ECE5984

# Orthogonal Frequency Division Multiplexing and Related Technologies

Fall 2007

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# OFDM Basics II

# Syllabus

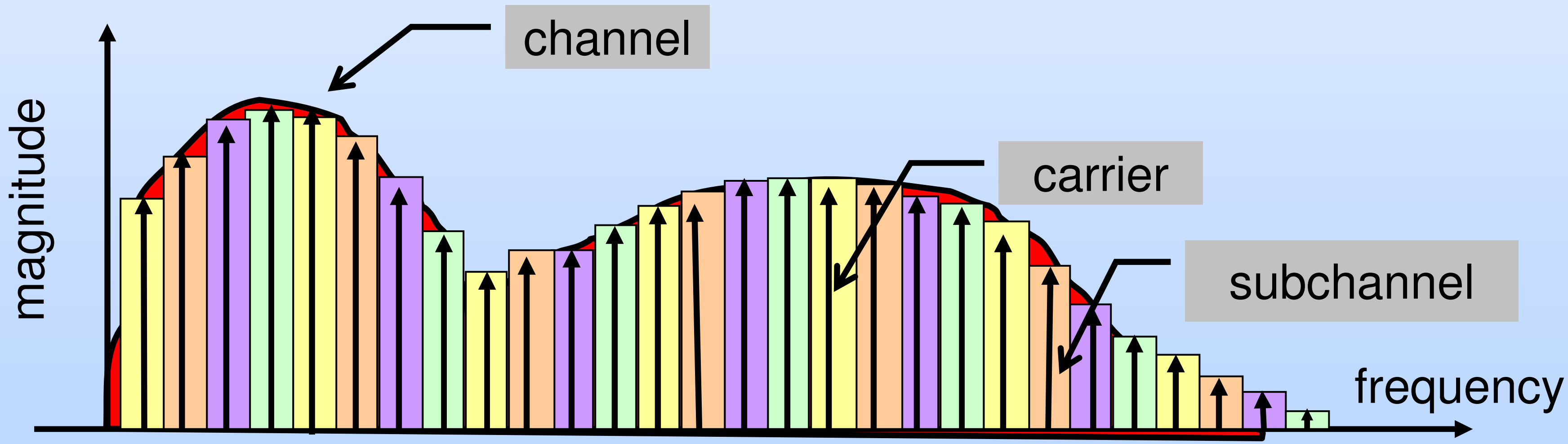
- **Wireless channels characteristics** (7.5%) **1**
  - wireless channel modeling and characteristics
    - Large scale and small scale models
    - Common channel models
    - Channel categories and parameter calculation.
    - **Prob. of error calculations**
- **OFDM Basics** (10%) **1**
  - History of OFDM
  - OFDM System model
  - Discrete-time signals & systems and DFT
  - Generation of subcarriers using the IFFT
  - Guard time, cyclic extension
  - Windowing
  - Choice of OFDM parameters & OFDM signal processing
  - Implementation complexity of OFDM versus single carrier modulation
- **Modulation and Coding** (10%) **2**
  - Linear and nonlinear modulation
  - Interleaving and channel coding
  - Optimal bit and power allocation
  - Adaptive modulation

# Matlab Assignment 1

- **Develop a WGN channel that accepts symbol input, adds noise to it with certain SNR and produces the noisy ouput.**
- **Develop a Flat Fading Channel following Rayleigh and ricean distribution**
- **Develop a Frequency selective channel following Rayleigh and ricean distribution**
  - All inputs to the channel are baseband signals.
  - Compare with the matlab functions (if exist)
  - Plot the probability of error vs SNR

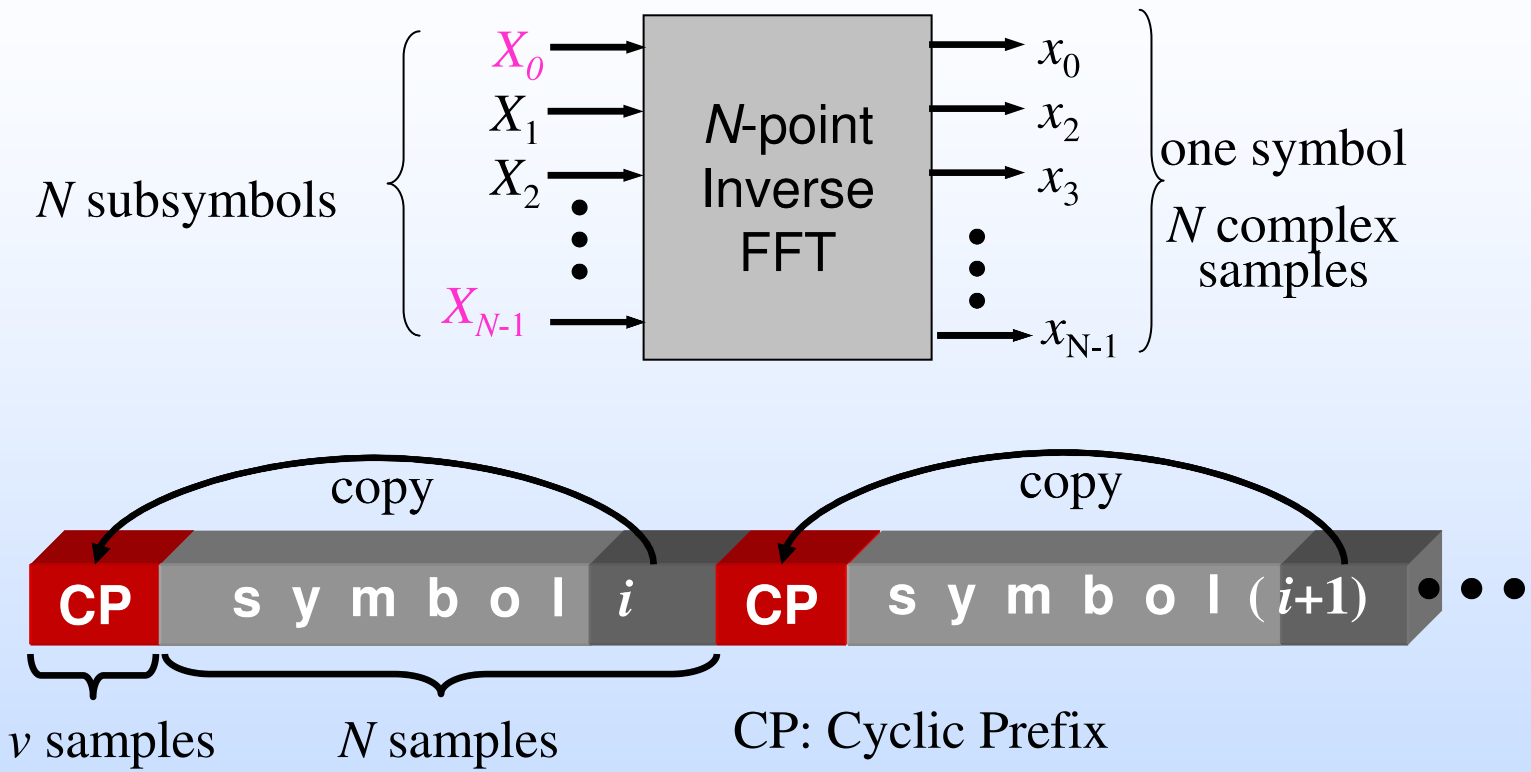
# Multicarrier Modulation

- **Divide broadband channel into narrowband subchannels**
  - No ISI in *subchannels* if constant gain in every subchannel and if ideal sampling
- **Orthogonal Frequency Division Multiplexing**
  - Based on the fast Fourier transform
  - Standardized for DAB, DVB-T, IEEE 802.11a, 802.16a, HyperLAN II
  - Considered for fourth-generation mobile communication systems




Subchannels are 312 kHz wide in 802.11a and HyperLAN II

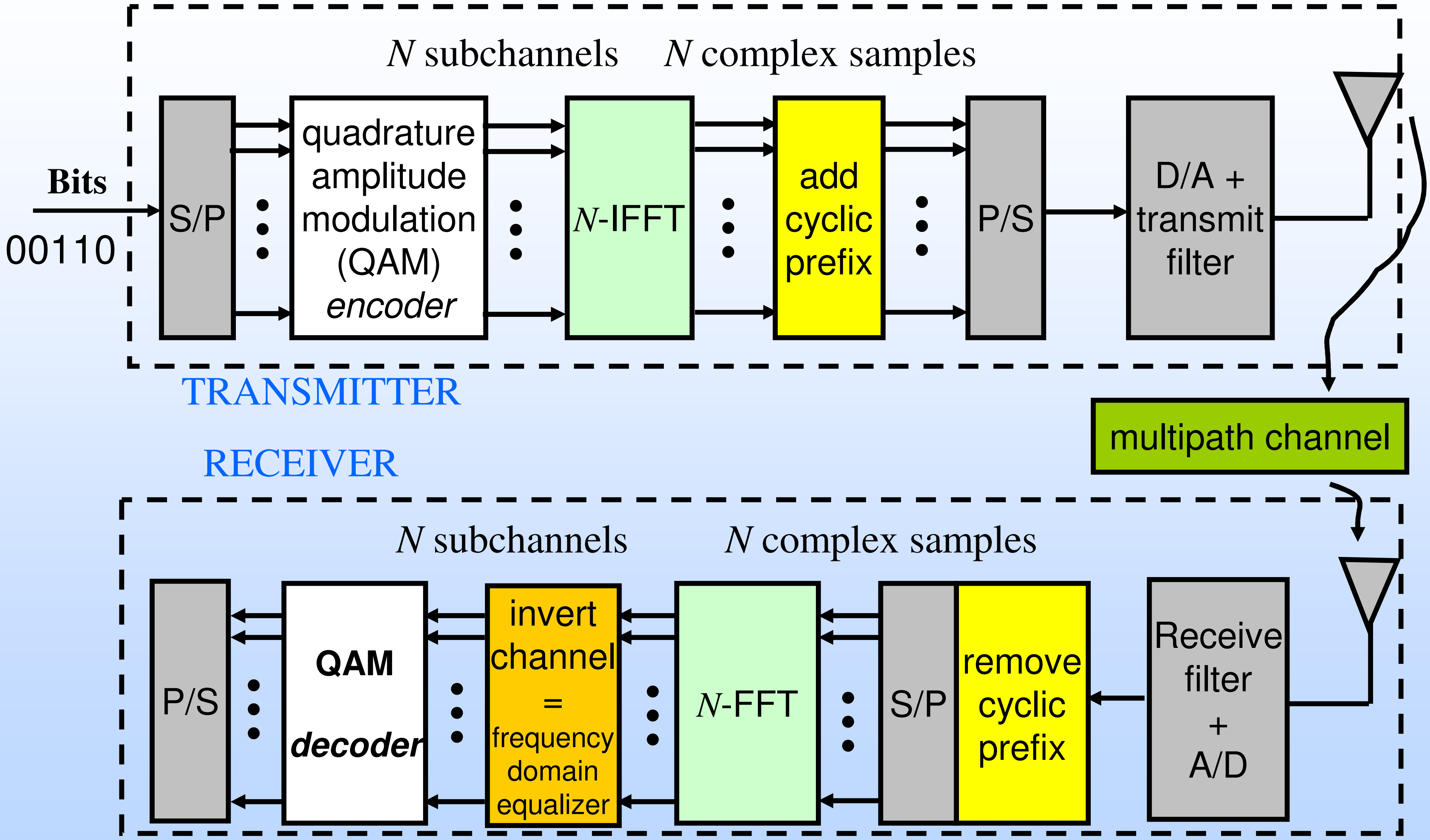
# An OFDM Symbol




- **Bandpass transmission allows for complex waveforms**
  - **Transmit: y(t) = Re{(*I*(*t*)+j *Q*(*t*)) exp(j2π $f_c$ *t*)}**

$$= I(t)\cos(2\pi f_c t) - Q(t)\sin(2\pi f_c t)$$

# An OFDM Modem



# Coded OFDM (COFDM)

- **Error correction is *necessary* in OFDM systems**
- **Forward error correction (FEC)**
  - Adds redundancy to data stream
  - Examples: convolutional codes, block codes
  - Mitigates the effects of bad channels
  - Reduces overall throughput according to the coding rate k/n
- **Automatic repeat request (ARQ)**
  - Adds error detecting ability to data stream
  - Examples: 16-bit cyclic redundancy code
  - Used to detect errors in an OFDM symbol
  - Bad packets are retransmitted (hopefully the channel changes)
  - Usually used with FEC
  - Minus: Ineffective in broadcast systems

# OFDM Mathematics

$$s(t) = \sum_{k=0}^{N-1} X_k e^{j2\pi f_k t} \qquad t \equiv [\,0, T_{os}]$$

Orthogonality Condition

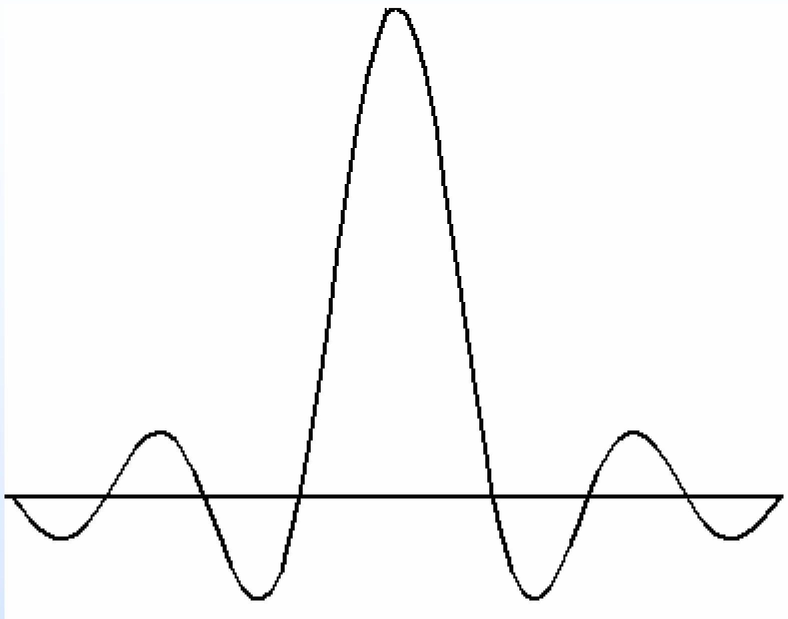
$$\int_0^{T_{os}} g_1(t).g_2^*(t)dt = 0$$

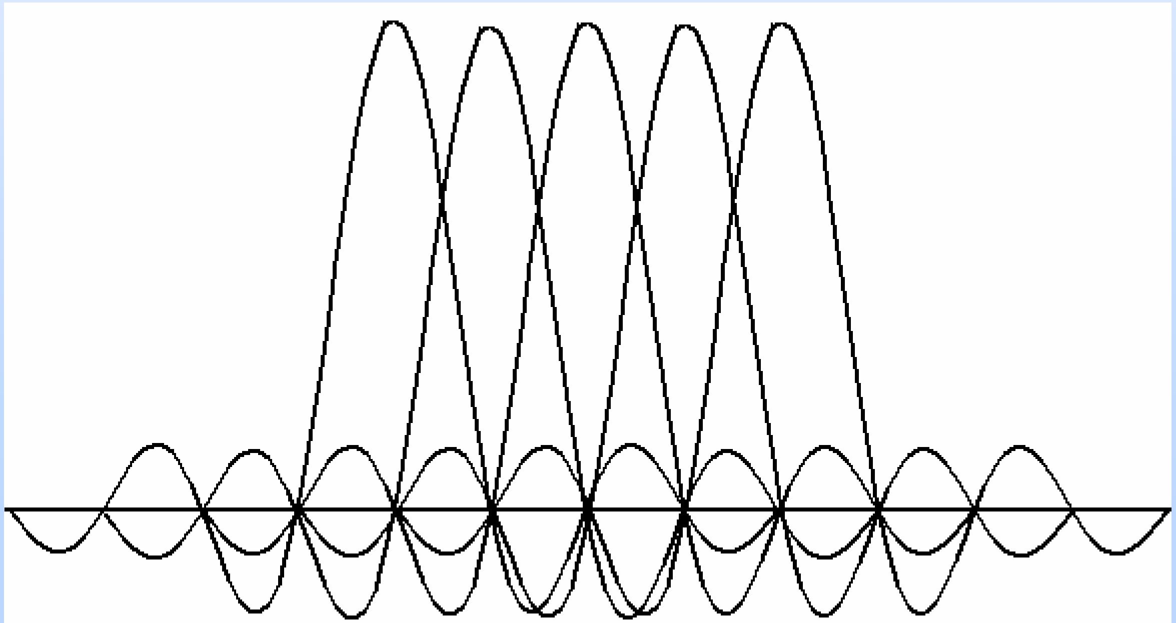
In our case

$$\int_0^{T_{os}} e^{j2\pi f_p t}.e^{-j2\pi f_q t}dt = 0$$

For $p \neq q$ Where $f_k = k/T_{os}$

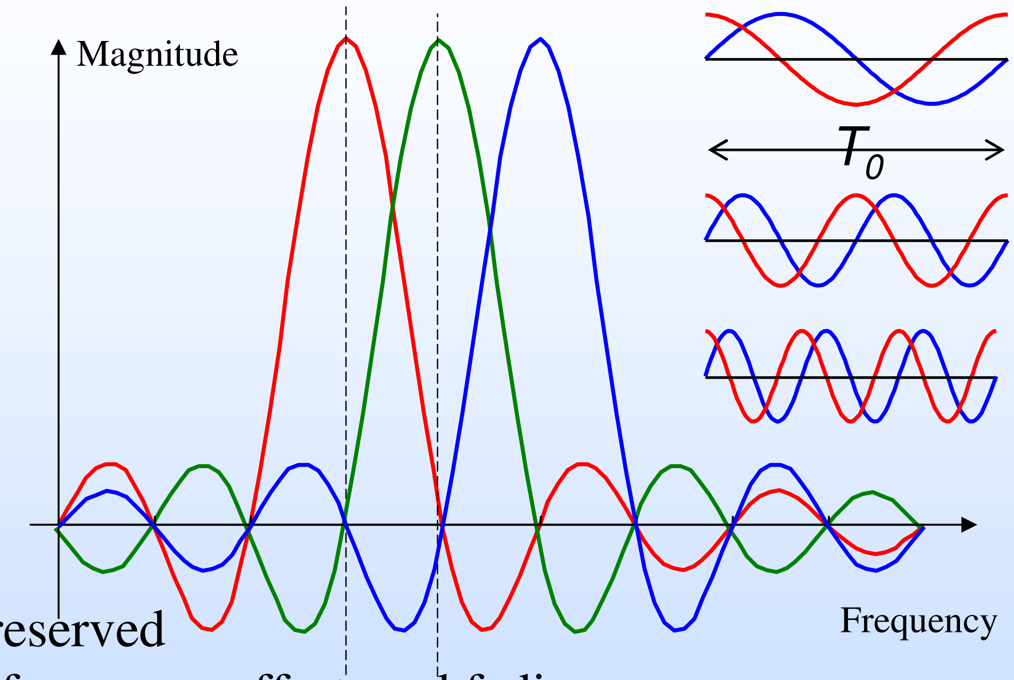
# Transmitted Spectrum





# Spectrum of the modulated data symbols

- **Rectangular Window of duration $T_0$**
- **Has a sinc-spectrum with zeros at $1/T_0$**
- **Other carriers are put in these zeros**
- **→ sub-carriers are orthogonal**

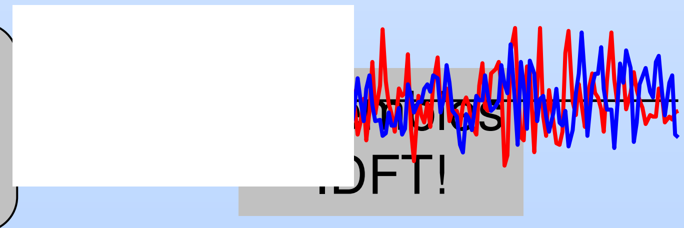



Subcarrier orthogonality must be preserved

Compromised by timing jitter, frequency offset, and fading.

$N$ sub-carriers:

$$s_{BB,k}(t) = w(t-kT) \sum_{i=0}^{N-1} x_{i,k} e^{j2\pi i \Delta f (t-kT)}$$



# OFDM terminology

- Orthogonal carriers referred to as subcarriers $\{f_i, i=0,....N-1\}$.
- OFDM symbol period $\{T_{os}=N \times T_s\}$.
- Subcarrier spacing $\Delta f = 1/T_{os}$.

# OFDM Signal

$$s(t) = \sum_{n=-\infty}^{\infty} \left( \sum_{k=0}^{N-1} X_{n,k}\, g_k(t - nT_{os}) \right)$$

$$g_k(t) = \begin{cases} e^{j2\pi f_k t} & t \equiv [\,0, T_{os}] \\ 0 & \text{Otherwise} \end{cases}$$

$$f_k = \frac{k}{T_{os}}$$

K=0,..........N-1

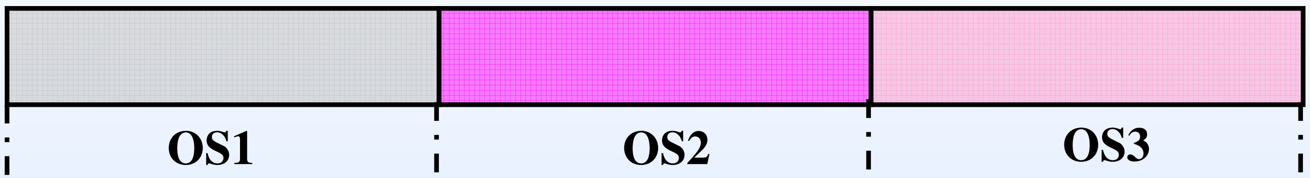
By sampling the low pass equivalent signal at a rate N times higher than the OFDM symbol rate $1/T_{os}$, OFDM frame can be expressed as:

$$F_n(m) = \sum_{k=0}^{N-1} X_{n,k} g_k(t - nT_{os}) \Big|_{t=(n+\frac{m}{N})T_{os}} \quad m = 0....N-1$$

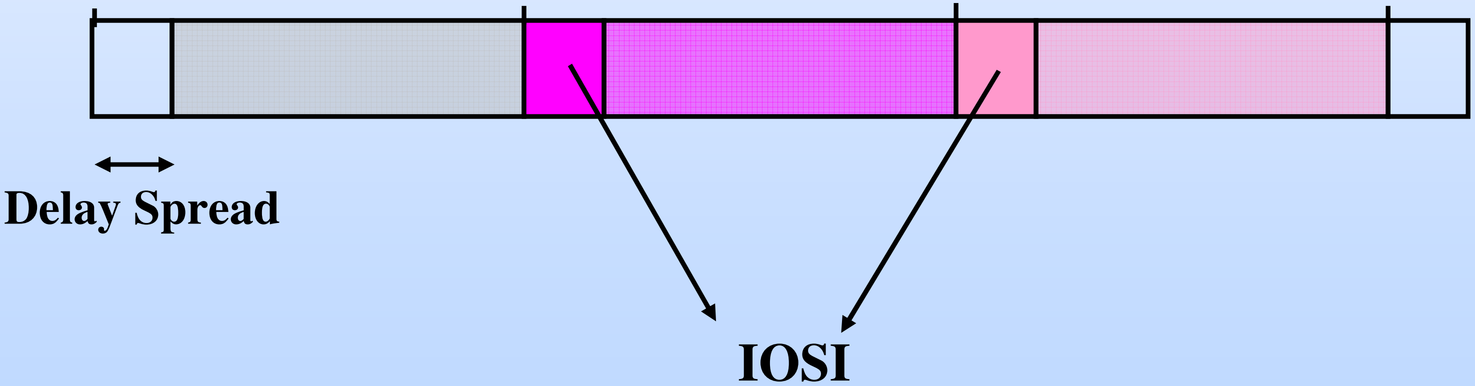
$$F_n(m) = \left( \sum_{k=0}^{N-1} X_{n,k} \, e^{j2\pi k \frac{m}{N}} \right) = N.IDFT\left\{ X_{n,k} \right\}$$

# Interference between OFDM Symbols

- **Transmitted Signal**

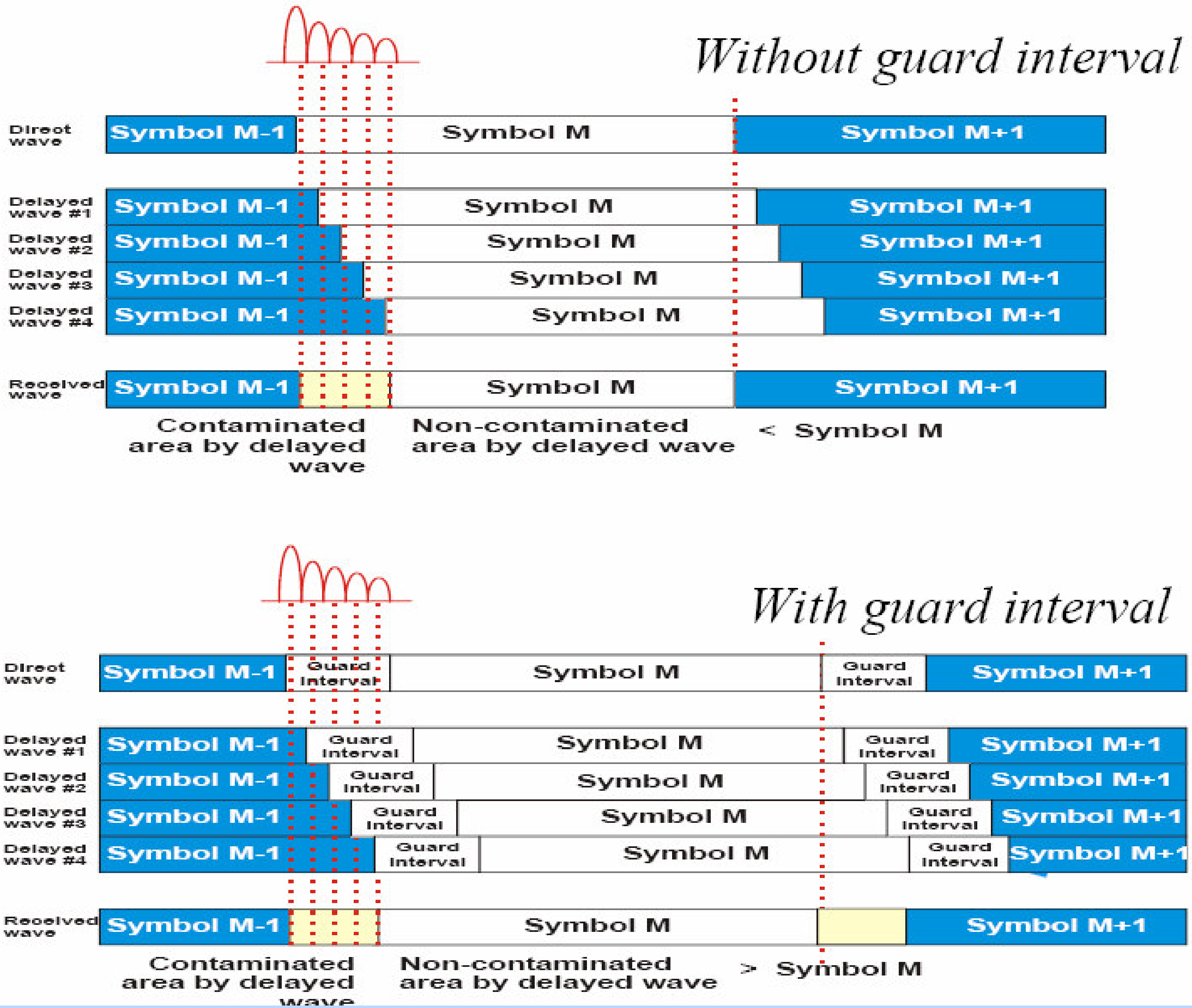



- **Due to delay spread ISI occurs**




- **Solution could be guard interval between OFDM symbols**

# Guard Interval



# Cyclic Prefix

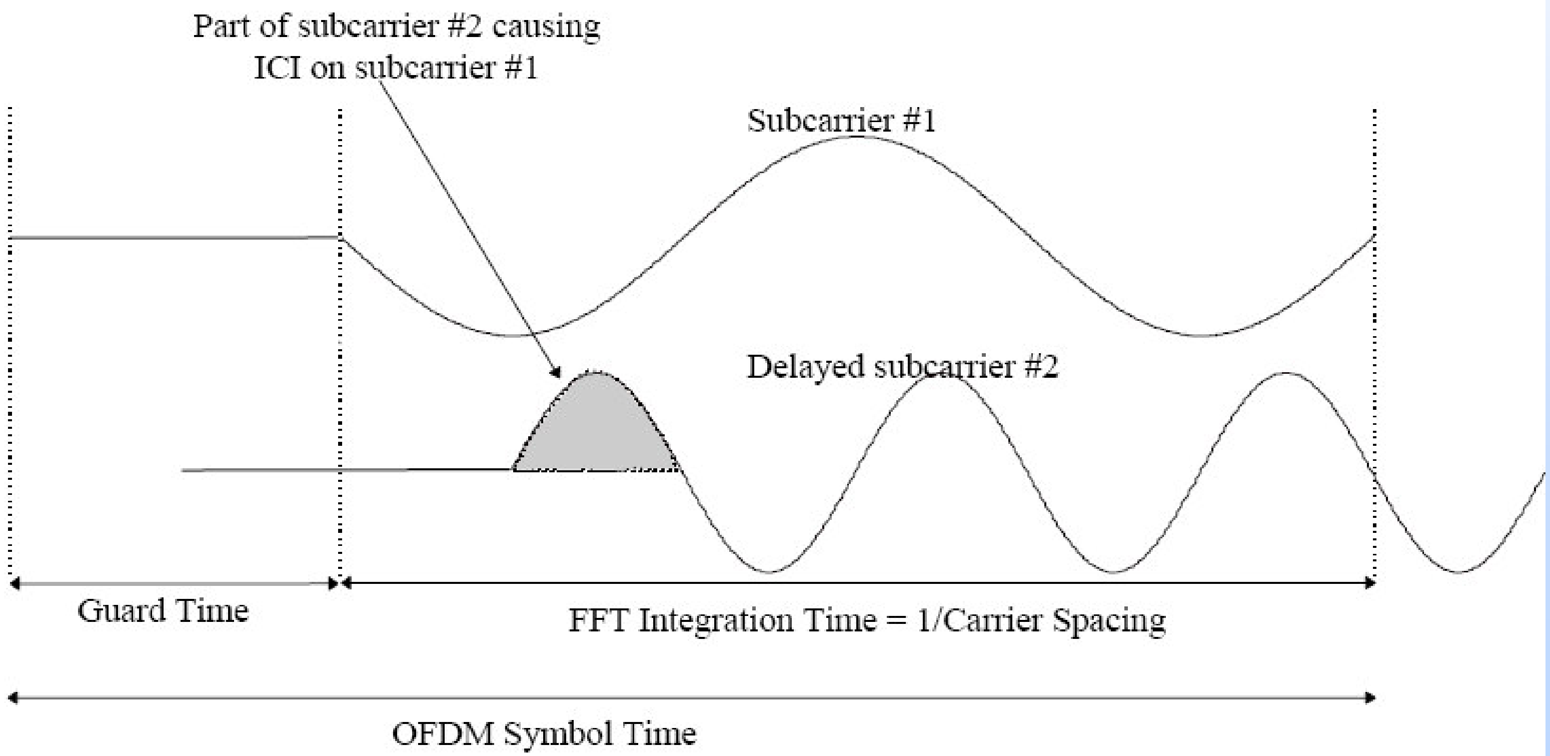
- Zeros used in the guard time can alleviate interference between OFDM symbols (IOSI problem).

- Orthogonality of carriers is lost when multipath channels are involved.

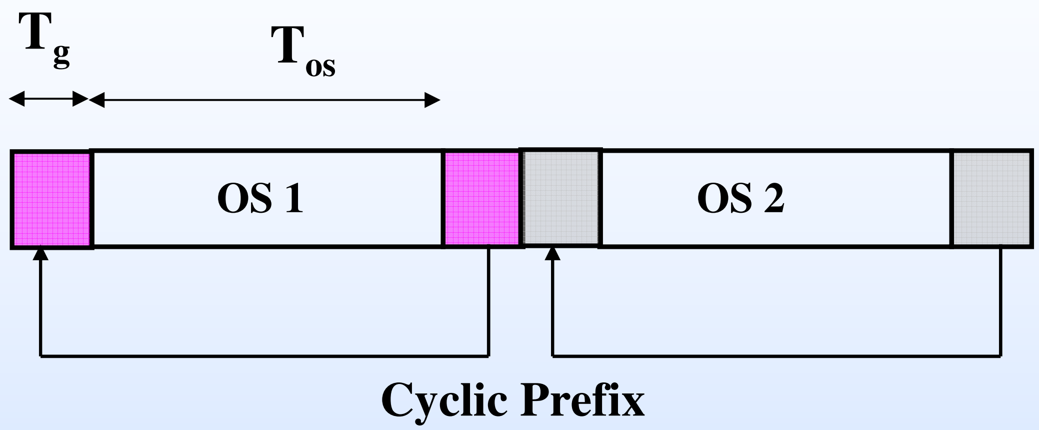
- Cyclic prefix can restore the orthogonality.
- Energy is wasted in the cyclic prefix samples.

# Guard time and Cyclic Prefix

Effect of multipath with zero signal in the guard time; the delayed subcarrier #2 causes inter carrier interference (ICI) on subcarrier #1 and vice-versa.



# Cyclic Prefix Illustration



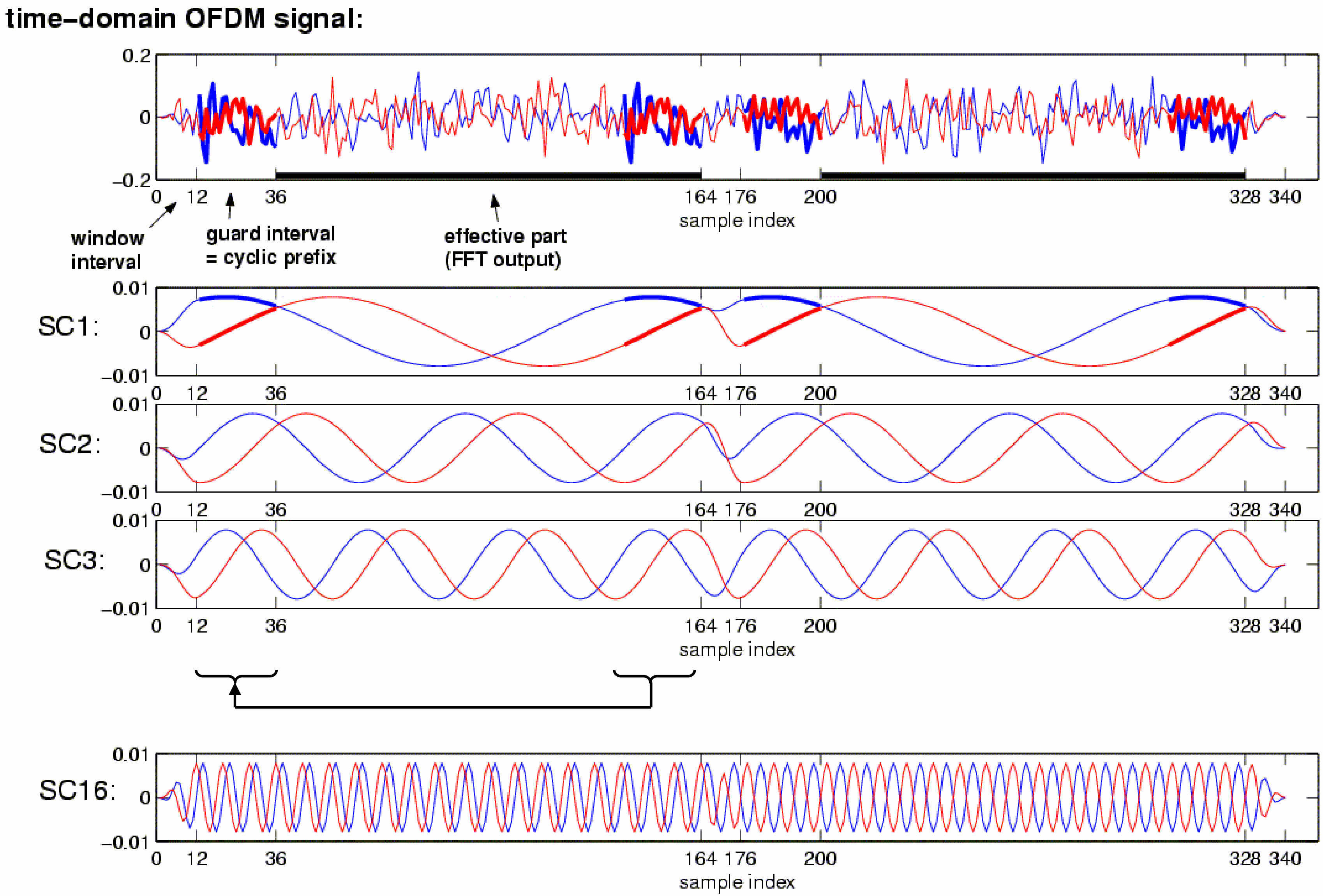

**OS1,OS2 - OFDM Symbols**

**$T_g$ - Guard Time Interval**

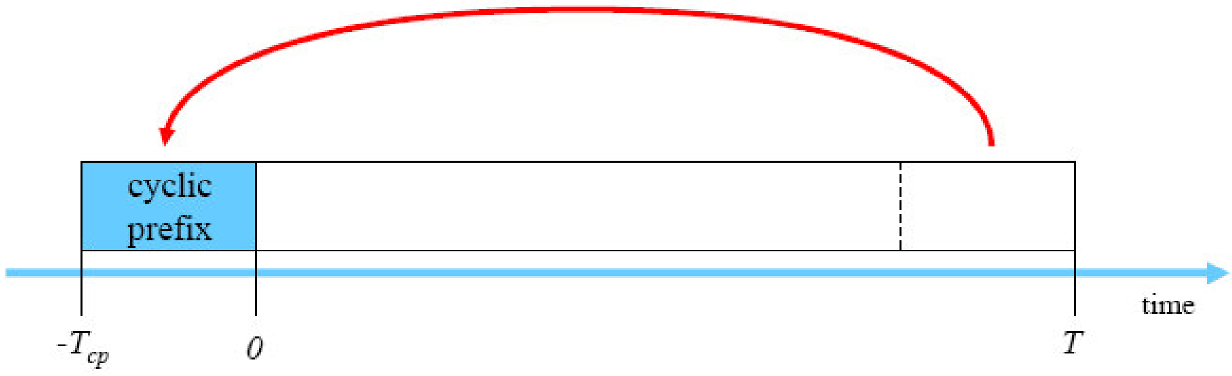
**$T_s$ - Data Symbol Period**

**$T_{os}$ - OFDM Symbol Period - N * $T_s$**

# Guard interval (2) - Cyclic extension



# Protection Against ICI & ISI Through CP




- ➔ Multipath delay spread causes ISI, and loss of orthogonality causes ISI.
- ➔ CP helps the signal to be protected against both ISI and ICI.
  - ❖ If CP is longer than maximum delay spread, then ISI will not occur.
  - ❖ CP ensures that delayed replicas of the OFDM symbol always have an integer number of cycles within the FFT interval. In this way, the orthogonality is preserved and no ICI is present in the received signal.
- ➔ CP causes loss of a partion of signal energy, given by

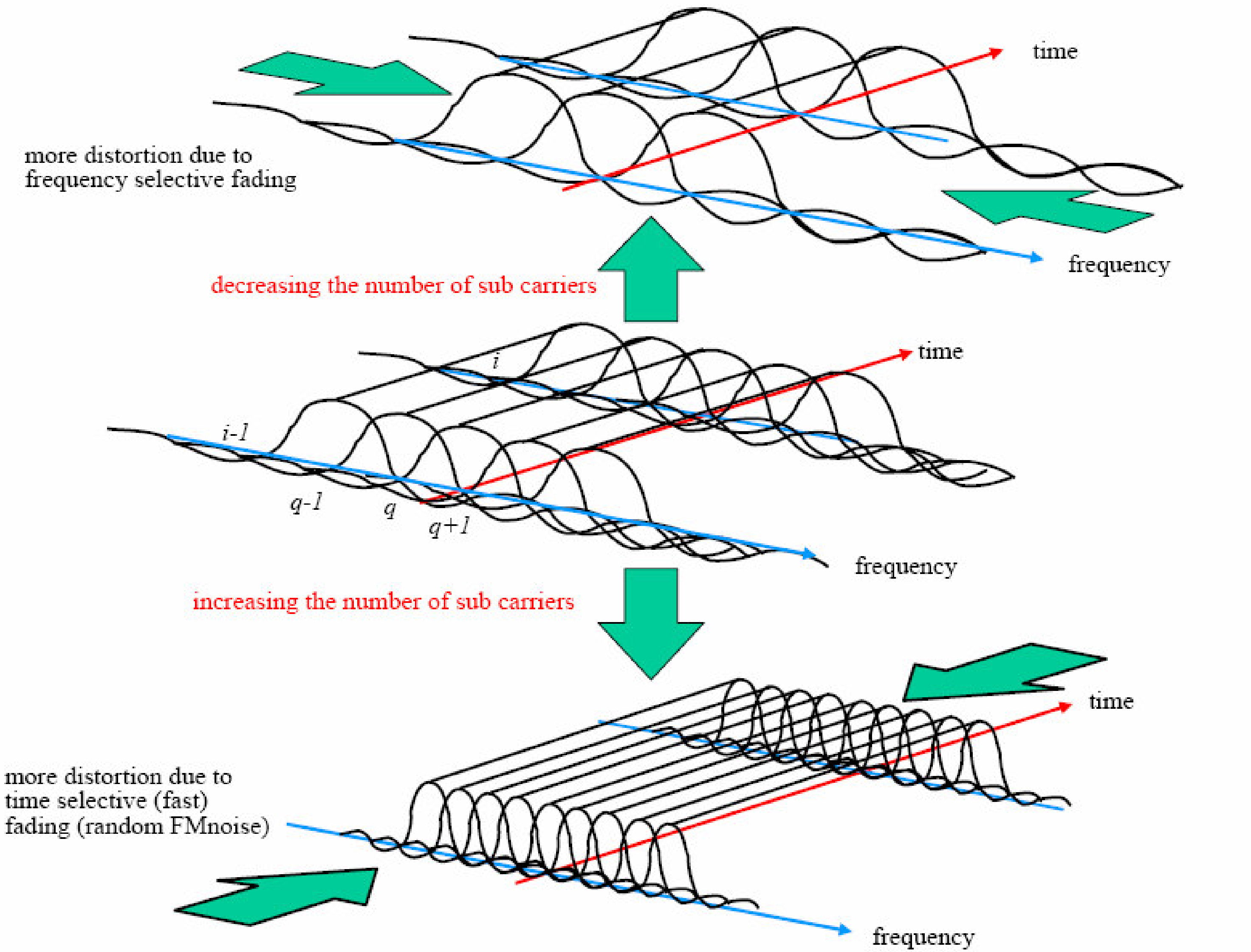
$$SNR_{loss\_CP} = -10\log_{10}\left(1-\frac{T_{CP}}{T_{symbol}}\right)$$

# OFDM System Design

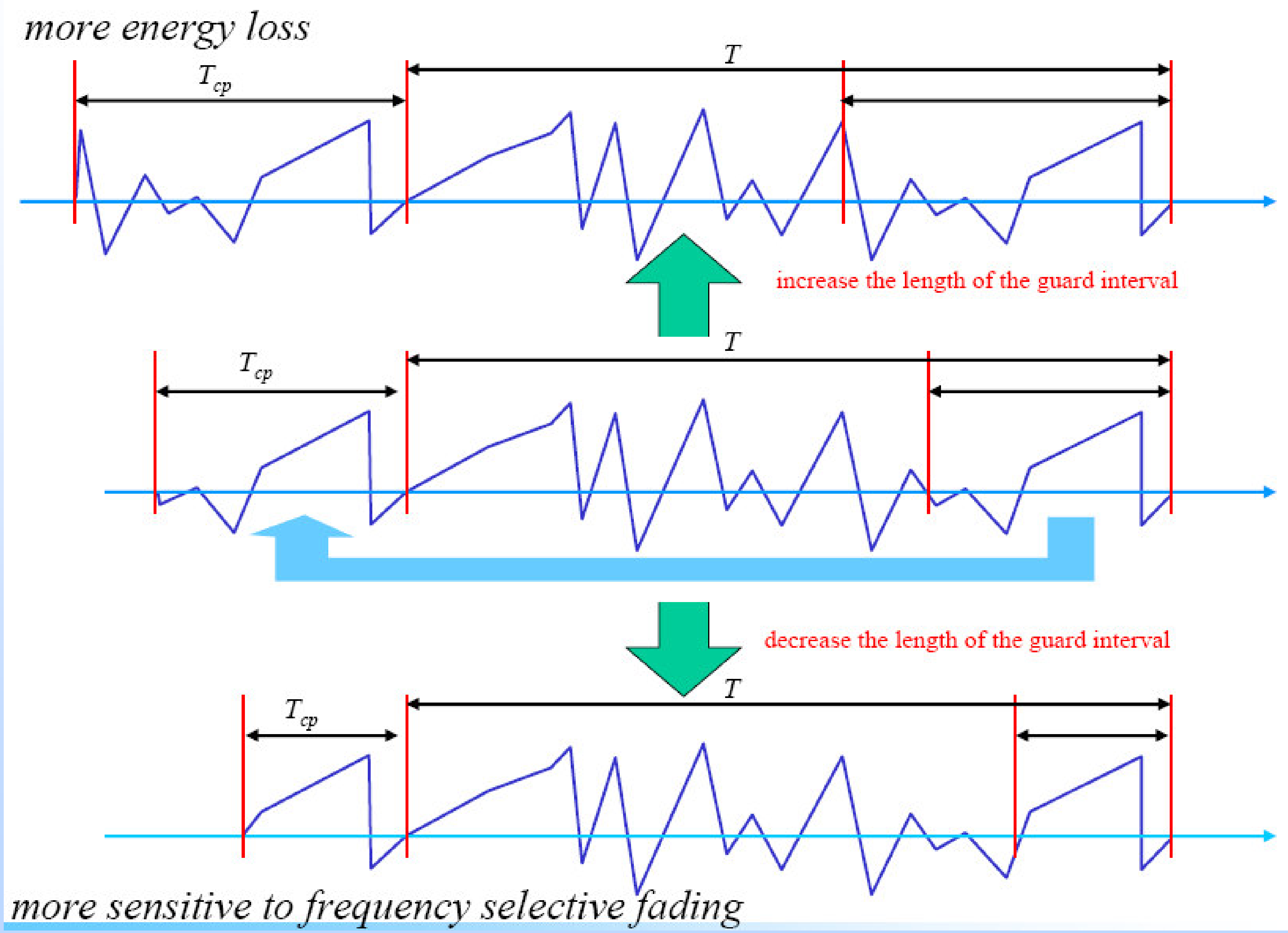
In OFDM scheme, when the transmission rate is given, the transmission performance becomes more sensitive to the time selectivity as the number of subcarriers increases because the wider symbol duration is less robust to the random FM noise, whereas it becomes poor as number of subcarriers decreases because the wider power spectrum of each subcarrier is less robust to the frequency selectivity.

On the other hand, the transmission performance becomes poor as the length of guard interval increases because the signal transmission in the guard duration introduces the power loss, whereas it becomes more sensitive to the frequency-selectivity as length of guard interval decreases because the shorter guard duration is less robust to the delay spread.

# Optimum in the number of sub carriers



# Optimum in the length of the guard interval



## OFDM Design Parameters

Parameters for designing an OFDM System:

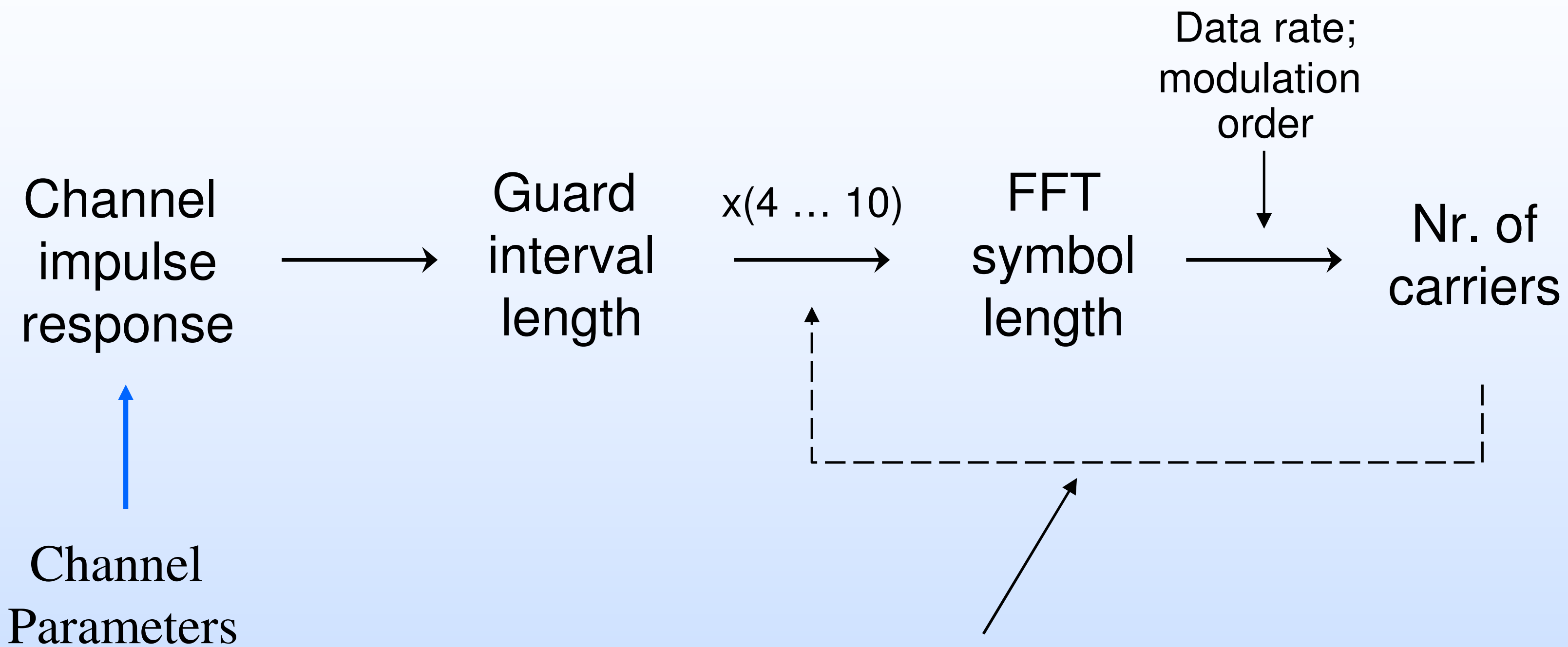
- number of subcarriers,
- guard time, symbol duration,
- subcarrier spacing,
- modulation type per subcarrier and
- the type of forward error correction coding

## OFDM System Requirements

Choice of parameters is influenced by system requirements:

- Available bandwidth,
- Required bit rate,
- Tolerable delay spread and
- Doppler values

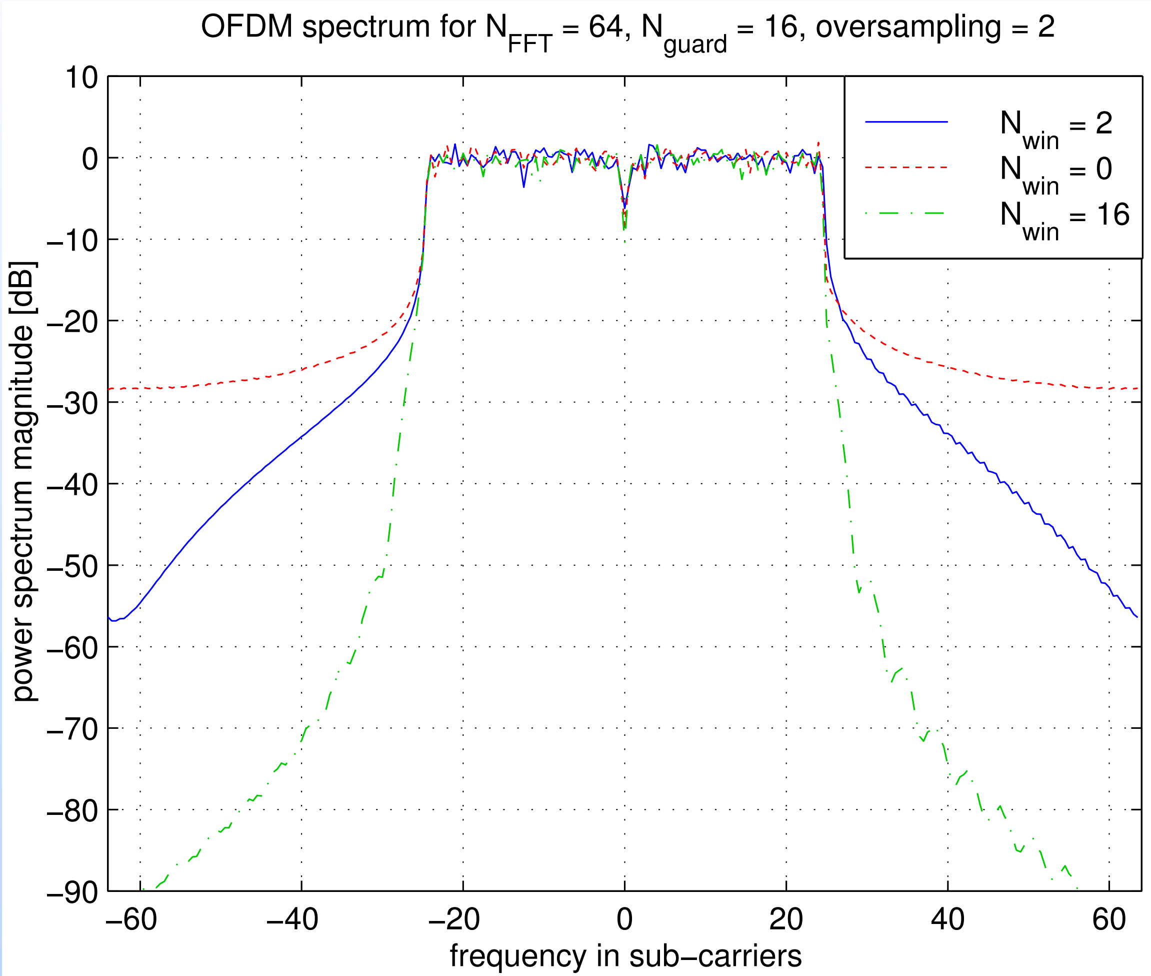
# Design of an OFDM System




*Other constraints:*

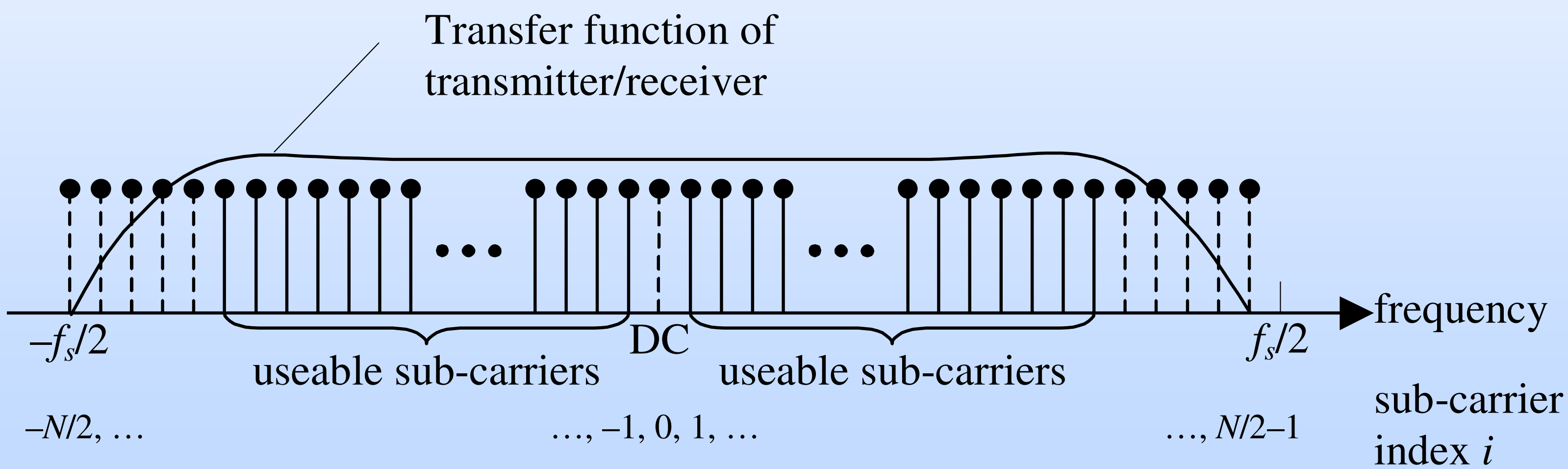
- Nr. of carriers should match FFT size and data packet length
- considering coding and modulation schemes

# Spectral Shaping by Windowing



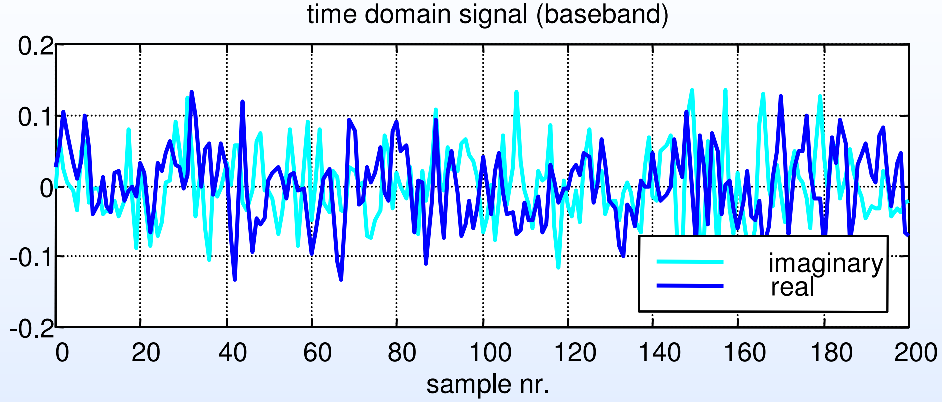
# OFDM Symbol Configuration

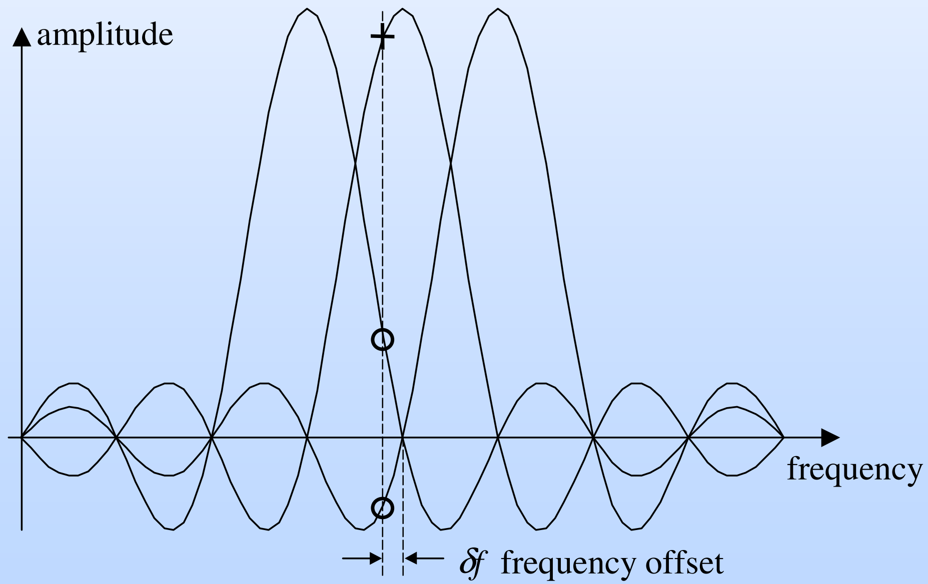
- **Not all FFT-points can be used for data carriers**
  - Lowpass filters for AD- and DA-conversion
    - oversampling required
  - DC offsets; carrier feedtrough; etc.



# Problems of OFDM (Research Topics)

- **Synchronization issues:**
  - **Time synchronization**
    - Find start of symbols
  - **Frequency synchr.**
    - Find sub-carrier positions
- **Non-constant power envelope**
  - Linear amplifiers needed
- **Channel estimation:**
  - To retrieve data
  - **Channel** is **time-variant**



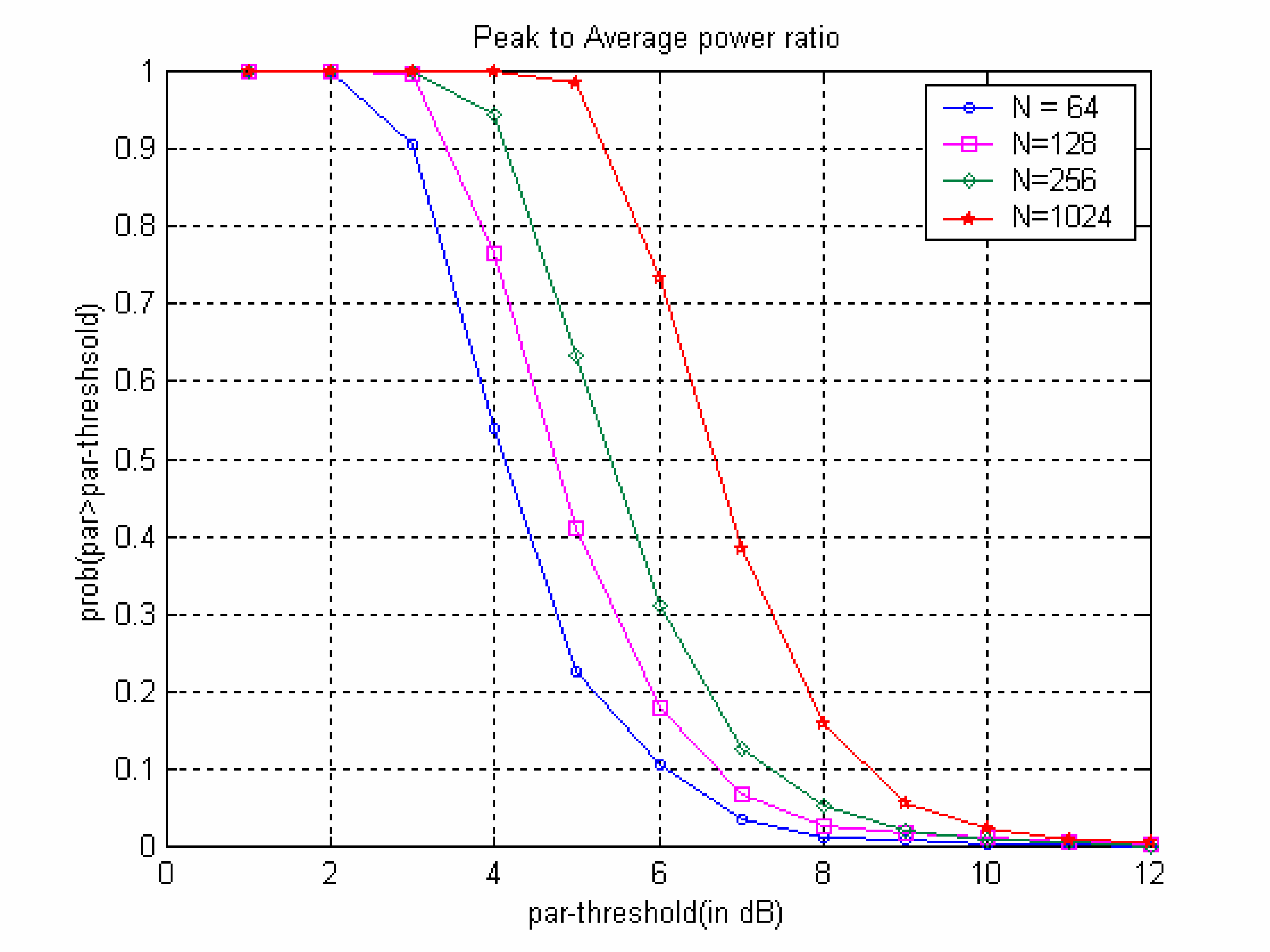
# Synchronization

- Timing and frequency offset can influence performance.
- Frequency offset can influence orthogonality of subcarriers.
- Loss of orthogonality leads to Inter Carrier Interference.

# Peak to Average Ratio

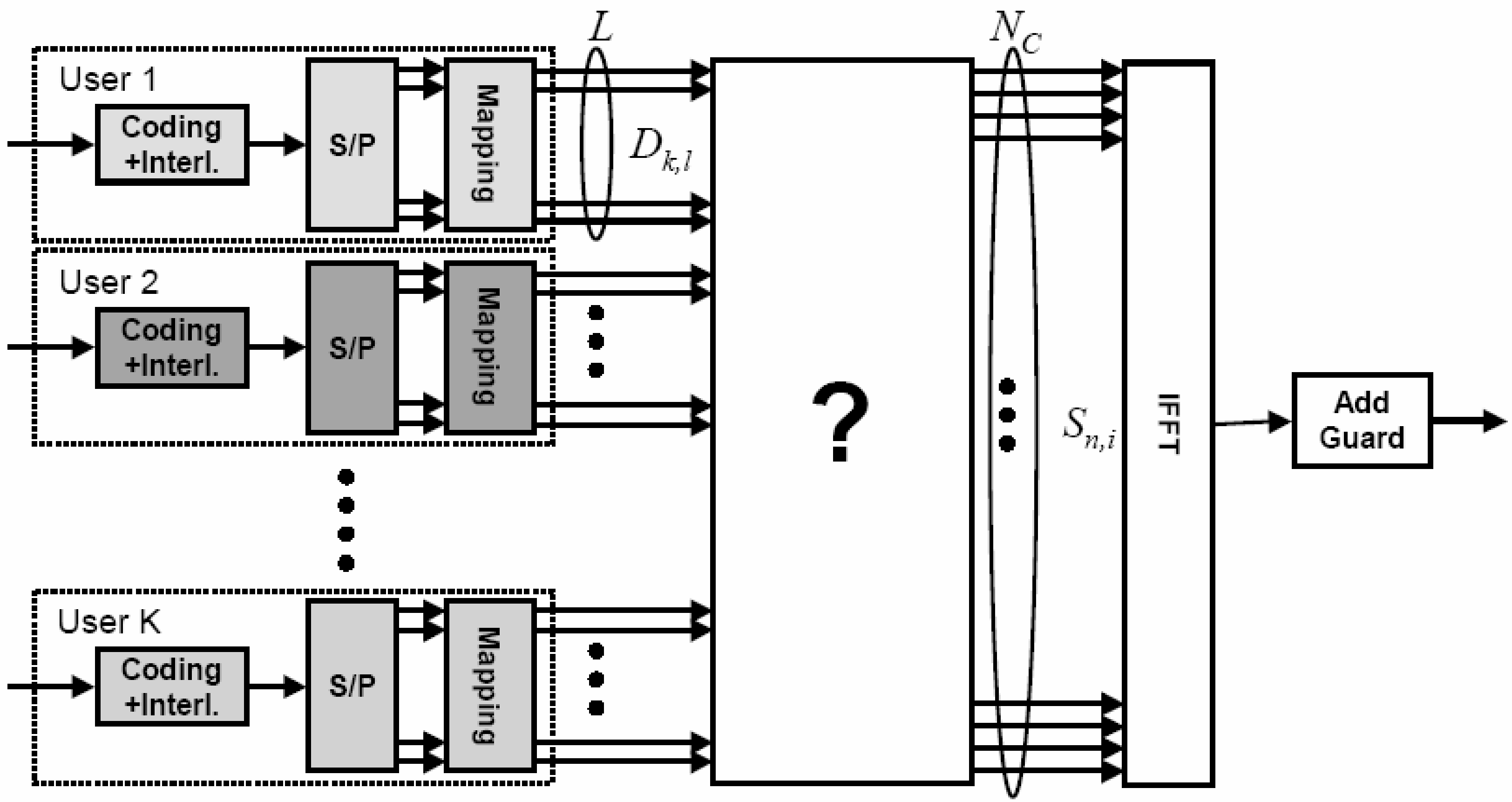
- Multicarrier signals have high PAR as compared to single carrier systems.
- PAR increases with the number of subcarriers.
- Affects power amplifier design and usage.

# Peak to Average Power Ratio

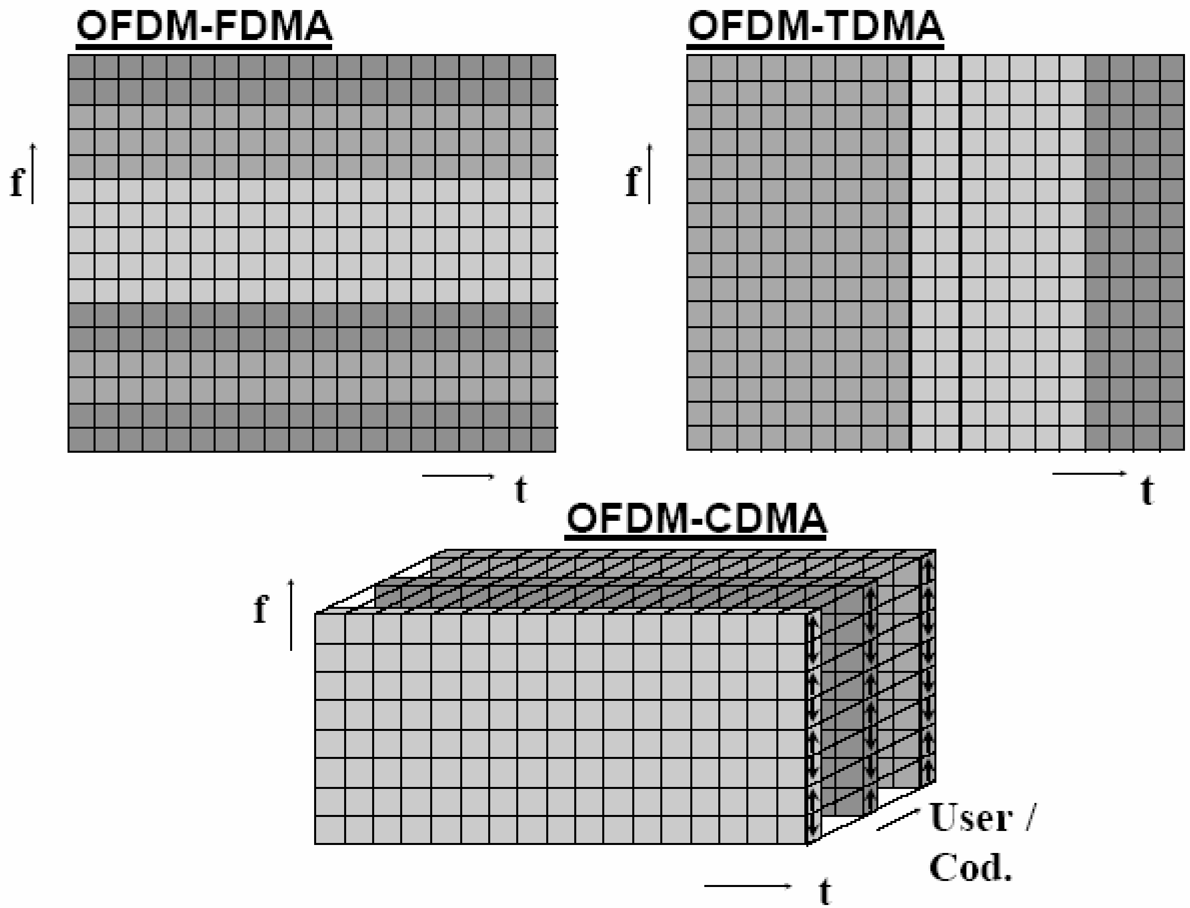


# OFDM for Communication Systems

**For a given OFDM system find a suitable multiple access scheme that maps the user data to a modulation block !**

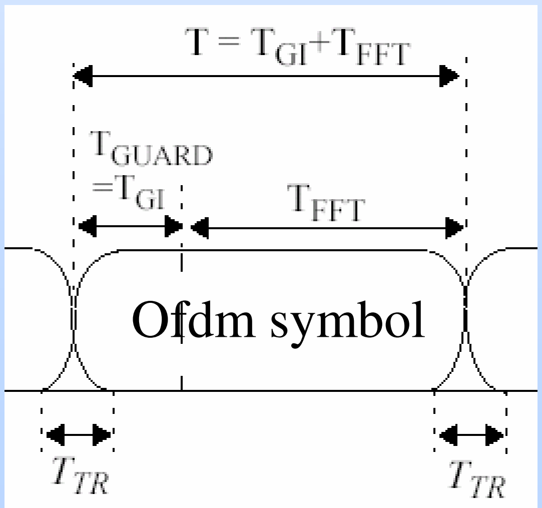


# OFDM Multiple Access Schemes

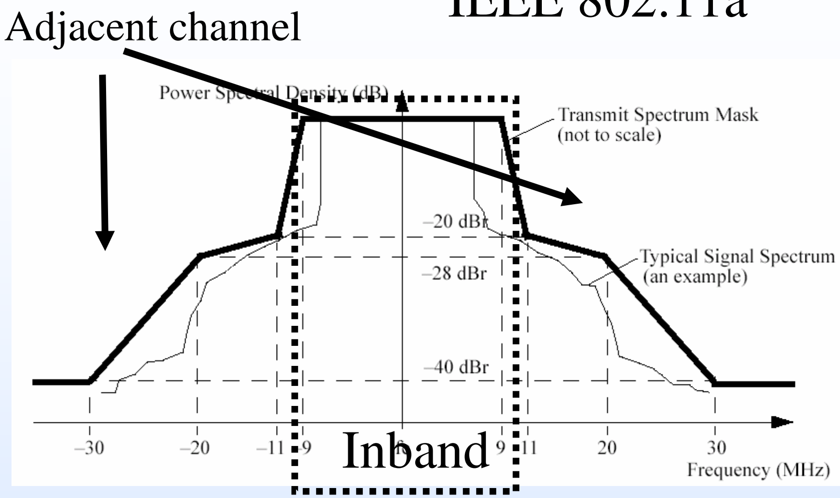


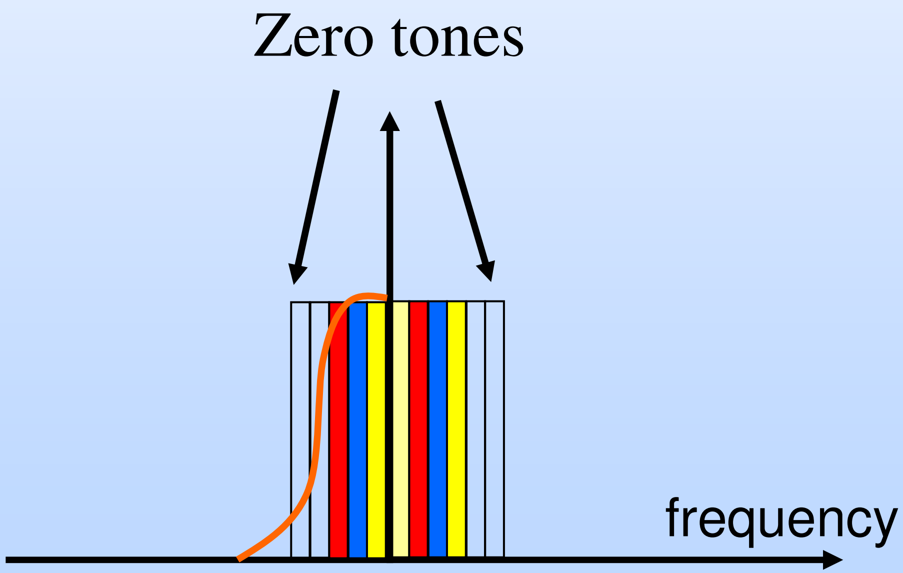
# Spectrum Shaping

- **FCC manages spectrum**
- **Specifies power spectral density mask**
  - Adjacent channel interference
  - Roll-off requirements
- **Implications to OFDM**
  - Zero tones on edge of band
  - Time domain windowing 'smoothes' adjacent symbols




IEEE 802.11a

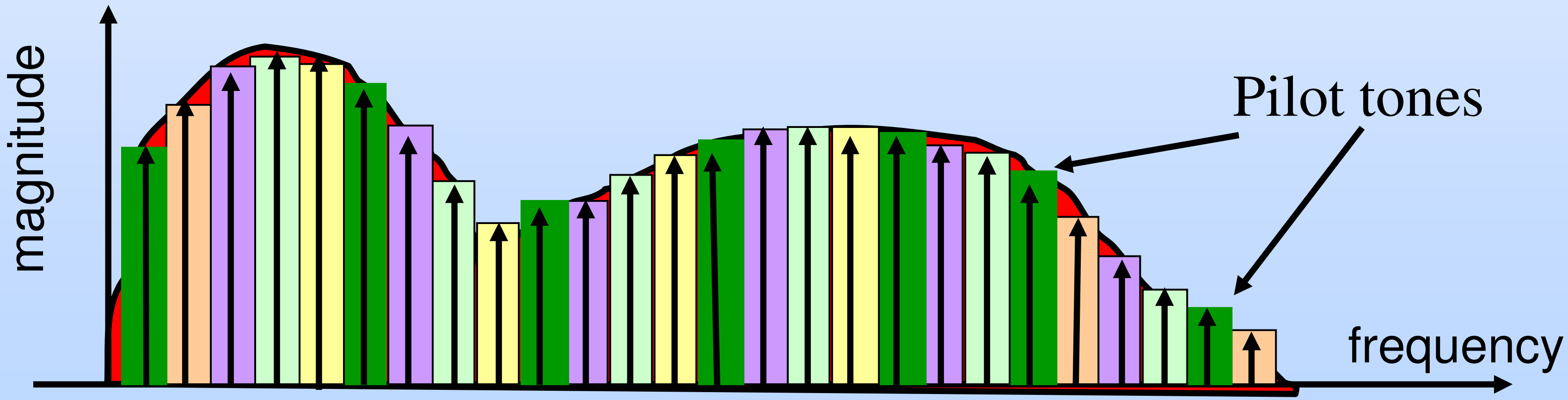



Reference: Std 802.11a

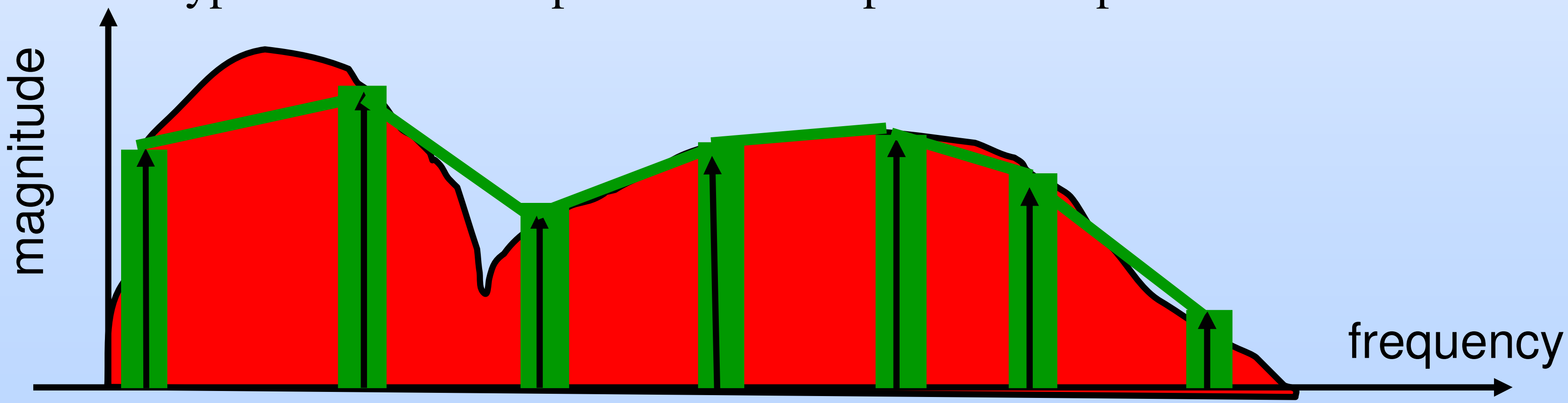
# Ideal Channel Estimation

- **Wireless channels change frequently ~ 10 ms**
- **Require frequent channel estimation**
- **Many systems use pilot tones – known symbols**
  - Given $s_k$, for $k = k_1, k_2, k_3, \ldots$ solve $x_k = \sum_{l=0}^{L} h_l e^{-j2\pi k l/N} s_k$ for $h_l$
  - Find $H_k = \sum_{l=0}^{L} h_l e^{-j2\pi k l / N}$ (significant computation)
- **More pilot tones**
  - Better noise resiliance
  - Lower throughput (pilots are *not* informative)



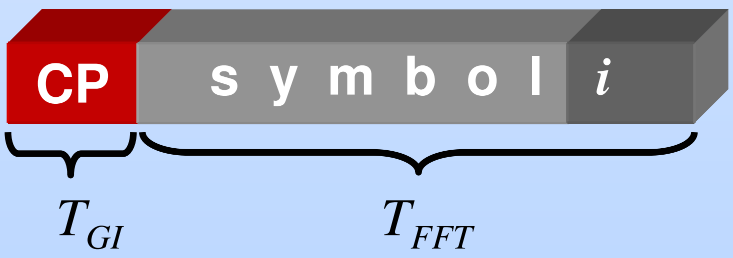
# Channel Estimation Via Interpolation

- **More efficient approach is interpolation**
- **Algorithm**
  - For each pilot $k_i$ find $H_{ki} = x_{ki} / s_{ki}$
  - Interpolate unknown values using interpolation filter
  - $H_m = \alpha_{m,1} H_{k1} + \alpha_{m,2} H_{k2} + \ldots$
- **Comments**
  - Longer interpolation filter: more computation, timing sensitivity
  - Typical 1dB loss in performance in practical implementation



# Case Study: IEEE 802.11a Wireless LAN

- **System parameters**
  - FFT size: 64
  - Number of tones used 52 (12 zero tones)
  - Number of pilots 4 (data tones = 52-4 = 48 tones)
  - Bandwidth: 20MHz
  - Subcarrier spacing : $\Delta_f$ = 20 MHz / 64 = 312.5 kHz
  - OFDM symbol duration: $T_{FFT} = 1/\Delta_f$ = 3.2us
  - Cyclic prefix duration: $T_{GI}$ = 0.8us
  - Signal duration: $T_{signal} = T_{FFT} + T_{GI}$



# Case Study: IEEE 802.11a WLAN

- Modulation: BPSK, QPSK, 16-QAM, 64-QAM
- Coding rate: 1 / 2, 2 / 3, 3 / 4
- FEC: *K*=7 (64 states) convolutional code

| Frequency Band (GHz) | Maximum Output Power (6dBi antenna gain) mW |
|---|---|
| 5.15 – 5.25 | 40 |
| 5.25 – 5.35 | 200 |
| 5.725-5.825 | 800 |