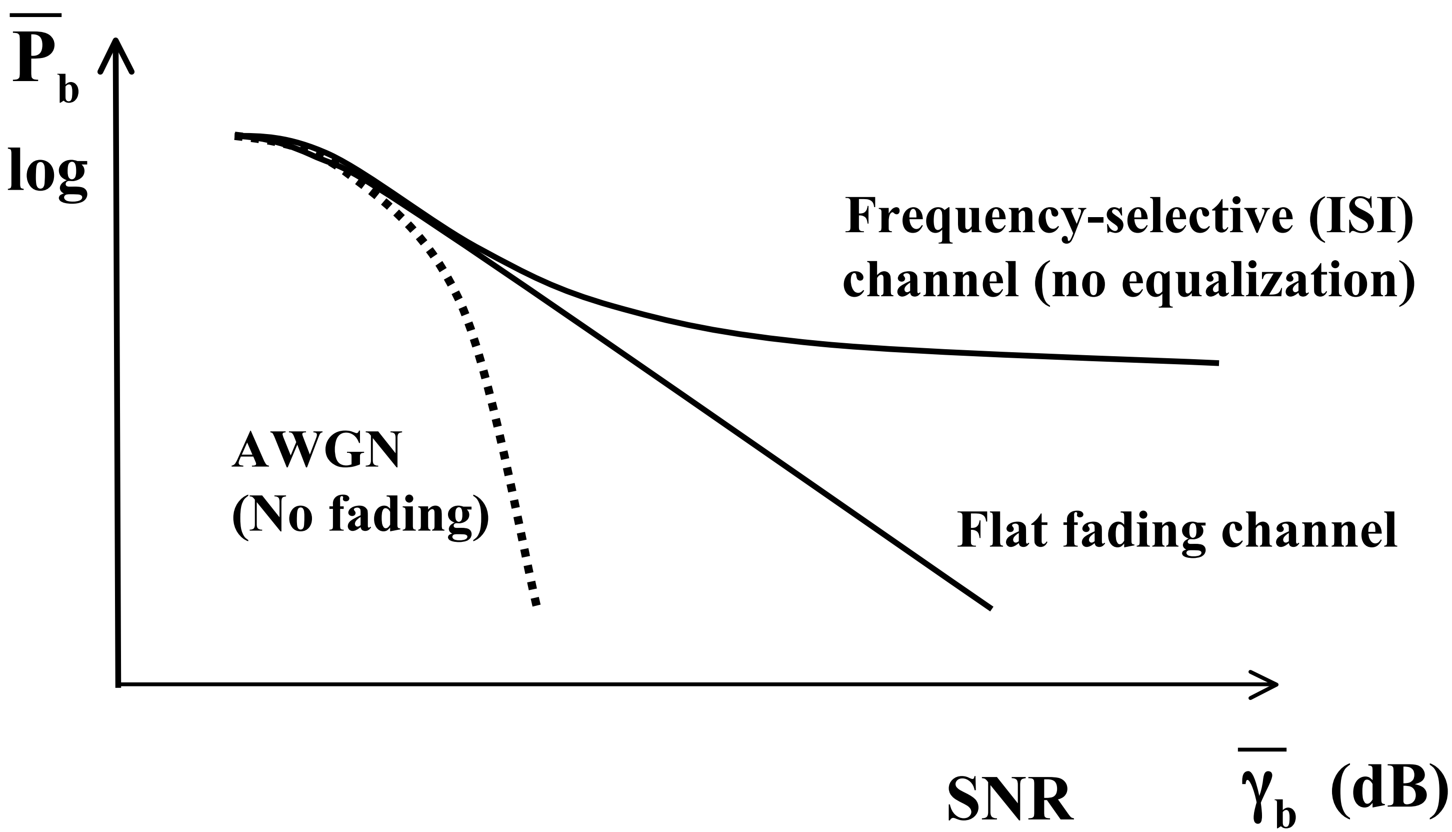
Simulation Results

$$d = \sigma_{T_M} / T_s$$

σ_{T_M}	R_s
2.5 μs	40 Kbaud
25 μs	4 Kbaud
50 ns	2 Mbaud



Typical BER Versus SNR Curves



Modulation for Major Standards

Second Generation

GSM:	GMSK
IS136:	π/4DQPSK
IS-95	BPSK/QPSK
PDC	π/4DQPSK

Third Generation

CDMA2000:	QPSK (DL), BPSK (UL) Phase I
	MPSK (DL), QPSK (UL) Phase II
W-CDMA:	QPSK (DL), BPSK (UL)

Modulation for Major Standards

Wireless LAN

802.11:	BPSK, QPSK
802.11a:	BPSK, QPSK, MQAM
802.11b	BPSK, QPSK
802.11g	BPSK, QPSK, MQAM

Short Range Wireless Network

ZigBee (802.15.4):	BPSK, OQPSK
Bluetooth(802.15.1):	GFSK
UWB (802.15.3) (proposal)	BPSK, QPSK

Main Points

- Fading greatly increases average P_s
  - Alternate Q function approach simplifies P_s calculation, especially its average value in fading
  - Moment Generating Function approach can be used effectively for average error probability calculations in fading
- Doppler spread only impacts differential modulation causing an irreducible error floor at low data rates
- Delay spread causes ISI and irreducible error floor or imposes limits on transmission rates
- Need to combat flat and frequency-selective fading
  - Focus of the rest of the course.