

ECE5984

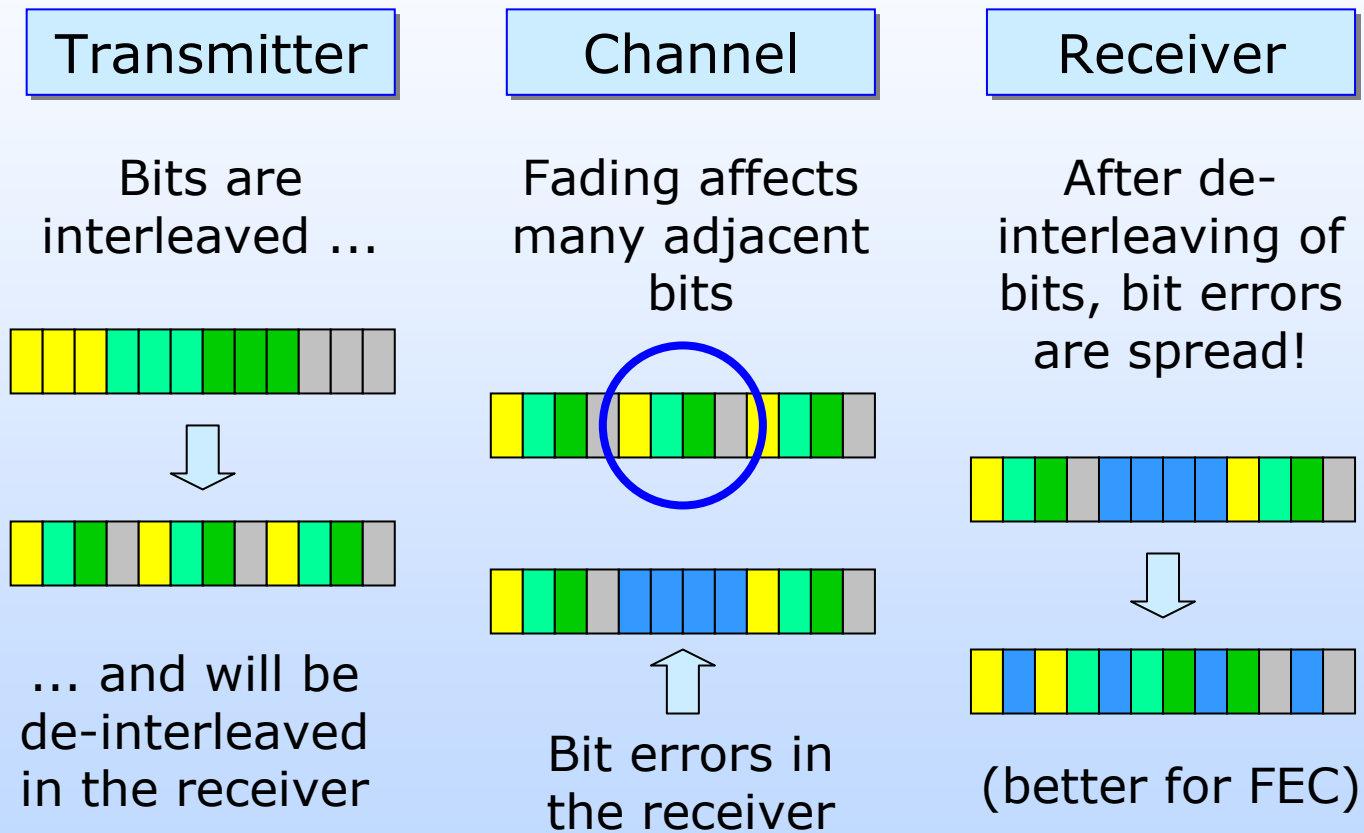
**Orthogonal Frequency Division Multiplexing and Related  
Technologies  
Fall 2007**

**Mohamed Essam Khedr**

**Adaptive Modulation in OFDM**

**<http://www.ccit.aast.edu/public/VT/Term4/>**

# Bit interleaving



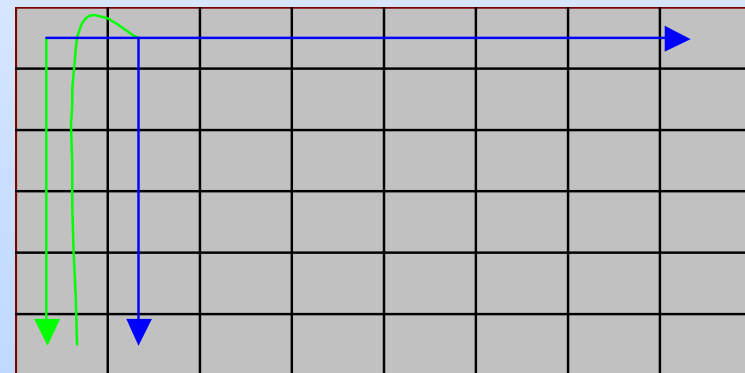
# Interleaving

- **Most codes are designed for AWGN channel. Performance rapidly degrades in frequency selective channels with correlated channels.**
- **Scatter error bursts**
- **Can be done in time and in frequency domains**

Interleaving basically a matrix which you write data in columns and read in rows

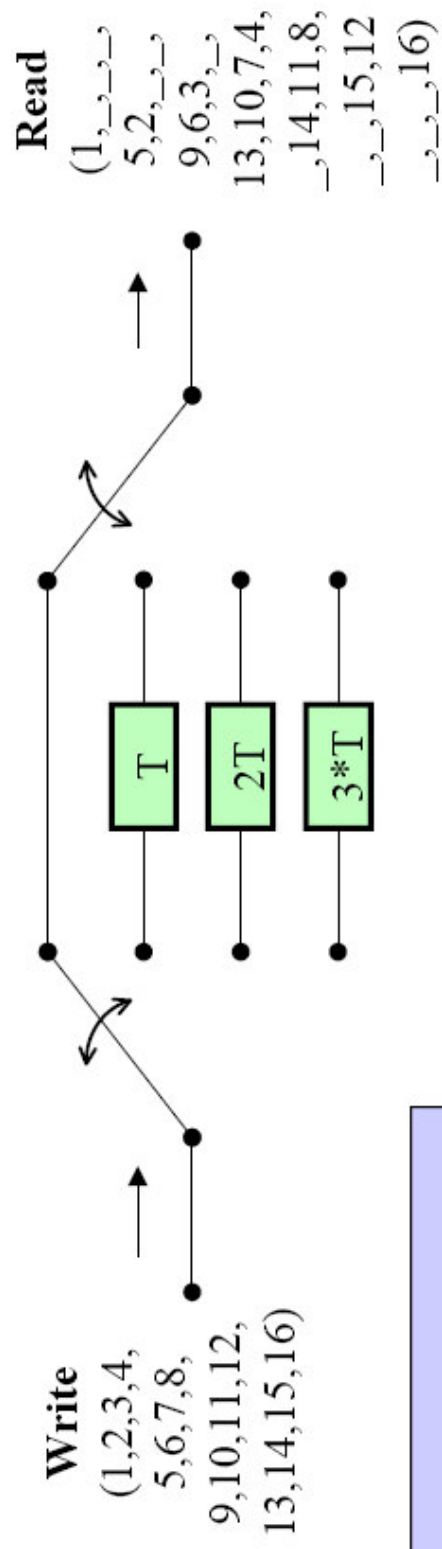
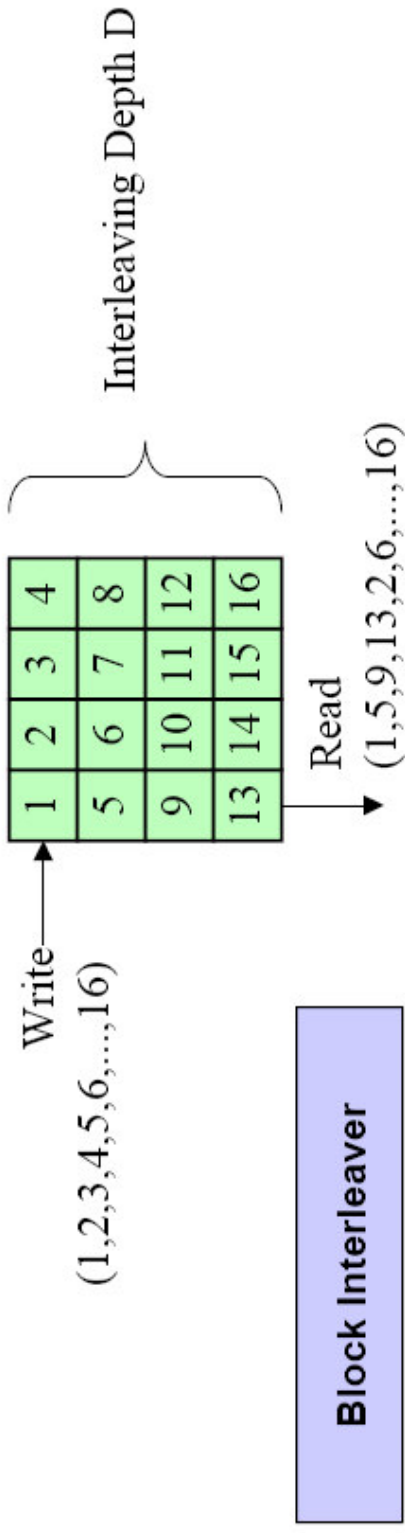
6x8 block interleaver shown

Has interleaving depth 48



# Interleaving (1)

VIII/27



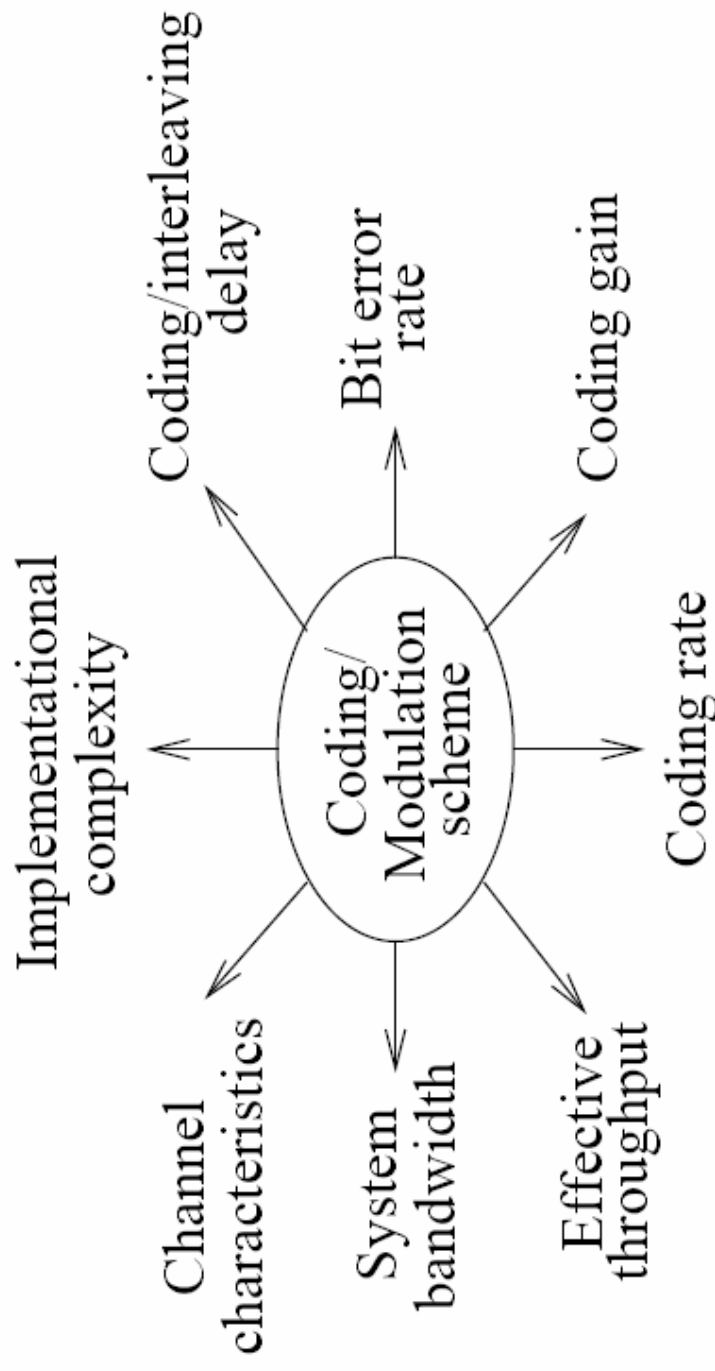


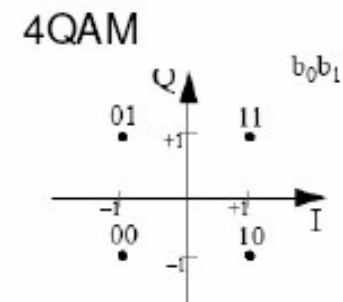
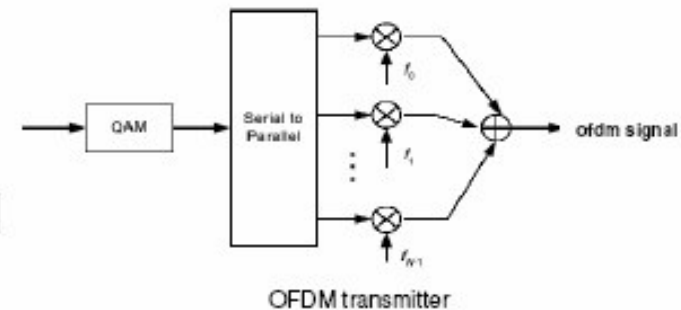
Figure 2: Factors affecting the design of channel coding, modulation and transceiver



# Homework

Due date: Tuesday 25-9-2007

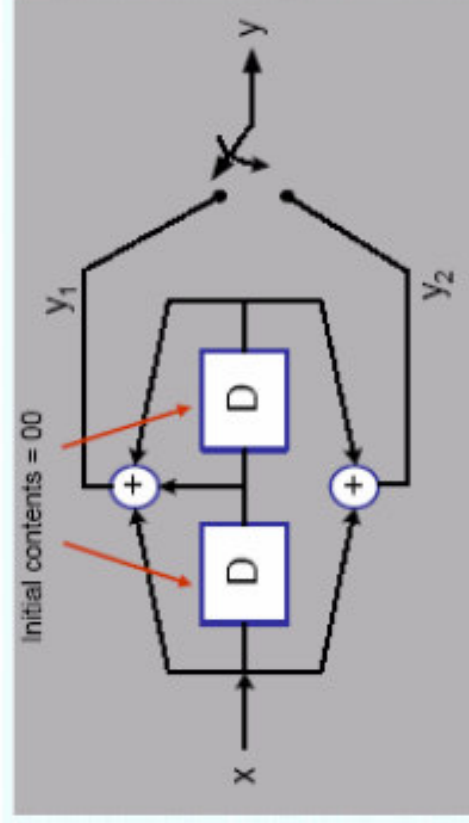
- Derive expression for OFDM-signal
- Use 4 subchannels and 4QAM
- Input data sequence:  
11 01 00 10
- Subcarrier frequencies are:  
 $-2f_c -1f_c 1f_c 2f_c$



🚩 Please use Viterbi algorithm to decode the received sequence:

$Z=[11\ 10\ 10\ 10\ 01]$

Please draw the trellis and state diagram



# Syllabus

- **Wireless channels characteristics (7.5%)** **1**
- **OFDM Basics (10%)** **1**
- **Modulation and Coding (10%)** **2**
  - Linear and nonlinear modulation
  - Interleaving and channel coding
  - Adaptive modulation, Optimal bit and power allocation



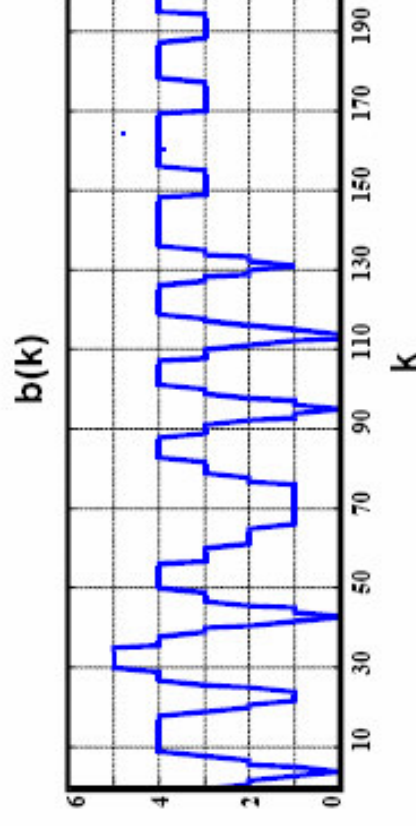
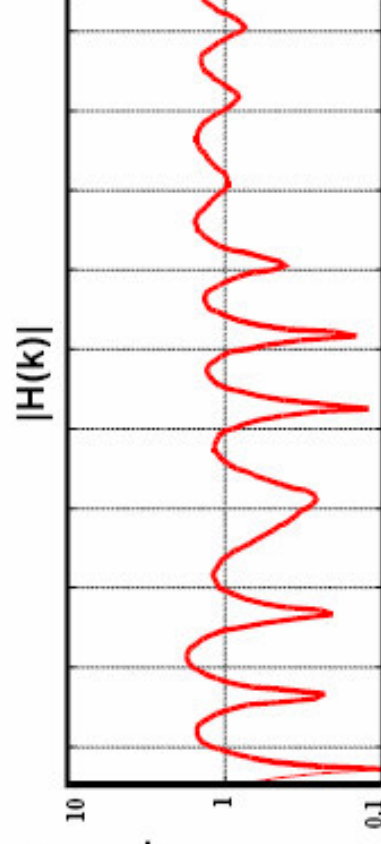
## Concept Behind Adaptive Modulation

- **The bit error probability of different OFDM subcarriers in a dispersive channels depends on the frequency domain channel transfer function.**
- **The overall BER can be improved if the deep faded subcarriers are identified and excluded from transmission**
  - Degradation in system throughput
- **This degradation can be avoided using adaptive modulation techniques**
  - Coding rate can also be adapted
- **Adaptation is done based on the transmitter's perception to the changes in the channel conditions in the forthcoming time slot**
  - Channel estimation is needed
  - Doppler problem : fast fading and slowly fading

## Adaptive Modulation (Bit Loading)

OFDM gives the opportunity to use:

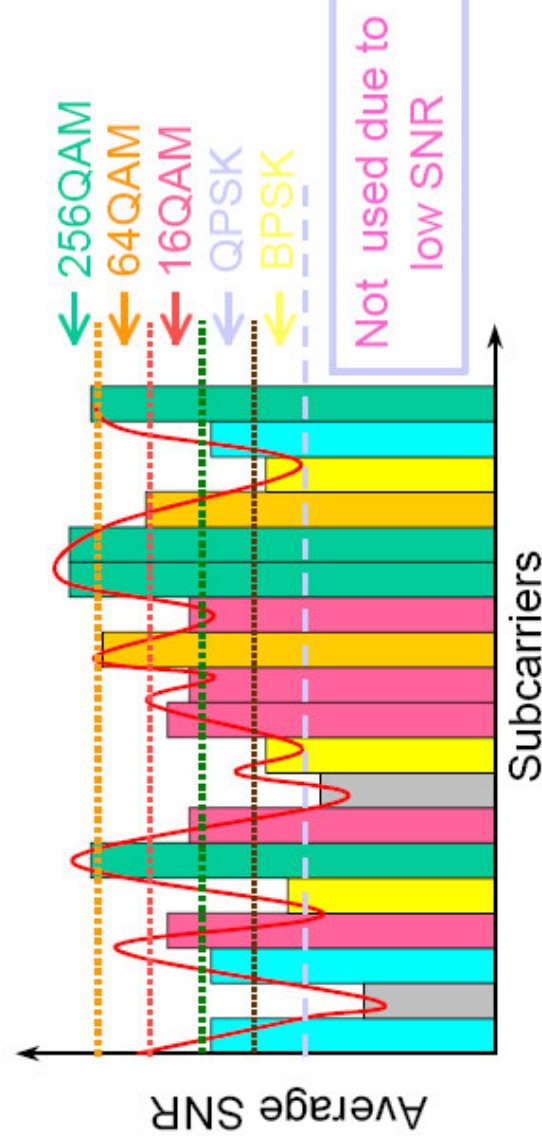
- different modulation schemes for each subchannel
- different power for each subchannel



### Adaptive Modulation

Adaptation to the channel transfer function using subchannel specific modulation schemes and power

## Adaptive Modulation (Bit Loading)



Algorithms: Chow, Cioffi and Bingham: capacity maximization

Fischer: Error probability minimization

Grünheid: simple blockwise loading algorithm

Hughes-Hartogs: sets target rate  $R$ , intensive searching

# Steps for Adaptation

- **Step one:**
  - Channel quality estimation. (better in slowly fading)
    - Open loop (reciprocal of the transfer function, eg: TDD)
    - Closed loop (side information, eg: FDD)
- **Step two:**
  - Choice of the appropriate parameters for the next transmission
    - Modulation and coding according to SNR
- **Step three:**
  - Signaling or blind detection of the employed parameters at the receiver side

## Goal of Adaptive Modulation

- The local signal to noise ratio

$$\gamma_n = |H_n|^2 \cdot \gamma_s$$

- Choose the appropriate modulation mode for transmission in each subcarrier given the local SNR.
- Modulation mode is not varied on a subcarrier-by-subcarrier basis to reduce complexity
  - Subbands and same modulation for each of these subbands

## Types of Adaptive Modulation

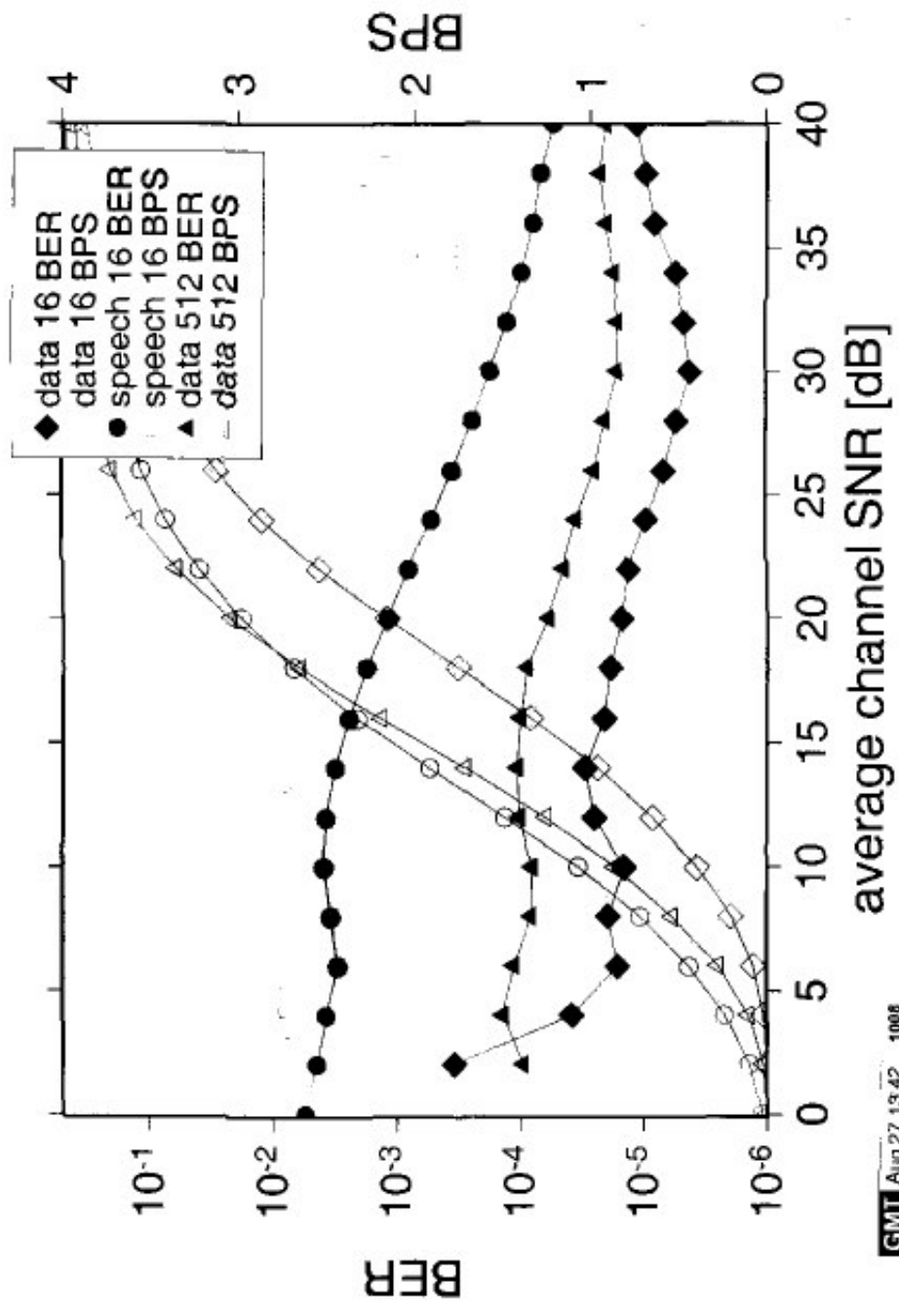
- Fixed threshold controlled algorithm
- Upper bound BER estimator
- Fixed throughput adaptation algorithm

## Fixed Threshold Controlled Algorithm

- The channel quality is assumed to be constant for all symbols in the time slot. → slowly varying channel.
- → all data symbols in the transmit time slot employ the same modulation mode chosen according to the predicted SNR.
- The SNR thresholds for a long term BER can be found using optimization procedure.

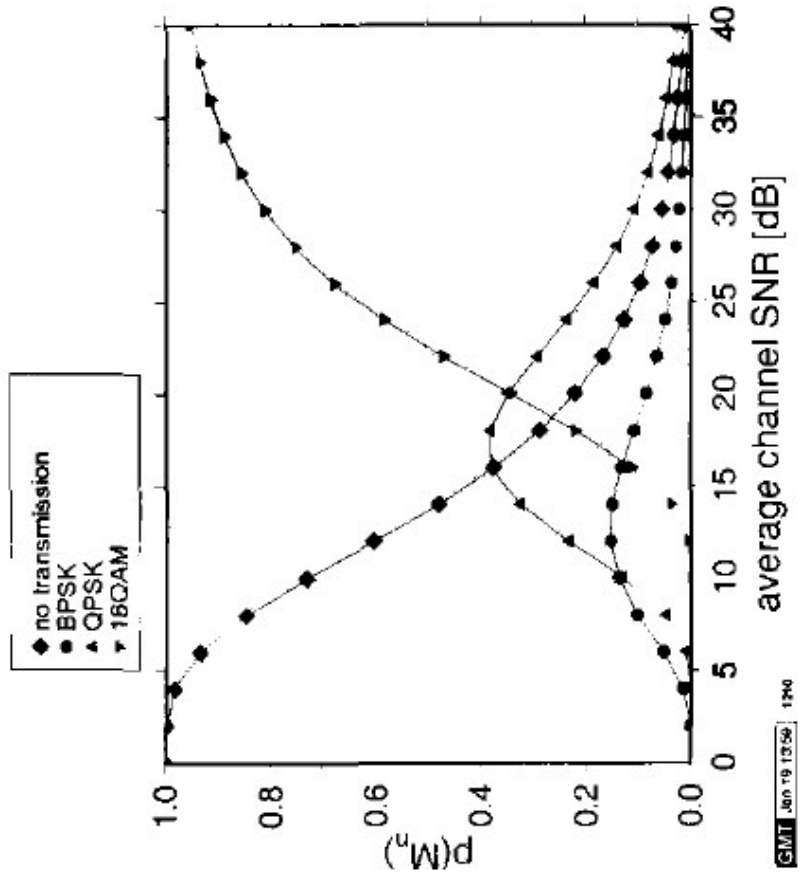
	$l_0$	$l_1$	$l_2$	$l_4$
speech system	$-\infty$	3.31	6.48	11.61
data system	$-\infty$	7.98	10.42	16.76

- OFDM is used in frequency selective channel that have channel quality varying across different subcarriers.

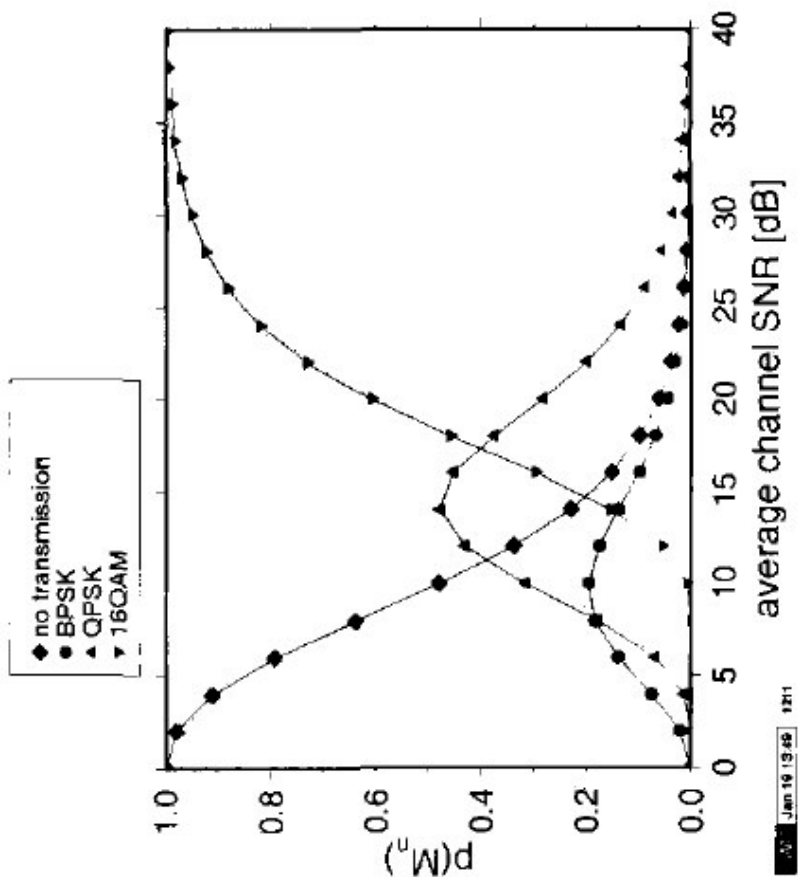


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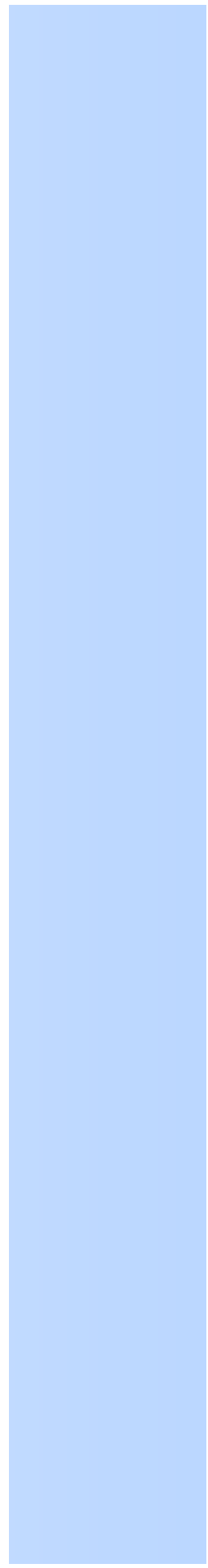
Figure 6.4: BER and BPS throughput performance of the 16 sub-carrier switching level adaptive OFDM modem employing BPSK, QPSK, 16-QAM and "no transmission" over the Rayleigh fading time dispersive channel of Figure 4.3 using the switching thresholds of Table 6.1



(a) 16 sub-bands



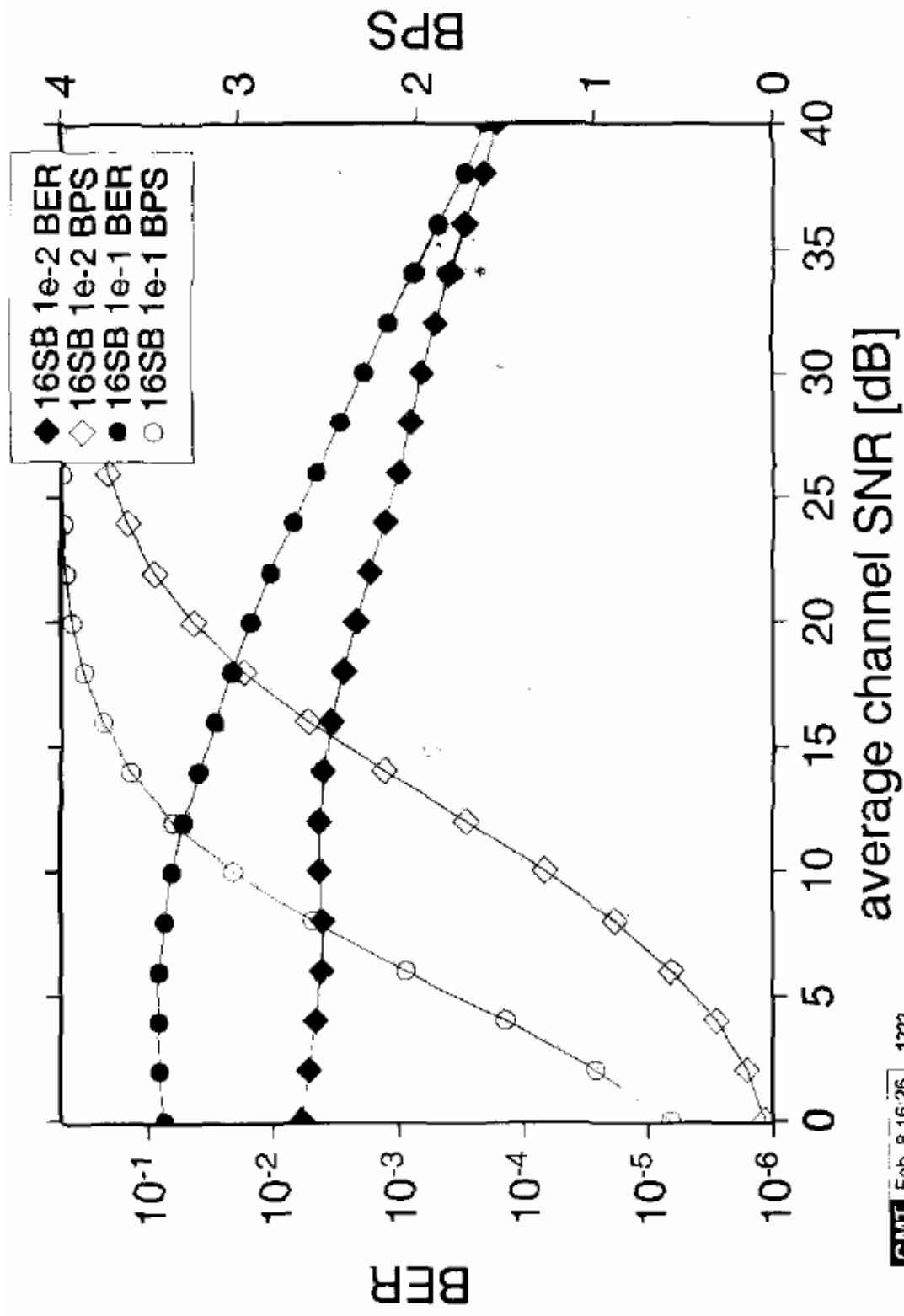
(b) subcarrier-by-subcarrier





## Sub-band BER Estimator Adaptation Algorithm

- The previous method has throughput penalty if used in an adaptive sub-band OFDM and the channel quality is non constant throughout each sub-band.
- Sub-band BER estimator adaptation algorithm takes into consideration the non-constant SNR values across the subcarriers in the  $j$ th sub-band
  - Calculating the expected overall P[E] for all available modulation modes in each sub-band  $\bar{p}_e(n) = 1/N_s \sum_j p_e(\gamma_j, M_n)$
  - For each sub-band, the modulation mode with the highest throughput and its estimated BER is below a given threshold is chosen.



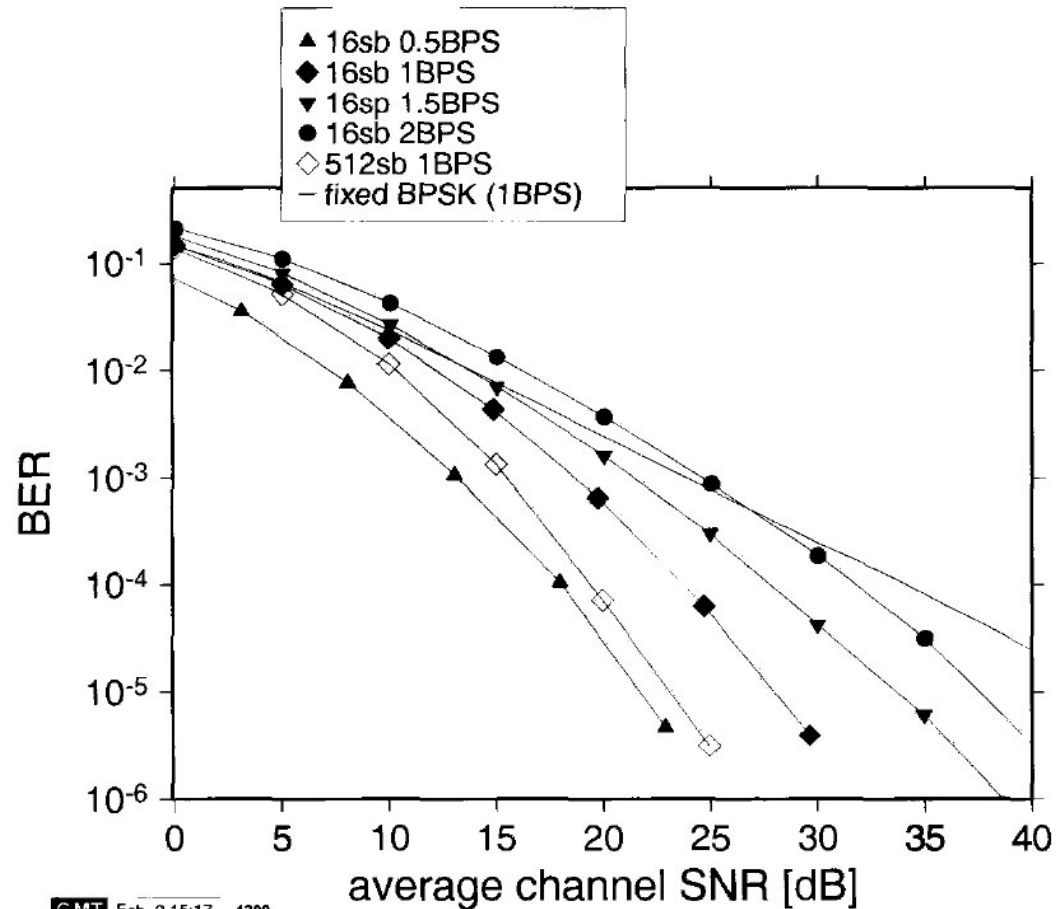
**6.6:** BER and BPS throughput performance of the 16 sub-band, 512 subcarrier BER estimator adaptive OFDM modem employing BPSK, QPSK, 16-QAM and “no transmission” over the Rayleigh fading time dispersive channel of Figure 4.3

# Constant Throughput Adaptive Modulation

- **The previous two methods are difficult to use for situations with variety constant rate applications.**
- **Constant Throughput Adaptive Modulation exploits the frequency selectivity of the channel while offering a constant bit rate.**
- **Sub-band adaptivity is assumed for simplification**
- **The algorithm is based on a cost function**
  - The cost function depends on the expected number of bit errors in each sub-band  $e_{n,s}$ .  $n$  is the sub-band index,  $s$  is the modulation index.
  - The cost function is calculated on the basis of the estimated channel transfer function and the number of bits transmitted per sub-band and per modulation  $b_{n,s}$
  - A set of cost functions are calculated for each sub-band and modulation index

$$c_{n,s} = \frac{e_{n,s+1} - e_{n,s}}{b_{n,s+1} - b_{n,s}}$$

- The modulation mode adaptation is performed by repeatedly searching for the block n having the lowest value of cost and incrementing its state variable.
- This is repeated until the total number of bits in the OFDM symbol reaches the target number of bits



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6.7: BER performance versus SNR for the 512 subcarrier, 16 sub-band constant throughput adaptive OFDM modem employing BPSK, QPSK, 16-QAM and “no transmission” in the Rayleigh fading time dispersive channel of Figure 4.3 for 0.5, 1, 1.5 and 2 bits per symbol (BPS) target throughput

# Capacity of Fading Channels

- **Three cases**
  - Fading statistics known
  - Fade value known at receiver
  - Fade value known at receiver and transmitter
- **Optimal Adaptation**
  - Vary rate and power relative to channel
  - Optimal power adaptation is water-filling
  - Exceeds AWGN channel capacity at low SNRs
  - Suboptimal techniques come close to capacity

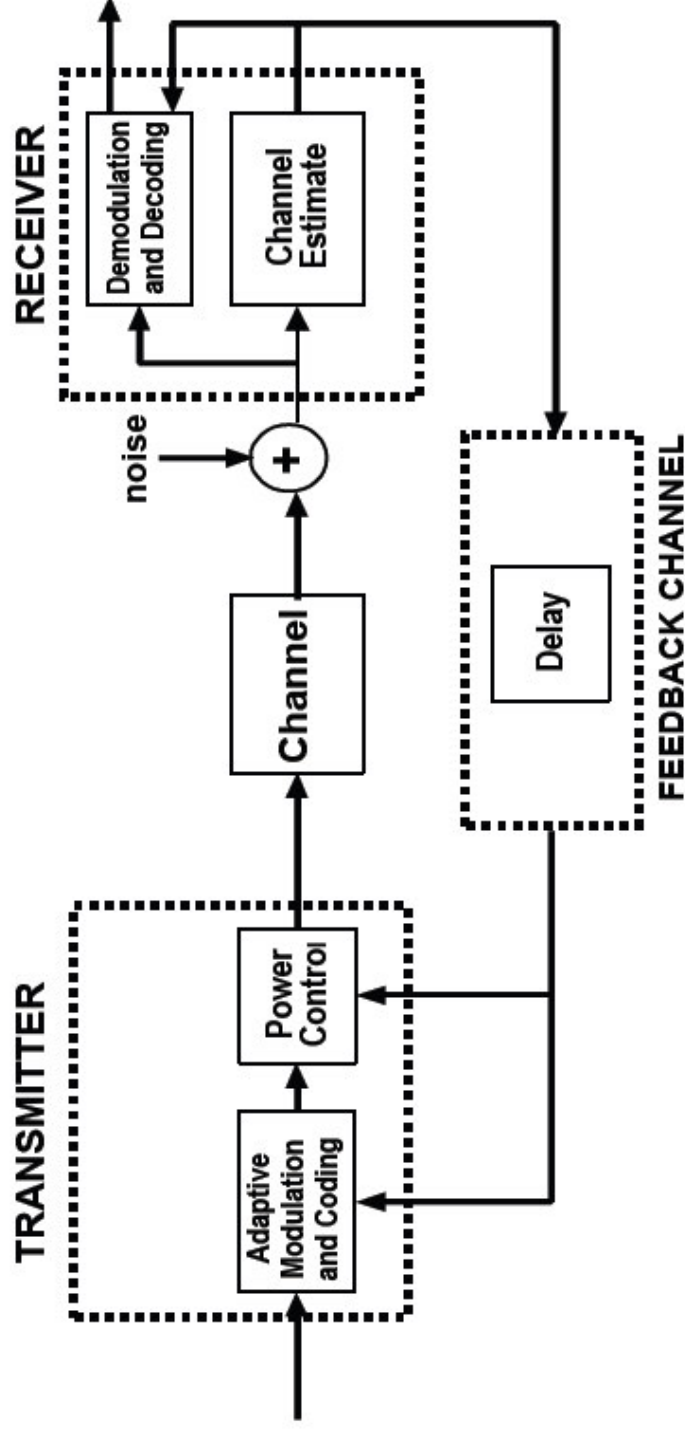
# Adaptive Modulation

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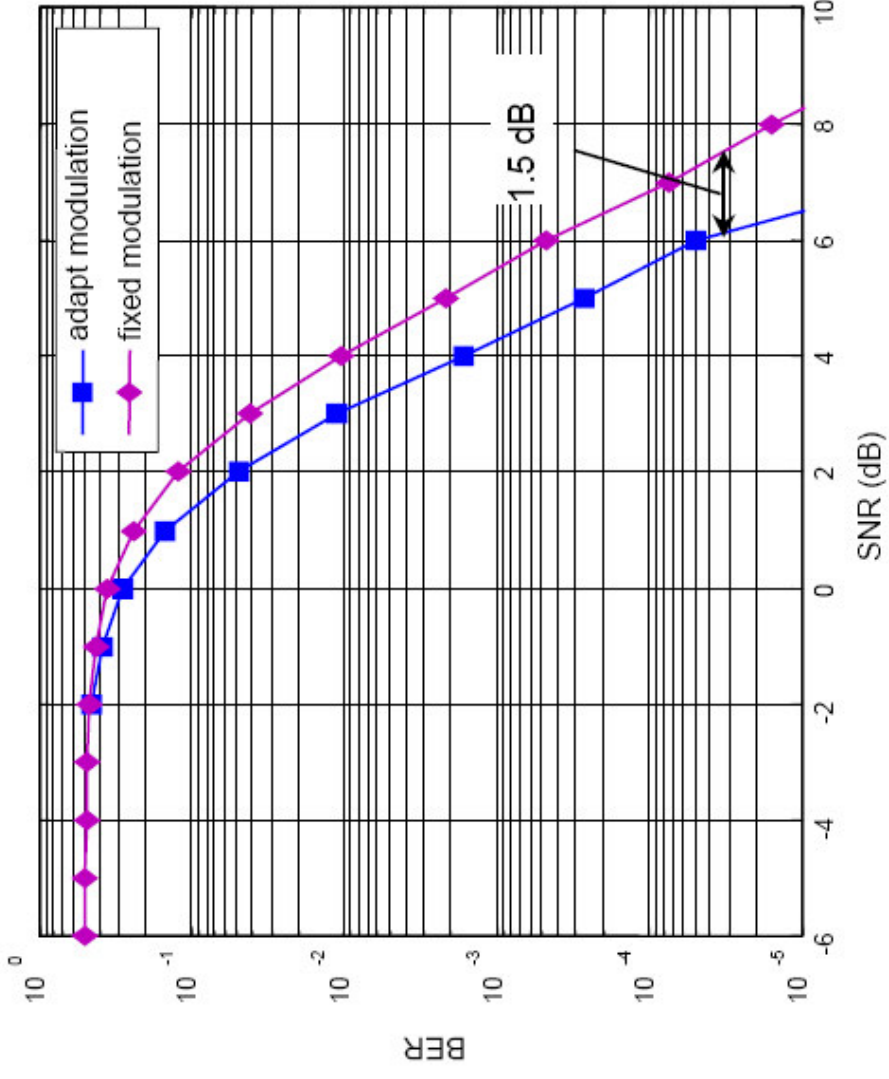
- Changing modulation relative to fading.  
Transmitter uses channel fade level to adapt.
  - Parameters to adapt:
    - Constellation size
    - Transmit power
    - Instantaneous BER
    - Symbol time
    - Coding rate/scheme
  - Optimization criterion:
    - Maximize throughput
    - Minimize average power
    - Minimize average BER
- Only 1-2 degrees of freedom needed for good performance*

# Adaptive Modulation



**Requires reliable feedback channel and accurate channel estimation**  
**Increases transmitter and receiver complexity**

- Adaptive modulation (average 2 bits per subcarrier)





# Adaptive Techniques

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- **Variable-Rate Techniques**
  - Data rate  $R(\gamma)$  is varied relative to channel gain  $\gamma$ 
    - λ Fix symbol rate  $R_s = 1/T_s$  and use multiple modulation schemes or increase constellation size
    - λ Fix modulation but change symbol rate (difficult to achieve because of variable bandwidth)
- **Variable-Power Techniques**
  - Transmit power is changed to compensate for SNR variations due to fading
  - Invert channel to maintain constant received SNR
- **Variable-Coding Techniques**

# Variable-Rate Variable-Power MQAM

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M-ary signaling: binary message sequence divided into words of length  $K$  bits, sent every  $T_s$  seconds

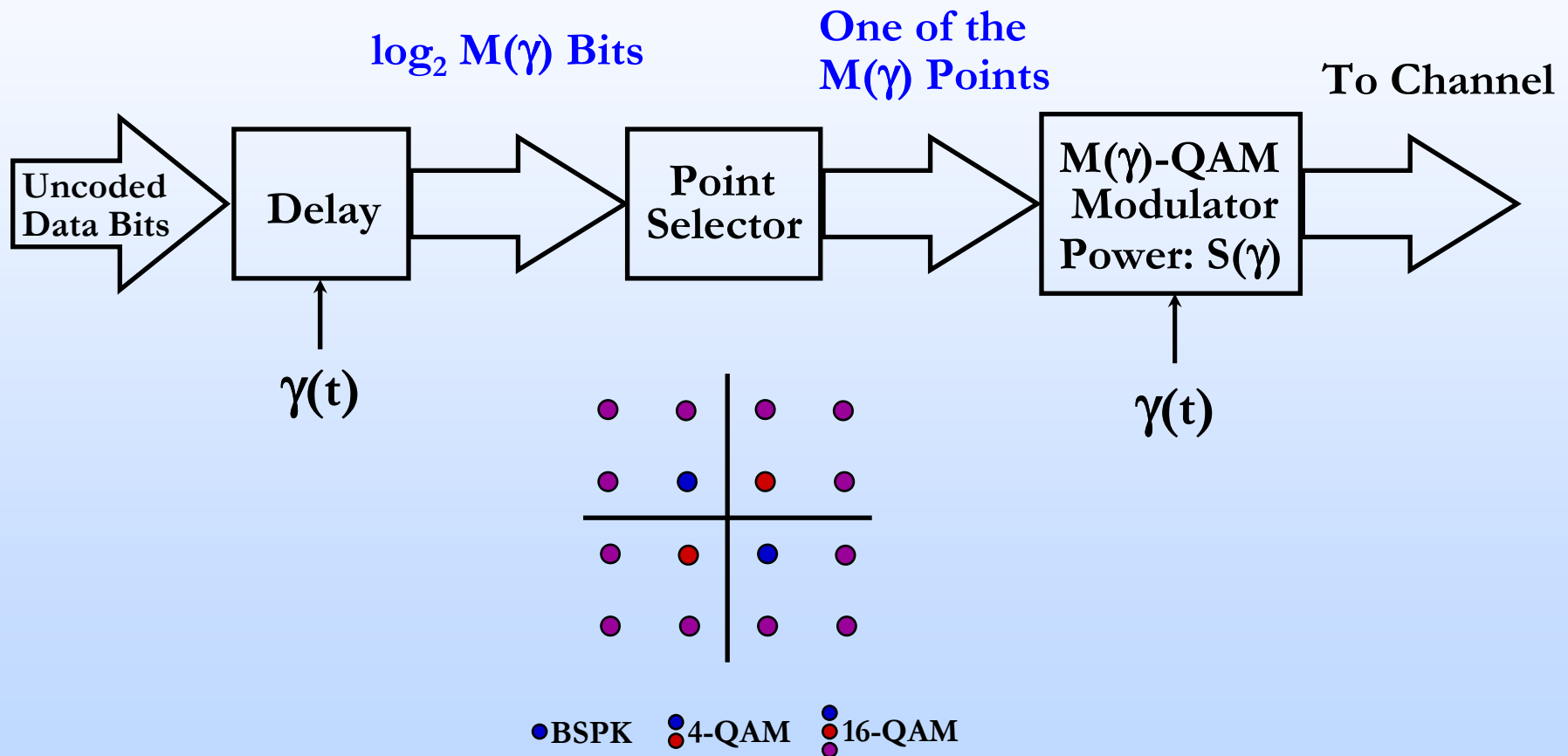
$$M = 2^K, \quad K = \log_2 M, \quad R = K/T_s \text{ bps}$$

$$\text{Let } B = 1/T_s \text{ (Nyquist),} \quad R = B \log_2 M,$$

$$\text{Spectral Efficiency:} \quad R/B = \log_2 M$$

**Goal:** Maximize spectral efficiency subject to a target  $P_b$ ,  
using Variable-Rate Variable-Power MQAM

## Variable-Rate Variable-Power MQAM



*Goal: Optimize  $S(\gamma)$  and  $M(\gamma)$  to maximize  $EM(\gamma)$*

# Optimization Formulation

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BER in non-fading AWGN channel with MQAM modulation,  
Coherent detection, SNR =  $\gamma$  :

$$P_b \leq 0.2e^{-1.5\gamma/(M-1)} \quad M \geq 4 \quad 0 \leq \gamma \leq 30dB$$

$$P_b(\gamma) \leq 0.2 \exp \left[ \frac{-1.5\gamma}{M-1} \frac{P(\gamma)}{\bar{P}} \right]$$

**Adaptive MQAM: Rate for fixed BER**

$$M(\gamma) = 1 + \frac{1.5\gamma}{-\ln(5BER)} \frac{P(\gamma)}{\bar{P}} = 1 + K\gamma \frac{P(\gamma)}{\bar{P}} \quad K = \frac{-1.5}{\ln(5P_b)} < 1$$

# Optimization Formulation

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## Rate and Power Optimization

$M(\gamma)$  is random. Spectral efficiency is maximized by  
Maximizing  $E[\log_2 M(\gamma)]$ :

$$\begin{aligned}\max_{P(\gamma)} E \log_2 [M(\gamma)] &= \max_{P(\gamma)} E \log_2 \left[ 1 + K\gamma \frac{P(\gamma)}{P} \right] \\ &= \max_{P(\gamma)} \int_0^{\infty} \log_2 \left( 1 + \frac{K\gamma P(\gamma)}{P} \right) p(\gamma) d\gamma\end{aligned}$$

Subject to average power:  $\int_0^{\infty} P(\gamma) p(\gamma) d\gamma = \bar{P}$

# Variable-Power Techniques

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**Maintains constant SNR  $\sigma$  at the receiver by adapting transmitting power. Adaptation is subject to maintaining given average power**

$$\int_0^{\infty} P(\gamma) p(\gamma) d\gamma = \bar{P} \qquad \frac{P(\gamma)}{P} \gamma = \sigma$$
$$\int_0^{\infty} \frac{P(\gamma)}{P} p(\gamma) d\gamma = \int_0^{\sigma} \frac{\sigma}{\gamma} p(\gamma) d\gamma = 1 \qquad \longrightarrow \qquad \sigma = \frac{1}{\mathbf{E}_{\gamma_0} \left[ \frac{1}{\gamma} \right]}$$

**$\sigma$  depends on  $p(\gamma)$ , which in turn depends on  $\bar{P}$ . For a given  $\bar{P}$  if  $\sigma > 1/\mathbf{E}[1/\gamma]$  target BER can not be met.**

Note that for Rayleigh fading where  $\gamma$  is exponentially distributed,  $E[1/\gamma] = \infty$ , so no target  $P_b$  can be met using channel inversion.

## Variable-Power Techniques

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**Fading can be inverted above a given cutoff  $\gamma_0$**

**Power adaptation based on truncated channel inversion:**

$$\frac{P(\gamma)}{\bar{P}} = \begin{cases} \frac{\sigma}{\gamma} & \gamma \geq \gamma_0 \\ 0 & \gamma < \gamma_0 \end{cases} \quad \mathbf{E}_{\gamma_0} \left[ \frac{1}{\gamma} \right] = \int_{\gamma_0}^{\infty} \frac{1}{\gamma} p(\gamma) d\gamma$$

**Fading can be inverted above a given cutoff value  $\gamma_0$  can be Based on a desired outage probability  $P_{\text{out}} = \text{pr}(\gamma < \gamma_0)$**

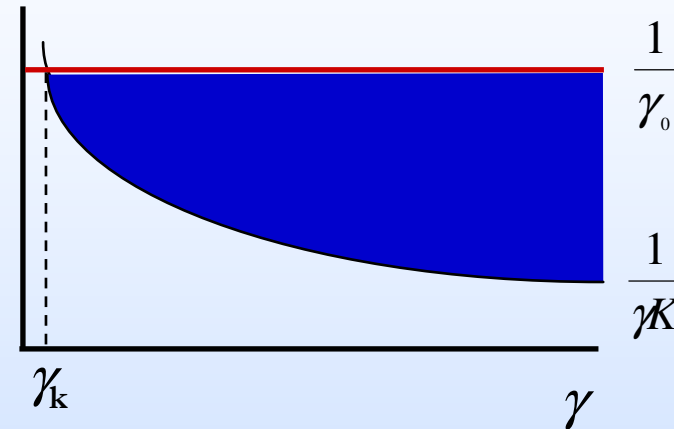
# Optimal Adaptive Scheme

- Power Water-Filling**

$$\frac{P(\gamma)}{\bar{P}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma K} & \gamma \geq \frac{\gamma_0}{K} = \gamma_K \\ 0 & \text{else} \end{cases}$$

instantaneous rate

$$\log_2 M(\gamma) = \log_2(\gamma/\gamma_K)$$



- Spectral Efficiency**

$$\frac{R}{B} = \int_{\gamma_K}^{\infty} \log_2 \left( \frac{\gamma}{\gamma_K} \right) p(\gamma) d\gamma$$

$$M = 2^K, \quad K = \log_2 M, \quad R = K/T_s \text{ bps}$$

$$\text{Let } B = 1/T_s \text{ (Nyquist),} \quad R = B \log_2 M,$$

$$\text{Spectral Efficiency:} \quad R/B = \log_2 M$$

*Equals Shannon capacity with an effective power loss of K.*



## Practical Adaptation Constraints

- **Constellation restriction**
- **Constant power restriction**
- **Constellation updates.**
- **Estimation error.**
- **Estimation delay.**
- **Lead to practical adaptive modulation schemes**

# Reading assignment

- **Chapter six up to section 6.2.6 from the textbook**
- **Paper: degrees of freedom in adaptive modulation a unified approach**
- **Paper: Variable-Rate Variable-Power MQAM for Fading Channels**
  - Write a report on these papers, due date: Thursday 27-9-2007