ECE5984

Orthogonal Frequency Division Multiplexing and Related Technologies

Fall 2007

Mohamed Essam Khedr

PAPR in OFDM

Major Learning Objectives

- **Upon successful completion of the course the student will be able to:**
- **√ Describe the complete architecture of an OFDM system, (serial to parallel, FFT/IFFT, Cyclic prefix, Modulation techniques, coding techniques)**
- **√ Evaluate the response of OFDM in Gaussian channels and fading channels.**
- **Define the problems associated of using multi-carrier in time varying channels and how to mitigate these problems.**
- **Design and analyze standards using OFDM such as IEEE 802.11a,g and IEEE 802.16**
- **Describe the principle mechanisms by which multiple access techniques are supported using OFDM.**
- **Able to categorize the different type of MC-CDMA and the degree of flexibility provided by each type.**
- **Able to simulate the basic and advanced techniques used in OFDM systems**

Syllabus

- **Analysis of OFDM systems** (15%) **2**
  - RF subsystems, amplifier classification and distortion
  - Crest factor (PAPR) reduction techniques
    - Pre-distortion & adaptive pre-distortion techniques
    - clipping
    - coding techniques
    - partial transmit sequences (PTS) & modified PTS v. selective mapping
    - nonlinear quantization (companding)
  - Phase noise and I&Q imbalance for QAM
  - Performance of OFDM in Gaussian channels √
  - Performance of OFDM in Wide-band channels √
- **Synchronization and Estimation** (15%) **2**
  - ICI and OISI problems √
  - Timing estimation
  - Frequency synchronization
  - Frequency error estimation algorithms
  - Carrier phase tracking
  - Frequency domain and time domain approaches for channel estimation √
    - coherent detection
    - differential detection

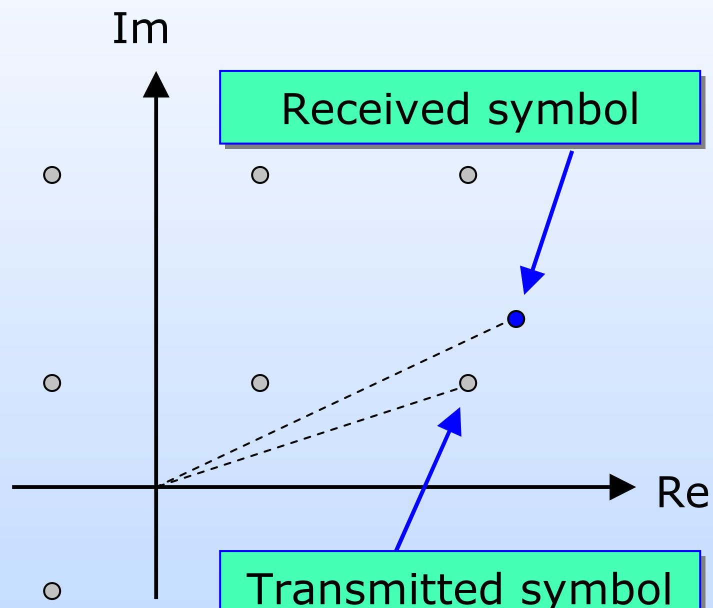
Pilots and Waterfilling concept

Clarifying Slides

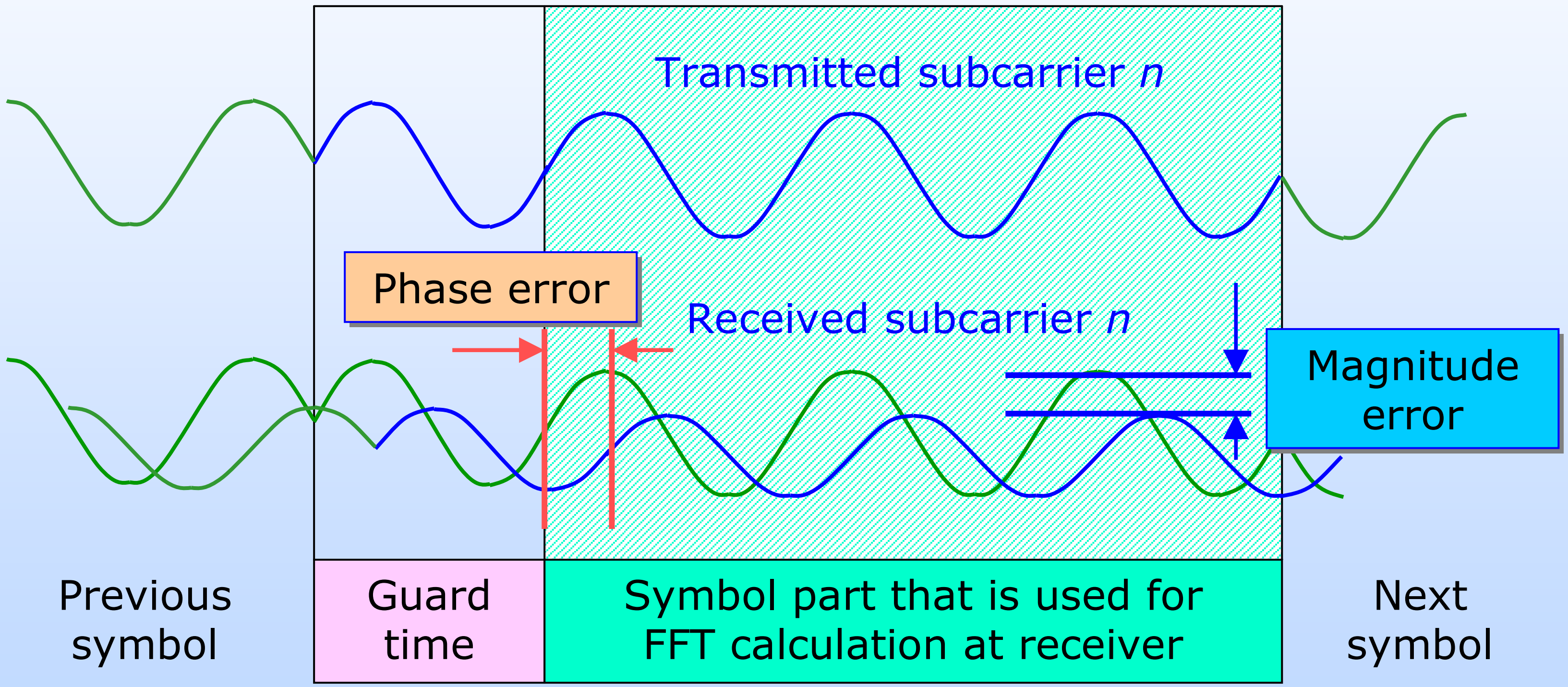
Task of pilot subcarriers

Pilot subcarriers contain signal values that are known in the receiver.

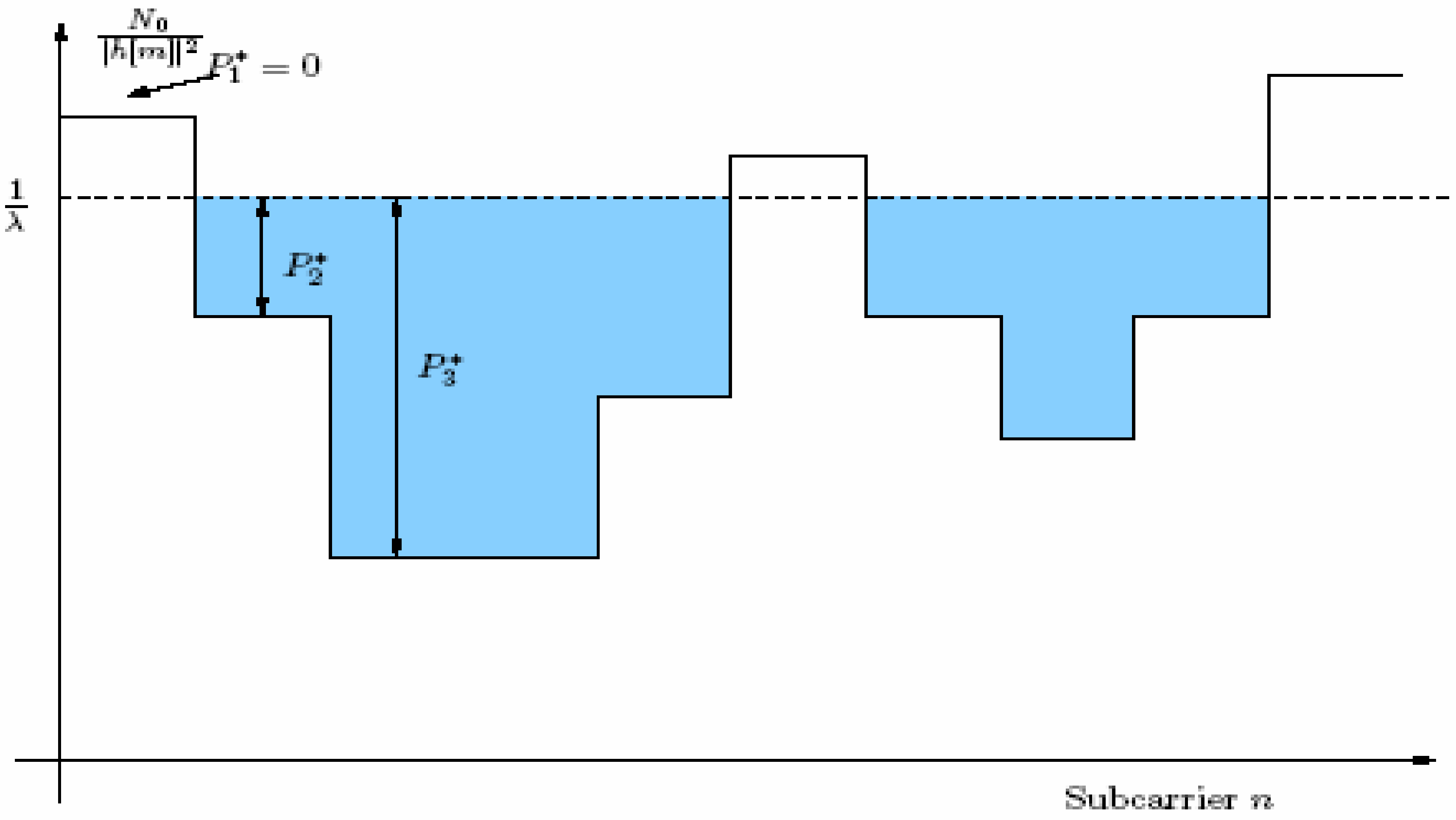
These pilot signals are used in the receiver for correcting the magnitude (important in QAM) and phase shift offsets of the received symbols (see signal constellation example on the right).



Transmitted and received subcarrier *n*



Waterfilling in Frequency Domain



Waterfilling Capacity

Suppose now transmitter has full channel knowledge.

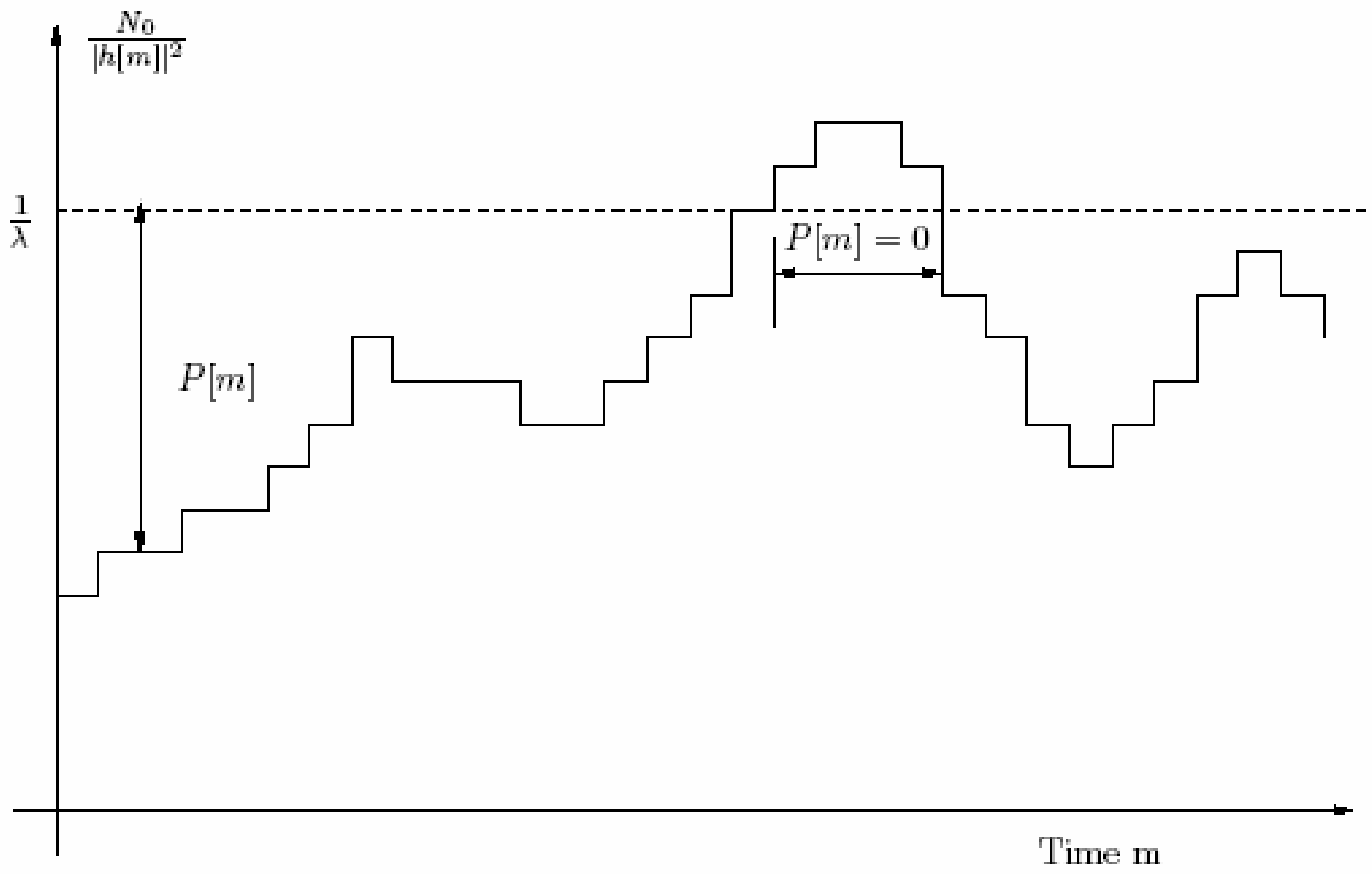
$$C = \mathcal{E}\left[\log\left(1 + \frac{P^*(h)|h|^2}{N_0}\right)\right]$$

where

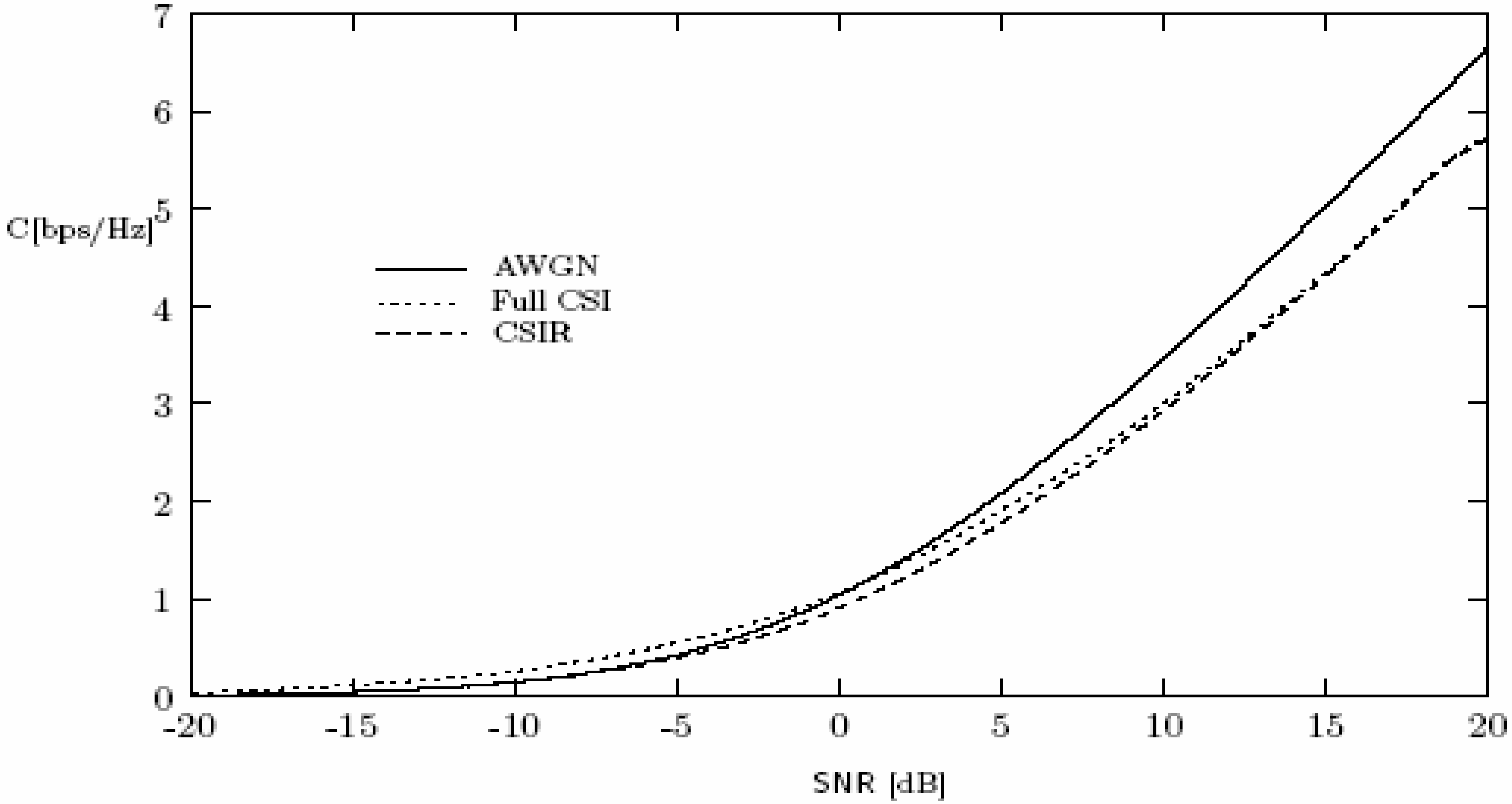
$$P^*(h) = \left(\frac{1}{\lambda} - \frac{N_0}{|h|^2}\right)^+.$$

is the waterfilling power allocation as a function of the fading state. and λ is chosen to satisfy the average power constraint.

Transmit More when Channel is Good

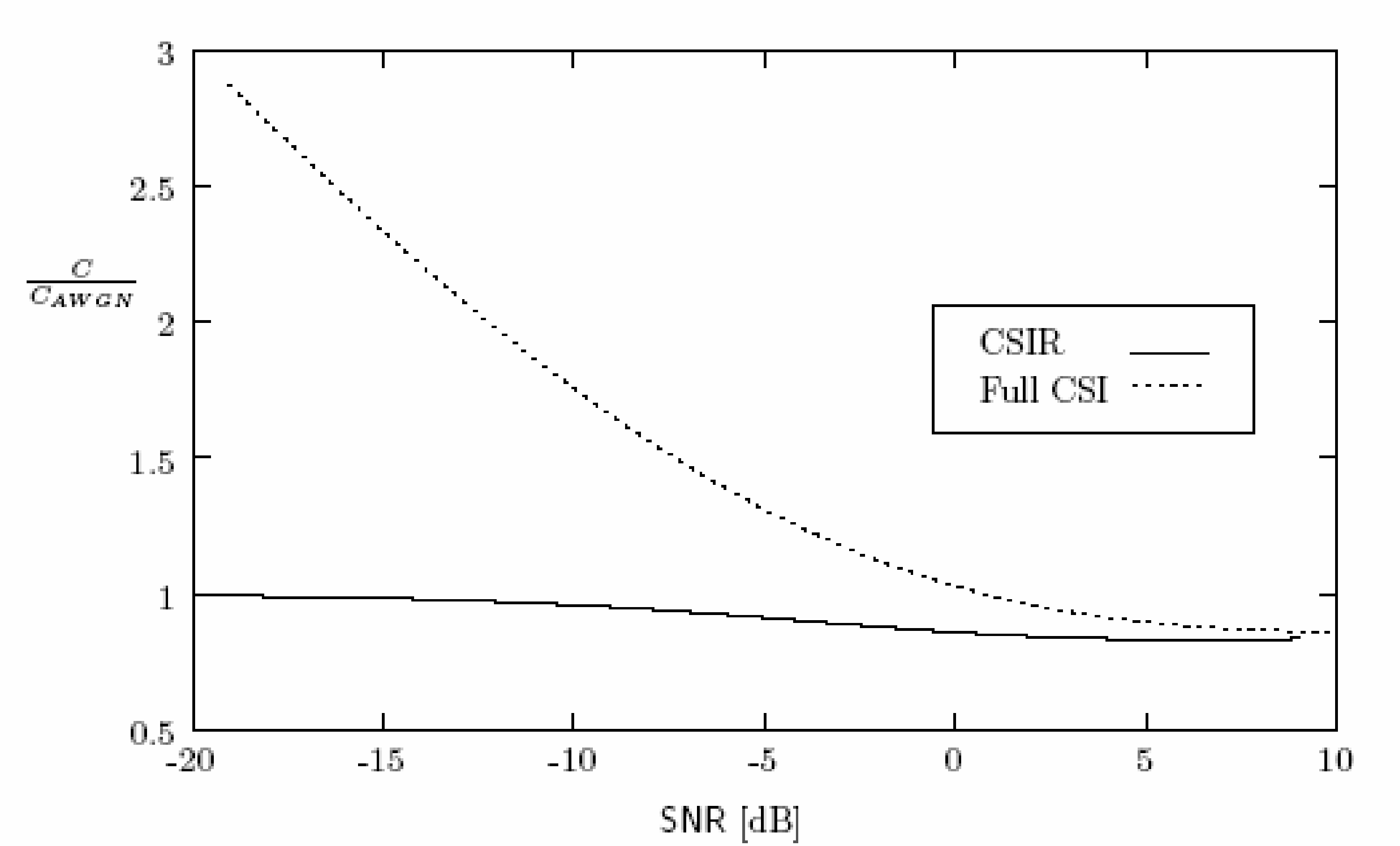


Performance



At high SNR, waterfilling does not provide any gain.

Performance: Low SNR

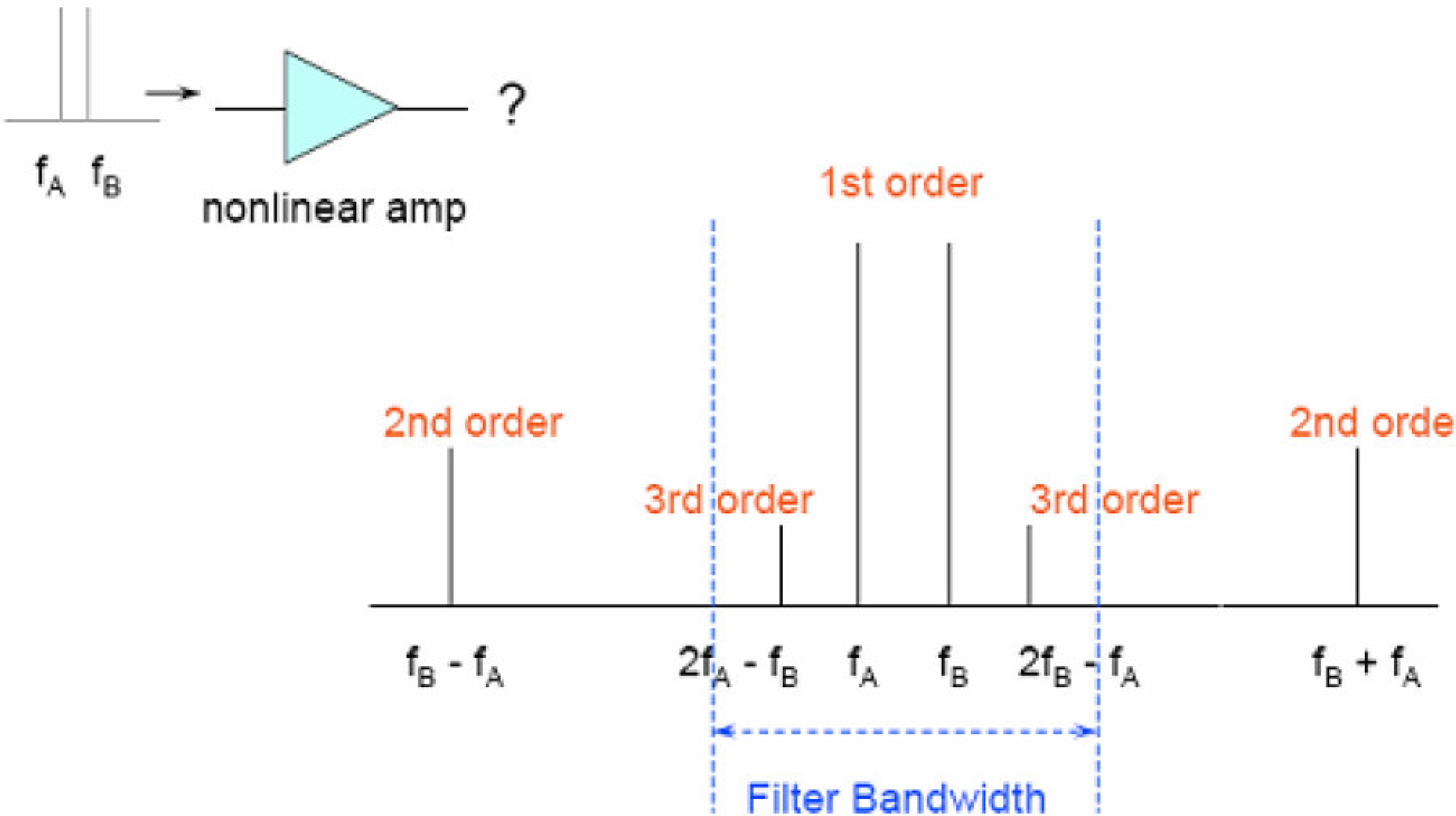


Waterfilling povides a significant power gain at low SNR.

Peak to Average Power Ratio

Definition of PAPR

- **PAPR & PAR: Peak-To-Average Power Ratio**
- **Crest factor of x(t): square root of PAR**
- Definition: $\mathbf{PAR = (\|x\|_\infty)^2 / E[(\|x\|_2)^2]}$
- OFDM signals have a higher Peak-to-Average Ratio (PAR) – often called a Peak-to-Average Power Ratio (PAPR) – than single carrier signals
- The reason for this is that in the time domain, a multicarrier signal is the sum of many narrowband signals.
  - **At some time instances, this sum is large, at other times it is small, which means that the peak value of the signal is substantially larger than the average value.**
  - **This high PAR is one of the most important implementation challenges that faces OFDM because it reduces the efficiency and hence increases the cost of the RF power amplifier, which is one of the most expensive components in the radio**



fA fB
nonlinear amp
?
1st order
2nd order
3rd order
3rd order
2nd order
fB - fA
2fA - fB
fA
fB
2fB - fA
fB + fA
Filter Bandwidth

What is PAPR

- **Definition**

$$PAPR = \frac{\max\left|x(t)\right|^2}{E\left\{\left|x(t)\right|^2\right\}} = N$$

$$x(t) = \left|\sum_{n=0}^{N-1} X_n \exp(j2\pi n\Delta f t)\right| \leq \sum_{n=0}^{N-1} \left|X_n\right| \left|\exp(j2\pi n\Delta f t)\right|$$

$$= \sum_{n=0}^{N-1} \left|X_n\right| = N$$

$$\Rightarrow \left|X_n\right|^2 = N^2$$

$$E\left\{\left|x(t)\right|^2\right\} = \frac{\sum\left|x(t)\right|^2}{N} = \frac{N^2}{N} = N$$

Peak to Average Power Ratio

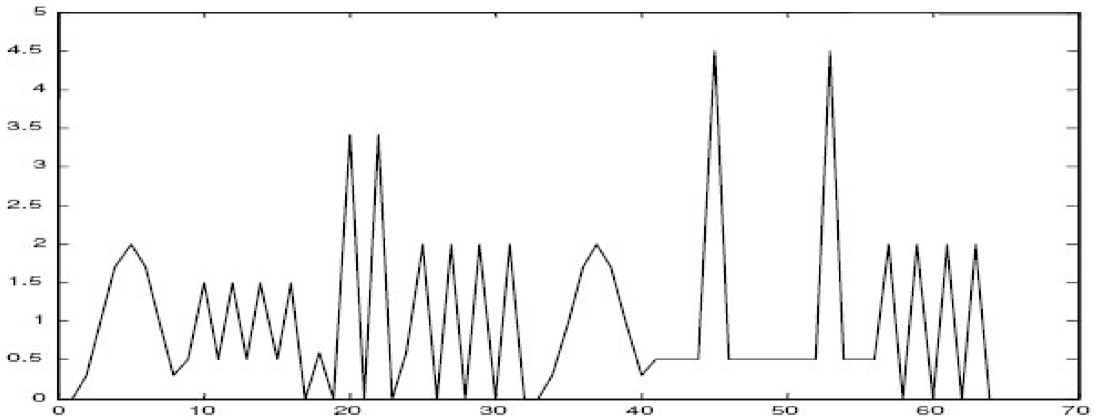


Figure 1: A sample OFDM symbol

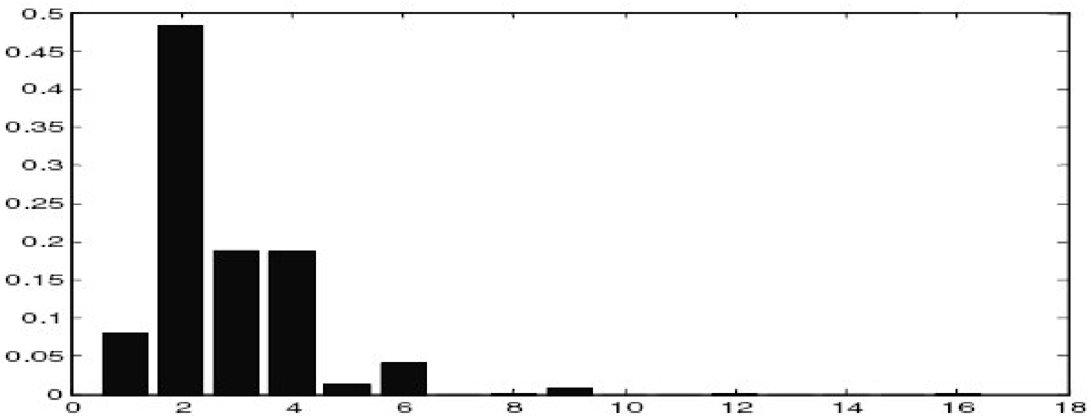


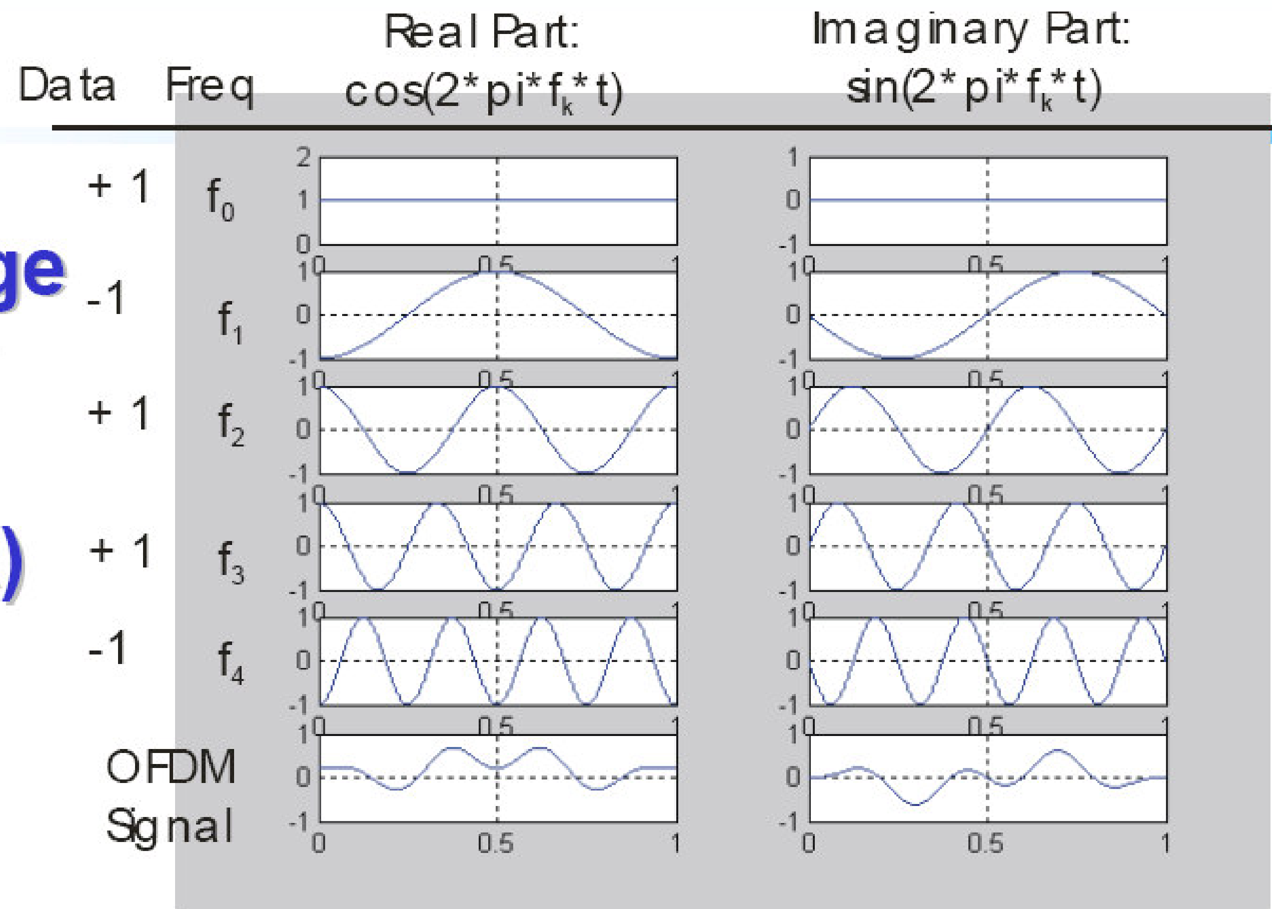
Figure 2: Statistics of OFDM PAPR values. $N = 64$

- Definition

$$R = \frac{\max_{0<t<T_{FFT}} |x(t)|^2}{P_{avg}} = N$$

- For $N = 128$, $R = 21$ dB.
- Large PAPR
  - In-band noise ⇒ increases BER
  - Spectral spreading ⇒ ICI
- PAPR could be large
  - We need to be very smart to solve this problem.
- Large PAPR does not appear frequently
  - Just ignore it!

Peak to Average Power Ratio (PAPR)

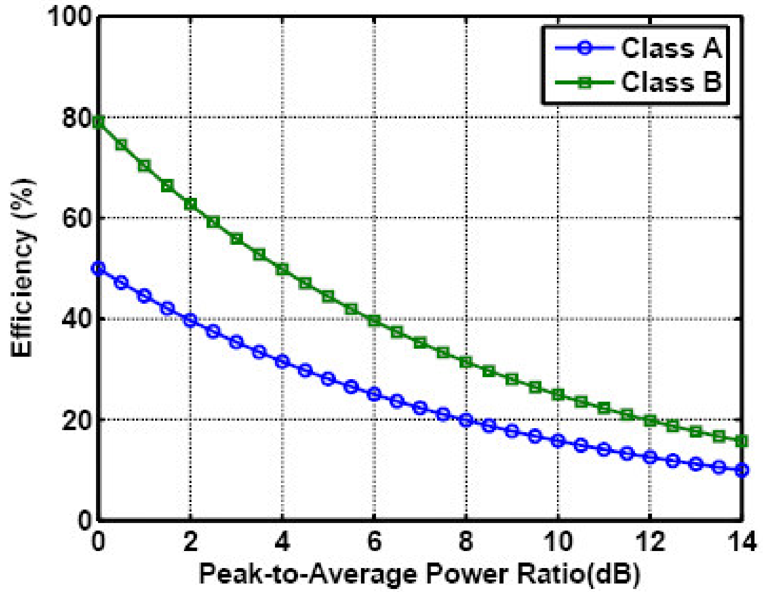


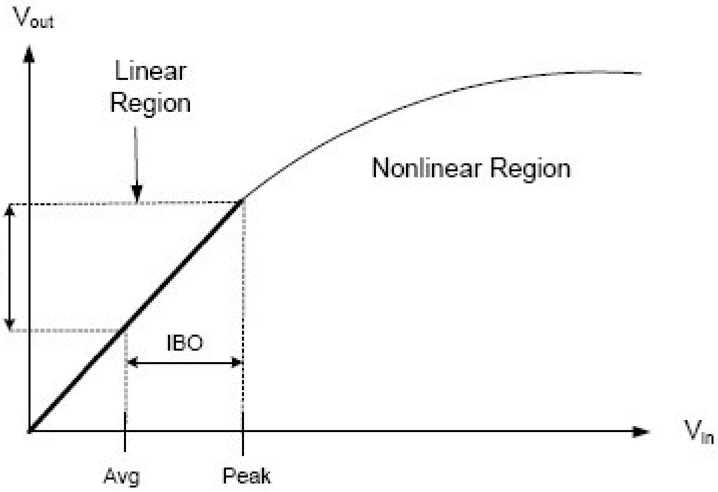
Peak-to-Average Power Ratio is the ratio between the maximum and average power amplitude

Large number of sub-carriers increase the probability that high PAPR will occur

When high PAPR occurs, the amplifier reach non-linear region, thus the amplifier efficiency is reduced (What does it mean?)

- When a high peak signal is transmitted through a nonlinear device such as a high power amplifier (HPA) or digital-to-analog converter (DAC), it generates out-of-band energy (spectral regrowth) and in-band distortion (constellation tilting and scattering).





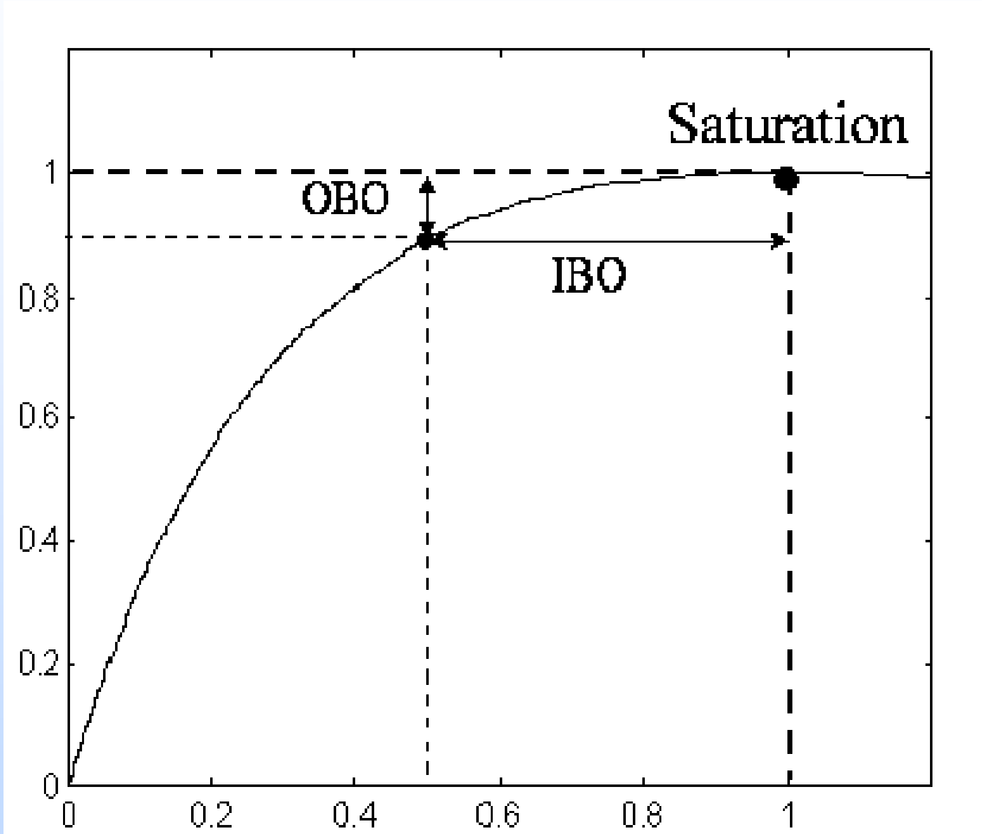
$$IBO = 10\log_{10}\frac{P_{inSat}}{\bar{P}_{in}},$$

Why PAPR happens(1/2)

- **Large peaks cause saturation in power amplifiers**

$$(\mathrm{IBO})_{dB} = 10Log_{10}\frac{P_{sat\_in}}{\overline{P}_{in}}$$

$$(\mathrm{OBO})_{dB} = 10Log_{10}\frac{P_{sat\_out}}{\overline{P}_{out}}$$



Complementary CDF of PAPR

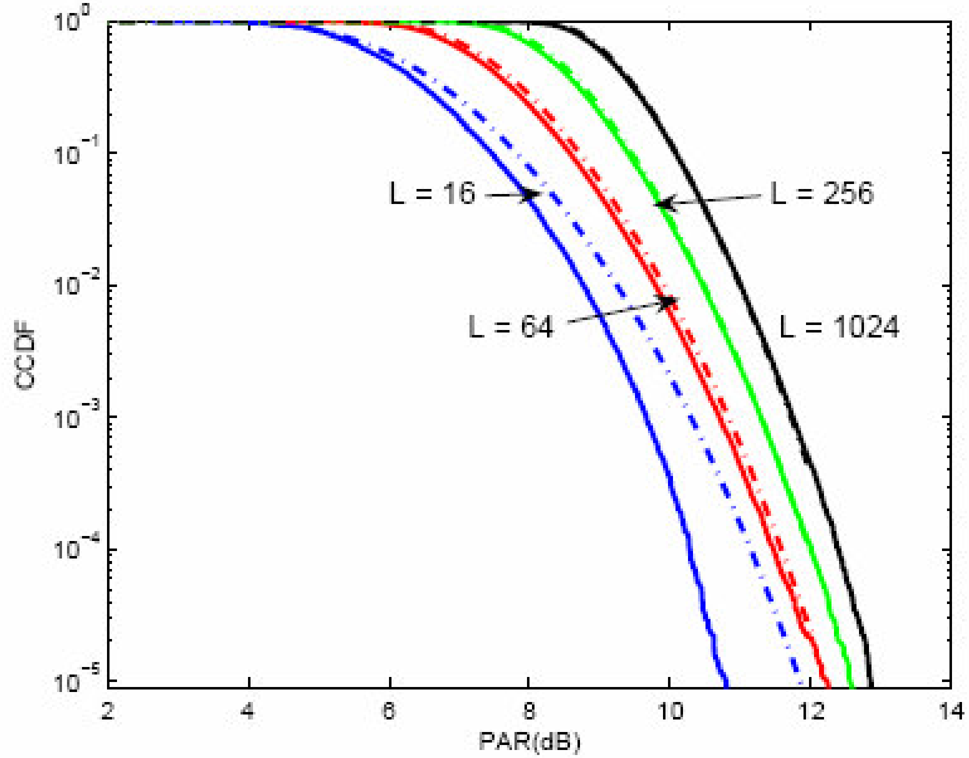
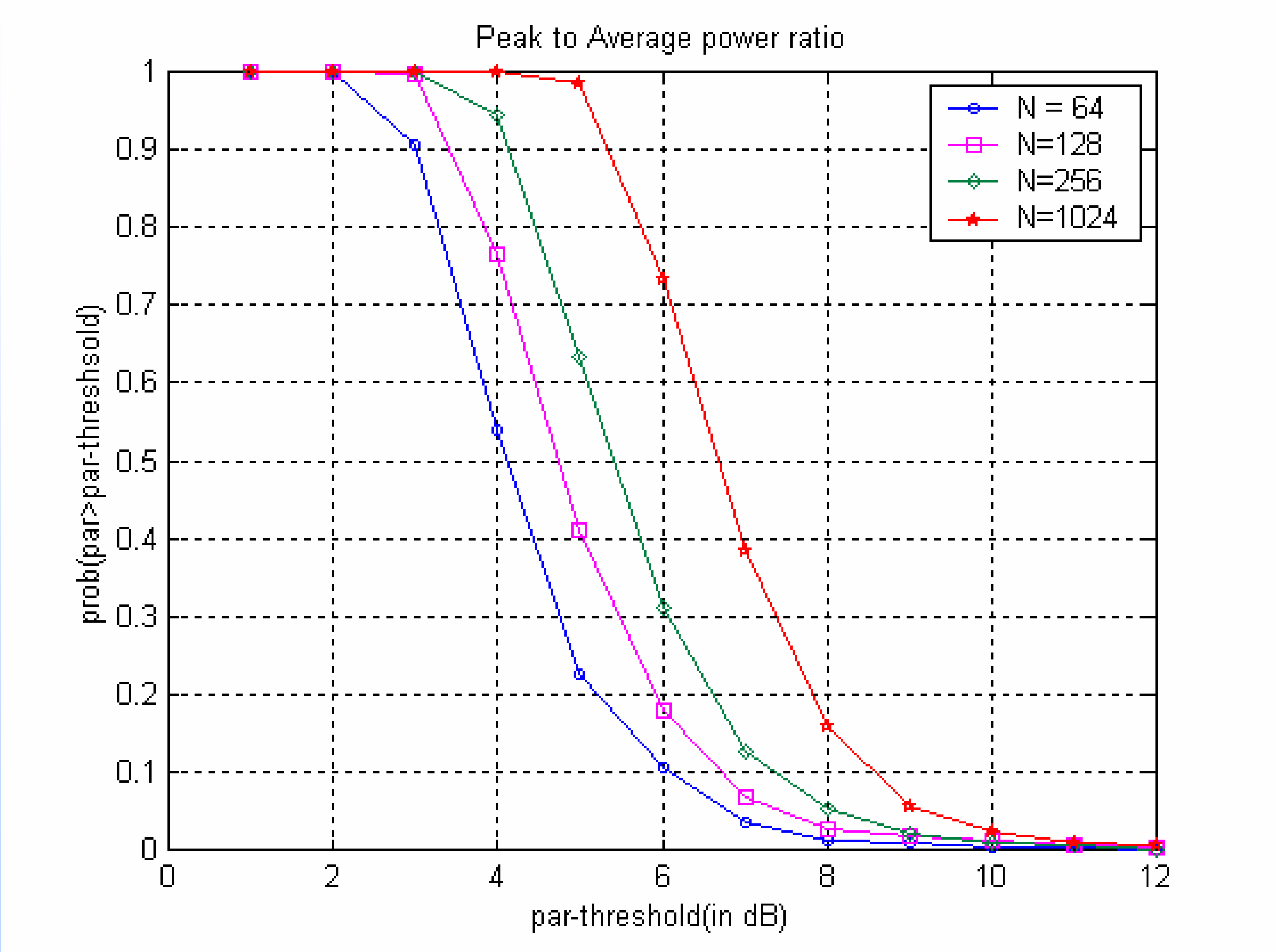


Figure 4.14: CCDF of PAR for QPSK OFDM system: $L = 16, 64, 256, 1024$. Solid line: simulation results, dotted line: approximation using $\beta = 2.8$.

Peak to Average Power Ratio

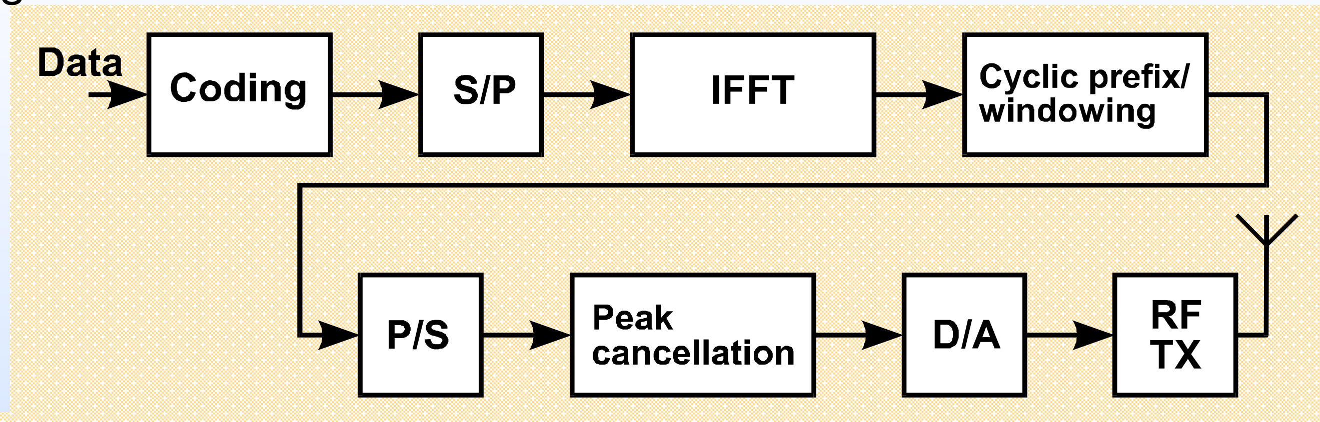


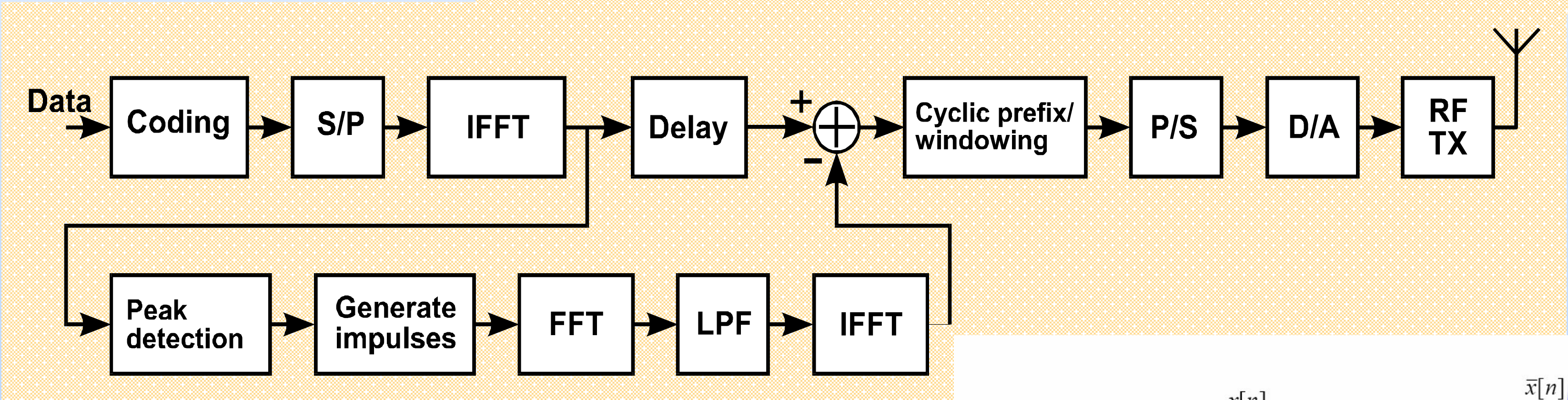
PAPR Effects

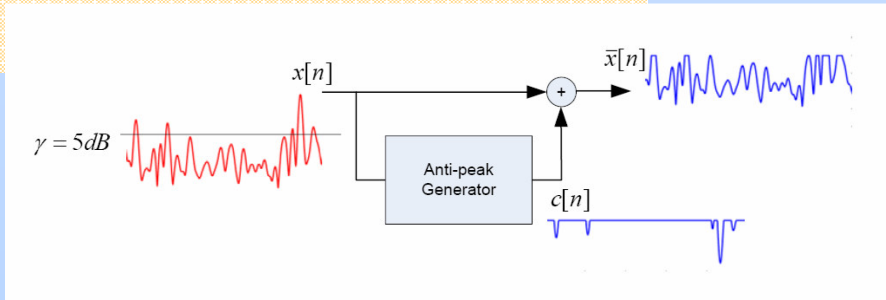
- **High Peak-to-Average Power Ratio (PAPR) of the transmitted signal result in:**
  - Clipping noise (limited quantization levels, rounding and truncation, during IFFT and FFT computation)
  - Nonlinear distortions of power amplifiers
  - BER performance degradation,
  - Energy spilling into adjacent channels,
  - Intermodulation effects on the subcarriers, warping of the signal constellation in each subchannel.

PAPR Reduction

- **PAPR reduction**
  - Clipping and windowing, recursive clipping
  - Reference signal subtraction







Clipping (1)

- Clipping is a distortion technique
- Process at transmitter only
- Limits the peak amplitude to a threshold value, if the amplitude is lower than this threshold, then leave undisturbed.
- Clipping ratio ‘cr’ is an important parameter, defined as:

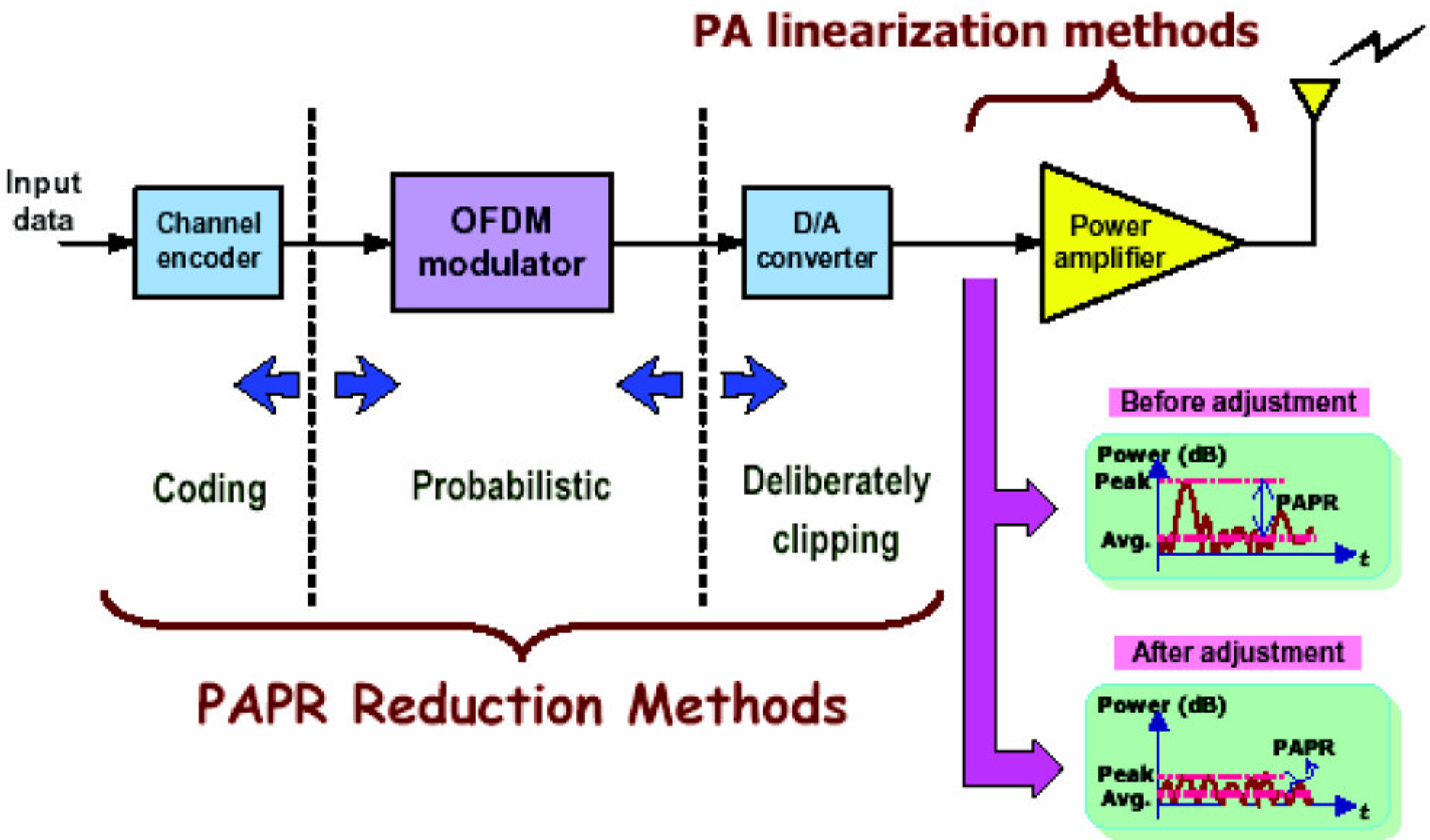
$$cr = (A/P_{avg})$$

where

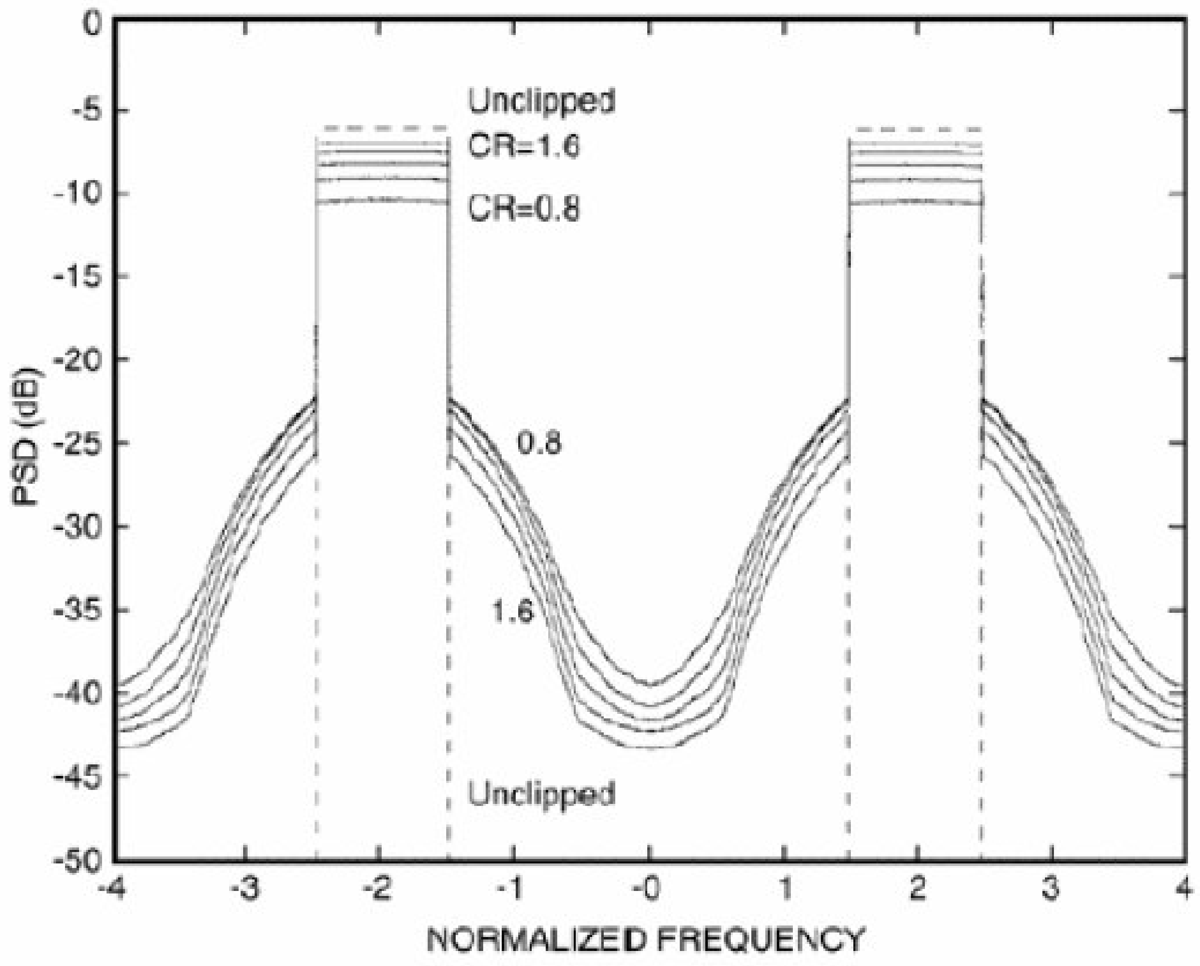
A = Clipping level

P_{avg} = Average Power

PAPR Reduction Methods

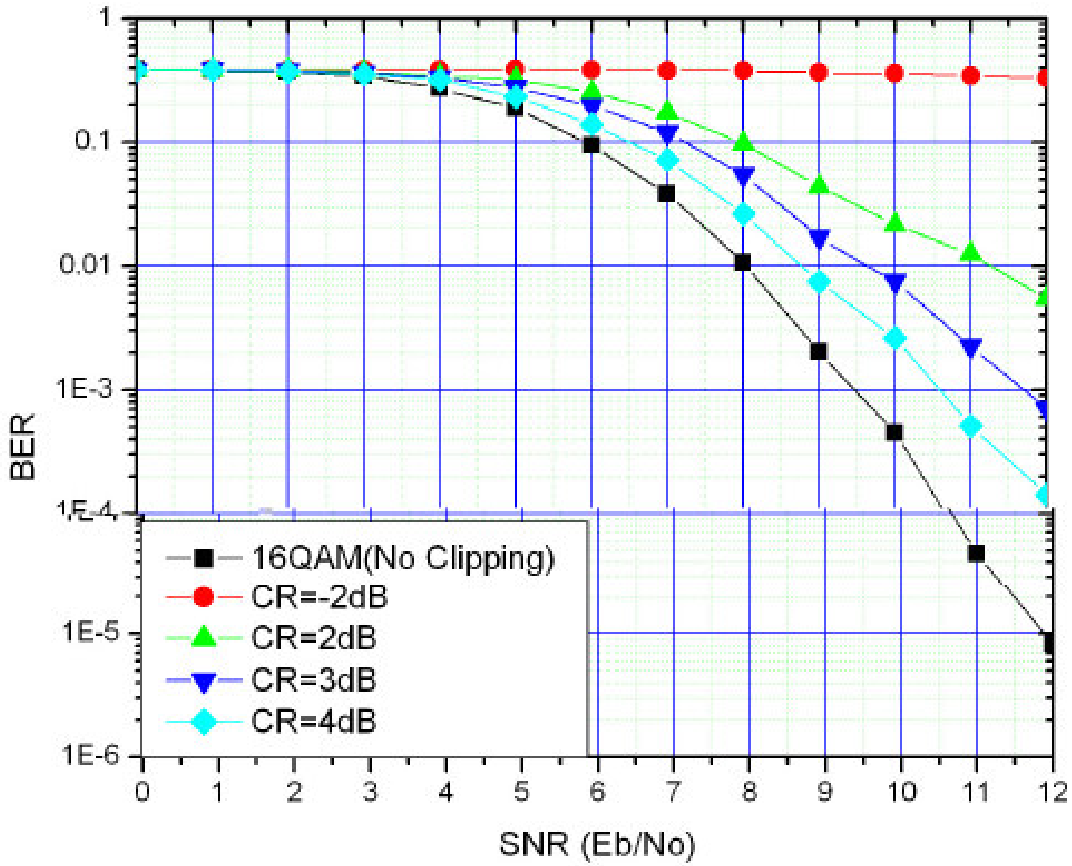


Power spectral density of unclipped vs clipped OFDM signals



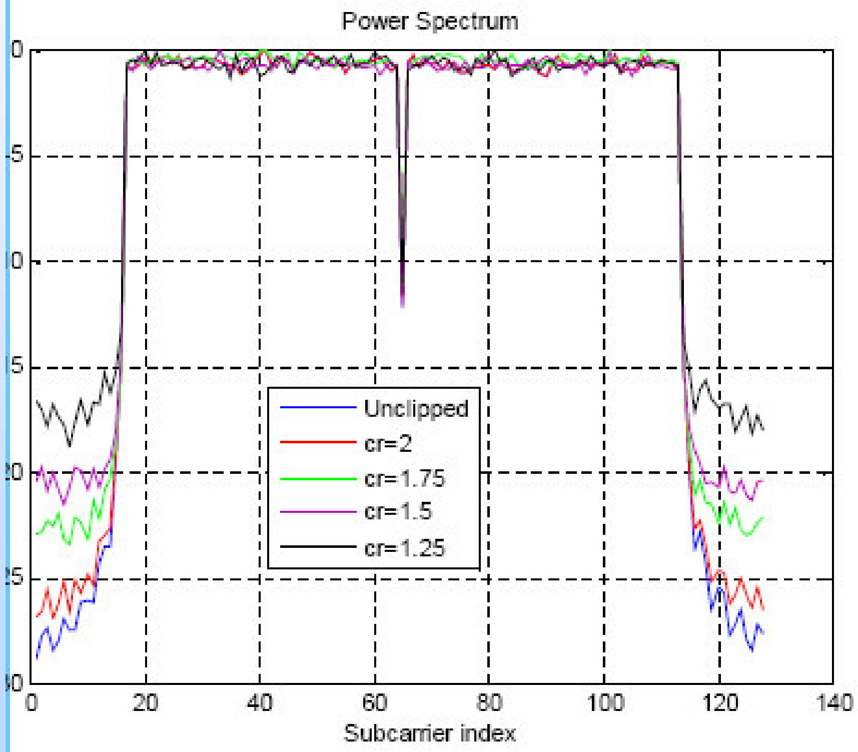
BER of Clipped OFDM Signals.

* Example

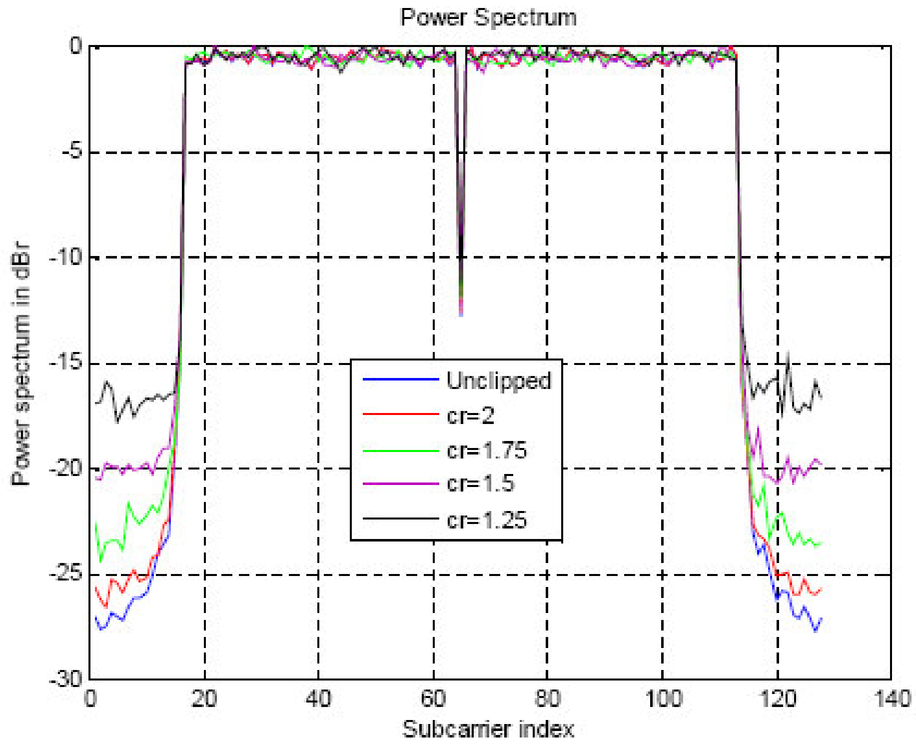


Power Spectrum

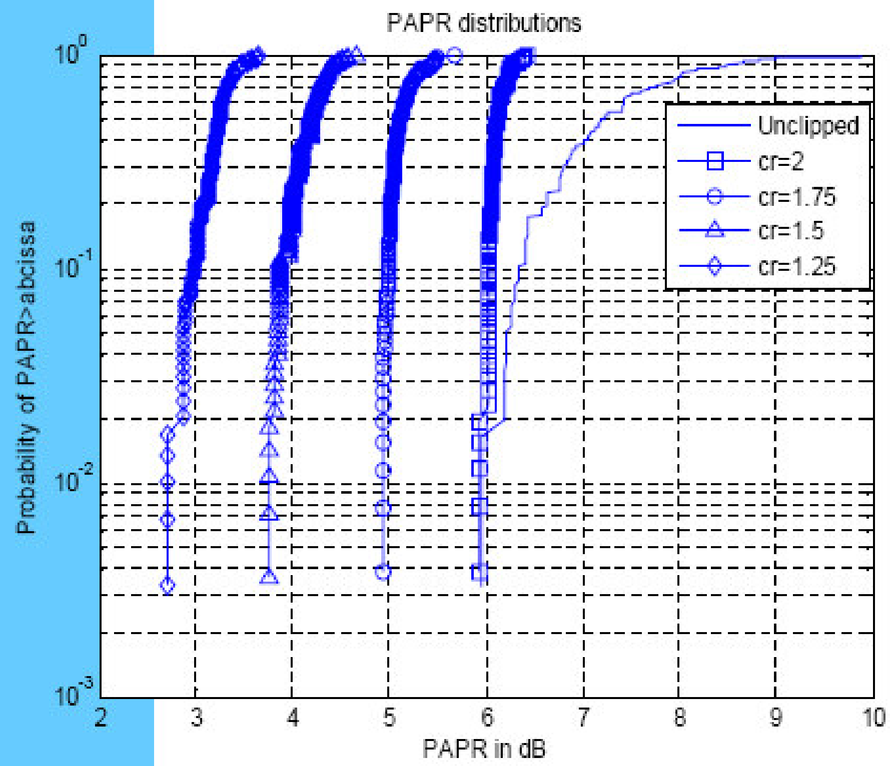
QPSK



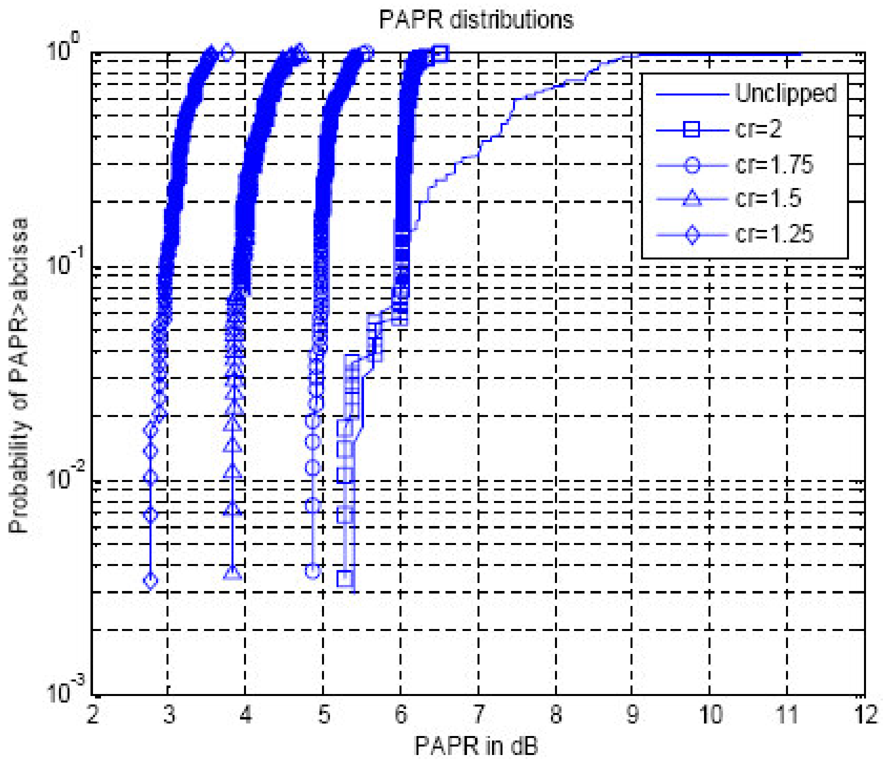
64QAM



QPSK

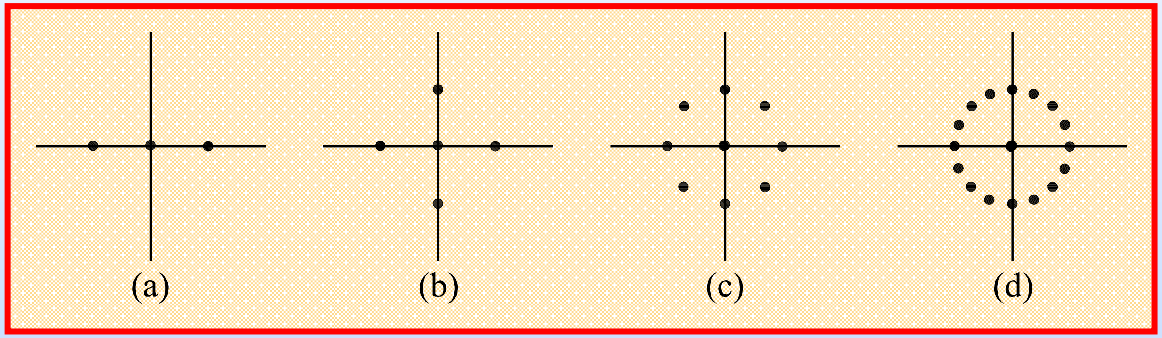

PAPR distributions
Probability of PAPR>abcissa
PAPR in dB
Unclipped
cr=2
cr=1.75
cr=1.5
cr=1.25

64QAM


PAPR distributions
Probability of PAPR>abcissa
PAPR in dB
Unclipped
cr=2
cr=1.75
cr=1.5
cr=1.25

PAPR Reduction, cont.

- Coding
  - Golay complementary codes
  - generalized Reed-Muller codes
- Parallel combinatory OFDM signaling



- Multiple signal representation
- Pre-distortion

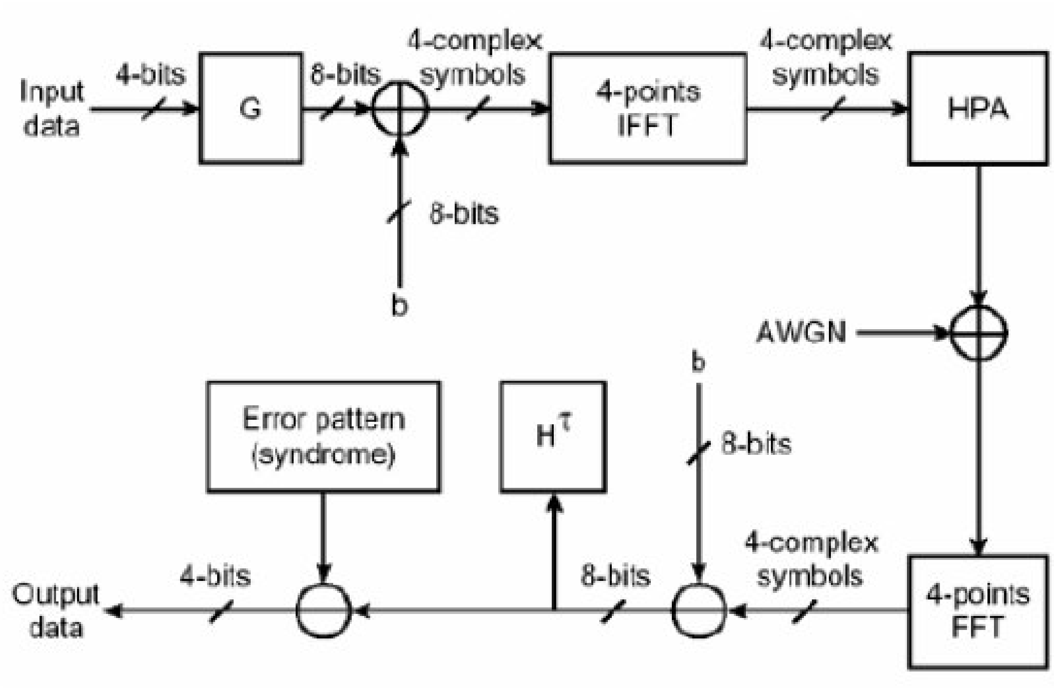
Data block X	PAPR (dB)	Data block X	PAPR (dB)
$[1,1,1,1]^T$	6.0	$[1,1,1,1]^T$	2.3
$[1,1,1,-1]^T$	2.3	$[1,1,1,1]^T$	3.7
$[1,1,-1,1]^T$	2.3	$[1,1,1,1]^T$	6.0
$[1,1,-1,-1]^T$	3.7	$[1,1,-1,-1]^T$	2.3
$[1,-1,1,1]^T$	2.3	$[-1,-1,1,1]^T$	3.7
$[1,-1,1,-1]^T$	6.0	$[-1,-1,1,-1]^T$	2.3
$[1,-1,-1,1]^T$	3.7	$[-1,-1,-1,1]^T$	2.3
$[1,-1,-1,-1]^T$	2.3	$[-1,-1,-1,-1]^T$	6.0

Table 1. *PAPR values of all possible data blocks for an OFDM signal with four subcarriers and BPSK modulation.*

PAP Reduction with Linear block code

- Linear block code
  - A block coding scheme provides error correction capability, and also achieves the minimum PAPR for the OFDM system utilizing QPSK modulation and 4 subcarriers.
  - Block coding approach : by selecting only those codewords with small PAPR. Well-designed block codes provide error correction capability.

- Block diagram of the OFDM signal with the proposed block



$$\mathbf{x} = \mathbf{a} \cdot \mathbf{G} + \mathbf{b} \pmod 2.$$

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \pmod 2$$

$$\mathbf{b} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \pmod 2$$

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \pmod 2$$

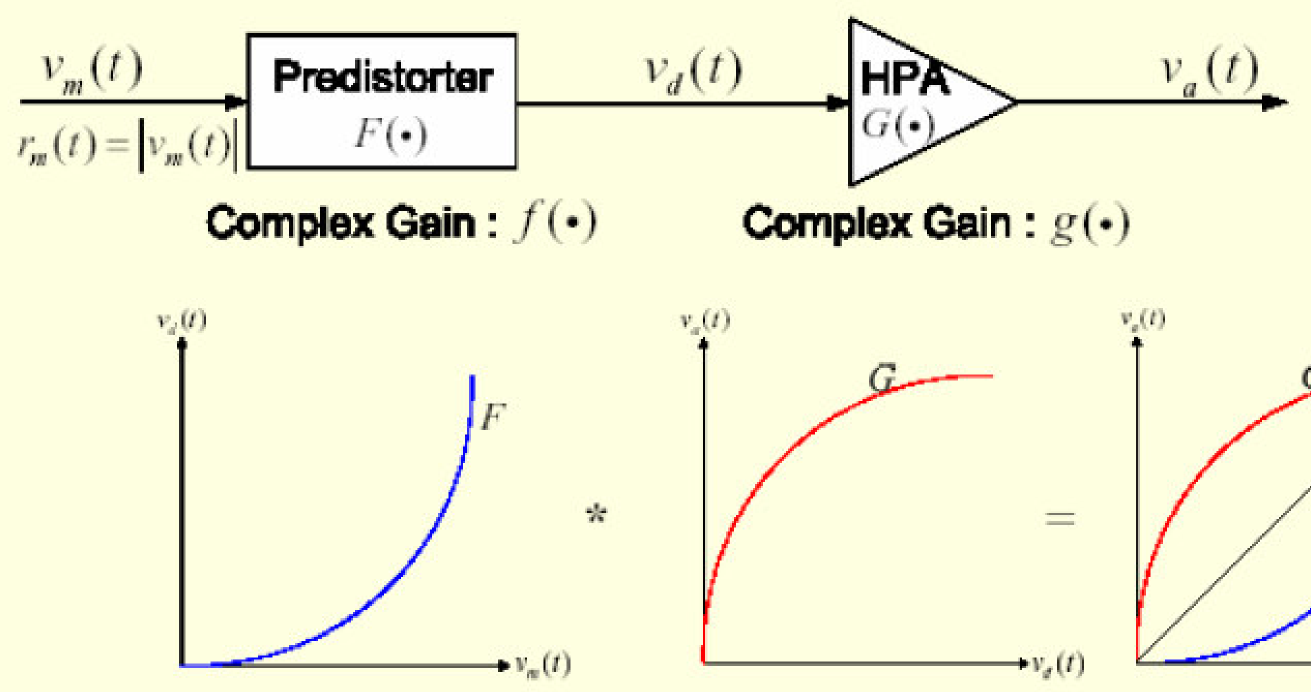
- The 8 bit vector x becomes 4 complex anti-podal symbols

$$y_i' = [2x_{2i} - 1] + j[2x_{2i+1} - 1] \quad (i = 0, 1, 2, 3)$$

Most of the decoding techniques for these codes require an exhaustive search so are feasible only for a small number of subcarriers. Moreover, it is difficult to maintain a reasonable coding rate in OFDM when the number of subcarriers grows large

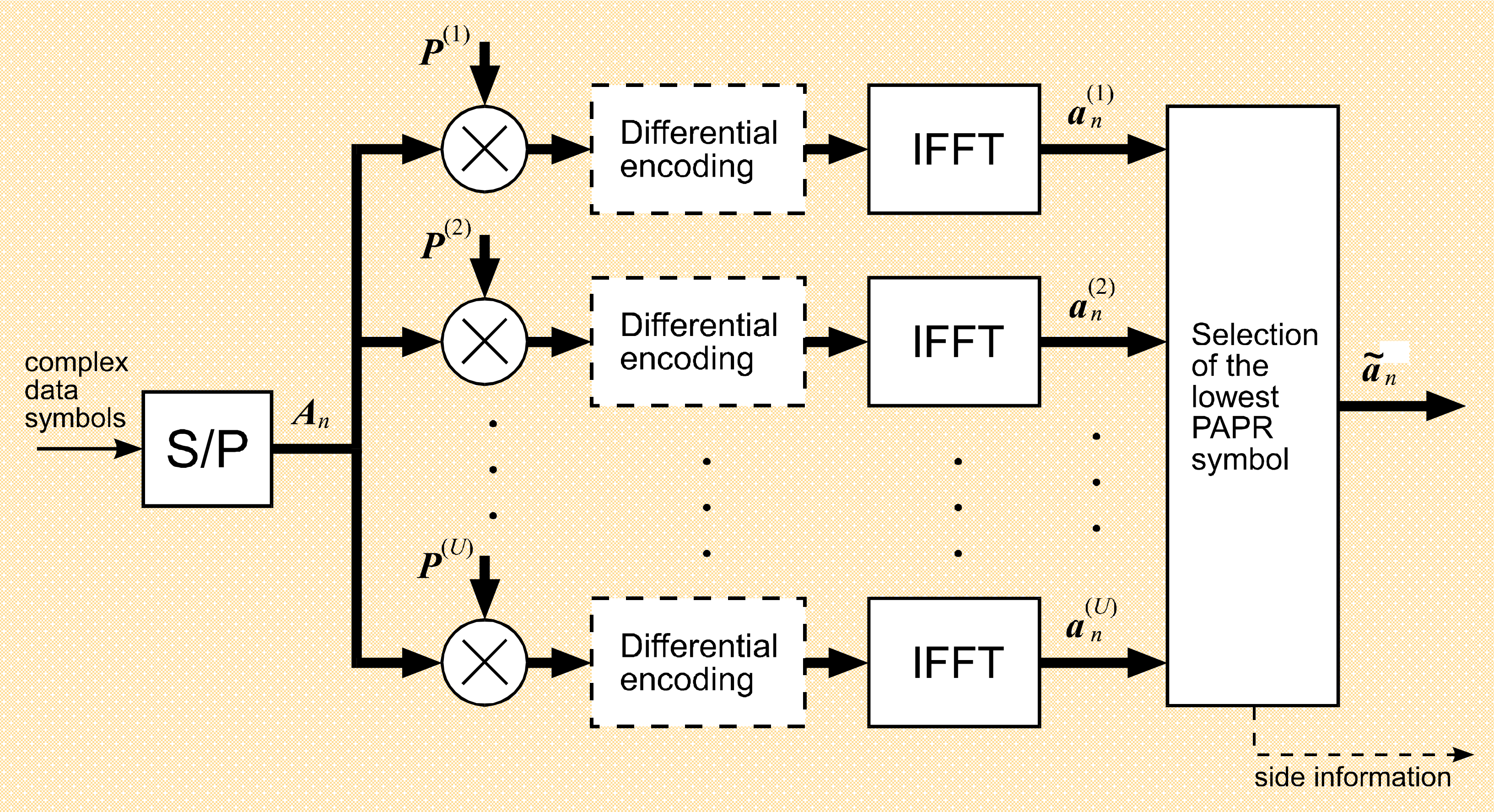
PAP Reduction with Predistortion

- Compander predistortion
  - •Try to predict the transfer function G(v) of the HPA and use its inverse function F(v) as predistortion. Hopefully the composite function is linearlized.
  - •Problem: Difficult to predict G which is device dependent and could be time varying
  - •Many adaptive methods to find the inverse function (LMS, LUT, etc)



PAPR Reduction - Multiple Signal Representation

- Selected Mapping



•Each block is multiplied symbol-by-symbol, before the IFFT operation, by one of pseudo-random but fixed set of vectors ri **whose elements are complex numbers with the amplitude equal to one and a random phase uniformly distributed between**

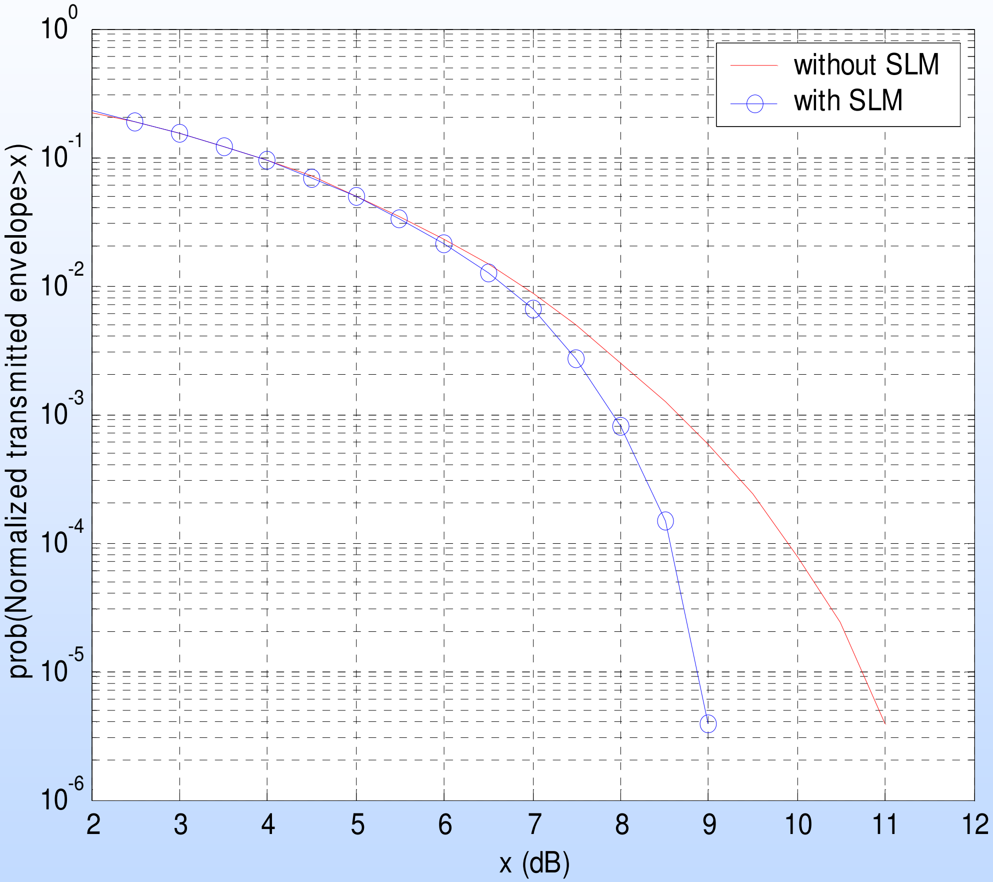
SLM method for OFDM

- **This method is based on generating M statistically independent transformed blocks for each data block and transmitting the one with the lowest PAPR.**

$$\Pr\{PAPR_{1,2,\ldots,M} > \lambda\} = \prod_{i=1}^{M} \Pr\{PAPR_i > \lambda\} = \Pr\{PAPR_1 > \lambda\}^M$$

- **It requires transmitting some side information about the identity of the selected block .**

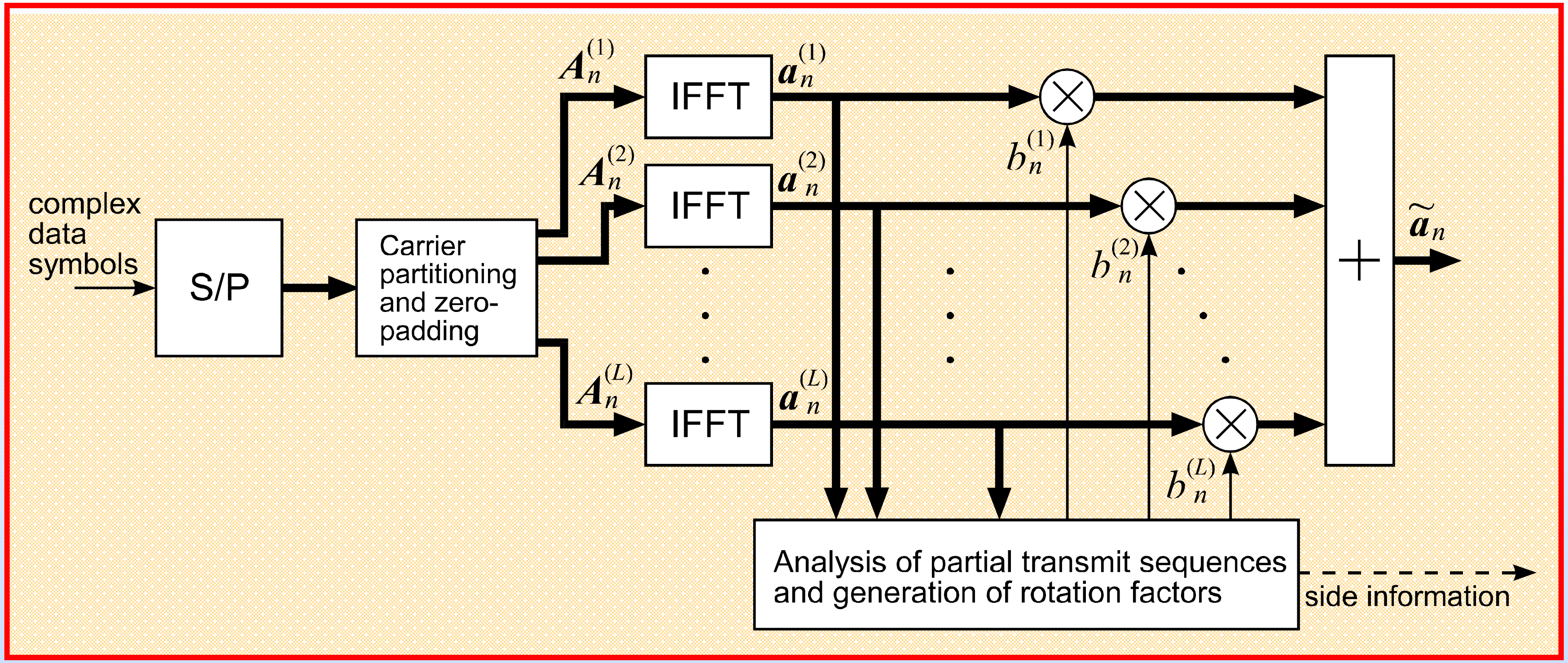
Improvement achieved by SLM on OFDM amplitude distribution



Example: Here, we show a simple example of the SLM technique for an OFDM system with eight subcarriers. We set the number of phase sequences to $U = 4$. The data block to be transmitted is denoted $\mathbf{X} = [1, -1, 1, 1, 1, -1, 1, -1]^T$ whose PAPR before applying SLM is 6.5 dB. We set the four phase factors as $\mathbf{B}^{(1)} = [1, 1, 1, 1, 1, 1, 1, 1]^T$, $\mathbf{B}^{(2)} = [-1, -1, 1, 1, 1, 1, 1, -1]^T$, $\mathbf{B}^{(3)} = [-1, 1, -1, 1, -1, 1, 1, 1]^T$, and $\mathbf{B}^{(4)} = [1, 1, -1, 1, 1, -1, 1, 1]^T$. Among the four modified data blocks $\mathbf{X}^{(u)}$, $u = 1, 2, 3, 4$, $\mathbf{X}^{(2)}$, has the lowest PAPR of 3.0 dB. Hence, $\mathbf{X}^{(2)}$ is selected and transmitted to the receiver. For this data block, the PAPR is reduced from 6.5 to 3.0 dB, resulting in a 3.5 dB reduction. In this case, the number of IDFT operations is 4 and the amount of side information is 2 bits. The amount of PAPR reduction may vary from data block to data block, but PAPR reduction is possible for all data blocks. □

PAPR Reduction - Multiple signal representation, cont.

- Partial Transmit Sequences



•The objective is to design an optimal phase for the subblock set that minimizes the PAR. The phase can then be corrected at the receiver.

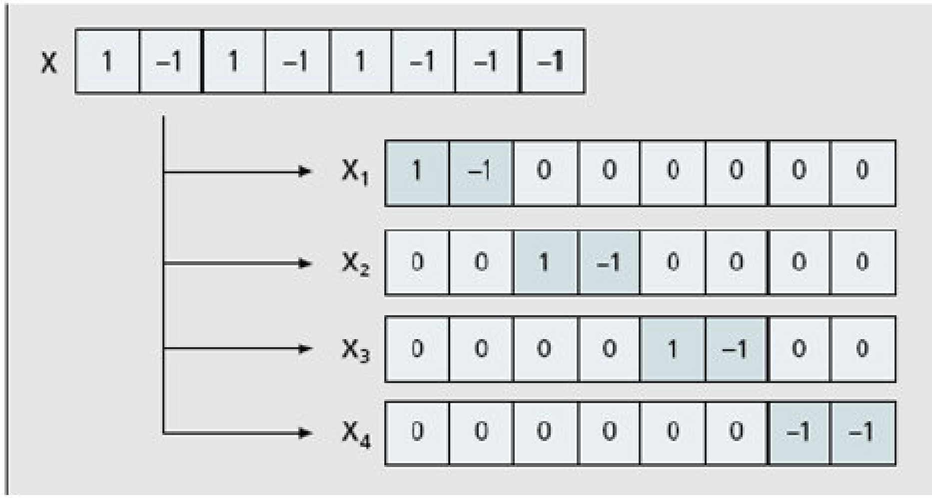
•An input data block of N symbols is partitioned into disjoint subblocks

•The subcarriers in each subblock are weighted by a phase factor for that subblock

•The phase factors are selected such that the PAPS of the combined signal is minimized

An example of PTS

Adjacent subblock partitioning in PTS



- FFT/IFFT is linear $\mathbf{x} = \mathbf{F}^*\mathbf{X} = \mathbf{F}^*(\sum_i \mathbf{X_i})$
- $\mathbf{x}_{pts} = \mathbf{F}^*(\sum_i p_i\mathbf{X_i}) = \sum_i(p_i\mathbf{F}^*\mathbf{X_i})$
- When the phase factors p_i are known in the receiver, $\mathbf{X}$ can be estimated.

How to solve the PAPR problem(1/6)

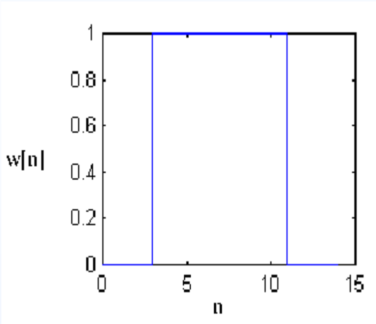
- **Signal distortion**
  - Clipping, Peak windowing, Peak cancellation
- **Coding**
  - Error correction, Use lower PAPR signals
- **Scrambling**
- Selected Mapping (SLM)
- The transmitter selects one favorable transmit signal from a set of sufficiently different signals which all represent the same information.
- □ Partial Transmit Sequence (PTS)
- The transmitter constructs its transmit signal with low PAR by coordinated addition of appropriately phase rotated signal parts.
- The difference between SLM and PTS is that the first applies independent scrambling rotations to all subcarriers, while the latter only applies scrambling rotations to group of subcarriers.

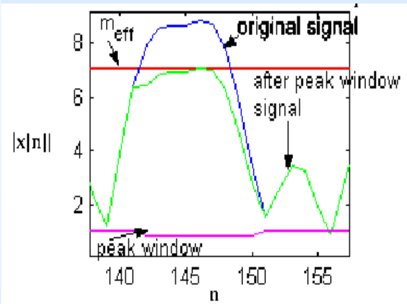
How to solve the PAPR problem(2/6)

- **Peak windowing**
  - Multiplied by certain window function
  - Rectangular, Kasier, Hanning, Hamming …..
- **Different window with same window length**
- **Same window with different window length**

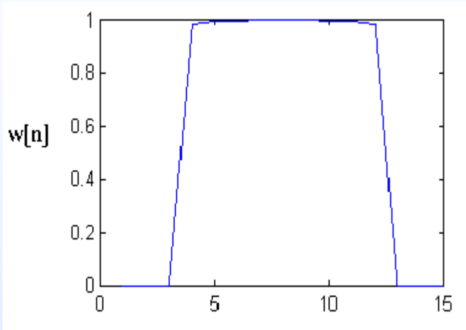
How to solve the PAPR problem(3/6)

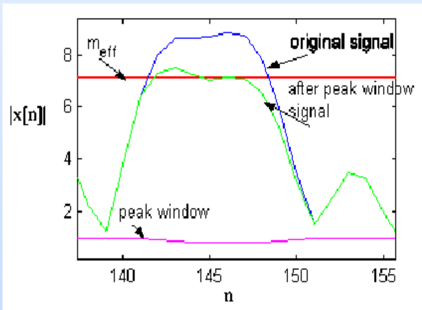
- **Different window with same window length (time domain)**



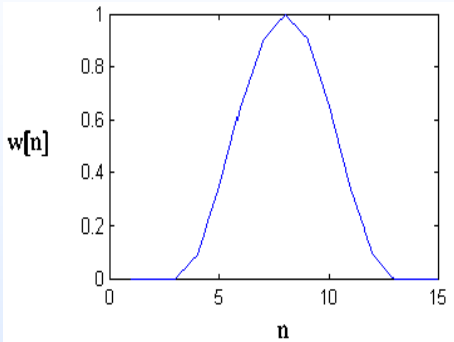


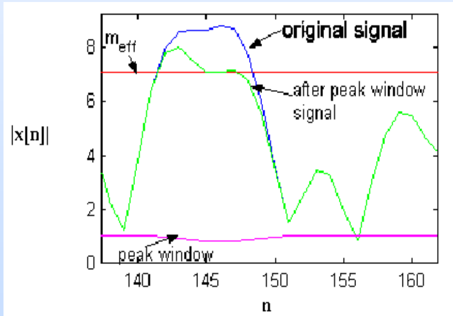
rectangular



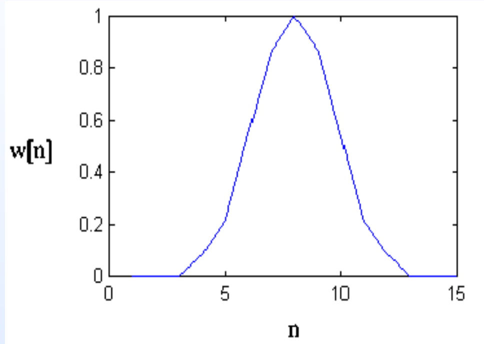


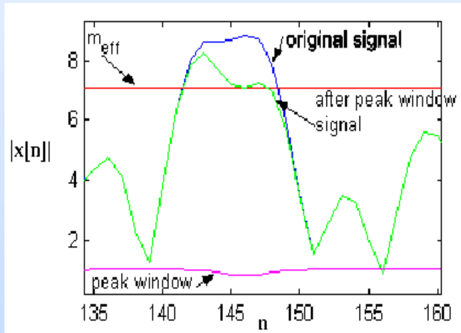
kaisar





hanning

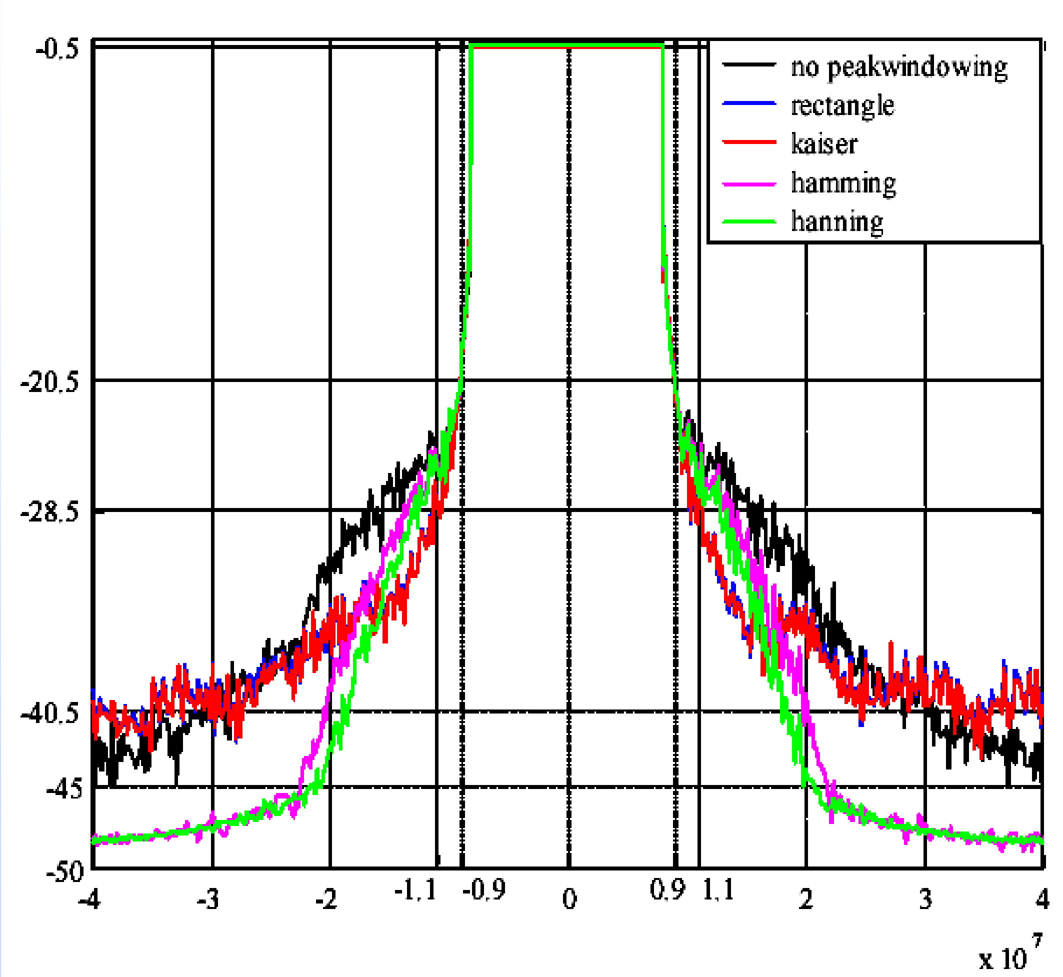




hamming

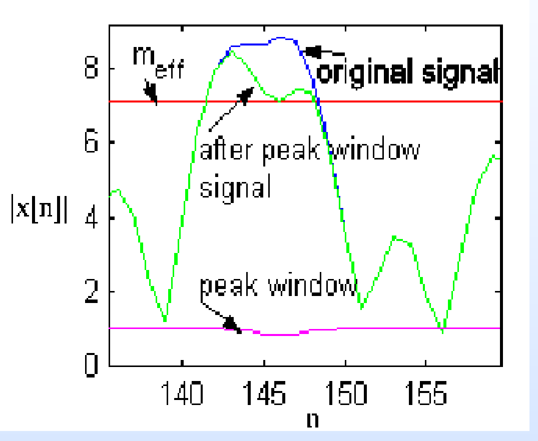
How to solve the PAPR problem(4/6)

- **Different window with same window length (frequency domain)**

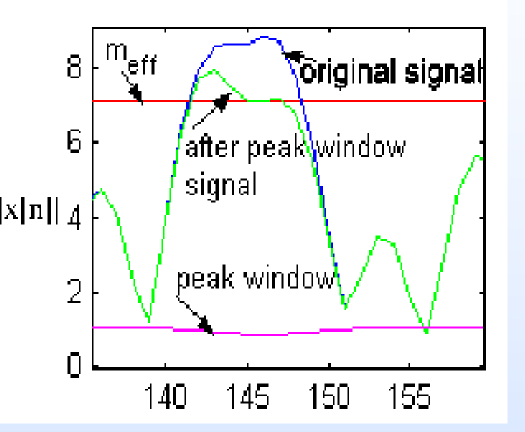


How to solve the PAPR problem(5/6)

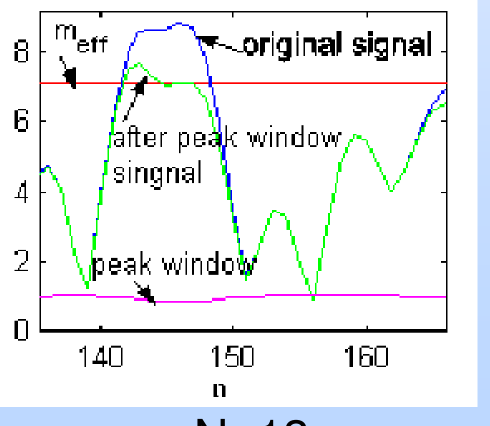
- **Same window with different window length-hamming window (time domain)**



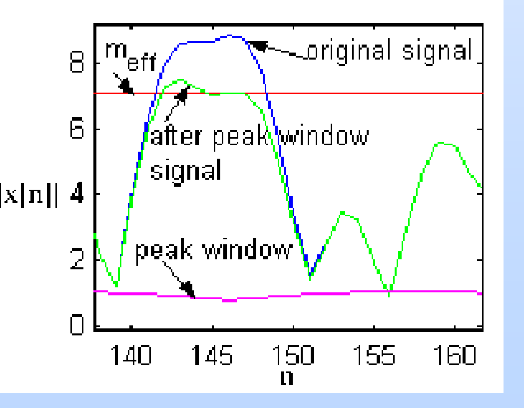
N=7



N=11



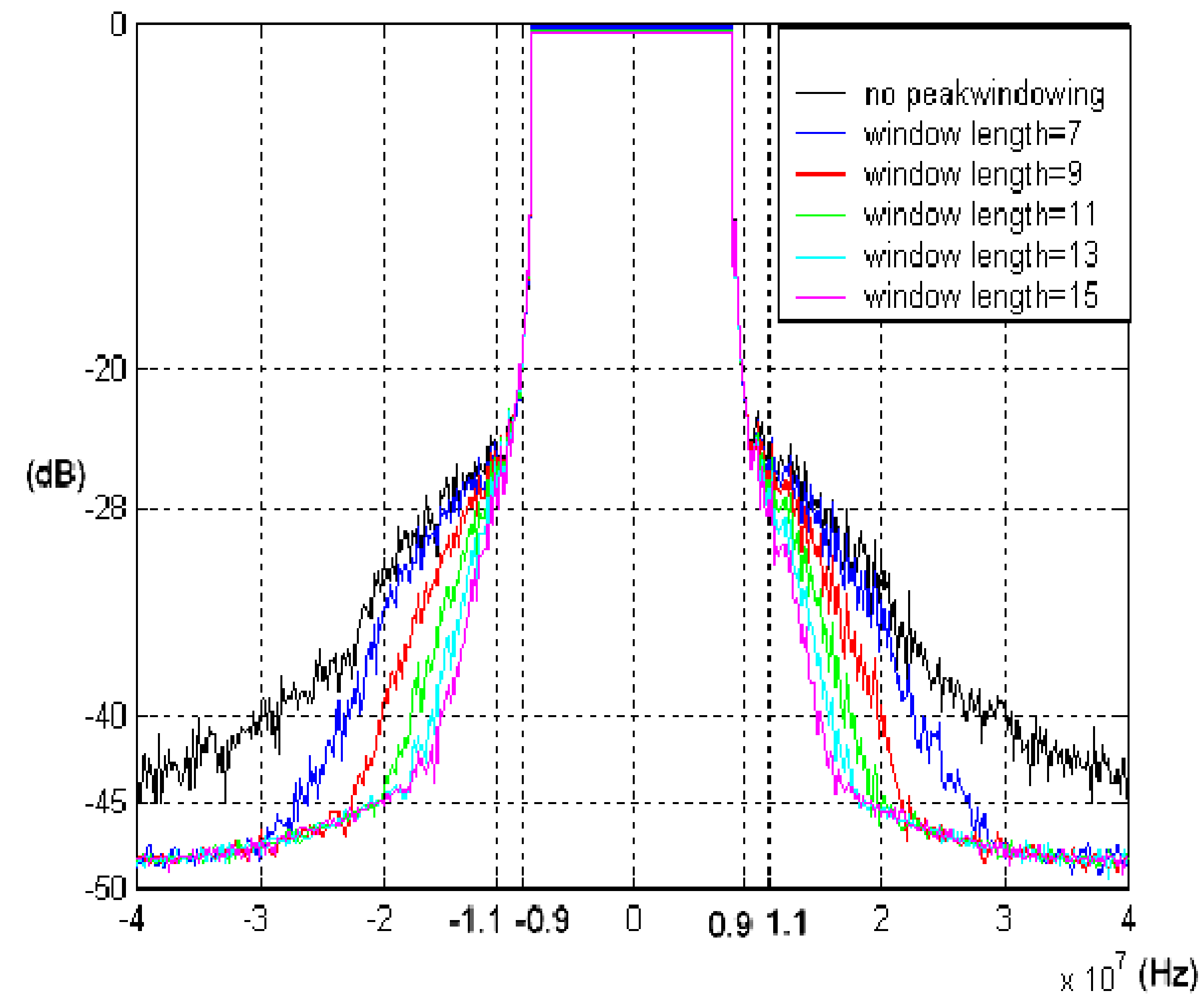
N=13



N=15

How to solve the PAPR problem(6/6)

- **Same window with different window length-hamming window (frequency domain)**



Comparison of PAPR reduction techniques.

	Distortionless	Power increase	Data rate loss	Requires processing at transmitter (Tx) and receiver (Rx
Clipping and filtering	No	No	No	Tx: Amplitude clipping, filtering Rx: None
Coding	Yes	No	Yes	Tx: Encoding or table search Rx: Decoding or table search
PTS	Yes	No	Yes	Tx: M IDFTs, W^{M-1} complex vector sums Rx: Side information extraction, inverse PTS
SLM	Yes	No	Yes	Tx: U IDFTs Rx: Side information extraction, inverse SLM
Interleaving	Yes	No	Yes	Tx: K IDFTs, $(K - 1)$ interleavings Rx: Side information extraction, inverse interleaving
TR	Yes	Yes	Yes	Tx: IDFTs, find value of PRCs Rx: Ignore non-data-bearing subcarriers
TI	Yes	Yes	No	Tx: IDFTs, search for maximum point in time, tones to be modified, value of p and q Rx: Modulo-D operation
ACE	Yes	Yes	No	Tx: IDFTs, projection onto "shaded area" Rx: None