ECE5984
Orthogonal Frequency Division Multiplexing and Related Technologies
Fall 2007

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PAPR in OFDM
Major Learning Objectives

- Upon successful completion of the course the student will be able to:
  - √ Describe the complete architecture of an OFDM system, (serial to parallel, FFT/IFFT, Cyclic prefix, Modulation techniques, coding techniques)
  - √ Evaluate the response of OFDM in Gaussian channels and fading channels.
  - **Define the problems associated of using multi-carrier in time varying channels and how to mitigate these problems.**
  - Design and analyze standards using OFDM such as IEEE 802.11a,g and IEEE 802.16
  - Describe the principle mechanisms by which multiple access techniques are supported using OFDM.
  - Able to categorize the different type of MC-CDMA and the degree of flexibility provided by each type.
  - Able to simulate the basic and advanced techniques used in OFDM systems
Syllabus

• **Analysis of OFDM systems (15%)**
  – RF subsystems, amplifier classification and distortion
  – Crest factor (PAPR) reduction techniques
    • Pre-distortion & adaptive pre-distortion techniques
    • clipping
    • coding techniques
    • partial transmit sequences (PTS) & modified PTS v. selective mapping
    • nonlinear quantization (companding)
  – Phase noise and I&Q imbalance for QAM
  – Performance of OFDM in Gaussian channels ✓
  – Performance of OFDM in Wide-band channels ✓

• **Synchronization and Estimation (15%)**
  – ICI and OISI problems ✓
  – Timing estimation
  – Frequency synchronization
  – Frequency error estimation algorithms
  – Carrier phase tracking
  – Frequency domain and time domain approaches for channel estimation ✓
    • coherent detection
    • differential detection
Pilots and Waterfilling concept
Clarifying Slides
Task of pilot subcarriers

Pilot subcarriers contain signal values that are known in the receiver.

These pilot signals are used in the receiver for correcting the magnitude (important in QAM) and phase shift offsets of the received symbols (see signal constellation example on the right).
Transmitted and received subcarrier $n$

- **Transmitted subcarrier $n$**
- **Received subcarrier $n$**
- **Phase error**
- **Guard time**
- **Symbol part that is used for FFT calculation at receiver**
- **Magnitude error**

Previous symbol | Guard time | Symbol part that is used for FFT calculation at receiver | Next symbol
Waterfilling in Frequency Domain
Suppose now transmitter has full channel knowledge.

\[ C = \mathcal{E} \left[ \log \left( 1 + \frac{P^*(h)|h|^2}{N_0} \right) \right] \]

where

\[ P^*(h) = \left( \frac{1}{\lambda} - \frac{N_0}{|h|^2} \right)^+ . \]

is the waterfilling power allocation as a function of the fading state. and \( \lambda \) is chosen to satisfy the average power constraint.
Transmit More when Channel is Good
At high SNR, waterfilling does not provide any gain.
Performance: Low SNR

Waterfilling provides a significant power gain at low SNR.
Peak to Average Power Ratio
Definition of PAPR

• PAPR & PAR: Peak-To-Average Power Ratio
• Crest factor of x(t): square root of PAR
• Definition: \( \text{PAR} = \frac{\|x\|_{\infty}^2}{\mathbb{E}[\|x\|_2^2]} \)
• OFDM signals have a higher Peak-to-Average Ratio (PAR) – often called a Peak-to-Average Power Ratio (PAPR) – than single carrier signals
• The reason for this is that in the time domain, a multicarrier signal is the sum of many narrowband signals.
  – At some time instances, this sum is large, at other times it is small, which means that the peak value of the signal is substantially larger than the average value.
  – This high PAR is one of the most important implementation challenges that faces OFDM because it reduces the efficiency and hence increases the cost of the RF power amplifier, which is one of the most expensive components in the radio
What is PAPR

- Definition

$$PAPR = \frac{\max |x(t)|^2}{E\{|x(t)|^2\}} = N$$

$$x(t) = \left| \sum_{n=0}^{N-1} X_n \exp(j2\pi n \Delta ft) \right| \leq \sum_{n=0}^{N-1} |X_n| |\exp(j2\pi n \Delta ft)|$$

$$= \sum_{n=0}^{N-1} |X_n| = N$$

$$\Rightarrow |X_n|^2 = N^2$$

$$E\{|x(t)|^2\} = \frac{\sum |x(t)|^2}{N} = \frac{N^2}{N} = N$$
Peak to Average Power Ratio

- **Definition**
  
  \[ R = \frac{\max_{0<t<T_{FFT}} |x(t)|^2}{P_{avg}} = N \]

- **For** \( N = 128 \), \( R = 21 \text{ dB} \).

- **Large PAPR**
  - In-band noise \( \Rightarrow \) increases BER
  - Spectral spreading \( \Rightarrow \) ICI

- **PAPR could be large**
  - We need to be very smart to solve this problem.

- **Large PAPR does not appear frequently**
  - Just ignore it!

**Figure 1:** A sample OFDM symbol

**Figure 2:** Statistics of OFDM PAPR values. \( N = 64 \)
Peak-to-Average Power Ratio (PAPR)

Peak-to-Average Power Ratio is the ratio between the maximum and average power amplitude.

Large number of sub-carriers increase the probability that high PAPR will occur.

When high PAPR occurs, the amplifier reach non-linear region, thus the amplifier efficiency is reduced (What does it mean?)
When a high peak signal is transmitted through a nonlinear device such as a high power amplifier (HPA) or digital-to-analog converter (DAC), it generates out-of-band energy (spectral regrowth) and in-band distortion (constellation tilting and scattering).

\[ IBO = 10 \log_{10} \frac{P_{inSat}}{P_{in}}, \]
Why PAPR happens (1/2)

- Large peaks cause saturation in power amplifiers

\[(\text{IBO})_{dB} = 10 \log_{10} \frac{P_{\text{sat in}}}{P_{\text{in}}}\]

\[(\text{OBO})_{dB} = 10 \log_{10} \frac{P_{\text{sat out}}}{P_{\text{out}}}\]
Figure 4.14: CCDF of PAR for QPSK OFDM system: $L = 16, 64, 256, 1024$. Solid line: simulation results, dotted line: approximation using $\beta = 2.8$. 
Peak to Average Power Ratio

![Graph showing the peak to average power ratio for different values of N: N = 64, N = 128, N = 256, N = 1024. The x-axis represents the par-threshold (in dB), and the y-axis represents the probability of exceeding the par-threshold.](image-url)
PAPR Effects

- High Peak-to-Average Power Ratio (PAPR) of the transmitted signal result in:
  - Clipping noise (limited quantization levels, rounding and truncation, during IFFT and FFT computation)
  - Nonlinear distortions of power amplifiers
  - BER performance degradation,
  - Energy spilling into adjacent channels,
  - Intermodulation effects on the subcarriers, warping of the signal constellation in each subchannel.
**PAPR Reduction**

- **PAPR reduction**
  - Clipping and windowing, recursive clipping
  - Reference signal subtraction
Clipping (1)

- Clipping is a distortion technique
- Process at transmitter only
- Limits the peak amplitude to a threshold value, if the amplitude is lower than this threshold, then leave undisturbed.
- Clipping ratio ‘cr’ is an important parameter, defined as:
  \[ \text{cr} = \frac{A}{P_{\text{avg}}} \]

where
A = Clipping level
\( P_{\text{avg}} = \) Average Power
Power spectral density of unclipped vs clipped OFDM signals
BER of Clipped OFDM Signals.

* Example
Power Spectrum

QPSK

64QAM
PAPR Reduction, cont.

- Coding
  - Golay complementary codes
  - Generalized Reed-Muller codes
- Parallel combinatory OFDM signaling

- Multiple signal representation
- Pre-distortion
<table>
<thead>
<tr>
<th>Data block $X$</th>
<th>PAPR (dB)</th>
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<th>PAPR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[1,1,1,1]^T$</td>
<td>6.0</td>
<td>$[1,1,1,1]^T$</td>
<td>2.3</td>
</tr>
<tr>
<td>$[1,1,1,-1]^T$</td>
<td>2.3</td>
<td>$[1,1,1,1]^T$</td>
<td>3.7</td>
</tr>
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Table 1. PAPR values of all possible data blocks for an OFDM signal with four subcarriers and BPSK modulation.
PAP Reduction with Linear block code

- Linear block code
- A block coding scheme provides error correction capability, and also achieves the minimum PAPR for the OFDM system utilizing QPSK modulation and 4 subcarriers.
- Block coding approach: by selecting only those codewords with small PAPR. Well-designed block codes provide error correction capability.
Most of the decoding techniques for these codes require an exhaustive search so are feasible only for a small number of subcarriers. Moreover, it is difficult to maintain a reasonable coding rate in OFDM when the number of subcarriers grows large.
PAP Reduction with Predistortion

- Compander predistortion
  - Try to predict the transfer function $G(v)$ of the HPA and use its inverse function $F(v)$ as predistortion. Hopefully the composite function is linearized.
  - Problem: Difficult to predict $G$ which is device dependent and could be time varying
  - Many adaptive methods to find the inverse function (LMS, LUT, etc)
Each block is multiplied symbol-by-symbol, before the IFFT operation, by one of pseudo-random but fixed set of vectors \( r_i \) whose elements are complex numbers with the amplitude equal to one and a random phase uniformly distributed between $[-\pi, \pi]$. 

Selected Mapping

Complex data symbols $A_n$ are multiplied symbol-by-symbol with each block, before the IFFT operation, by one of pseudo-random but fixed set of vectors $r_i$. The resulting complex signal is then subjected to the IFFT operation to produce the output $\tilde{a}_n$. 

The process is repeated for multiple signal representations, each with a different amplitude $a_n$. The output of the IFFT operation is then selected based on the lowest PAPR symbol, resulting in the final output $\tilde{a}_n$. 

- $P^{(1)}$ represents the first signal representation
- $P^{(2)}$ represents the second signal representation
- $P^{(U)}$ represents the $U$th signal representation

The diagram illustrates the process of PAPR reduction through selected mapping, where each block is multiplied by a pseudo-random vector before the IFFT operation.
SLM method for OFDM

- This method is based on generating $M$ statistically independent transformed blocks for each data block and transmitting the one with the lowest PAPR.

$$\Pr\{PAPR_{1,2,\ldots,M} > \lambda\} = \prod_{i=1}^{M} \Pr\{PAPR_i > \lambda\} = \Pr\{PAPR_1 > \lambda\}^M$$

- It requires transmitting some side information about the identity of the selected block.
Example: Here, we show a simple example of the SLM technique for an OFDM system with eight subcarriers. We set the number of phase sequences to $U = 4$. The data block to be transmitted is denoted $\mathbf{X} = [1, -1, 1, 1, -1, 1, -1]^{T}$ whose PAPR before applying SLM is 6.5 dB. We set the four phase factors as $\mathbf{B}^{(1)} = [1, 1, 1, 1, 1, 1, 1, 1]^{T}$, $\mathbf{B}^{(2)} = [-1, -1, 1, 1, 1, 1, -1]^{T}$, $\mathbf{B}^{(3)} = [-1, 1, -1, 1, -1, 1, 1, 1]^{T}$, and $\mathbf{B}^{(4)} = [1, 1, -1, 1, 1, -1, 1, 1]^{T}$. Among the four modified data blocks $\mathbf{X}^{(u)}$, $u = 1, 2, 3, 4$, $\mathbf{X}^{(2)}$, has the lowest PAPR of 3.0 dB. Hence, $\mathbf{X}^{(2)}$ is selected and transmitted to the receiver. For this data block, the PAPR is reduced from 6.5 to 3.0 dB, resulting in a 3.5 dB reduction. In this case, the number of IDFT operations is 4 and the amount of side information is 2 bits. The amount of PAPR reduction may vary from data block to data block, but PAPR reduction is possible for all data blocks.
The objective is to design an optimal phase for the subblock set that minimizes the PAR. The phase can then be corrected at the receiver.
An input data block of N symbols is partitioned into disjoint subblocks.
The subcarriers in each subblock are weighted by a phase factor for that subblock.
The phase factors are selected such that the PAPS of the combined signal is minimized.
An example of PTS

Adjacent subblock partitioning in PTS

- FFT/IFFT is linear $x = F^*X = F^*(\sum_i X_i)$
- $x_{pts} = F^*(\sum_i p_i X_i) = \sum_i (p_i F^*X_i)$
- When the phase factors $p_i$ are known in the receiver, $X$ can be estimated.
How to solve the PAPR problem (1/6)

- **Signal distortion**
  - Clipping, Peak windowing, Peak cancellation

- **Coding**
  - Error correction, Use lower PAPR signals

- **Scrambling**

- **Selected Mapping (SLM)**
  - The transmitter selects one favorable transmit signal from a set of sufficiently different signals which all represent the same information.

- **Partial Transmit Sequence (PTS)**
  - The transmitter constructs its transmit signal with low PAR by coordinated addition of appropriately phase rotated signal parts.

- The difference between SLM and PTS is that the first applies independent scrambling rotations to all subcarriers, while the latter only applies scrambling rotations to group of subcarriers.
How to solve the PAPR problem (2/6)

• Peak windowing
  – Multiplied by certain window function
  – Rectangular, Kasier, Hanning, Hamming …..

• Different window with same window length
• Same window with different window length
How to solve the PAPR problem (3/6)

- Different window with same window length (time domain)

```plaintext
rectangular  kaisar  hanning  hamming
```
How to solve the PAPR problem (4/6)

- Different window with same window length (frequency domain)
How to solve the PAPR problem (5/6)

- Same window with different window length - hamming window (time domain)

\[ N = 7 \quad N = 11 \quad N = 13 \quad N = 15 \]
How to solve the PAPR problem (6/6)

- Same window with different window length - hamming window (frequency domain)
## Comparison of PAPR reduction techniques.

<table>
<thead>
<tr>
<th>Technique</th>
<th>Distortionless</th>
<th>Power increase</th>
<th>Data rate loss</th>
<th>Requires processing at transmitter (Tx) and receiver (Rx)</th>
</tr>
</thead>
</table>
| Clipping and filtering | No             | No             | No             | Tx: Amplitude clipping, filtering  
Rx: None                                                                                         |
| Coding          | Yes            | No             | Yes            | Tx: Encoding or table search  
Rx: Decoding or table search                                                                   |
| PTS             | Yes            | No             | Yes            | Tx: $M$ IDFTs, $W^{M-1}$ complex vector sums  
Rx: Side information extraction, inverse PTS                                                     |
| SLM             | Yes            | No             | Yes            | Tx: $U$ IDFTs  
Rx: Side information extraction, inverse SLM                                                    |
| Interleaving    | Yes            | No             | Yes            | Tx: $K$ IDFTs, $(K - 1)$ interleavings  
Rx: Side information extraction, inverse interleaving                                            |
| TR              | Yes            | Yes            | Yes            | Tx: IDFTs, find value of PRCs  
Rx: Ignore non-data-bearing subcarriers                                                           |
| TI              | Yes            | Yes            | No             | Tx: IDFTs, search for maximum point in time, tones to be modified, value of $p$ and $q$  
Rx: Modulo-$D$ operation                                                                            |
| ACE             | Yes            | Yes            | No             | Tx: IDFTs, projection onto “shaded area”  
Rx: None                                                                                           |