ECE5984

Orthogonal Frequency Division Multiplexing and Related Technologies

Fall 2007

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Channel Estimation

Matlab Assignment #1
Thursday 4 October 2007

- **Develop an OFDM system with the following components**
  - S/P
  - Mapping model (modulation techniques)
  - Coding model (conv, turbo)
  - IFFT
  - CP
  - Channel (Gaussian, SFFF channel)
  - Mapping decoding
  - Decoding model
  - FFT
  - CP removal
  - Channel Estimation (later)
- **Input :pulse shaping, Number of subcarriers, symbol rate, BW, CP ratio**
- **Output: Signal in time, spectrum, BER, ICI (later), ISI (later)**

HW 2 1/2
Due Thursday 4 December

Example 4.1 *A certain wideband wireless channel has a delay spread of* $1\mu sec$*. We assume that in order to overcome ISI, that* $T_s \geq 10\tau$.

1. *What is the maximum bandwidth allowable in this system?*
2. *If multicarrier modulation is used, and we desire a 5 MHz bandwidth, what is the required number of subcarriers?*

Single-Carrier Vs Multi-Carrier HW 2/2

Design Parameters for outdoor channel	Required data rate	1Mbps
	RMS delay spread, σ	10 µs
	Channel Coherence bandwidth, $B_c = 1/5\ \sigma$	20 kHz
	Frequency selectivity condition	$\sigma > T_{symbol}/10$
Single Carrier Approach	Symbol duration, T_{symbol}	
	Frequency selectivity	
	Comment on ISI	
Multicarrier approach	Total number of carriers	128
	Data rate per carrier	
	Symbol duration per carrier	
	Frequency selectivity	
	Comment on ISI	

Major Learning Objectives

- Upon successful completion of the course the student will be able to:
- Describe the complete architecture of an OFDM system, (serial to parallel, FFT/IFFT, Cyclic prefix, Modulation techniques, coding techniques)
- Evaluate the response of OFDM in Gaussian channels and fading channels.
- Design and analyze standards using OFDM such as IEEE 802.11a,g and IEEE 802.16
- Define the problems associated of using multi-carrier in time varying channels and how to mitigate these problems.
- Describe the principle mechanisms by which multiple access techniques are supported using OFDM.
- Able to categorize the different type of MC-CDMA and the degree of flexibility provided by each type.
- Able to simulate the basic and advanced techniques used in OFDM systems

Syllabus

- **Analysis of OFDM systems** (15%) **2**
  - RF subsystems, amplifier classification and distortion
  - Crest factor (PAPR) reduction techniques
    - Pre-distortion & adaptive pre-distortion techniques
    - clipping
    - coding techniques
    - partial transmit sequences (PTS) & modified PTS v. selective mapping
    - nonlinear quantization (companding)
  - Phase noise and I&Q imbalance for QAM
  - Performance of OFDM in Gaussian channels
  - Performance of OFDM in Wide-band channels
- **Synchronization and Estimation** (15%) **2**
  - ICI and OISI problems √
  - Timing estimation
  - Frequency synchronization
  - Frequency error estimation algorithms
  - Carrier phase tracking
  - Frequency domain and time domain approaches for channel estimation
    - coherent detection
    - differential detection

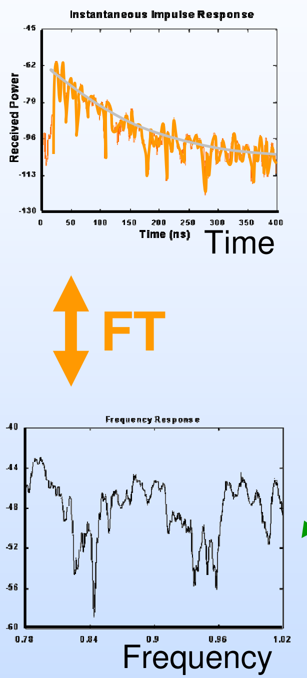
Our OFDM System Assumptions

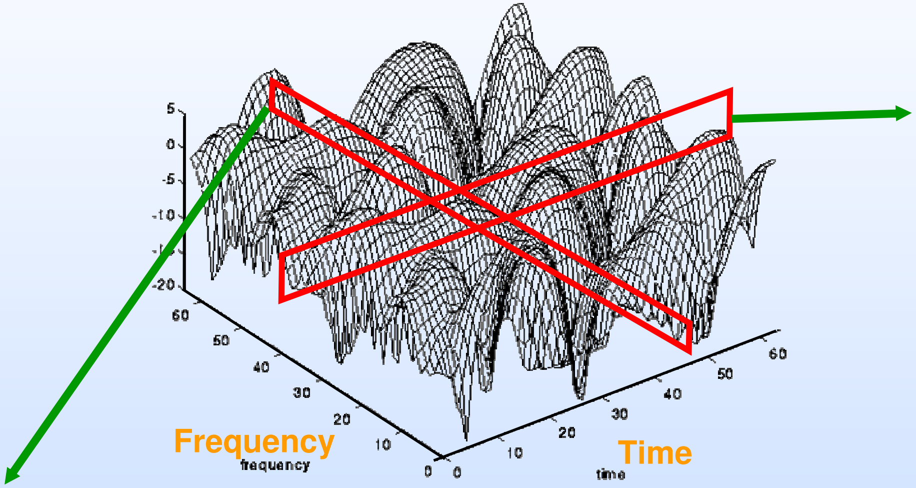
- Usage of cyclic Prefix
- Impulse response of the channel shorter than Cyclic Prefix.
- Slow fading effects so that the channel is time-invariant over the symbol interval.
- Rectangular Windowing of the transmitted pulses
- Perfect Synchronization of transmitter and receiver
- Additive, white, Gaussian channel noise

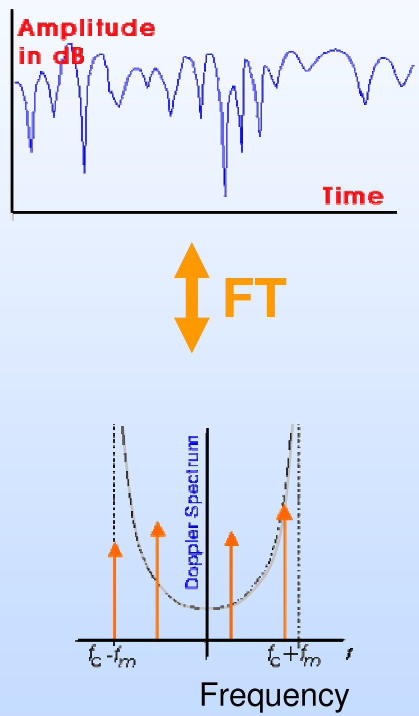
The Mobile Multipath Channel

•Delay spread

•Doppler spread







Channel Estimation

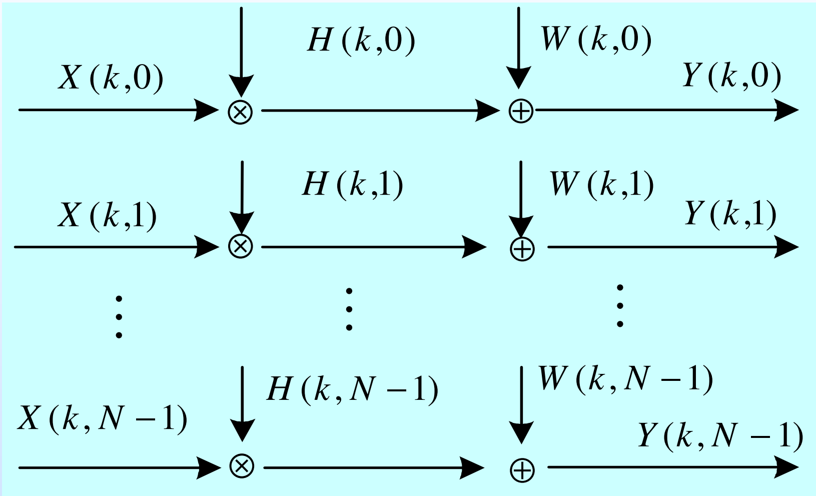
- **Interpolation, or filtering. Assume that known symbols X(n,k) were transmitted in various positions (tones (n) or blocks (k)). Estimate H(n,k) and then interpolate to n',k'.**

Channel Estimation Types

- **Parametric vs non-parametric**
- **Frequency and time correlation**
- **Training vs blind**
- **Adaptive vs non-adaptive**

- **Parametric → based on a channel model**
- **Non-parametric → based on measurements.**
- **Correlation → estimation is based on previous estimates**
- **Training→ well known symbols**
- **Blind→ based on the statistical properties of the signal**
- **Adaptive → estimation algorithm modified with the channel variations**

OFDM System Model



$$\mathbf{Y_k} = \mathbf{X_k^{N\times N}} \mathbf{H_k} + \mathbf{W_k} = \mathbf{H_k^{N\times N}} \mathbf{X_k} + \mathbf{W_k}$$

System Architecture-2

1 Input to Time Domain

$$x(n) = IDFT\ \{X(k)\}$$

$$n = 0,1,2,\ldots, N-1$$

2 Guard Interval

$$x_f(n) = \begin{cases} x(N+n), & n = -N_g, -N_g+1, \ldots, -1 \\ x(n), & n = 0,1,\ldots, N-1 \end{cases}$$

3 Channel

$$y_f = x_f(n) \otimes h(n) + w(n)$$

4 Guard Removal

$$y(n) = y_f(n) \quad n = 0,1,\ldots, N-1$$

5 Output to Frequency Domain

$$Y(k) = DFT\ \{y(n)\}$$

$$k = 0,1,2,\ldots, N-1$$

6 Output

Channel | ICI | AWGN

$$Y(k) = X(k)H(k) + I(k) + W(k)$$

$$k = 0,1,\ldots, N-1$$

7 Channel Estimation

Estimated Channel

$$X_e(k) = \frac{Y(k)}{H_e(k)} \quad k = 0,1,\ldots, N-1$$

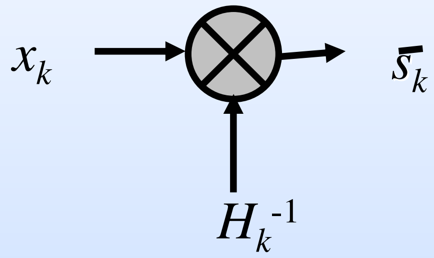
Frequency Domain Equalization

- For the k^{th} carrier:

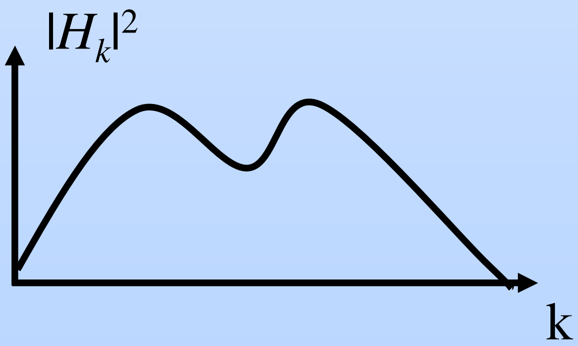
$$x_k = H_k\, s_k + v_k$$

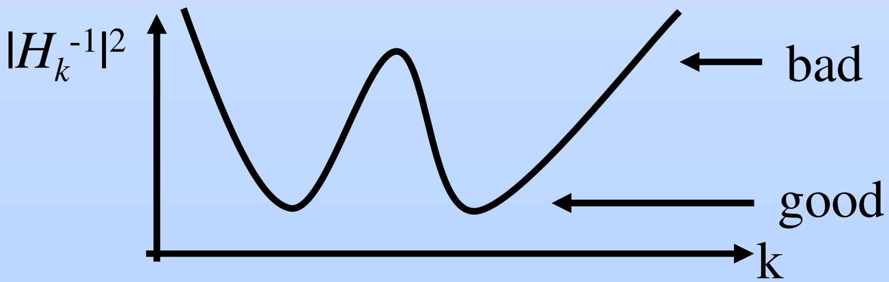
where $H_k = \sum_n h_k(nT_s) \exp(j2\pi\, k\, n\, /\, N)$ where $n = 0, \ldots,. N\text{-}1$

- Frequency domain equalizer



- Noise enhancement factor $\hat{\sigma}_k^2 = \sigma_k^2 |H_k^{-1}|^2$





Channel Estimation

- OFDM uses variations of Quadrature Amplitude Modulation (QAM) schemes for symbol mapping which require a coherent detection method in the receiver.
  - **Naturally, data detection in coherent OFDM receivers require an accurate (or near accurate) estimate of Channel State Information (CSI).**
- There are two major kinds of channel estimators that are found in literature:
  - **Pilot assisted.**
  - **blind estimation.**
  - **A mixture of these two, where a blind method with limited training symbols is used, is called semi-blind technique.**

Types of Channel Estimation

- Traditional one-dimensional channel estimation techniques for the OFDM systems can be summarized as follows:
  - **Least Squares (LS)**
  - **Minimum Mean Squared Error (MMSE)**
  - **Linear MMSE (LMMSE).**
- LS estimators are very simple to constitute, but they suffer from MSE in low SNR conditions.
- MMSE, based on time domain estimations, are high complexity estimators that provide good performance in sampled-spaced channels, but limited performance in non-sample spaced channels and high SNR conditions.
- LMMSE provides good performance in both sampled and non-sampled channels

Channel State Information

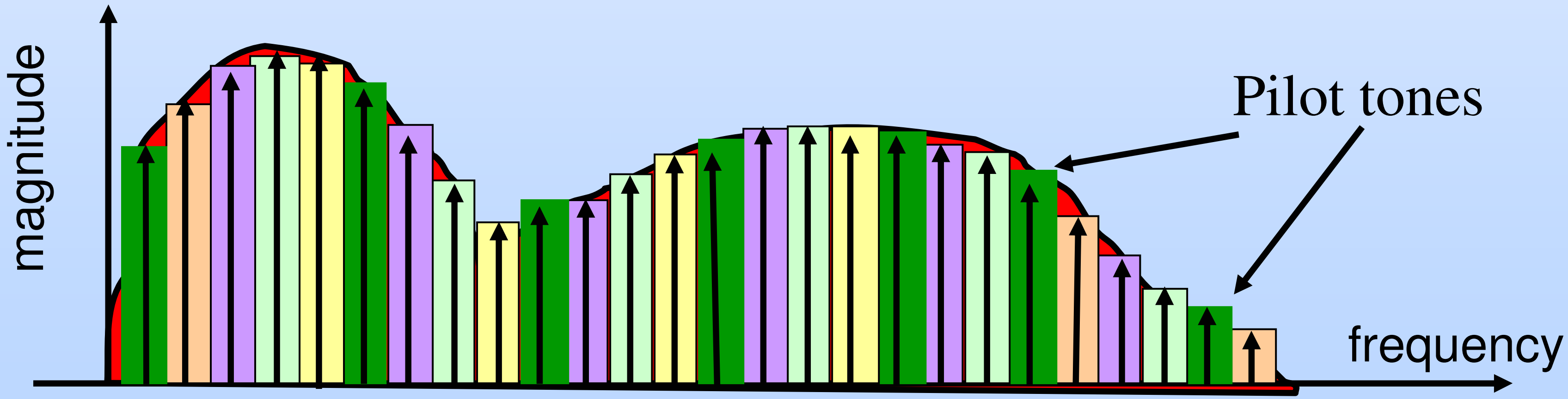
- In OFDM systems, the Doppler effects are kept smaller by making sure that the symbol duration is much smaller compared to the channel coherence time.
  - **In this case, the channel attenuations at successive symbol durations experience sufficiently higher time correlation.**
- Similarly, if subcarrier spacing is chosen in a way that the spacing is much smaller than the coherence bandwidth of the channel
  - **The channel attenuations at the adjacent subcarriers will be highly frequency correlated.**
- So, the estimator can exploit both of these two correlation properties

Channel State Information

- Channel estimation of a SISO-OFDM system can be done by using complete training symbols after certain OFDM data symbols, or by inserting some training pilot tones in every OFDM symbol.
- In the first case, the CSI is estimated with the training symbol and interpolated for the consecutive symbol before the next training symbol appears.
  - **This technique renders unacceptable results when the channel variation time is comparable to OFDM symbol duration.**
- The second method is suitable in these kinds of fast varying channels.
- The CSI is estimated for all the pilot tones using the pilot subcarriers from that particular symbol and later CSI for all other subcarriers are obtained by interpolation.
- In that way, perfect or near perfect estimates are achievable. But the cost is paid in significant throughput reduction.

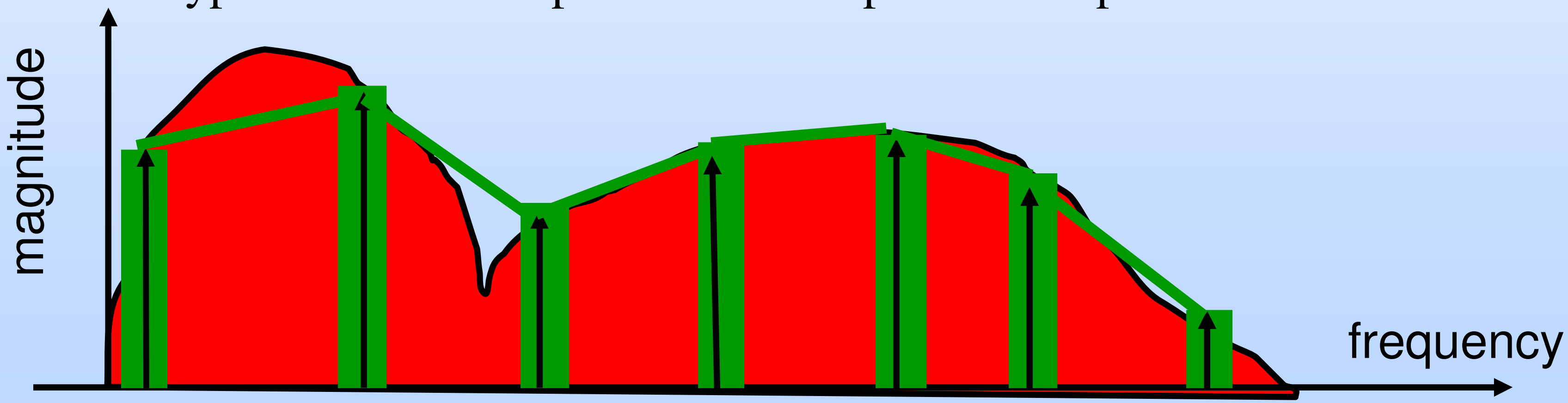
Ideal Channel Estimation

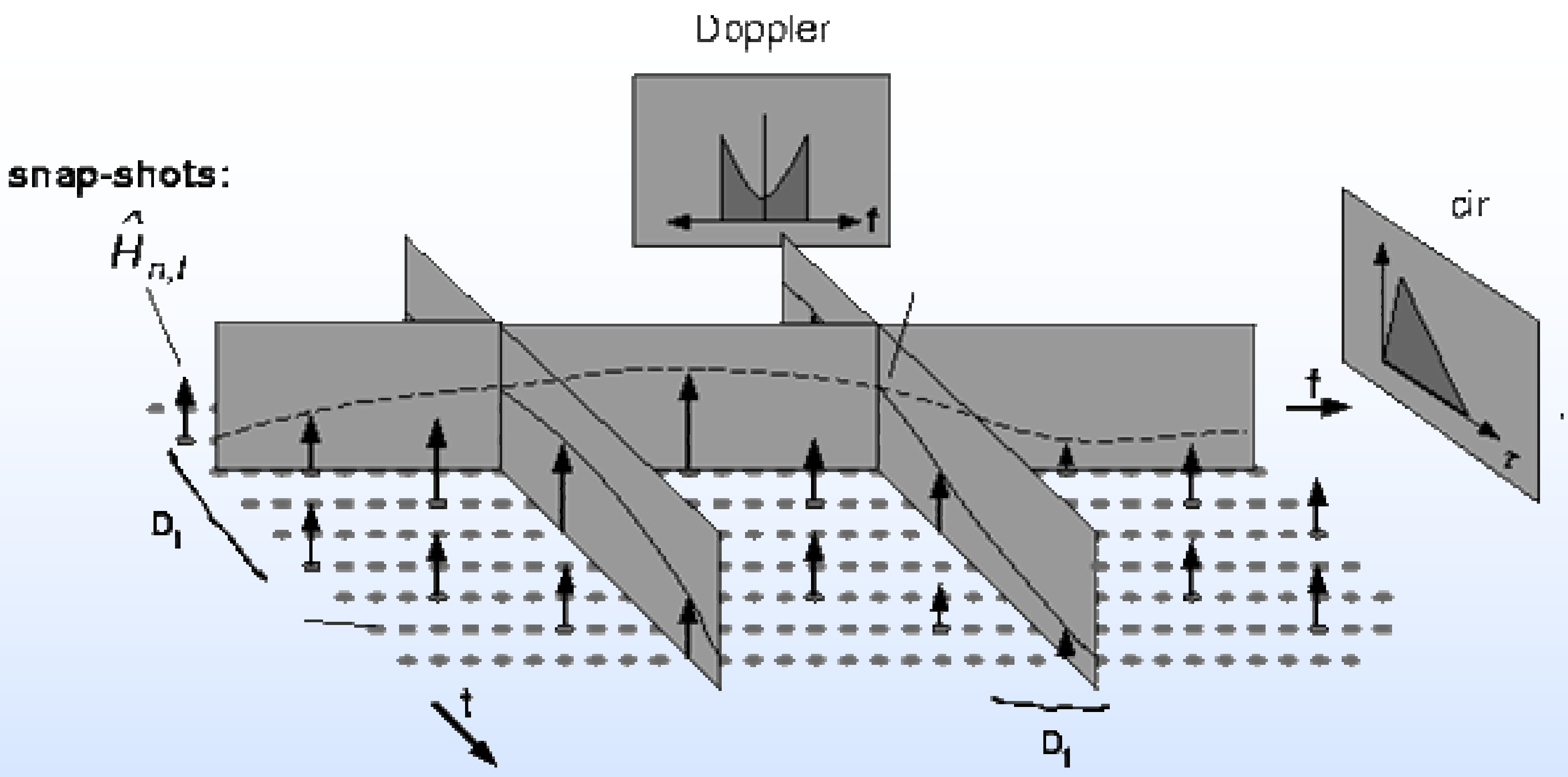
- **Wireless channels change frequently ~ 10 ms**
- **Require frequent channel estimation**
- The attenuations of the pilot symbols are measured and the attenuations of the data symbols between these pilot symbols are typically estimated/interpolated using time correlation property of fading channel
- **Many systems use pilot tones – known symbols**
  - Given s_k, for $k = k_1, k_2, k_3, \ldots$ solve $x_k = \sum_{l=0}^{L} h_l \, e^{-j2\pi k l/N} s_k$ for h_l
  - Find $H_k = \sum_{l=0}^{L} h_l \, e^{-j2\pi k l / N}$ (significant computation)
- **More pilot tones**
  - Better noise resilience
  - Lower throughput (pilots are *not* informative)



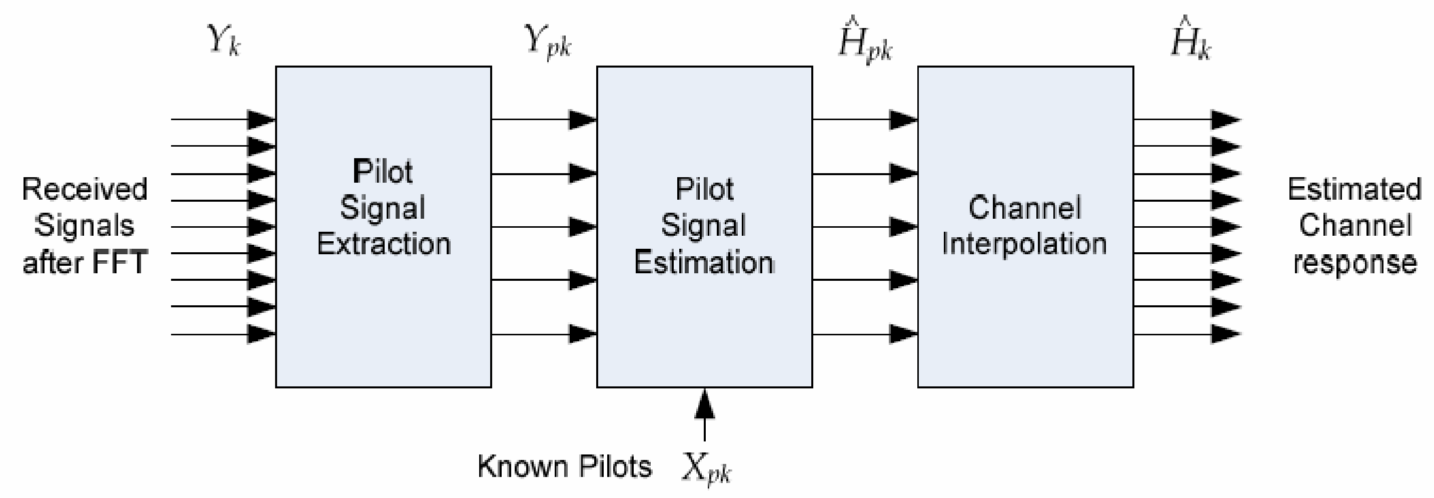
Channel Estimation Via Interpolation

- **More efficient approach is interpolation**
- **Algorithm**
  - For each pilot k_i find $H_{ki} = x_{ki} / s_{ki}$
  - Interpolate unknown values using interpolation filter
  - $H_m = \alpha_{m,1} H_{k1} + \alpha_{m,2} H_{k2} + \ldots$
- **Comments**
  - Longer interpolation filter: more computation, timing sensitivity
  - Typical 1dB loss in performance in practical implementation

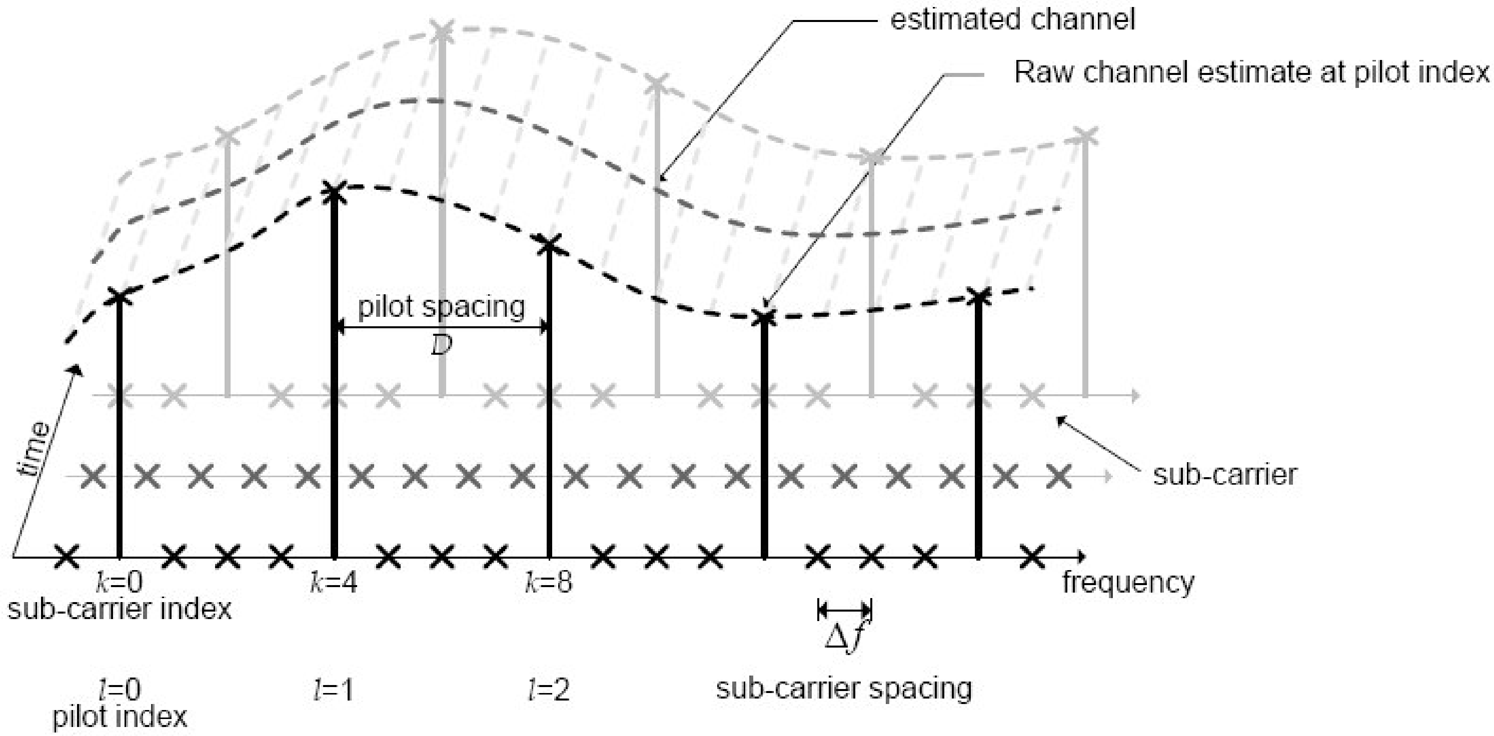




Doppler
snap-shots:
$\hat{H}_{n,l}$
f
cir
f
τ
D
t
D



Y_k
Y_{pk}
$\hat{H}_{pk}$
$\hat{H}_k$
Received Signals after FFT
Pilot Signal Extraction
Pilot Signal Estimation
Channel Interpolation
Estimated Channel response
Known Pilots X_{pk}



Channel estimation in OFDM is a two-dimensional (2-D) problem i.e., channel needs to be estimated in time-frequency domain as illustrated in Figure 4-6. Hence 2-D methods could be applied to estimate the channel from pilots. However, due to the computational complexity of 2-D estimators, the scope of channel estimators can be limited to one-dimensional (1-D). The idea behind 1-D estimators is to estimate the channel in one dimension (say frequency) and later estimate the channel in the second dimension (say time), thus obtaining a 2-D channel estimate.

Channel Estimation algorithm LS

- In a matrix form, the observed symbols after the DFT operation in the receiver can be written as $\bar{r} = \bar{X}\bar{h} + \bar{n}$
- where the diagonal matrix $\overline{X}$ contains the transmitted symbols on its diagonal (either known pilot symbols or receiver decisions of information symbols which are assumed to be correct), the channel attenuations of one OFDM symbol (i.e. Fourier transform of h(t) evaluated at the frequency f_k) is collected in vector $\overline{h}$ and the vector $\overline{r}$ contains the observed outputs of the DFT.
- If we maximize the channel estimates in the Least-Square (LS) sense: maximize $\left\|\bar{r} - \bar{X}\hat{\bar{h}}\right\|^2$ for all possible $\hat{h}$

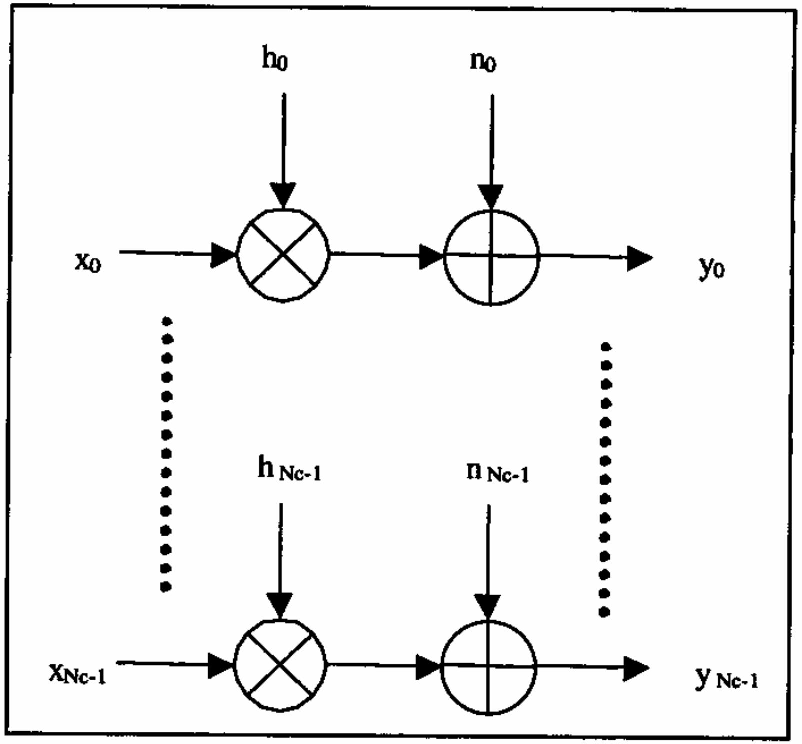
$$\hat{h}_{ls} = \bar{X}^{-1}\bar{r} = \begin{bmatrix} \frac{r_0}{X_0} & \frac{r_1}{X_1} & \cdots & \frac{r_{N-1}}{X_{N-1}} \end{bmatrix}^T$$

- This is a straight forward estimation technique where the received symbol on each subcarrier is divided by the transmitted symbol to obtain the estimate.

Least Squares Estimator

$$H_{LS}(m,k) \equiv \frac{Y(m,k)}{X(m,k)}$$

•Disadvantage: Poor performance, due to oversimplified channel model. Does not take into account correlations of the channel.

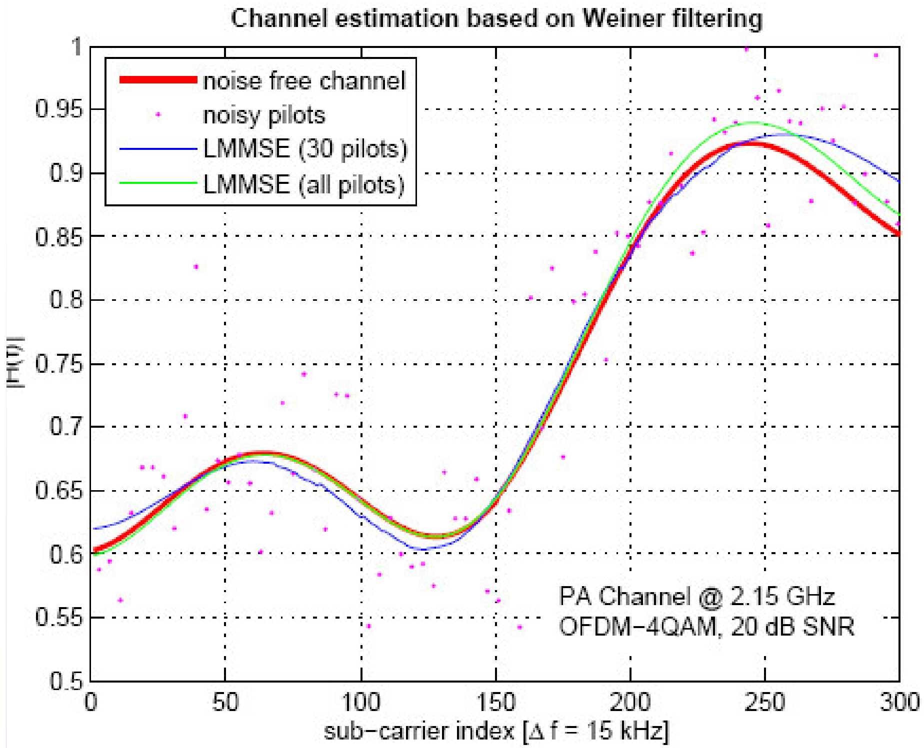


Channel Estimation algorithm LMMSE

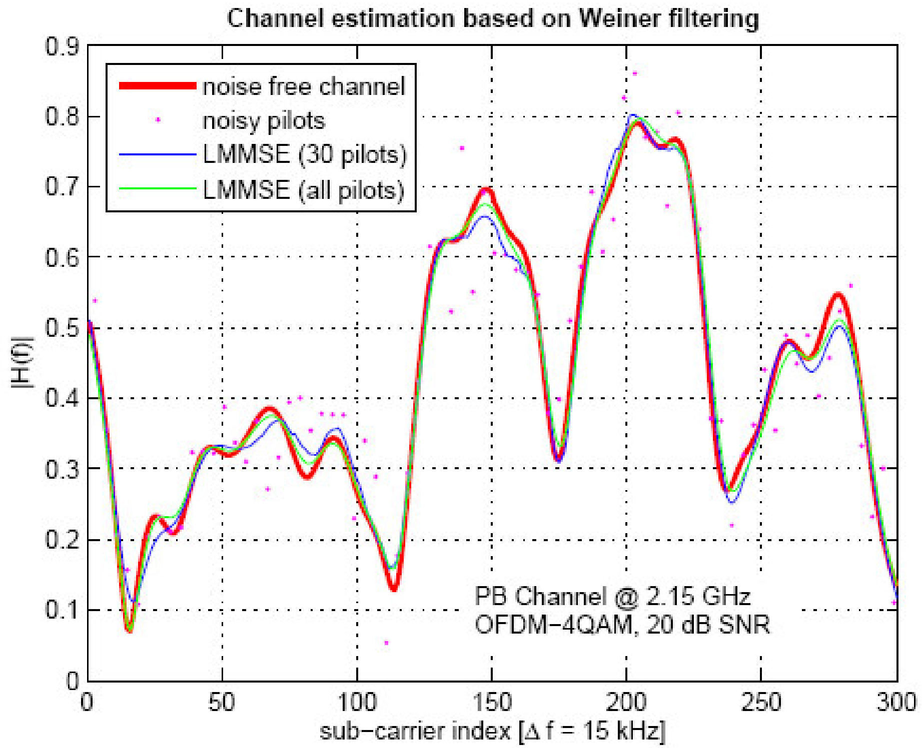
- Minimize the mean square error between the actual channel response and the estimated one by linear transformation to H_{LS}
- The optimal Linear Minimum Mean-Square Error (LMMSE) estimate of $\hat{\bar{h}}$ by
- (minimizing $E\left(\left\|\hat{\bar{h}} - \bar{h}\right\|^2\right)$ for all possible linear estimators $\hat{\bar{h}}$)

$$\hat{\bar{h}}_{lmmse} = \bar{A}\hat{\bar{h}}_{ls}$$

$$\bar{A} = \bar{R}_{hhls}\bar{R}^{-1}_{hlshls} = \bar{R}_{hh}\left(\bar{R}_{hh} + \sigma_n^2\left(\bar{X}\bar{X}^H\right)^{-1}\right)^{-1} \text{ and } \bar{R}_{hh} = E\left(\bar{h}\bar{h}^H\right)$$



(a) PA channel (high coherence bandwidth)

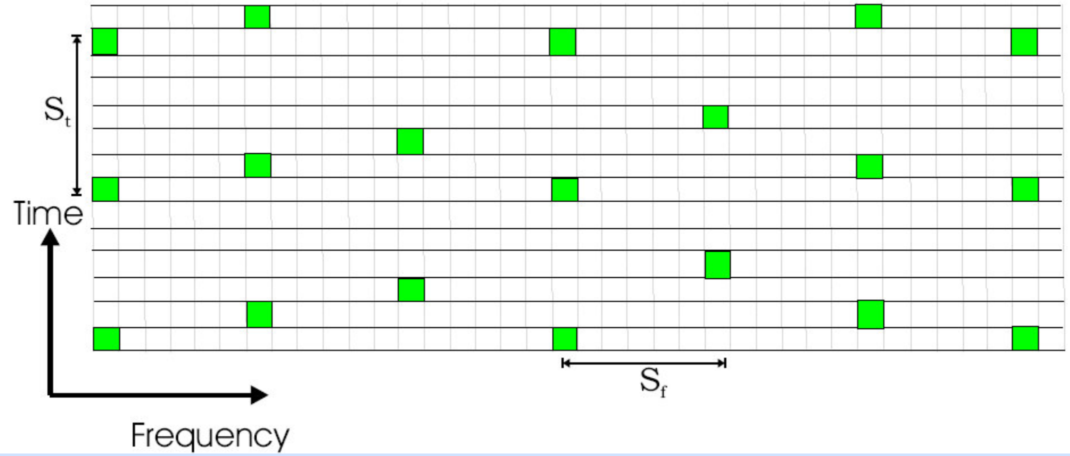


(b) PB channel (low coherence bandwidth)

Figure 4-9. Channel estimation using Wiener filtering (LMMSE method)

Design of Pilot Based Channel Estimator

- There are mainly two problems in designing channel estimators for wireless OFDM systems.
  - **The first problem concerns the choice of how pilots should be inserted.**
  - **The second problem is the design of the estimator as a low complexity with good channel tracking ability.**
- The pilot symbols should be inserted properly, so that it successfully estimates the frequency response of the channel. The difference between two consecutive pilot symbols in time and frequency domain, S_t and S_f respectively, can be represented as



$$S_t \leq \frac{1}{B_{doppler}} \quad \text{and} \quad S_f \leq \frac{1}{\tau_{max}}$$

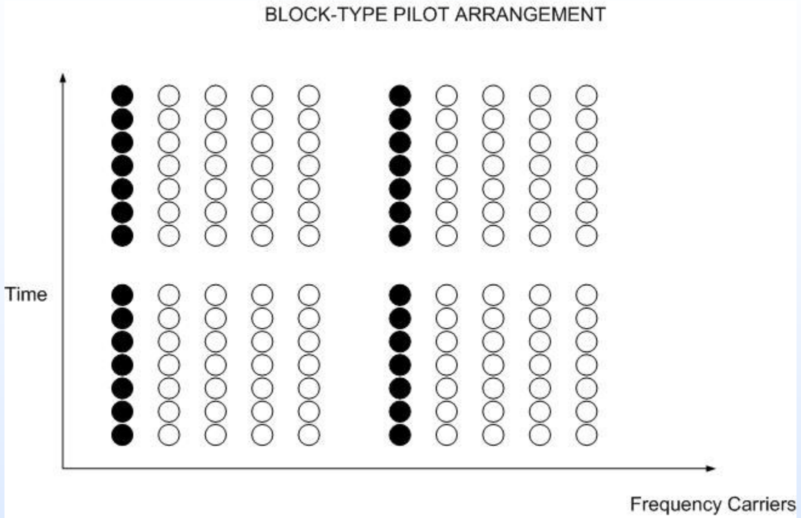
Pilot Symbol Assisted Modulation

- N_p pilot symbols P_i are transmitted in the subcarriers within the total OFDM symbol bandwidth of N subcarriers.
- At the receiver, the channel transfer function at the pilot subcarriers is estimated from the received samples

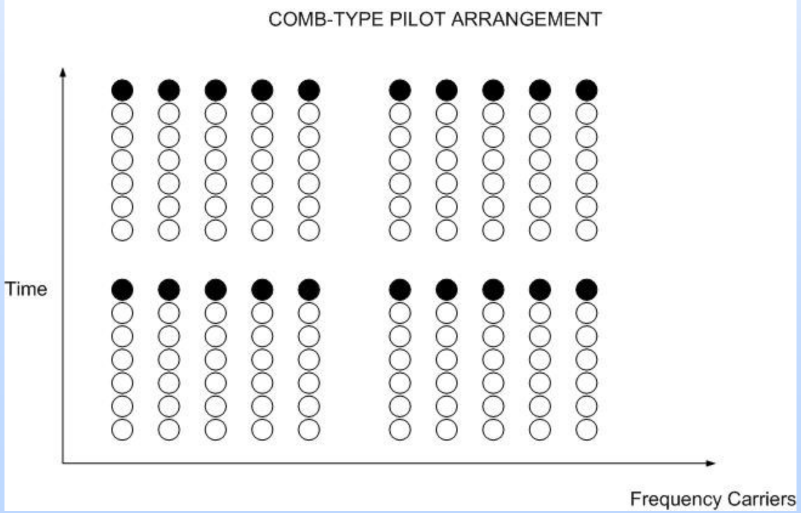
$$\hat{H}(p_i) = r(p_i)/P_i.$$

- The second step, the values of the channel transfer function are estimated for the unknown data symbols by interpolation using the abovementioned equation.
- The placement of the pilots and the interpolation technique will influence the quality of the channel estimation

Pilot Arrangement



- **Block Type**
  - All sub-carriers reserved for pilots with a specific period



- **Comb Type**
  - Some sub-carriers are reserved for pilots for each symbol

Linear Interpolation

$$\hat{H}(n) = \hat{H}(p_i) + \frac{\hat{H}(p_{i+1}) - \hat{H}(p_i)}{p_{i+1} - p(i)} \cdot (n - p_i) \quad \text{for} \quad p_i \leq n \leq p_{i+1}.$$

- **H(f) is the FT of the h(t).**
  - In order to sample H(f) according to sampling theorem, the maximum pilot spacing Δp in OFDM symbol is

$$\Delta p \leq \frac{N}{2\tau/T_s} \Delta f.$$

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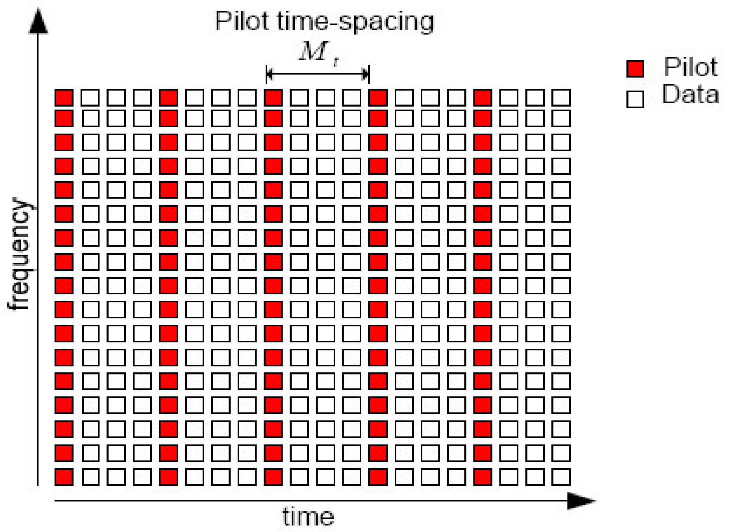
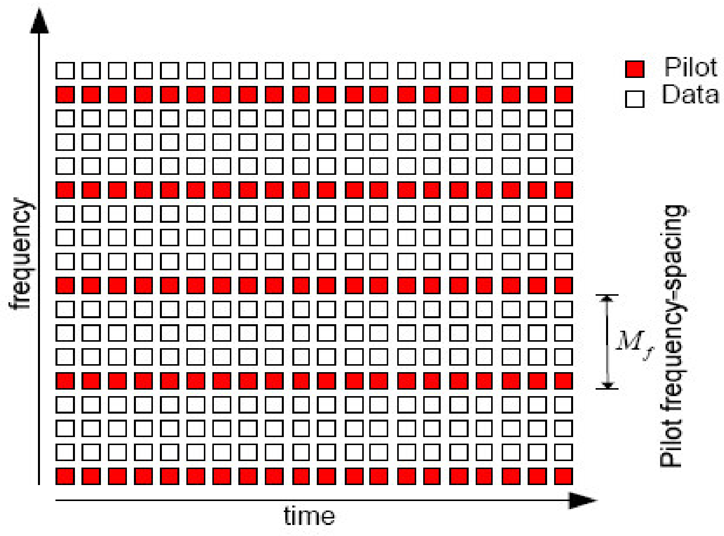
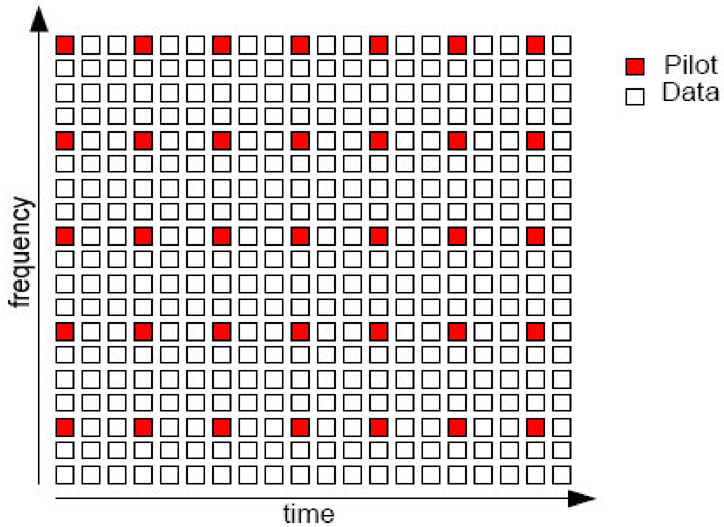


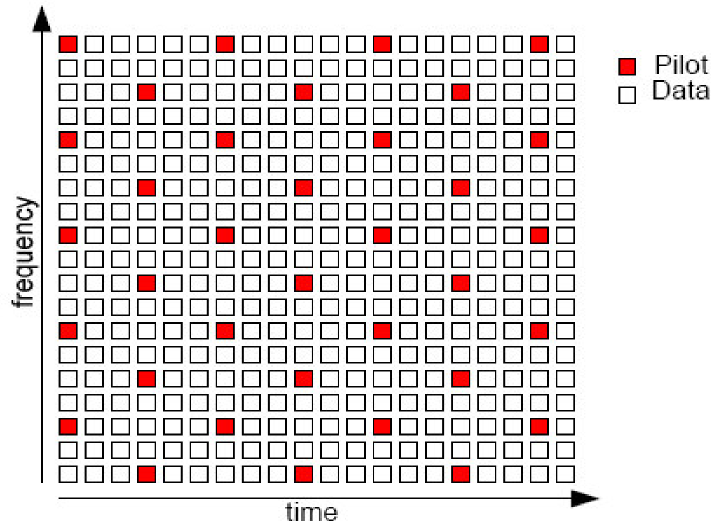
(a) All frequency-time spaced pilot allocation



(b) Frequency spaced - all time pilot allocation



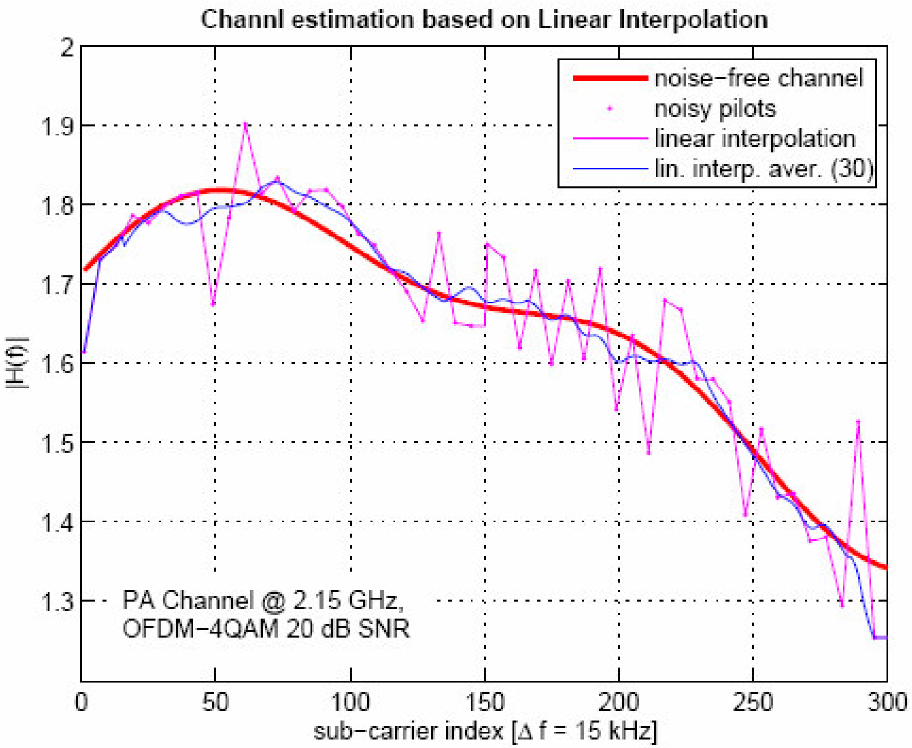
(c) Frequency spaced - time spaced pilot allocation



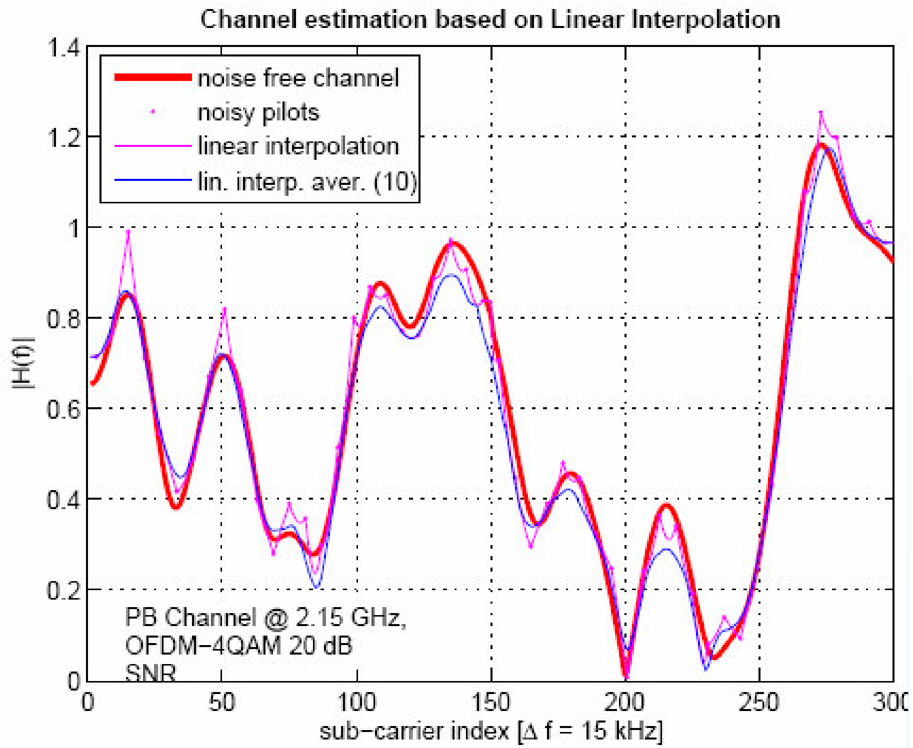
(d) Frequency spaced - time spaced pilot allocation

Figure 4-4. Different possibilities for pilot allocation

Figure 4-7 illustrates channel estimation based on 1-D frequency-domain linear interpolation method for a high coherence bandwidth channel (Figure 4-7a) and low coherence bandwidth channel (Figure 4-7b). It is evident that simply using linear interpolation on raw channel estimates provides inferior channel estimate than using post averaging after linear interpolation. Thirty and ten sub-carriers were used for post averaging after linear interpolation in Figure 4-7a and Figure 4-7b respectively. Every 6th sub-carrier was used as a pilot and QPSK modulation was used in downlink. The carrier frequency was set to 2.15 GHz, the mobile speed was set to 120 km/h and the SNR over the channel was set to 20 dB.



(a) PA channel (high coherence bandwidth)



(b) PB channel (low coherence bandwidth)

Time domain Channel Estimation using Training Sequence

- Conventional estimation schemes send a stream of transmitted symbols with a modulation scheme known to the receiver, and the receiver analyzes the effect of the channel on the known symbols by observing the deviations on the received known symbols.
- The transmission of training symbols reduces the spectral efficiency of the system

$$R_{1,k} = H_k L_k + W_{l,k}$$

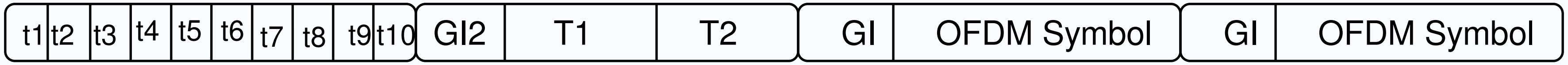
$$\hat{H}_k = \frac{1}{2}(R_{1,k} + R_{2,k}) L_k^*$$

$$\hat{H}_k = \frac{1}{2}(H_k L_k + W_{1,k} + H_k L_k + W_{2,k}) L_k^*$$

$$\hat{H}_k = H_k |L_k|^2 + \frac{1}{2}(W_{1,k} + W_{2,k}) L_k^*$$

$$= H_k + \frac{1}{2}(W_{1,k} + W_{2,k}) L_k^*$$

802.11a System Specification



Short training sequence:
AGC and frequency offset

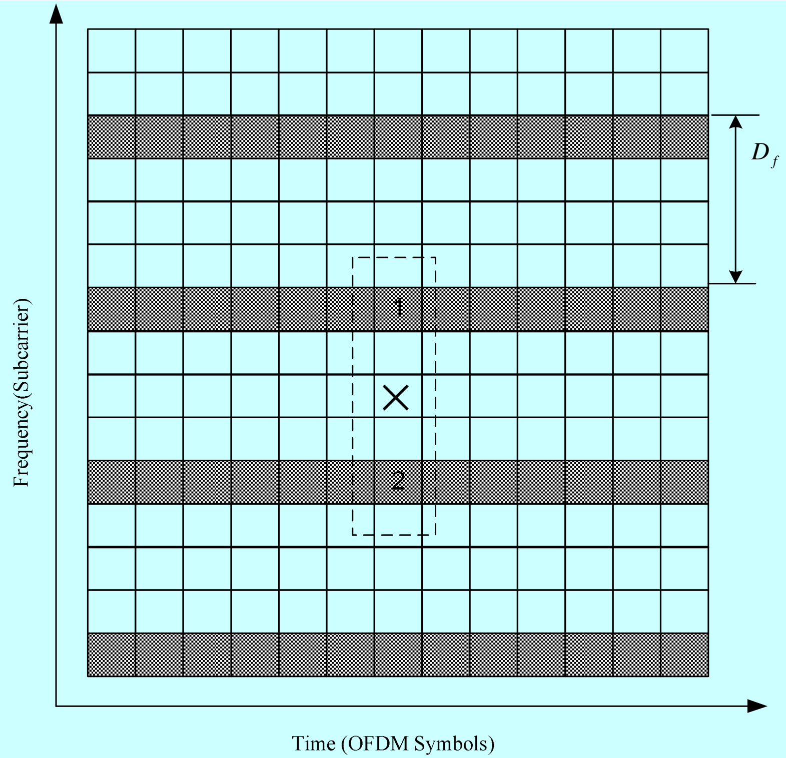
Long training sequence:
Channel estimation

- **Sampling (chip) rate: 20MHz**
- **Chip duration: 50ns**
- **Number of FFT points: 64**
- **FFT symbol period: 3.2µs**
- **Cyclic prefix period: 16 chips or 0.8µs**
  - Typical maximum indoor delay spread < 400ns
  - OFDM frame length: 80 chips or 4µs
  - FFT symbol length / OFDM frame length = 4/5
- **Modulation scheme**
  - QPSK: 2bits/sample
  - 16QAM: 4bits/sample
  - 64QAM: 6bits/sample
- **Coding: rate ½ convolutional code with constraint length 7**

Channel Estimation Algorithms

- Linear Interpolation
- Second Order Interpolation
- Maximum Likelihood (Least Square in time domain)
- Linear Minimum Square Error

Linear Interpolator (I)



- **Use two piloted grid closest to the grid needed to be estimated in LI; M pilot grids during the same symbol for MMSE and ML**

$$\hat{H}(k,m) = \hat{H}_P(k,m_1)a + \hat{H}_P(k,m_2)(1-a)$$

$$where \qquad a = d_1 / D_f, \quad d_1 < D_f$$

Weighted Linear Interpolator (II)

- **Extending linear to weighted linear**

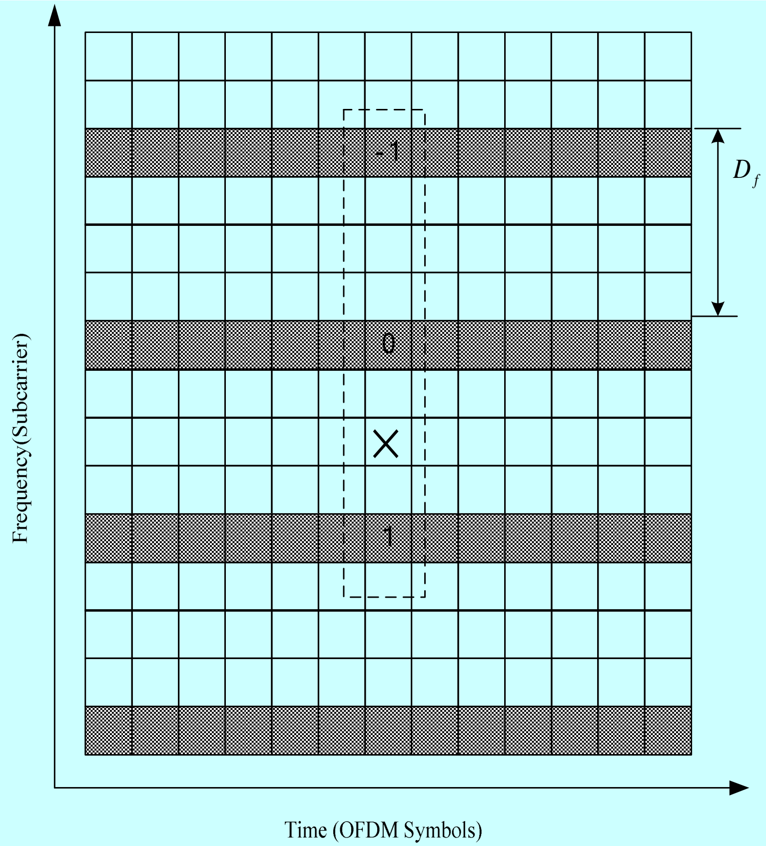
$$g_\Delta(m,n)=\begin{cases} a_1 \bullet \dfrac{2^{-\left|n-K_f\right|+1}}{3(1-2^{-K_f})} & m=1,\quad 1\le n\le 2K_f \\ a_m \bullet \dfrac{2^{n-K_f}}{(1-2^{-K_f})} & m>1,\quad 1\le n\le K_f \\ (1-a_m)\bullet\cdot\dfrac{2^{K_f-n+1}}{(1-2^{K_f})} & m>1,\quad K_f<n\le 2K_f \end{cases}$$

Where

$$a_m=\frac{D_f-m+1}{D_f}\qquad m>1$$

$$\sum_{n=1}^{2K_f} g_\Delta(m,n)=1\qquad 1\le m\le D_f$$

Second Order Interpolation (I)



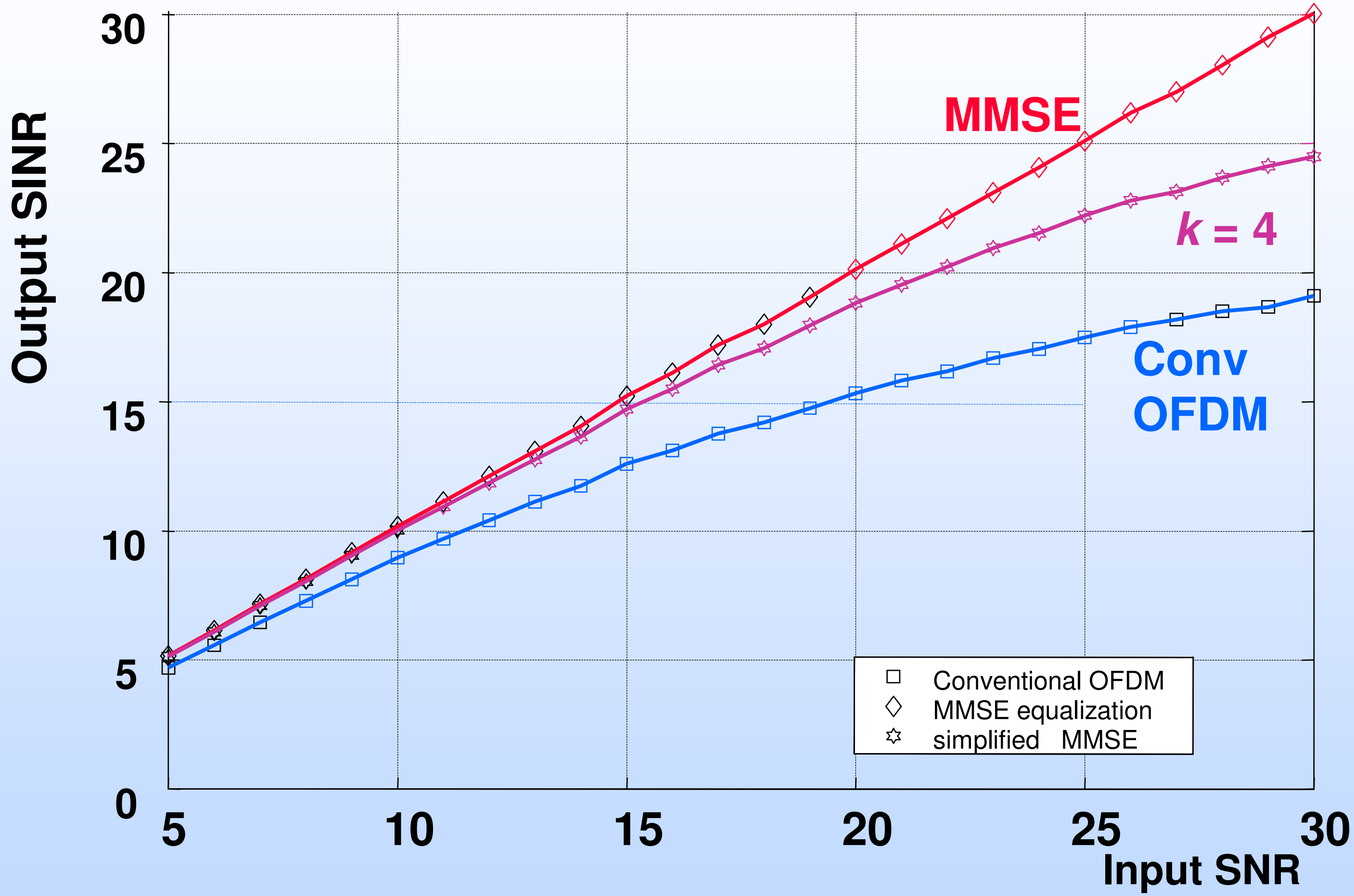
- **Use two piloted grid closest to the grid needed to be estimated in LI; M pilot grids during the same symbol for MMSE and ML**

$$H(k,(m-1)\times D_f + l) =$$

$$= c_1 \hat{H}_p(k,m-1) + c_0 \hat{H}_p(k,m) + c_{-1} \hat{H}_p(k,m+1)$$

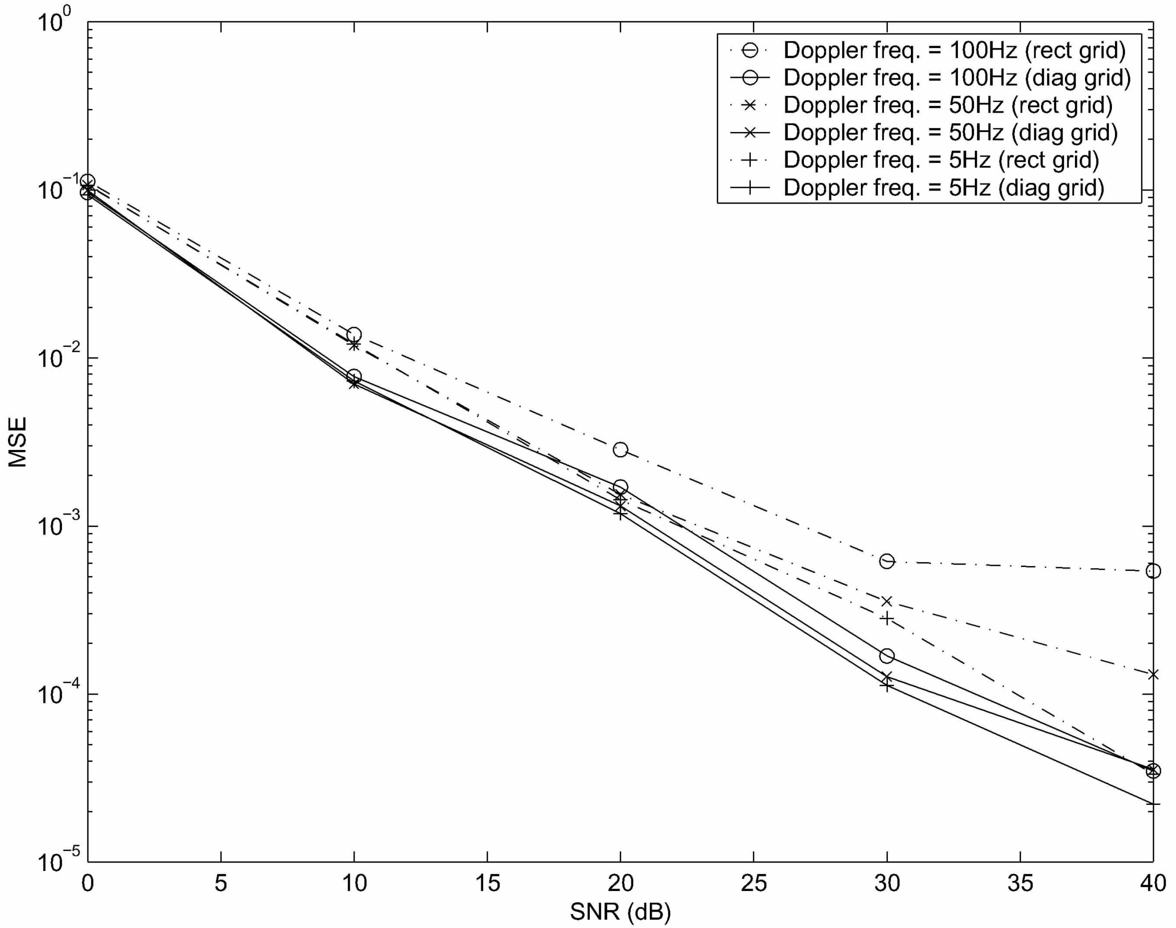
$$where \begin{cases} c_1 = \dfrac{\alpha(\alpha-1)}{2}, \\ c_0 = -(\alpha-1)(\alpha+1), \qquad \alpha = l/N \\ c_{-1} = \dfrac{\alpha(\alpha+1)}{2}, \end{cases}$$

Performance of (Simplified) Matrix Inversion

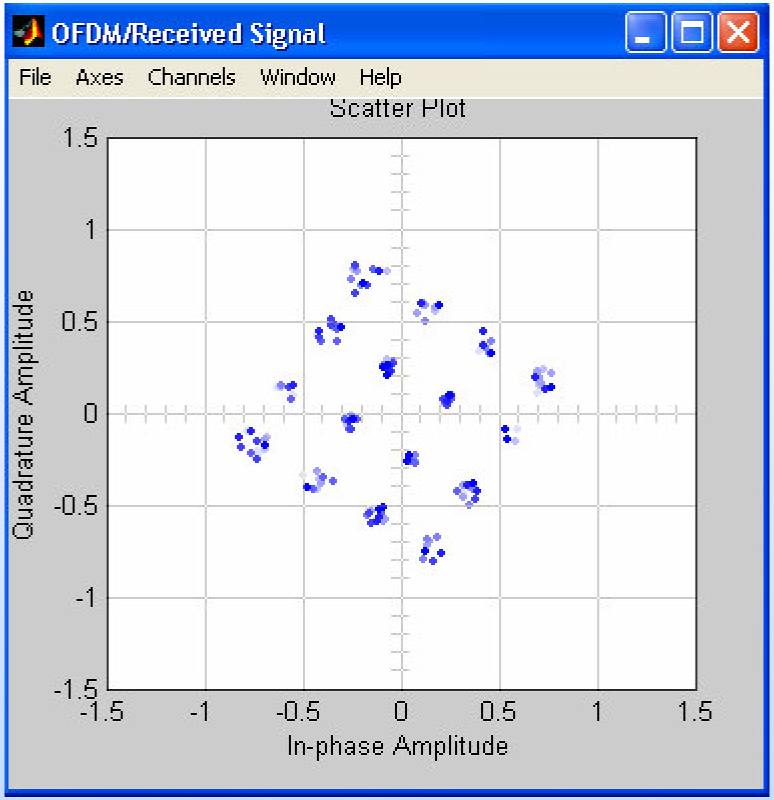


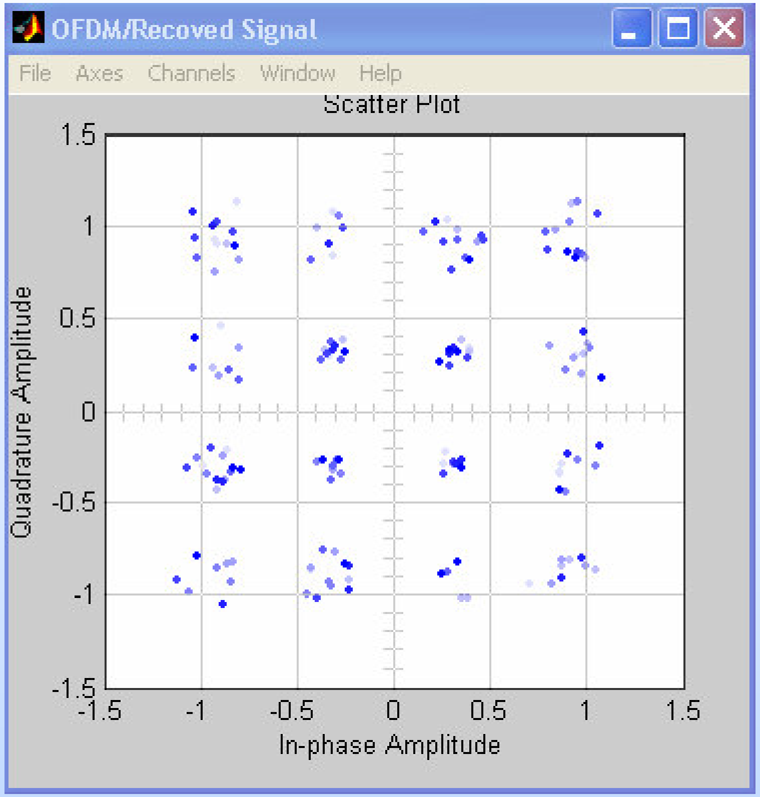
- N = 64, v = 200 km/h, f_c = 17 GHz, T_{RMS} = 1 µs, sampling at T = 1µs.
- $f_{Doppler}$ = 3.15 kHz, Subc. spacing f_{sr} = 31.25 kHz:
- Compare to DVB-T: v = 140 km/h, f_c = 800MHz: $f_{doppler}$ = 100 Hz while f_{sr} = 1.17 kHz

MSE vs SNR for different grid



Received and Recovered Signals





Received signal phases are distorted by multi-path fading