ECE5984

Orthogonal Frequency Division Multiplexing and Related Technologies

Fall 2007

Mohamed Essam Khedr

OFDM Basics I

Textbook

- **OFDM and MC-CDMA for broadband multi-user communications by Lajos Hanzo et al**

- **Additional Readings:**

- **Richard van Nee and Ramjee Prasad, OFDM for Wireless Multimedia Communications, Artech House: 2000 (ISBN: OR90065306)**
- **Orthogonal Frequency Division Multiplexing for Wireless Communications by Ye (Geoffrey) Li (Editor), Gordon L. Stuber (Editor), ISBN 0387290958**
- **Ahmad Bahai and Burton Saltzberg, Multi-Carrier Digital Communications: Theory and Applications of OFDM, Plenum Publishing Corporation: 1999, ISBN: 0306462966.**

Syllabus

- **Wireless channels characteristics** (7.5%) **1**
  - wireless channel modeling and characteristics
    - Large scale and small scale models
    - Common channel models
    - Channel categories and parameter calculation.
    - **Prob. of error calculations**
- **OFDM Basics** (10%) **1**
  - History of OFDM
  - OFDM System model
  - Discrete-time signals & systems and DFT
  - Generation of subcarriers using the IFFT
  - Guard time, cyclic extension
  - Windowing
  - Choice of OFDM parameters & OFDM signal processing
  - Implementation complexity of OFDM versus single carrier modulation
- **Modulation and Coding** (10%) **2**
  - Linear and nonlinear modulation
  - Interleaving and channel coding
  - Optimal bit and power allocation
  - Adaptive modulation

Syllabus

- **Analysis of OFDM systems** (15%) **2**
  - RF subsystems, amplifier classification and distortion
  - Crest factor (PAPR) reduction techniques
    - Pre-distortion & adaptive pre-distortion techniques
    - clipping
    - coding techniques
    - partial transmit sequences (PTS) & modified PTS v. selective mapping
    - nonlinear quantization (companding)
  - Phase noise and I&Q imbalance for QAM
  - Performance of OFDM in Gaussian channels
  - Performance of OFDM in Wide-band channels
- **Synchronization and Estimation** (15%) **2**
  - ICI and OISI problems
  - Timing estimation
  - Frequency synchronization
  - Frequency error estimation algorithms
  - Carrier phase tracking
  - Frequency domain and time domain approaches for channel estimation
    - coherent detection
    - differential detection

Syllabus

- **Multi-user OFDM Techniques** (10%) 2
  - Adaptive modulations in OFDM
  - Power and bit allocations in OFDM
  - Scalable OFDM
  - Flash OFDM
- **Diversity** (7.5%) **1**
  - Limits of capacity in fading environments
  - Channel models for multiple-input-multiple-output (MIMO) system
  - Receiver diversity techniques
  - Transmit diversity techniques and design criteria for fading channels
  - Block, trellis and layered space-time codes
- **Multi-carrier CDMA** (10%) 1
  - MC-CDMA versus DS-CDMA
  - MC-CDMA versus orthogonal frequency division multiple access (OFDMA)
  - OFDMA and MC-CDMA performance evaluation in wide-band channels

Syllabus

- **Physical and Medium Access Control (MAC) for IEEE 802.11 Networks** (7.5%) **1**
  - Physical modeling of 802.11 networks
  - MAC system architecture
  - Frame exchange with RTS/CTS
  - Power management
  - Synchronization
- **Physical and Medium Access Control (MAC) for IEEE 802.16 Networks** (7.5%) 1
  - Physical modeling of 802.16 networks
  - MAC system architecture
  - QoS guarantees in Wimax
  - Power management
  - Synchronization

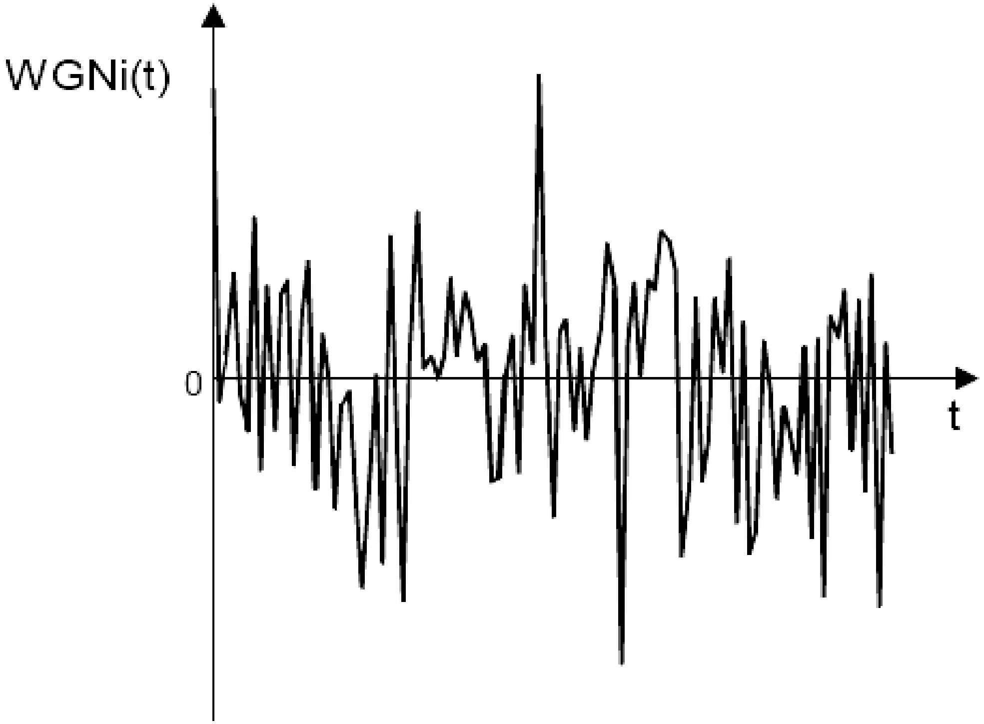
Grading

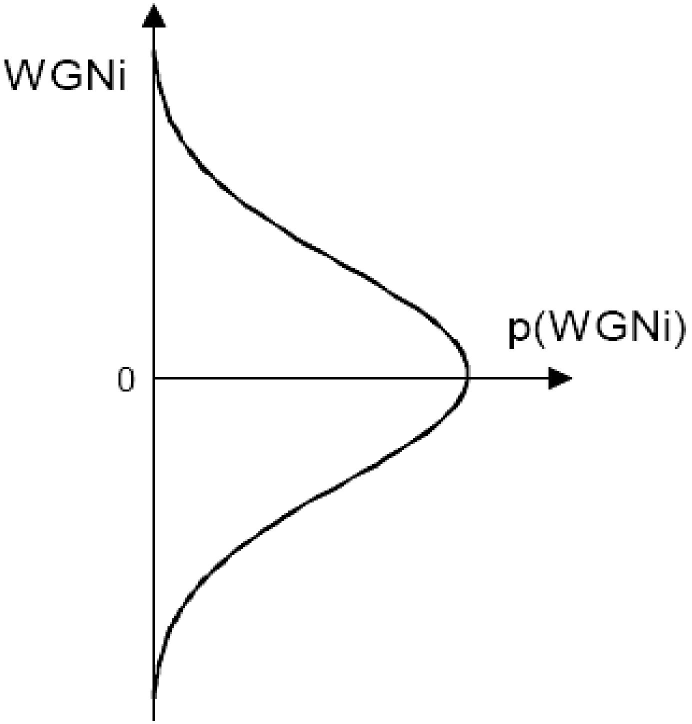
Type of assignment	**Percent of Grade**
Home works	20%
Matlab Assignments	20%
Midterm	20%
Final project presentation and term paper	20%
Final Exam	20%

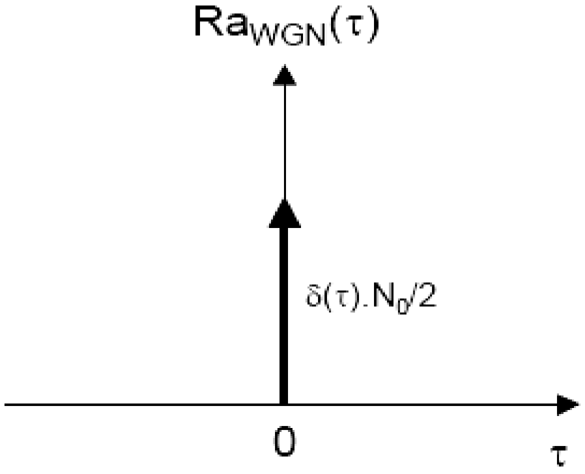
Matlab Assignment 1

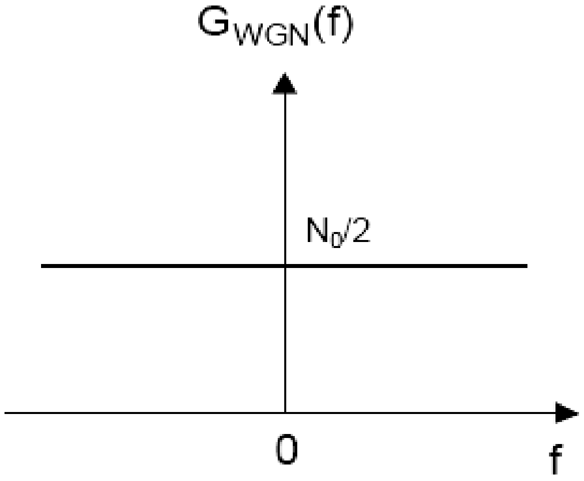
- **Develop a WGN channel that accepts symbol input, adds noise to it with certain SNR and produces the noisy ouput.**
- **Develop a Flat Fading Channel following Rayleigh and ricean distribution**
- **Develop a Frequency selective channel following Rayleigh and ricean distribution**
  - All inputs to the channel are baseband signals.
  - Compare with the matlab functions (if exist)
  - Plot the probability of error vs SNR

Lecture one Fading channels

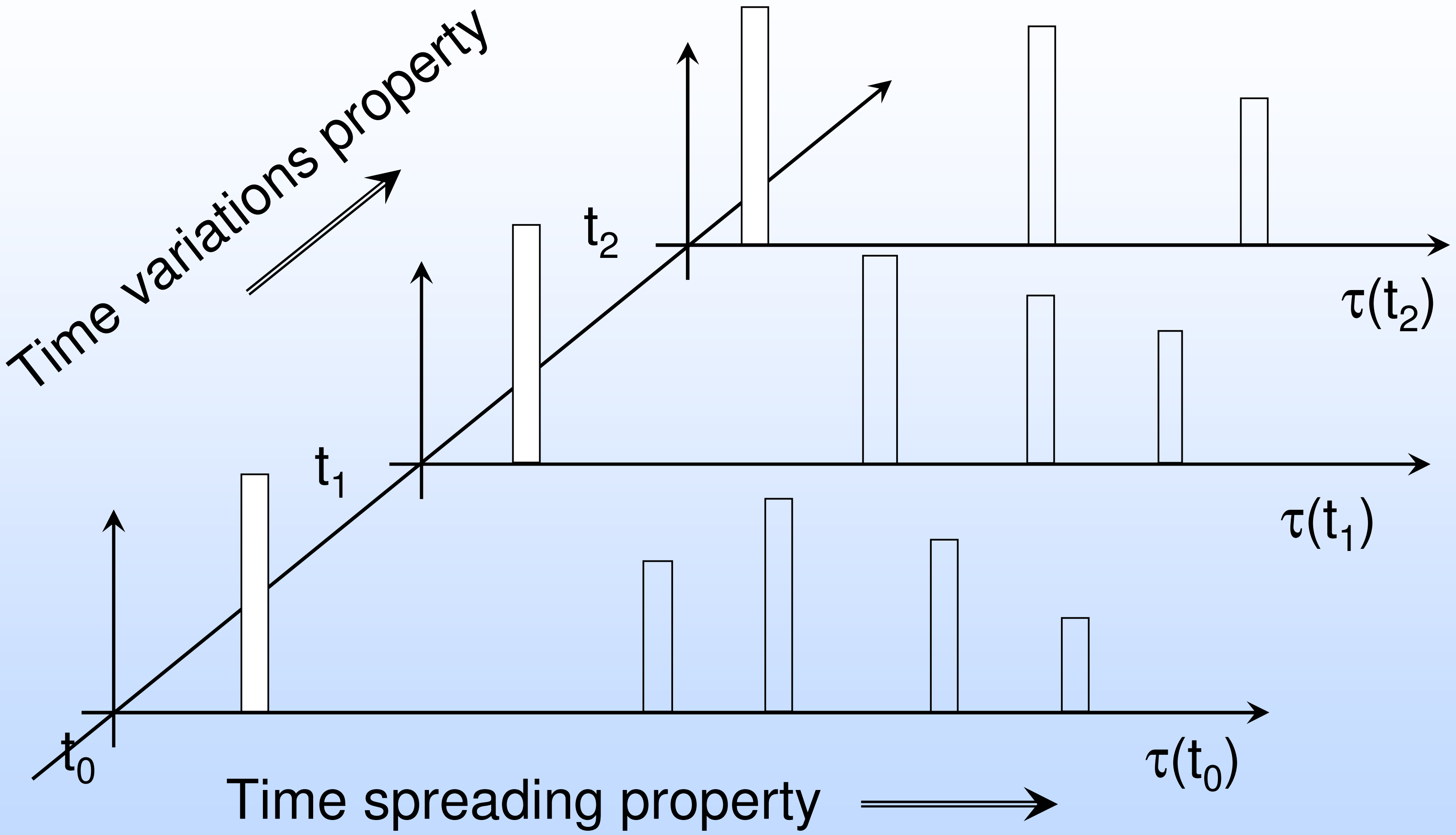








Impulse Response Characterization

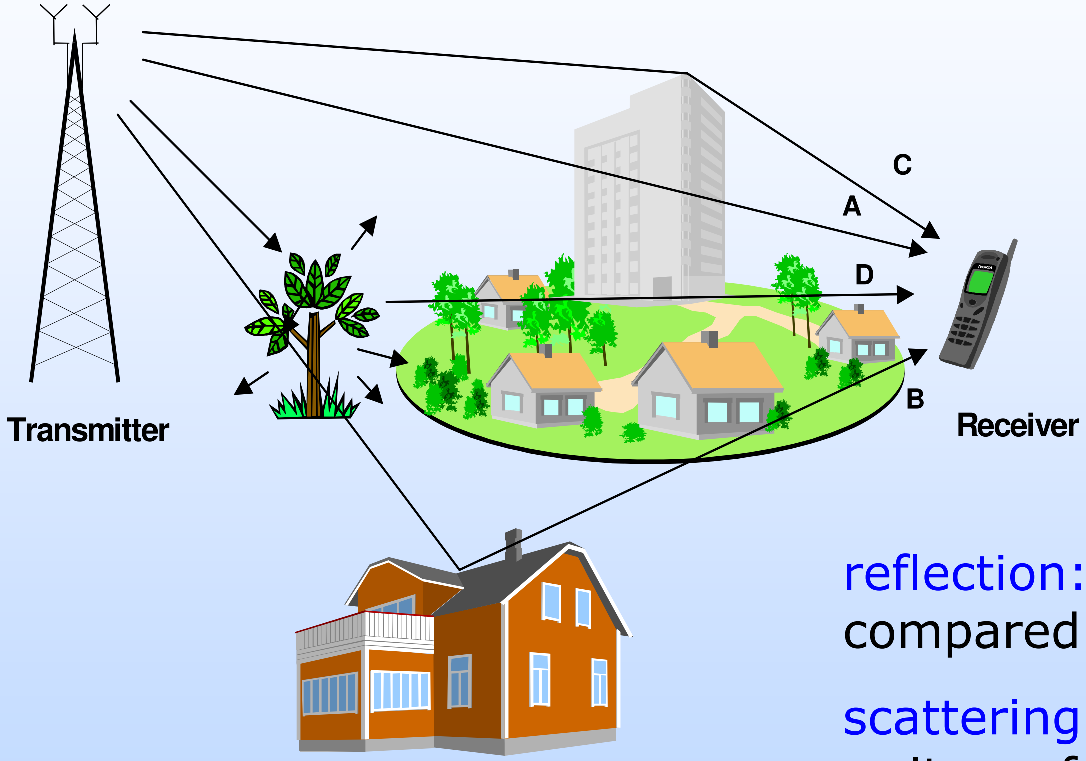

Time variations property

Time spreading property

- Impulse response: Time-spreading : multipath

  and time-variations: time-varying environment

Propagation mechanisms

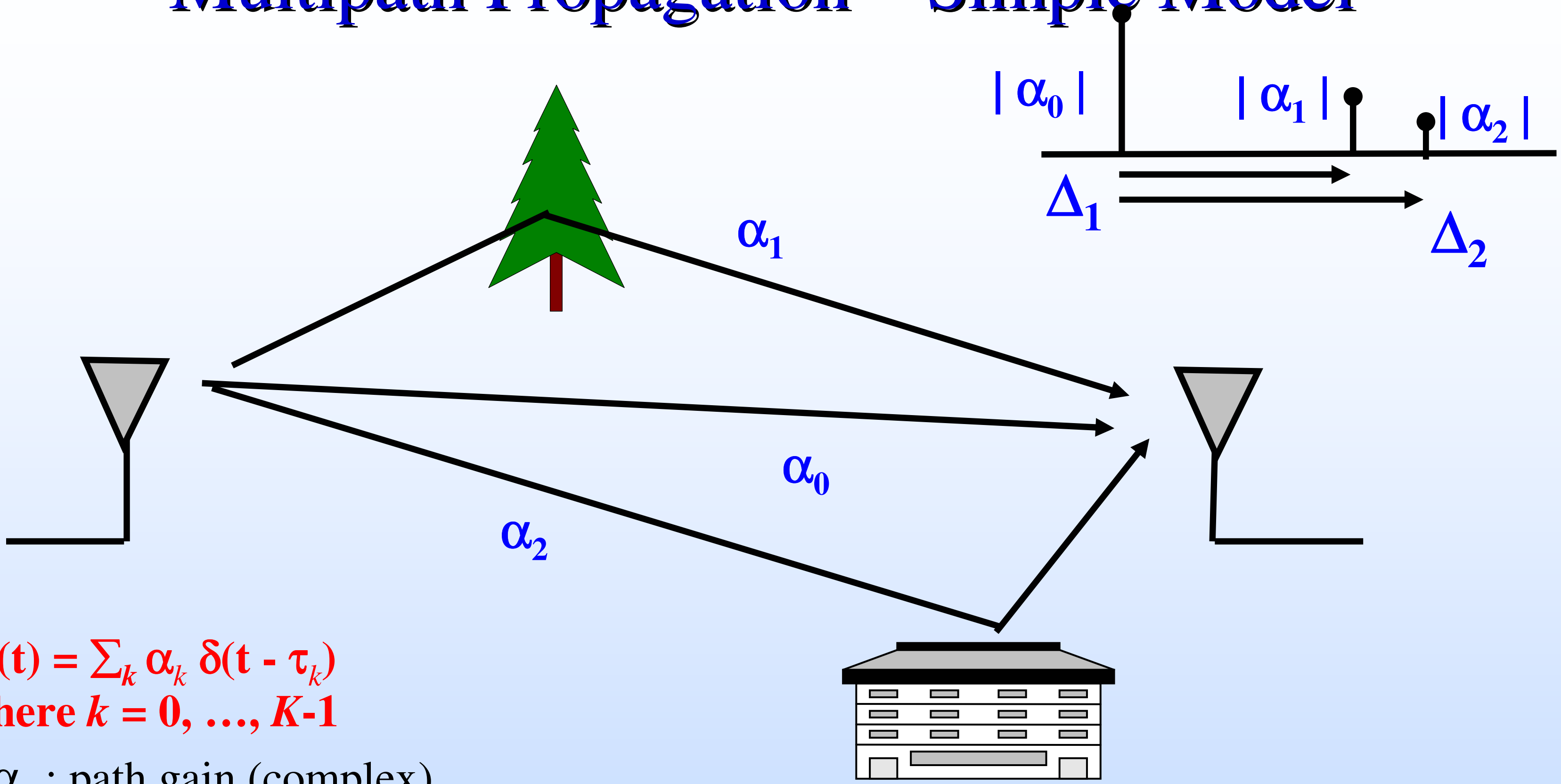


A: free space
B: reflection
C: diffraction
D: scattering

reflection: object is large compared to wavelength

scattering: object is small or its surface irregular

Multipath Propagation – Simple Model



- **$h_c(t) = \sum_k \alpha_k \delta(t - \tau_k)$**
  where $k = 0, \dots, K-1$

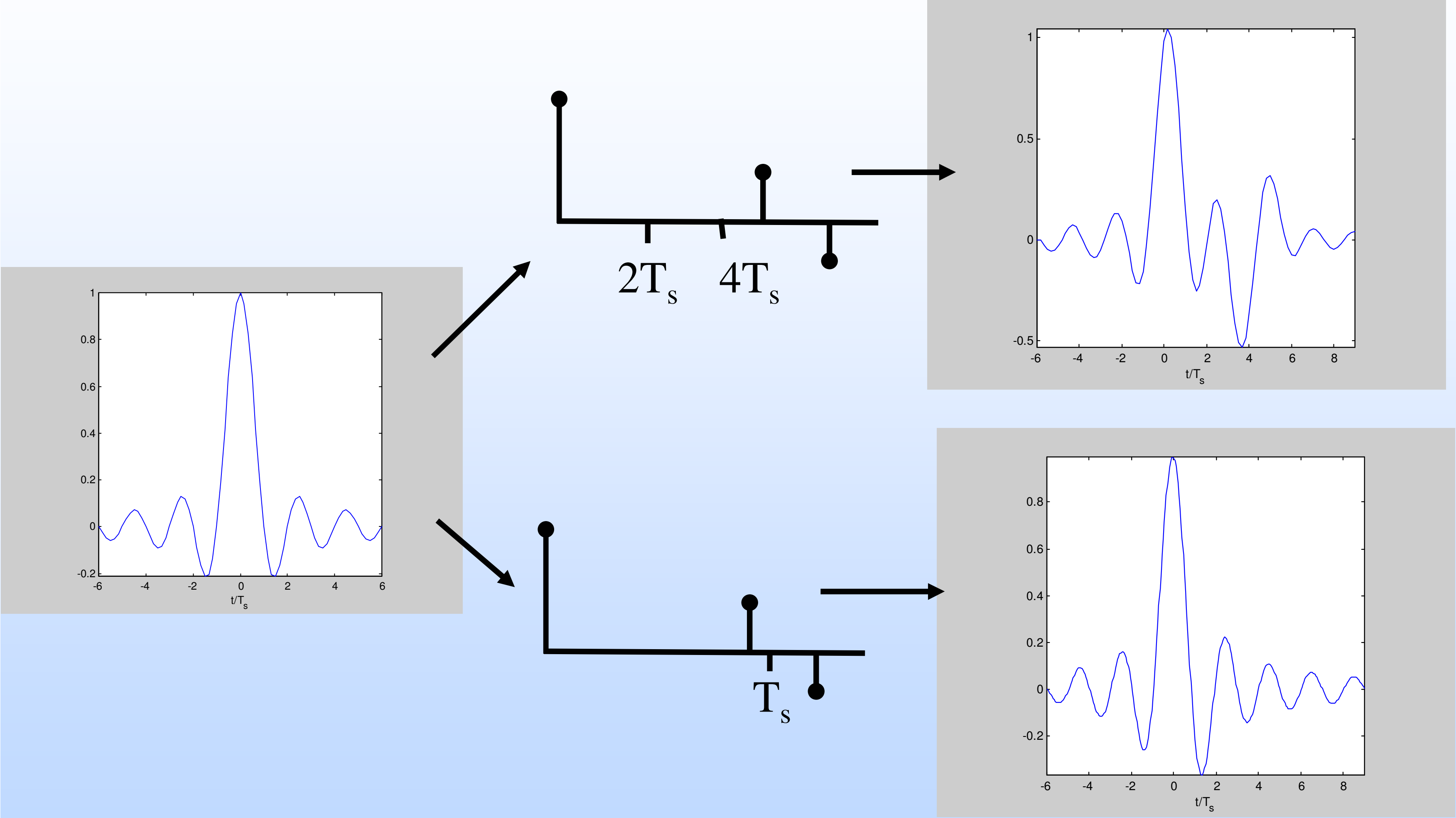
  α_k : path gain (complex)

  $\tau_0 = 0$ normalize relative delay of first path

  $\Delta_k = \tau_k - \tau_0$ difference in time-of-flight

$$h(\tau,t)=\sum_{i=0}^{L-1} \underbrace{a_i(t)}_{\text{path attenuation}}\, \underbrace{e^{j\phi_i(t)}}_{\text{path phase}}\, \underbrace{\delta(\tau-\tau_i)}_{\text{path delay}}$$

Impact of Multipath: Delay Spread & ISI

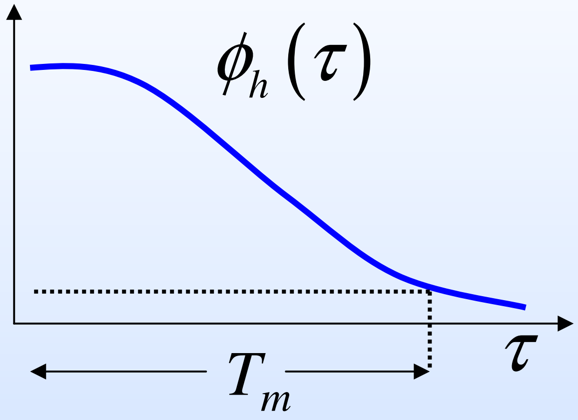


Max delay spread = effective number of symbol periods occupied by channel

⇒ **Requires equalization to remove resulting ISI**

Stochastical (WSSUS) channel variables

Maximum delay spread: T_m



Maximum delay spread may be defined in several ways.

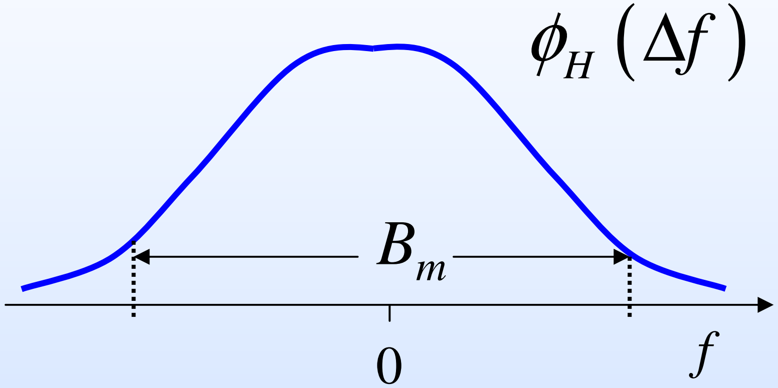
For this reason, the RMS delay spread is often used instead:

$$\sigma_\tau = \sqrt{\frac{\int \tau^2 \phi_h(\tau)\,d\tau}{\int \phi_h(\tau)\,d\tau} - \left[\frac{\int \tau \phi_h(\tau)\,d\tau}{\int \phi_h(\tau)\,d\tau}\right]^2}$$

Stochastical (WSSUS) channel variables

Coherence bandwidth
of channel:

$$B_m \approx 1/T_m$$



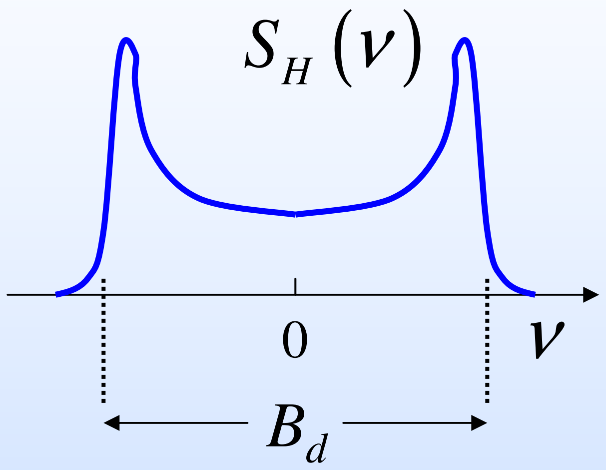
Implication of
coherence bandwidth:

If two sinusoids (frequencies) are spaced much less apart than B_m , their fading performance is similar.

If the frequency separation is much larger than B_m , their fading performance is different.

Stochastical (WSSUS) channel variables

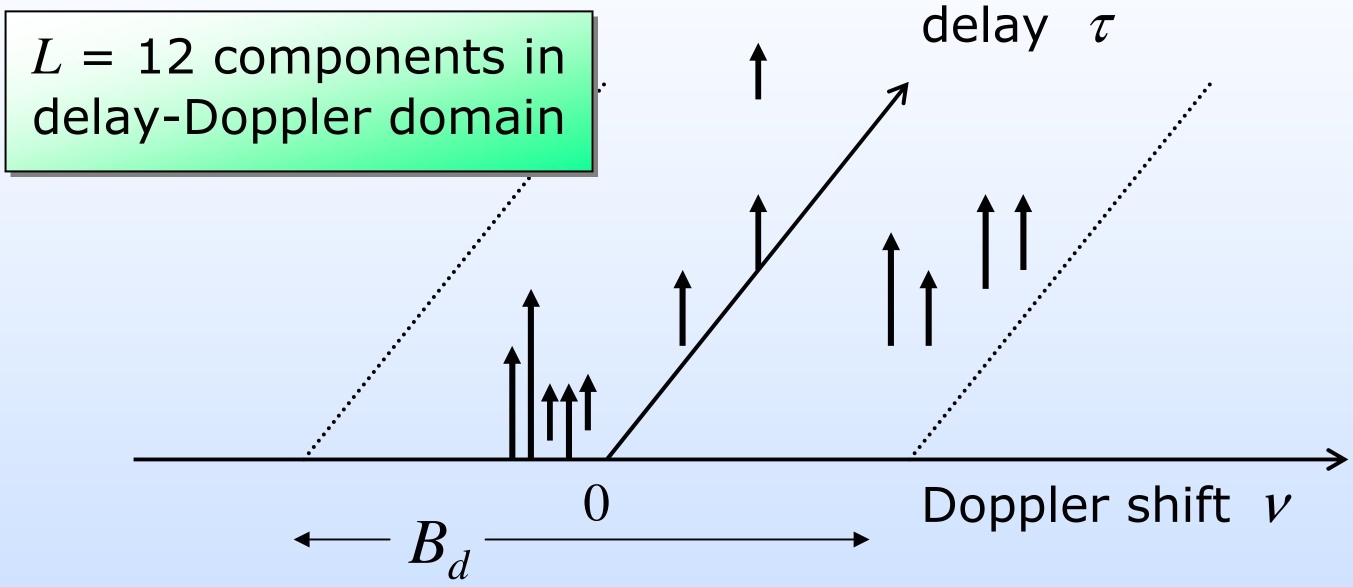
Maximum Doppler spread: B_d



The Doppler spectrum is often U-shaped (like in the figure on the right). The reason for this behaviour is the relationship

$$\nu = \frac{V}{\lambda}\cos\alpha = f_d \cos\alpha$$

Delay - Doppler spread of channel



L = 12 components in delay-Doppler domain

$$h(\tau,t)=\sum_{i=0}^{L-1} a_i(t)\,e^{j(2\pi\nu_i t+\phi_i)}\,\delta(\tau-\tau_i)$$

Statistical Models

- Design and performance analysis based on statistical ensemble of channels rather than specific physical channel.

- Rayleigh flat fading model: many small scattered paths

$$h_\ell[m] \approx \sum_i a_i e^{-j2\pi f_c \tau_i}$$

Complex circular symmetric Gaussian .

$$h[m] \sim \mathcal{N}(0, \frac{1}{2}) + j\mathcal{N}(0, \frac{1}{2}) \sim \mathcal{CN}(0, 1)$$

- Rician model: 1 line-of-sight plus scattered paths

$$h[m] \sim \sqrt{\kappa} + \mathcal{CN}(0, 1)$$

Fading distributions (Rayleigh)

In a flat fading channel, the (time-variant) CIR reduces to a (time-variant) complex channel coefficient:

$$c(t) = a(t)\,e^{j\phi(t)} = x(t) + j\,y(t) = \sum_i a_i(t)\,e^{j\phi_i(t)}$$

When the quadrature components of the channel coefficient are independently and Gaussian distributed, we get:

$$p(a) = \frac{a}{\sigma^2}\,e^{-a^2/2\sigma^2} \qquad\qquad p(\phi) = \frac{1}{2\pi}$$

Rayleigh distribution

Uniform distribution

Fading distributions (Rice)

In case there is a strong (e.g., LOS) multipath component in addition to the complex Gaussian component, we obtain:

$$c\left(t\right)=a_0+a\left(t\right)e^{j\phi\left(t\right)}=a_0+\sum_i a_i\left(t\right)e^{j\phi_i\left(t\right)}$$

From the joint (magnitude and phase) pdf we can derive:

$$p\left(a\right)=\frac{a}{\sigma^2}e^{-\left(a^2+a_0{}^2\right)/2\sigma^2}I_0\left(\frac{aa_0}{\sigma^2}\right)$$

Rice distribution

Modified Bessel function of first kind and order zero

Channel Classification

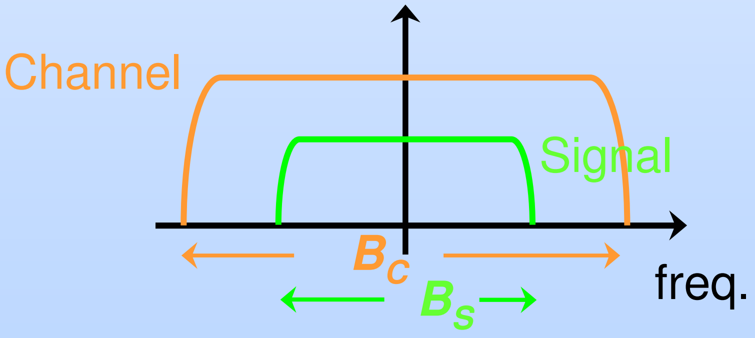
- Based on Time-Spreading

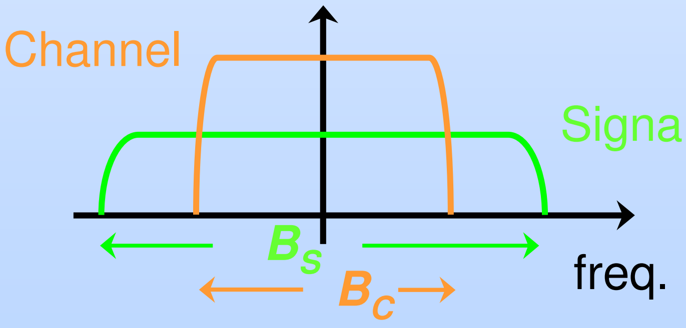
Flat Fading

1. $B_S < B_C \Leftrightarrow T_m < T_s$
2. Rayleigh, Ricean distrib.
3. Spectral char. of transmitted signal preserved

Frequency Selective

1. $B_S > B_C \Leftrightarrow T_m > T_s$
2. Intersymbol Interference
3. Spectral chara. of transmitted signal not preserved
4. Multipath components resolved





Channel Classification

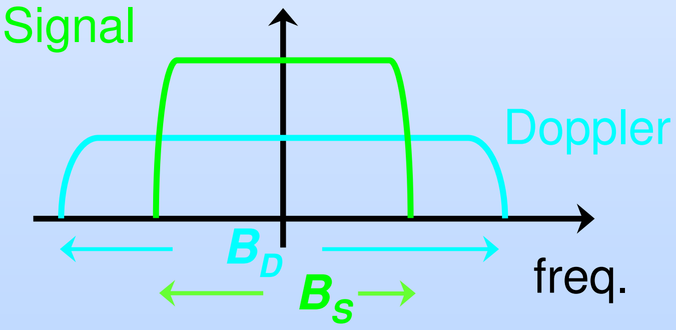
- Based on Time-Variations

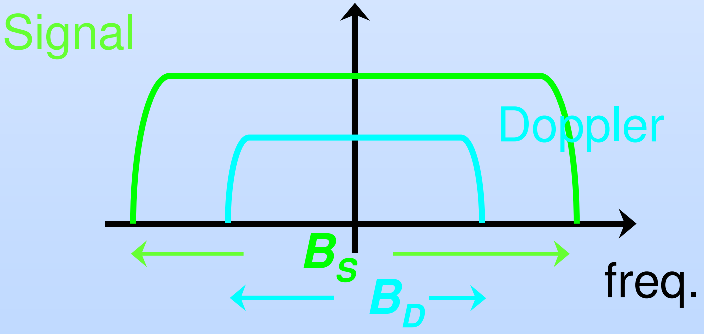
Fast Fading

1. High Doppler Spread
2. $1/B_d \cong T_C < T_s$

Slow Fading

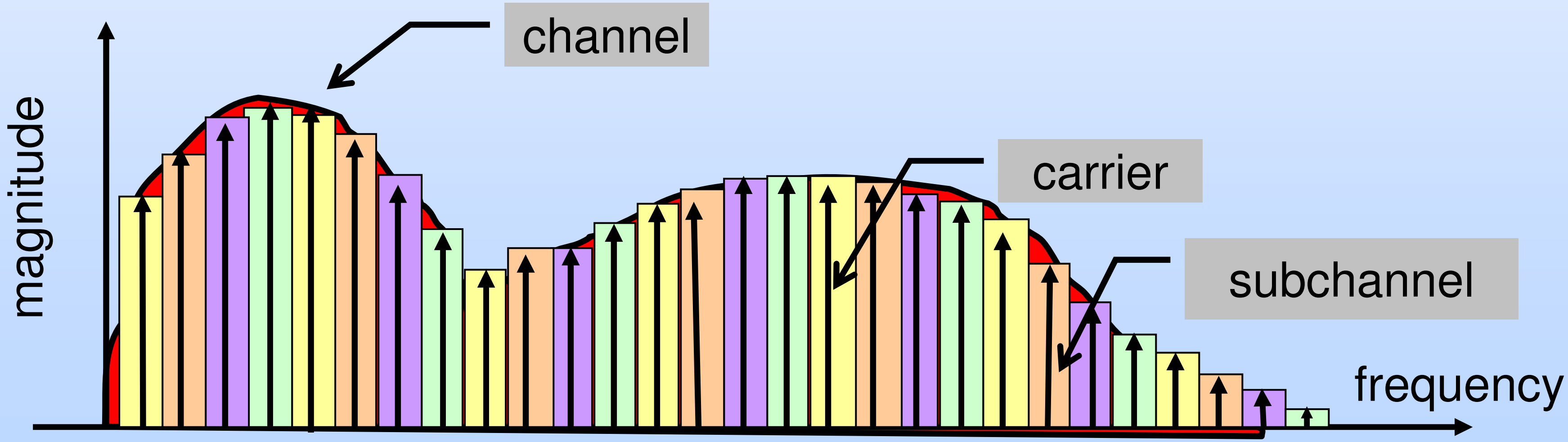
1. Low Doppler Spread
2. $1/B_d \cong T_C > T_s$





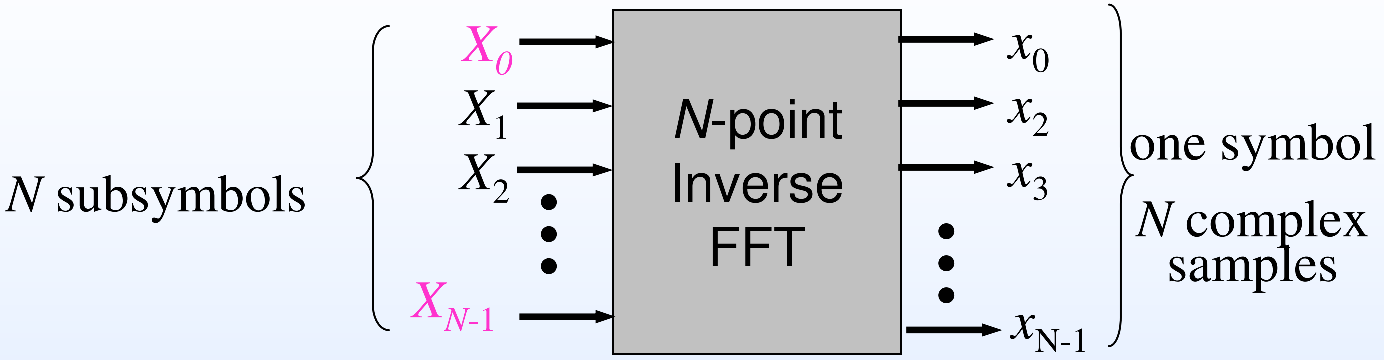
Multicarrier Modulation

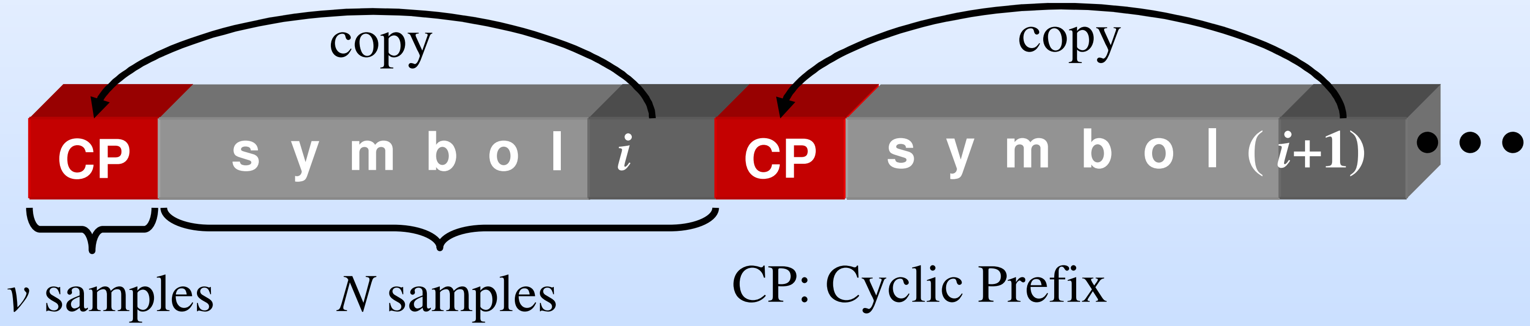
- **Divide broadband channel into narrowband subchannels**
  - No ISI in *subchannels* if constant gain in every subchannel and if ideal sampling
- **Orthogonal Frequency Division Multiplexing**
  - Based on the fast Fourier transform
  - Standardized for DAB, DVB-T, IEEE 802.11a, 802.16a, HyperLAN II
  - Considered for fourth-generation mobile communication systems



Subchannels are 312 kHz wide in 802.11a and HyperLAN II

An OFDM Symbol





CP: Cyclic Prefix

- **Bandpass transmission allows for complex waveforms**
  - **Transmit:** $y(t) = \mathrm{Re}\{(I(t)+j\,Q(t))\exp(j2\pi f_c t)\}$

$$= I(t)\cos(2\pi f_c t) - Q(t)\sin(2\pi f_c t)$$

Introduction to OFDM

- Basic idea
  - » Using a large number of parallel narrow-band sub-carriers instead of a single wide-band carrier to transport information
- Advantages
  - » Very easy and efficient in dealing with multi-path
  - » Robust again narrow-band interference
- Disadvantages
  - » Sensitive to frequency offset and phase noise
  - » Peak-to-average problem reduces the power efficiency of RF amplifier at the transmitter
- Adopted for various standards
  - – DSL, 802.11a, DAB, DVB

OFDM can achieve large delay spread tolerance at high bit rates by:

Converting single bit stream in *N* parallel bit streams

- Symbol duration is increased, so relative delay spread decreases
- Each parallel bit stream is modulated on one of *N subcarriers*

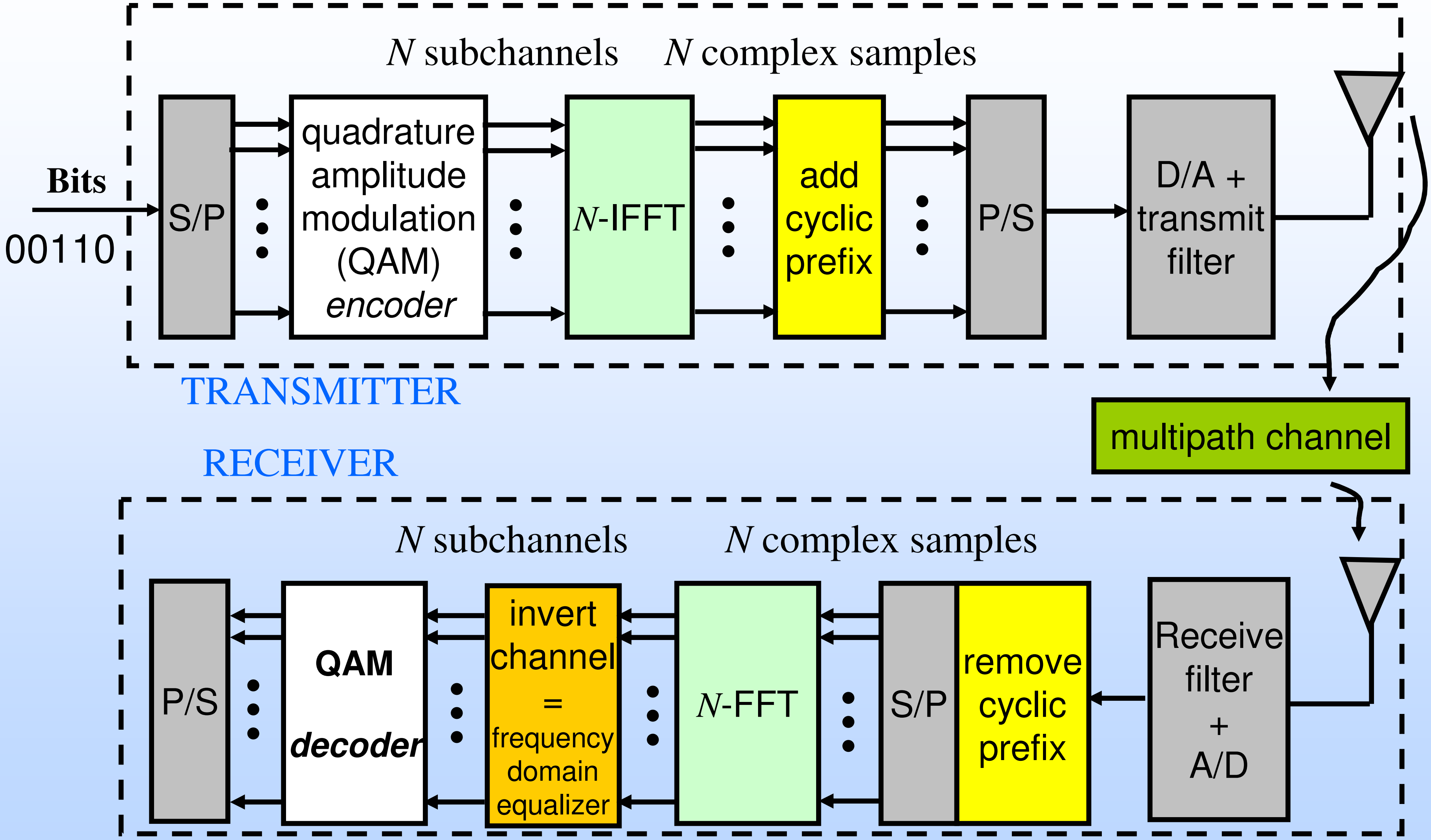
Adding a guard time to each OFDM symbol

- Inter Symbol Interference (ISI) is avoided
- Guard loss is made small (<1 dB) by choosing *N* large enough

Use FEC coding to correct for subcarriers in deep fades

- In a multipath channel, subcarriers have different amplitudes
- By using coding, performance is determined by average received power rather than lowest subcarrier power

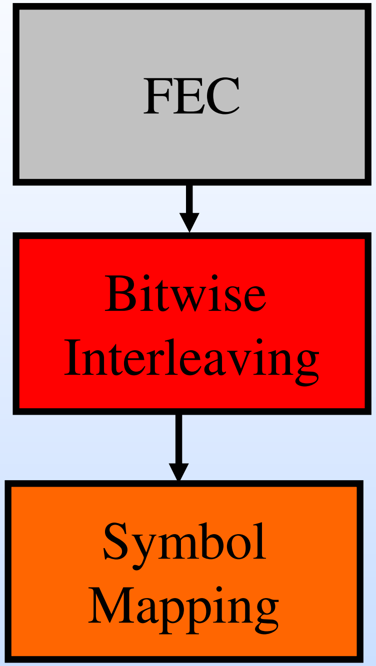
An OFDM Modem



Coded OFDM (COFDM)

- **Error correction is *necessary* in OFDM systems**
- **Forward error correction (FEC)**
  - Adds redundancy to data stream
  - Examples: convolutional codes, block codes
  - Mitigates the effects of bad channels
  - Reduces overall throughput according to the coding rate k/n
- **Automatic repeat request (ARQ)**
  - Adds error detecting ability to data stream
  - Examples: 16-bit cyclic redundancy code
  - Used to detect errors in an OFDM symbol
  - Bad packets are retransmitted (hopefully the channel changes)
  - Usually used with FEC
  - Minus: Ineffective in broadcast systems

Typical Coded OFDM Encoder



FEC

Bitwise Interleaving

Symbol Mapping

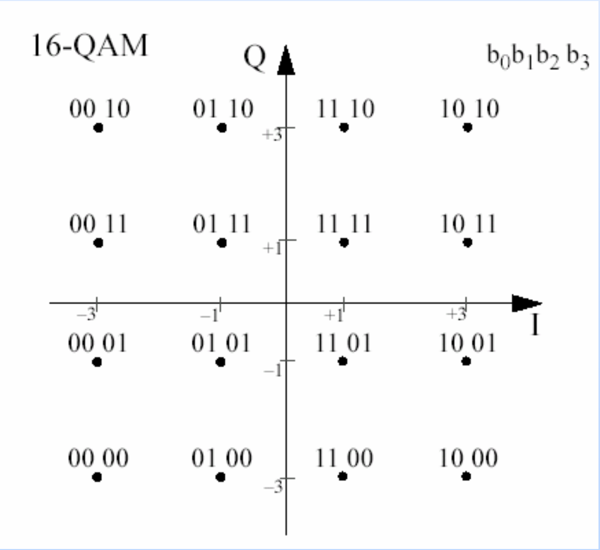
- Reed-Solomon and/or convolutional code

Data bits	Parity bits

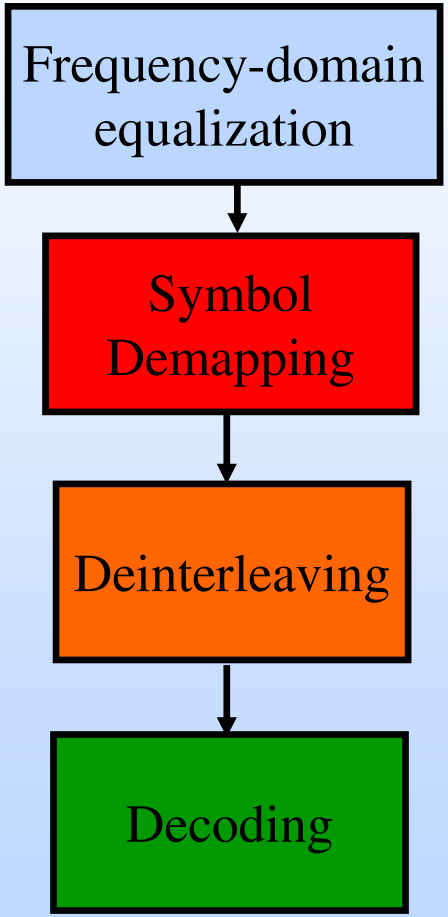
Rate 1/2

- Intersperse coded and uncoded bits

- Map bits to symbols



Typical Coded OFDM Decoder



Frequency-domain equalization

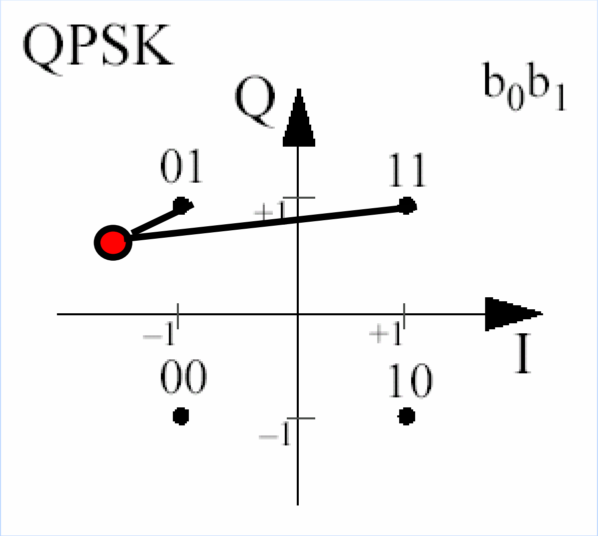
Symbol Demapping

Deinterleaving

Decoding

- **Symbol demapping**
  - Produce soft estimate of each bit
  - Improves decoding

$$\begin{aligned} L(b_k) &= \log\left(\frac{Pr(x|b_k=1)}{Pr(x|b_k=0)}\right) \\ &\approx \log\left(\frac{\max_{s_k} Pr(x|b_k=1, s_k)}{\max_{s_k} Pr(x|b_k=0, s_k)}\right) \end{aligned}$$



OFDM Mathematics

$$s(t) = \sum_{k=0}^{N-1} X_k e^{j2\pi f_k t} \qquad t \equiv [\,0, T_{os}]$$

Orthogonality Condition

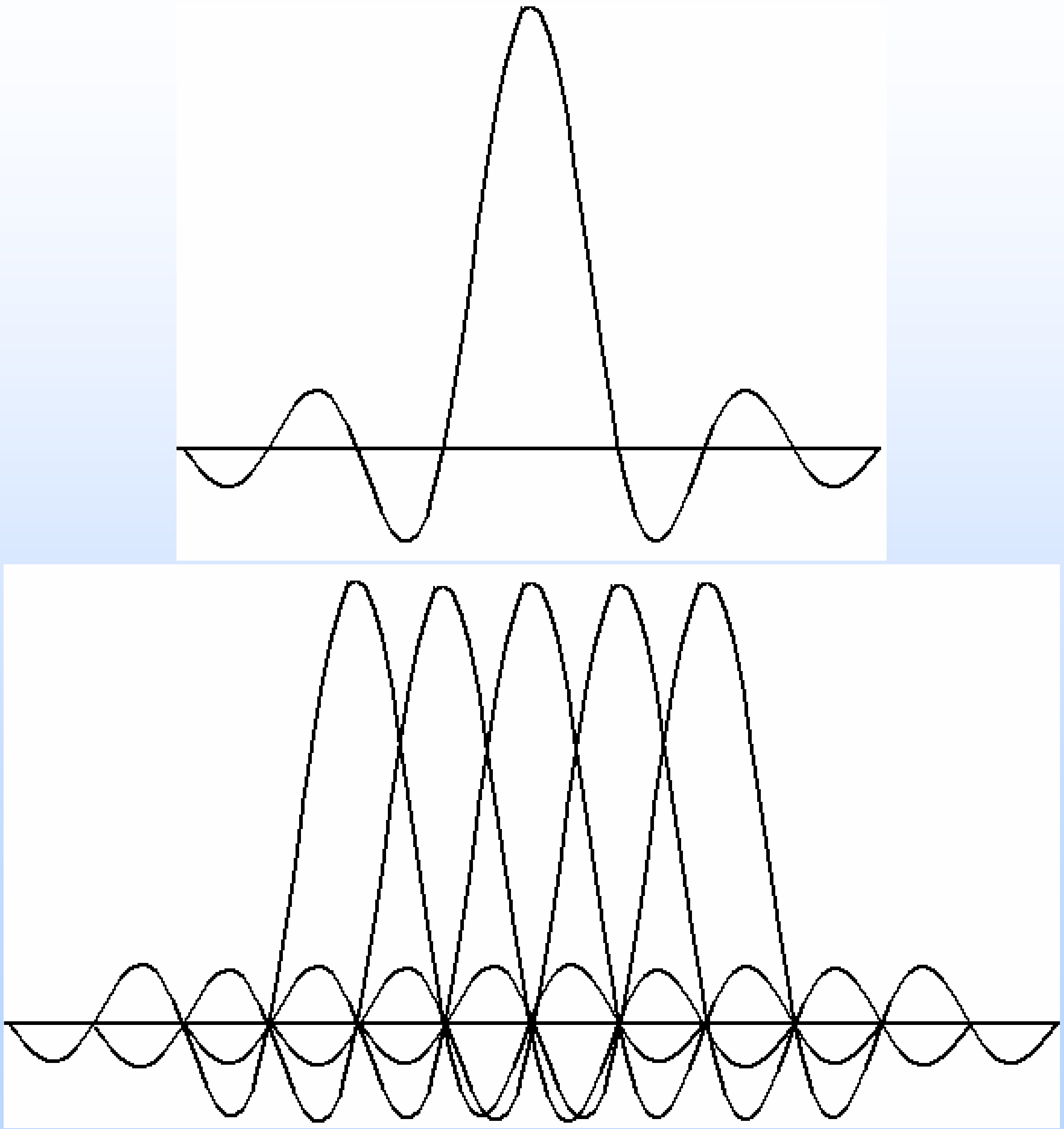
$$\int_0^{T_{os}} g_1(t) . g_2^*(t) dt = 0$$

In our case

$$\int_0^{T_{os}} e^{j2\pi f_p t} . e^{-j2\pi f_q t} dt = 0$$

For $p \neq q$ Where $f_k = k/T_{os}$

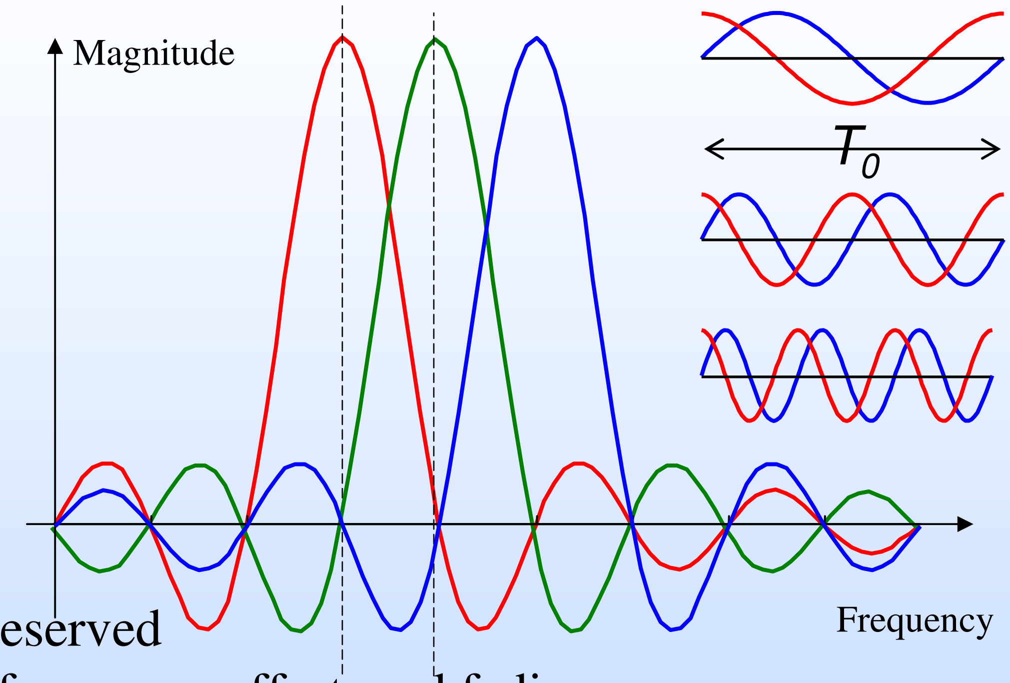
Transmitted Spectrum



Spectrum of the modulated data symbols

- **Rectangular Window of duration T_0**
- **Has a sinc-spectrum with zeros at $1/T_0$**

- **Other carriers are put in these zeros**
- **→ sub-carriers are orthogonal**

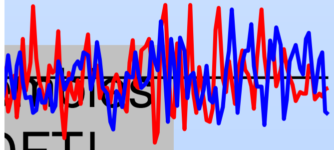


Subcarrier orthogonality must be preserved

Compromised by timing jitter, frequency offset, and fading.

N sub-carriers:

$$s_{BB,k}(t) = w(t-kT)\boxed{\sum_{i=0}^{N-1} x_{i,k} e^{j2\pi i\Delta f(t-kT)}}$$



DFT!

OFDM terminology

- Orthogonal carriers referred to as subcarriers $\{f_i,i=0,....N-1\}$.
- OFDM symbol period $\{T_{os}=N \text{ x } T_s\}$.
- Subcarrier spacing $\Delta f = 1/T_{os}$.

OFDM and FFT

- Samples of the multicarrier signal can be obtained using the IFFT of the data symbols - a key issue.

- FFT can be used at the receiver to obtain the data symbols.

- No need for ‘N’ oscillators,filters etc.

- Popularity of OFDM is due to the use of IFFT/FFT which have efficient implementations.

OFDM Signal

$$s(t) = \sum_{n=-\infty}^{\infty} \left(\sum_{k=0}^{N-1} X_{n,k} g_k (t - nT_{os}) \right)$$

$$g_k(t) = \begin{cases} e^{j2\pi f_k t} & t \equiv [0, T_{os}] \\ 0 & \text{Otherwise} \end{cases}$$

$$f_k = \frac{k}{T_{os}}$$

K=0,..........N-1

By sampling the low pass equivalent signal at a rate N times higher than the OFDM symbol rate 1/T_{os}, OFDM frame can be expressed as:

$$F_n(m) = \sum_{k=0}^{N-1} X_{n,k} g_k(t - nT_{os}) \Big|_{t=(n+\frac{m}{N})T_{os}} \qquad m = 0....N-1$$

$$F_n(m) = \left(\sum_{k=0}^{N-1} X_{n,k}\, e^{j2\pi k \frac{m}{N}} \right) = N.IDFT\left\{ X_{n,k} \right\}$$

Interpretation of IFFT&FFT

- **IFFT at the transmitter & FFT at the receiver**
- **Data symbols modulate the spectrum and the time domain symbols are obtained using the IFFT.**
- **Time domain symbols are then sent on the channel.**
- **FFT at the receiver to obtain the data.**

Assume the information data sequence be $X(k)$, k=0,1…N-1, then the transmitted signal $s(t)$ can be expressed as

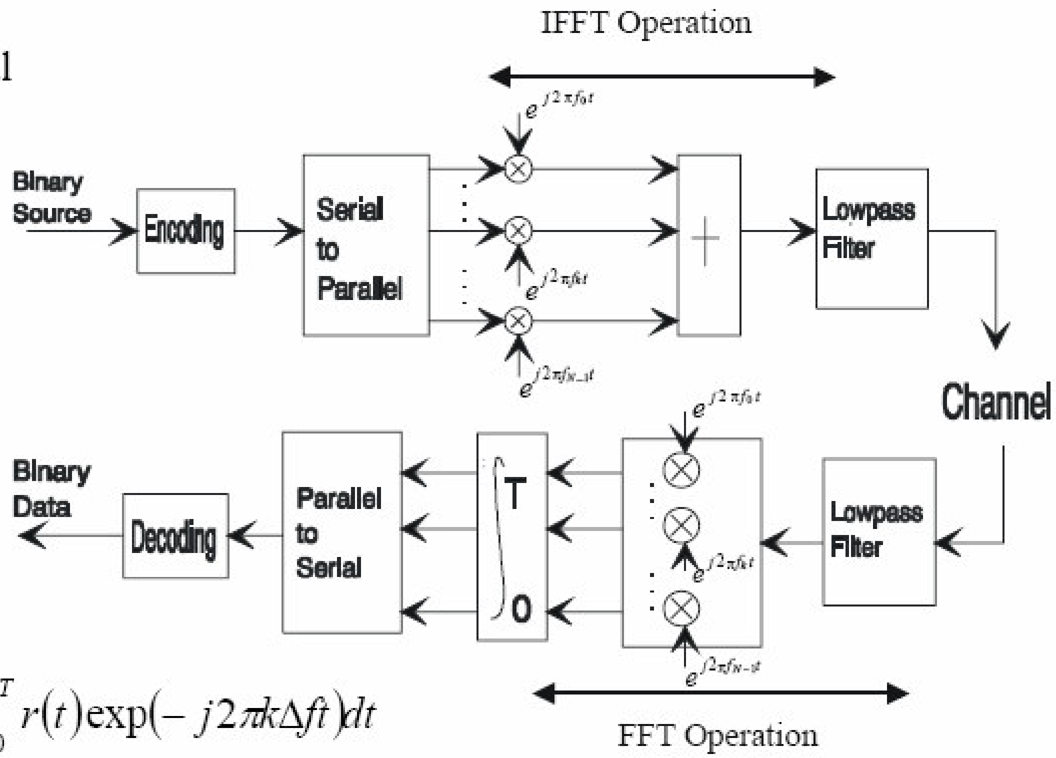
$$s(t)=\sum_{p=0}^{N-1} X(p)\exp\left(j2\pi f_p t\right)$$

$$=\sum_{p=0}^{N-1} X(p)\exp\left(j2\pi p\Delta f t\right),$$

$$valid \quad for \quad 0 \le t < T;\ 0 \le p \le N-1.$$

The received signal $r(t)$ is

$$r(t)=s(t)*h(t)+w(t)$$



The received on subcarrier k_0 is $Y(k)=\frac{1}{T}\int_0^T r(t)\exp(-j2\pi k\Delta f t)dt$

Assume $h(t)$=1, $w(t)$=0 (ideal channel), then $Y(k)$ becomes

$$Y(k)=\frac{1}{T}\int_0^T \sum_{p=0}^{N-1} X(p)\exp\left(j2\pi p\Delta f t\right)\exp\left(-j2\pi k\Delta f t\right)$$

Transmitted data can be fully recovered from above euqation, if $P = k$ in $Y(K)$.