ECE5984

Orthogonal Frequency Division Multiplexing and Related Technologies

Fall 2007

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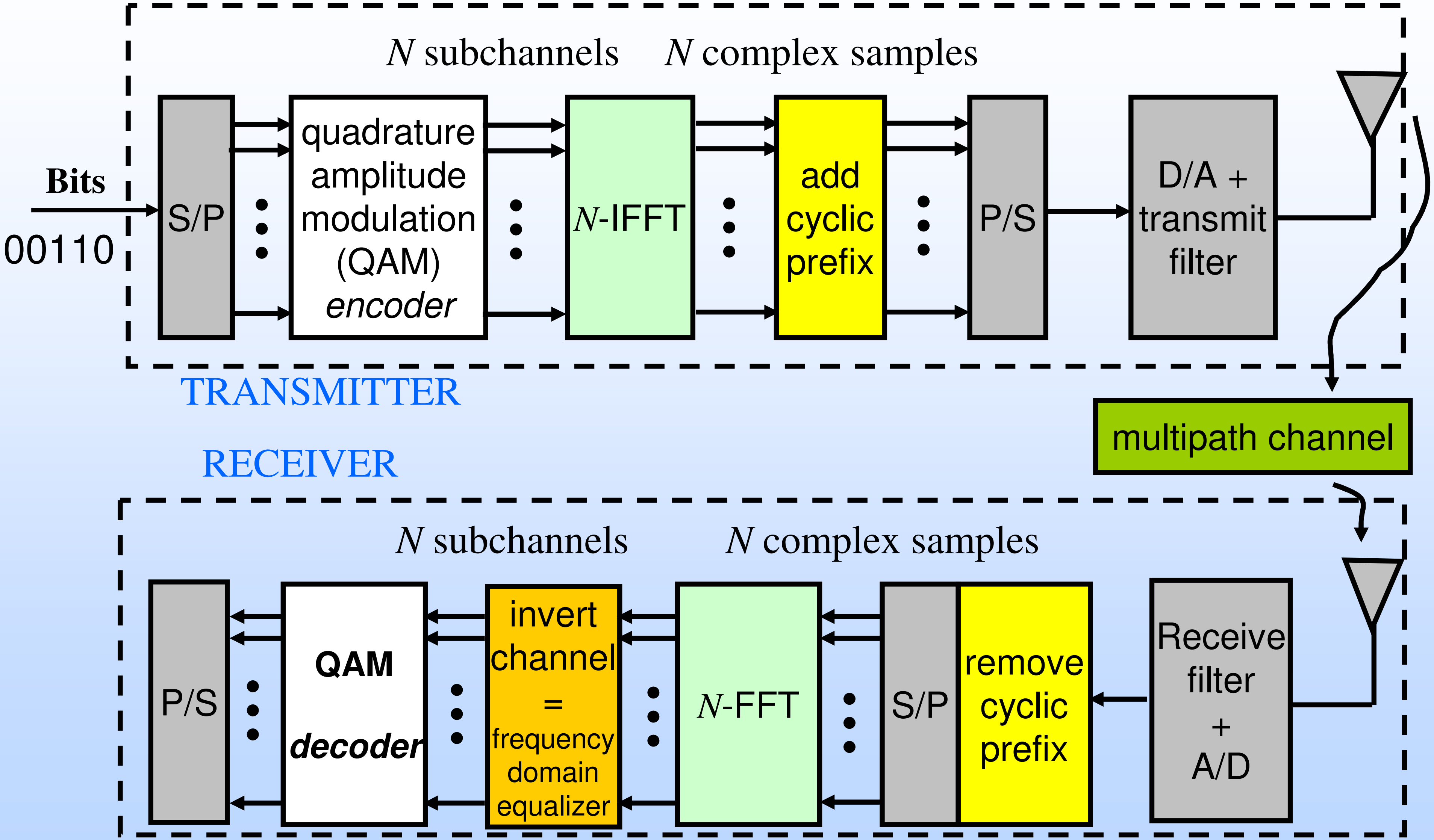
Modulation (Mapping) in OFDM

Syllabus

- **Wireless channels characteristics** (7.5%) **1**
- **OFDM Basics** (10%) **1**
- **Modulation and Coding** (10%) **2**
  - Linear and nonlinear modulation
  - Interleavin\g and channel coding
  - Optimal bit and power allocation
  - Adaptive modulation

An OFDM Modem



N subchannels *N* complex samples

Bits

00110

S/P

quadrature amplitude modulation (QAM) *encoder*

N-IFFT

add cyclic prefix

P/S

D/A + transmit filter

TRANSMITTER

multipath channel

RECEIVER

N subchannels *N* complex samples

P/S

QAM

decoder

invert channel = frequency domain equalizer

N-FFT

S/P

remove cyclic prefix

Receive filter + A/D

OFDM Mathematics

$$s(t) = \sum_{k=0}^{N-1} X_k e^{j2\pi f_k t} \qquad t \equiv [\,0, T_{os}]$$

Orthogonality Condition

$$\int_0^{T_{os}} g_1(t).g_2^*(t)dt = 0$$

In our case

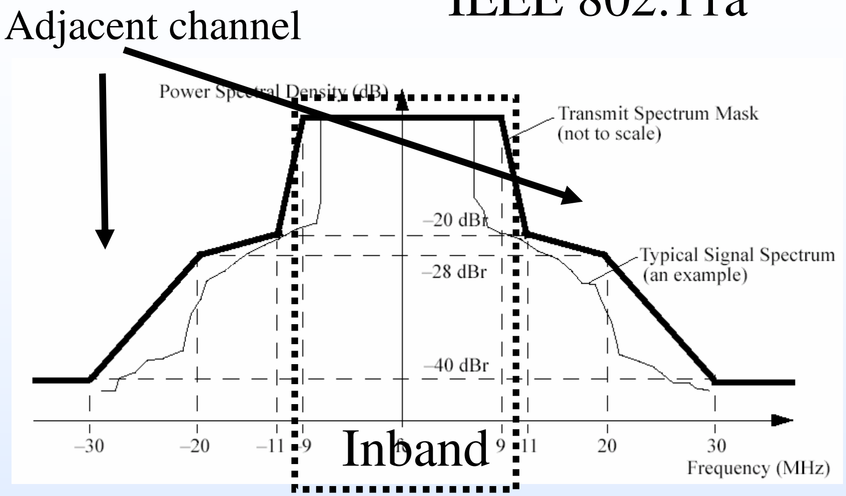
$$\int_0^{T_{os}} e^{j2\pi f_p t}.e^{-j2\pi f_q t}dt = 0$$

For $p \neq q$ Where $f_k = k/T_{os}$

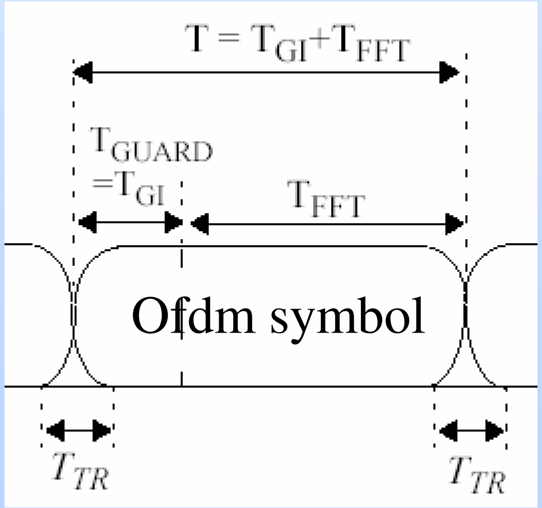
Spectrum Shaping

- **FCC manages spectrum**
- **Specifies power spectral density mask**
  - Adjacent channel interference
  - Roll-off requirements
- **Implications to OFDM**
  - Zero tones on edge of band
  - Time domain windowing ‘smoothes’ adjacent symbols

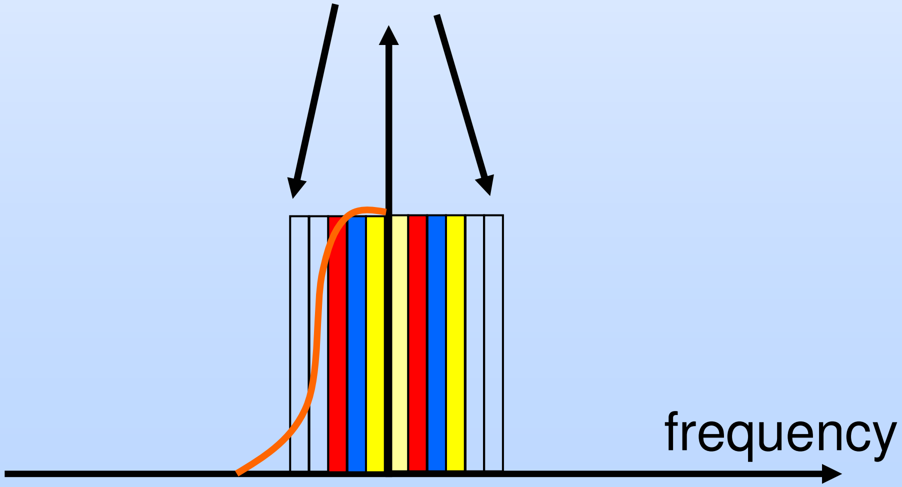
IEEE 802.11a


Adjacent channel

Inband


Ofdm symbol

Zero tones


frequency

Reference: Std 802.11a

Mitigating Multipath effects

- **Channel estimation required**
- **Training based methods**
  - Tradeoffs in overhead, complexity, and delays
- **Linear equalizers can completely eliminate ISI, but this may enhance noise.**
- **Decision feedback (nonlinear) equalizers can improve performance.**

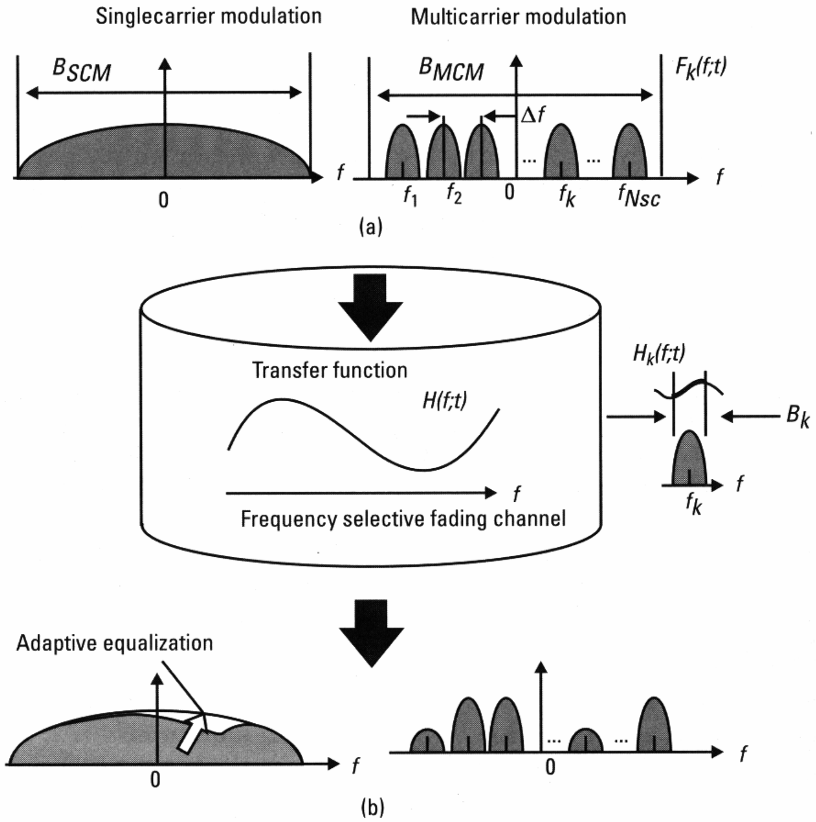


Figure 3.1 Comparison of SCM and MCM: (a) frequency spectra of transmitted signals; and (b) frequency spectra of received signals.

Equalization

- **Digital Equalizer**

$$H_{eq}(z) = w_0 + w_1 z^{-1} + \ldots + w_n z^{-n}$$

- **Criterion for coefficient choice**
  - Minimize BER (Hard to solve for $\{w\}$s)
  - Eliminate ISI (Zero forcing, enhances noise)
  - Minimize MSE between d_n and d_n

Discrete Random Variables and Probability Quick Revision

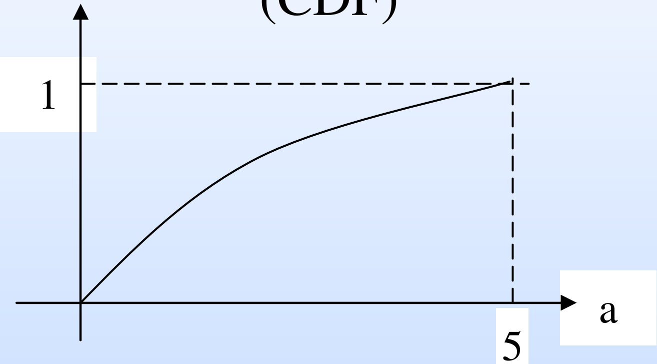
- ***Random variable*** **X assumes a value as a function from outcomes of a process which can not be determined in advance.**
- ***Sample space*** **S of a random variable is the set of all possible values of the variable X.**
- **Ω: set of all outcomes and divide it into elementary events, or** ***states***

$$\sum_{\{x\}} p(x) = 1 \qquad 1 \geq p(x) \geq 0$$

Continuous Random Variables

Cummulative distribution function

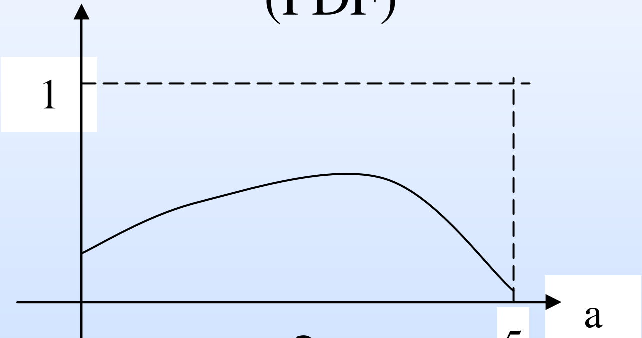
(CDF)



$$F_X(\alpha) = \Pr(X \le \alpha)$$

Probability density function

(PDF)



$$f_X(\alpha) = \frac{\partial}{\partial \alpha} F_X(\alpha)$$

$$\int_{-\infty}^{\infty} f_X(\alpha)\partial\alpha = 1$$

$$F_X(\infty) = ?$$

$$F_X(-\infty) = ?$$

Ensemble Average

- **Mean:**
  - Continuous

$$\langle X \rangle = \mathrm{E}\{X\} = \int_{-\infty}^{\infty} \alpha \, f_X(\alpha) \partial\alpha$$

  - Discrete

$$\langle X \rangle = \sum_k \alpha_k \Pr(X = \alpha_k)$$

- **Variance**

$$\sigma^2 = \langle X^2 \rangle - \left(\langle X \rangle \right)^2$$

$$\sigma_x^2 = \mathrm{E}\left\{ \left(X - \langle X \rangle \right)^2 \right\} = \int_{-\infty}^{\infty} \left(\alpha - \langle X \rangle \right)^2 f_X(\alpha) \partial\alpha$$

Correlation & Covariance

- **Crosscorrelation**
- **Covariance**

$$r_{XY} = \mathrm{E}\{\mathrm{XY}^*\}$$

$$c_{XY} = \mathrm{E}\left\{\left(X\text{-}\langle X\rangle\right)\left(Y-\langle Y\rangle\right)^*\right\} = \mathrm{E}\left\{XY^*\right\} - \langle X\rangle\langle Y^*\rangle$$

- **If <X> or <Y> equal zero, correlation equals covariance**

- **Note that X^* means the complex conjugate of X**

$$X^* = (X_r + jX_i)^* = X_r - jX_i$$

$$j = \sqrt{-1}$$

Random Process

- **X, Y need not be separate events**
- **X, Y can be samples of process observed at different instants t_1, t_2**

$$X = Z(t_1) \qquad r_{XY} = \mathrm{E}\{XY^*\}$$

$$Y = Z(t_2) \qquad R_Z(t_1,t_2) = \mathrm{E}\{Z(t_1)Z^*(t_2)\}$$

$$c_{XY} = \mathrm{E}\{(X\text{-}\langle X\rangle)(Y-\langle Y\rangle)^*\}$$

$$C_Z(t_1,t_2) = \mathrm{E}\{(Z^*(t_1)-\langle Z(\mathrm{t}_1)\rangle)(Z(t_2)-\langle Z(t_2)\rangle)^*\}$$

Independence vs. Uncorrelatedness

- R.V.s independent

$$f_{XY}(\alpha,\beta) = f_X(\alpha) f_Y(\beta)$$

- Uncorrelated (weaker condition), when

$$r_{XY} = \mathrm{E}\left\{XY^*\right\} = \mathrm{E}\left\{X\right\}\mathrm{E}\left\{Y^*\right\} = \left\langle X\right\rangle\left\langle Y^*\right\rangle$$

- R.V. X, Y uncorrelated if covariance is zero.

$$c_{XY} = \mathrm{E}\left\{\left(X\text{-}\left\langle X\right\rangle\right)\left(Y-\left\langle Y\right\rangle\right)^*\right\} = \mathrm{E}\left\{XY^*\right\} - \left\langle X\right\rangle\left\langle Y^*\right\rangle$$

- Independent R.V. always uncorrelated.
- Uncorrelated R.V. may not be independent!

Major Channel Effects

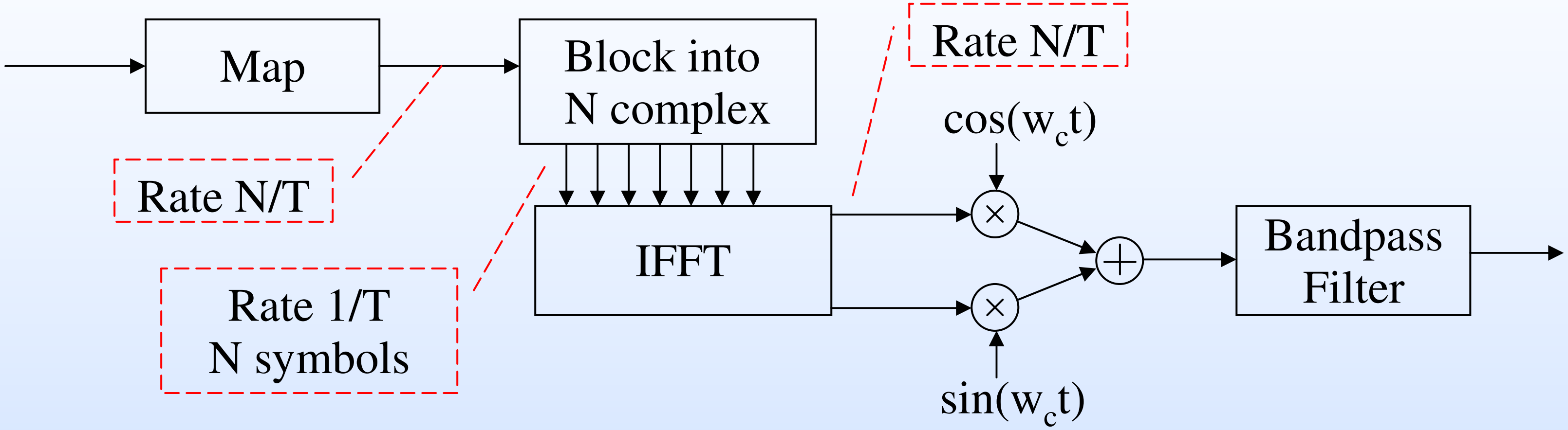
- **Propagation Loss: attenuation, also called path loss**

$$\frac{P_R}{P_T} = G_T G_R \left(\frac{\lambda}{4\pi d} \right)^2$$

- **Gaussian Noise and Interference: Time variant nature due to mobility of objects in an environment**
- **Time Dispersion: multiple reflections due to obstacles leading to multipaths**
- **Doppler Effects: Time variant nature due to mobility of objects in an environment**

Transmitters

Multicarrier System –Wireless (Complex Transmission)



Map

Block into N complex

Rate N/T

Rate 1/T N symbols

IFFT

Rate N/T

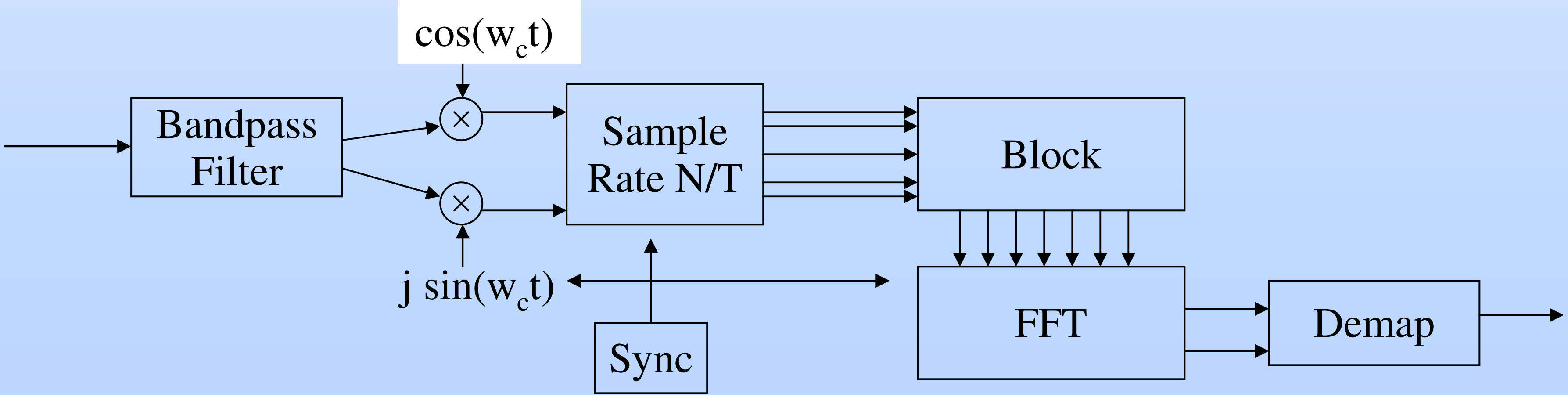
$\cos(w_c t)$

$\sin(w_c t)$

Bandpass Filter

Receivers

Multicarrier System –Wireless (Complex Transmission)



$\cos(w_c t)$

Bandpass Filter

Sample Rate N/T

Block

j $\sin(w_c t)$

Sync

FFT

Demap

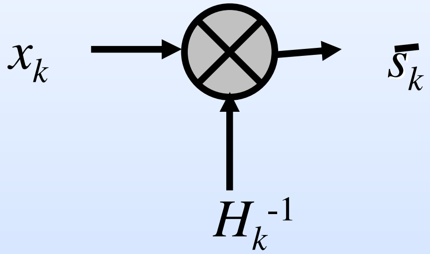
Frequency Domain Equalization

- For the k^{th} carrier:

$$x_k = H_k\, s_k + v_k$$

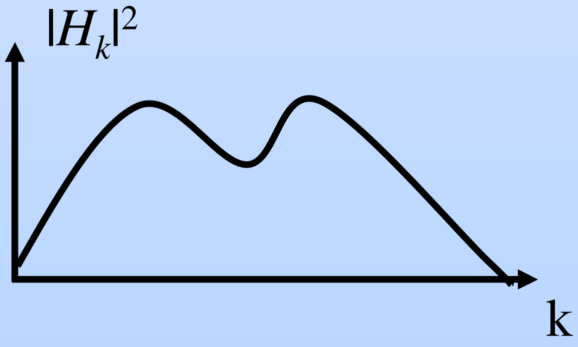
where $H_k = \sum_n h_k(nT_s) \exp(j2\pi\, k\, n\, /\, N)$ where $n = 0, \ldots,. N\text{-}1$

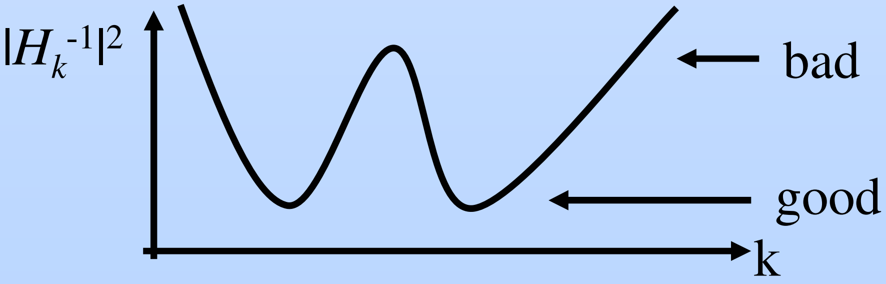
- Frequency domain equalizer



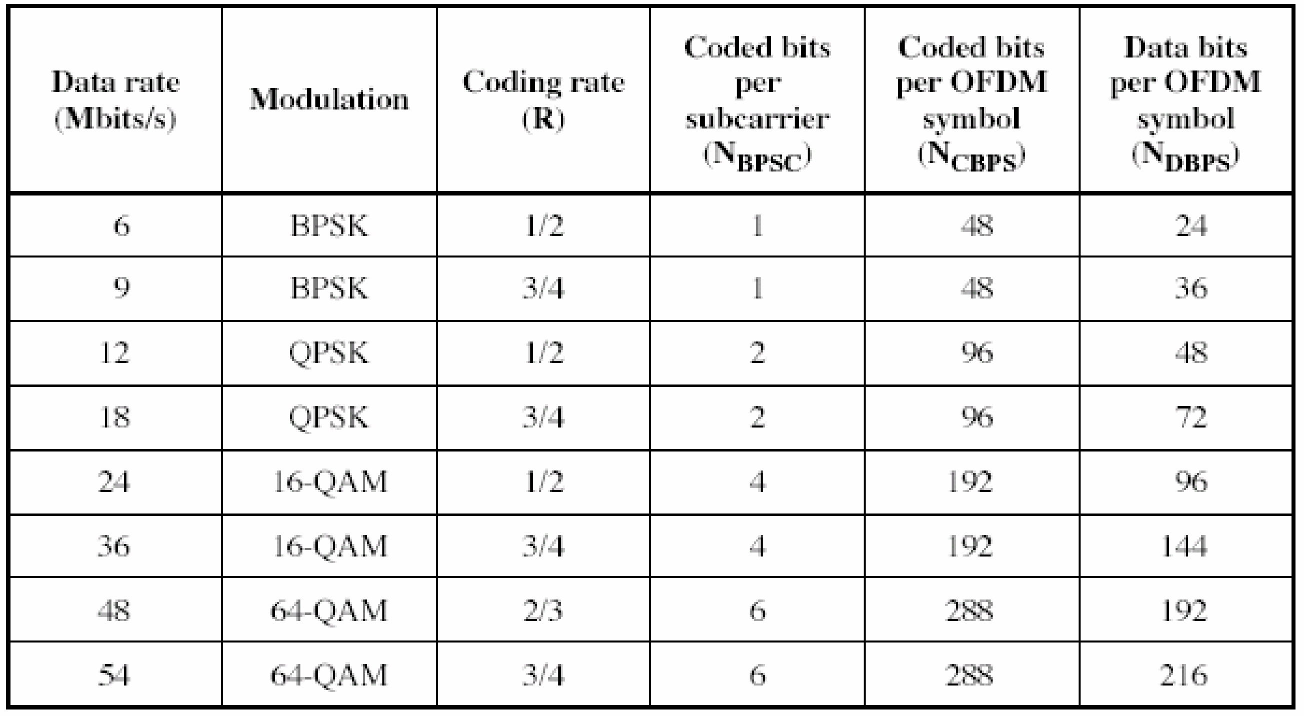
- Noise enhancement factor

$$\hat{\sigma}_k^2 = \sigma_k^2 |H_k^{-1}|^2$$





Example: IEEE 802.11a



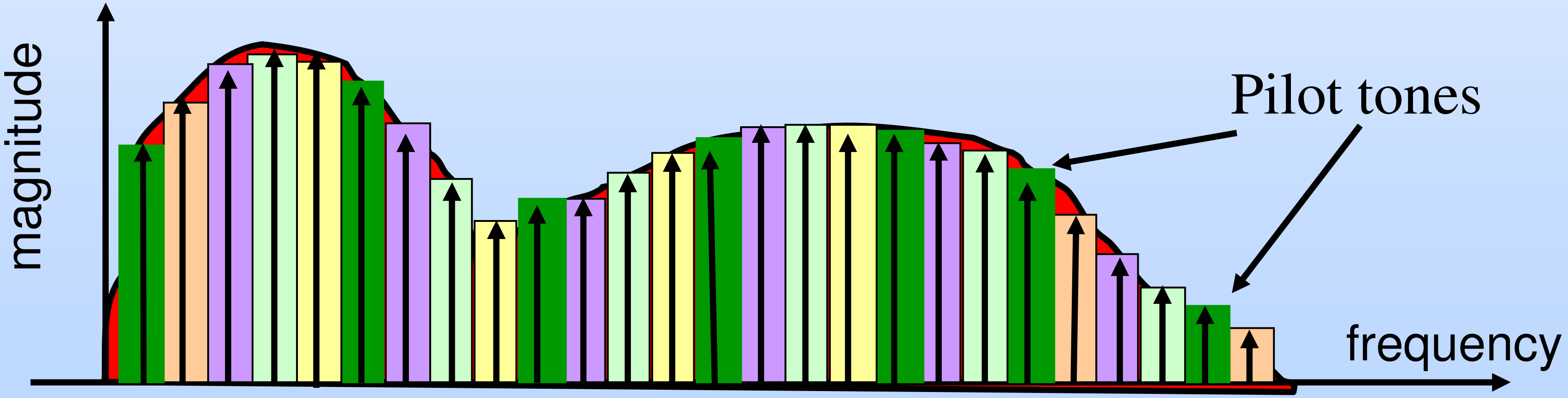
Data rate (Mbits/s)	Modulation	Coding rate (R)	Coded bits per subcarrier (N_{BPSC})	Coded bits per OFDM symbol (N_{CBPS})	Data bits per OFDM symbol (N_{DBPS})
6	BPSK	1/2	1	48	24
9	BPSK	3/4	1	48	36
12	QPSK	1/2	2	96	48
18	QPSK	3/4	2	96	72
24	16-QAM	1/2	4	192	96
36	16-QAM	3/4	4	192	144
48	64-QAM	2/3	6	288	192
54	64-QAM	3/4	6	288	216

- **IEEE 802.11 employs adaptive modulation**
  - Code rate & modulation depends on distance from base station
  - Overall data rate varies from 6 Mbps to 54 Mbps

Reference: IEEE Std 802.11a-1999

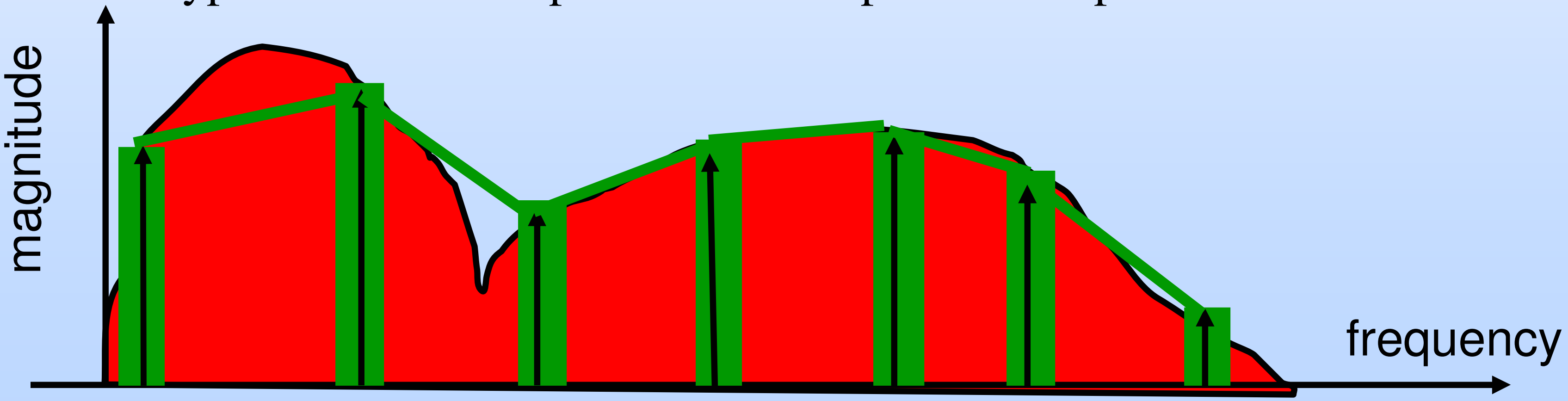
Ideal Channel Estimation

- **Wireless channels change frequently ~ 10 ms**
- **Require frequent channel estimation**
- **Many systems use pilot tones – known symbols**
    - Given s_k, for $k = k_1, k_2, k_3, \ldots$ solve $x_k = \sum_{l=0}^{L} h_l \, e^{-j2\pi k \, l/N} s_k$ for h_l
    - Find $H_k = \sum_{l=0}^{L} h_l \, e^{-j2\pi k \, l \, / \, N}$ (significant computation)
- **More pilot tones**
    - Better noise resilience
    - Lower throughput (pilots are *not* informative)



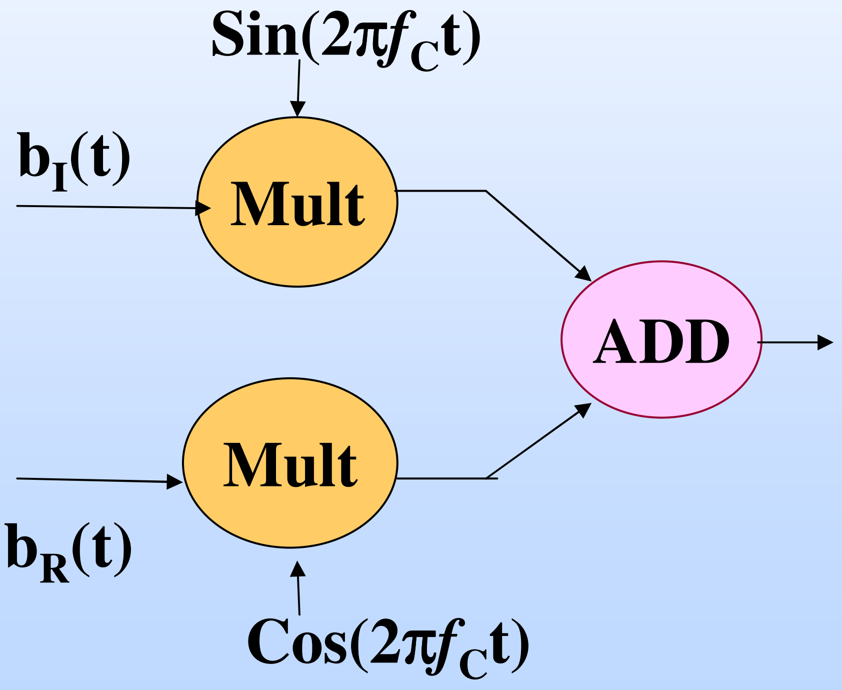
Channel Estimation Via Interpolation

- **More efficient approach is interpolation**
- **Algorithm**
  - For each pilot k_i find $H_{ki} = x_{ki} / s_{ki}$
  - Interpolate unknown values using interpolation filter
  - $H_m = \alpha_{m,1} H_{k1} + \alpha_{m,2} H_{k2} + \ldots$
- **Comments**
  - Longer interpolation filter: more computation, timing sensitivity
  - Typical 1dB loss in performance in practical implementation



Vector modulator & complex baseband

- **Independently modulate** $\cos(2\pi f_C t)$ **&** $\sin(2\pi f_C t)$ **and sum.**
- **Coherent demodulator for ‘cos’ transmission blind to ‘sin’ trans. and vice-versa.**



$\mathrm{Sin}(2\pi f_C t)$

$b_I(t)$

Mult

ADD

Mult

$b_R(t)$

$\mathrm{Cos}(2\pi f_C t)$

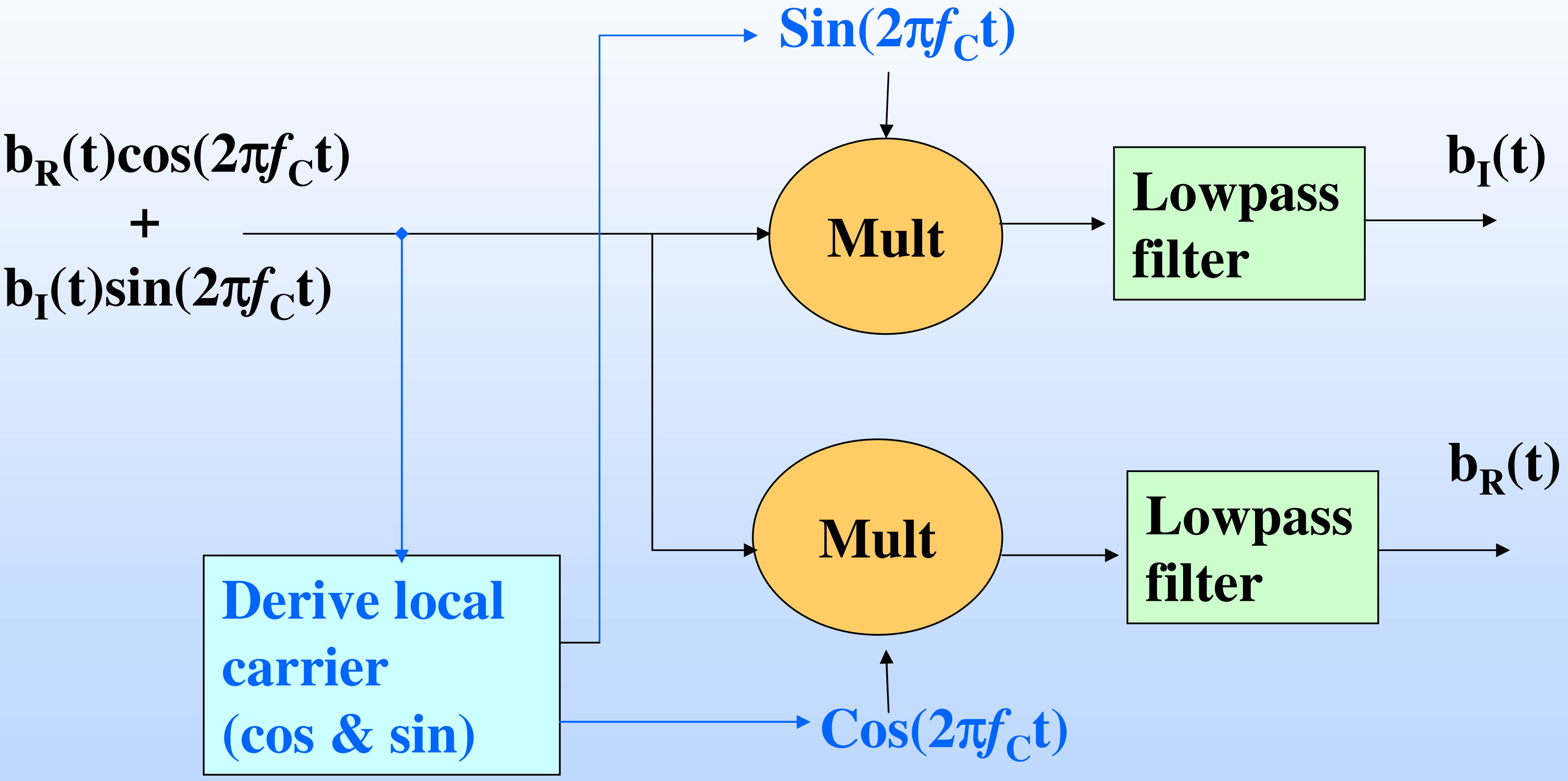
- “2 channels for price of 1”
- Still single carrier

- Complex baseband:

  $b(t) = b_I(t) + jb_R(t)$

- More about this later

Vector demodulator



$b_R(t)\cos(2\pi f_C t)$
$+$
$b_I(t)\sin(2\pi f_C t)$

$\mathrm{Sin}(2\pi f_C t)$

Mult

Lowpass filter

$b_I(t)$

Mult

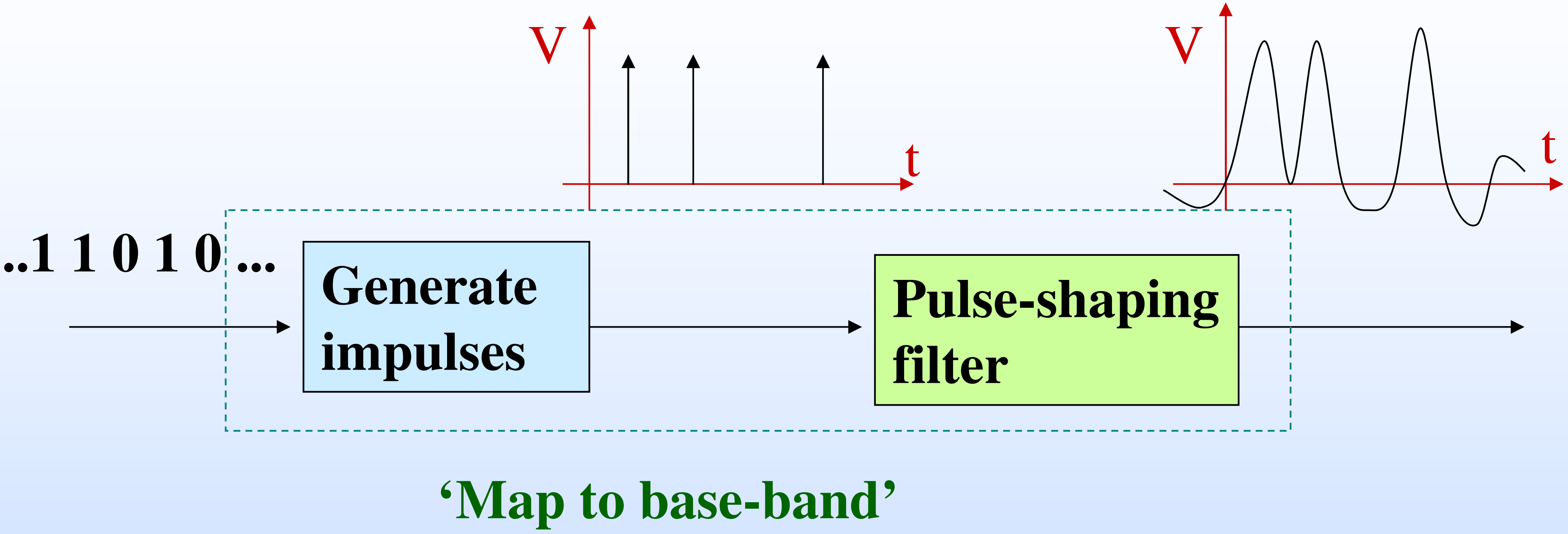
Lowpass filter

$b_R(t)$

Derive local carrier (cos & sin)

$\mathrm{Cos}(2\pi f_C t)$

Mapping bit stream to base-band



..1 1 0 1 0 ...

Generate impulses

Pulse-shaping filter

‘Map to base-band’

- Stream of impulses produced according to bits & approach

  e.g. for unipolar: unit impulse for ‘1’ & zero for ‘0’.
- Pass impulse stream through pulse shaping filter.
- Impulses & filter may be analogue or digital (generally digital)

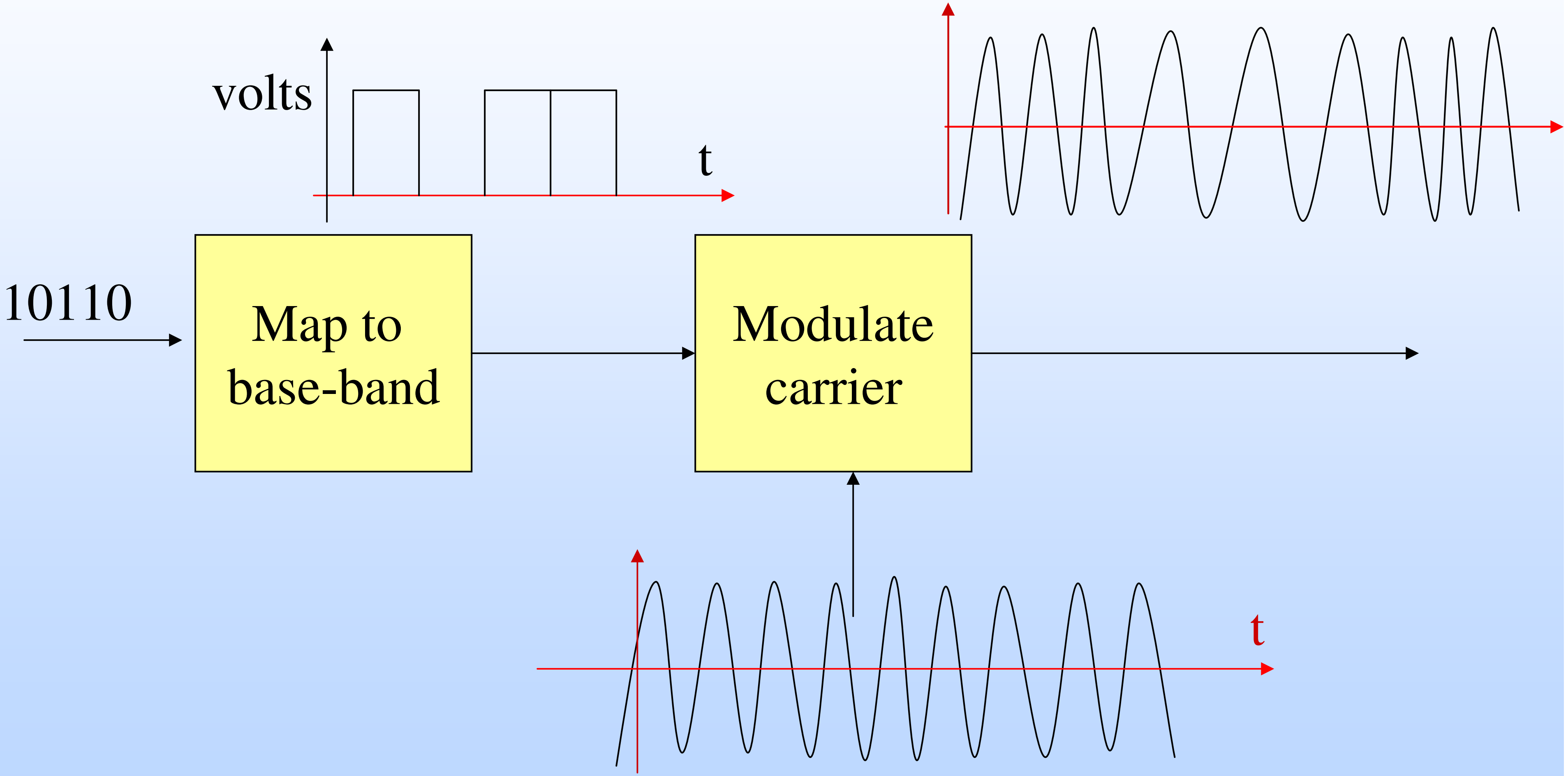
Techniques for digital transmission

- Can modulate amplitude, frequency &/or phase of $\cos(2\pi f_C t)$.
- These 3 forms of modulation when used independently give us

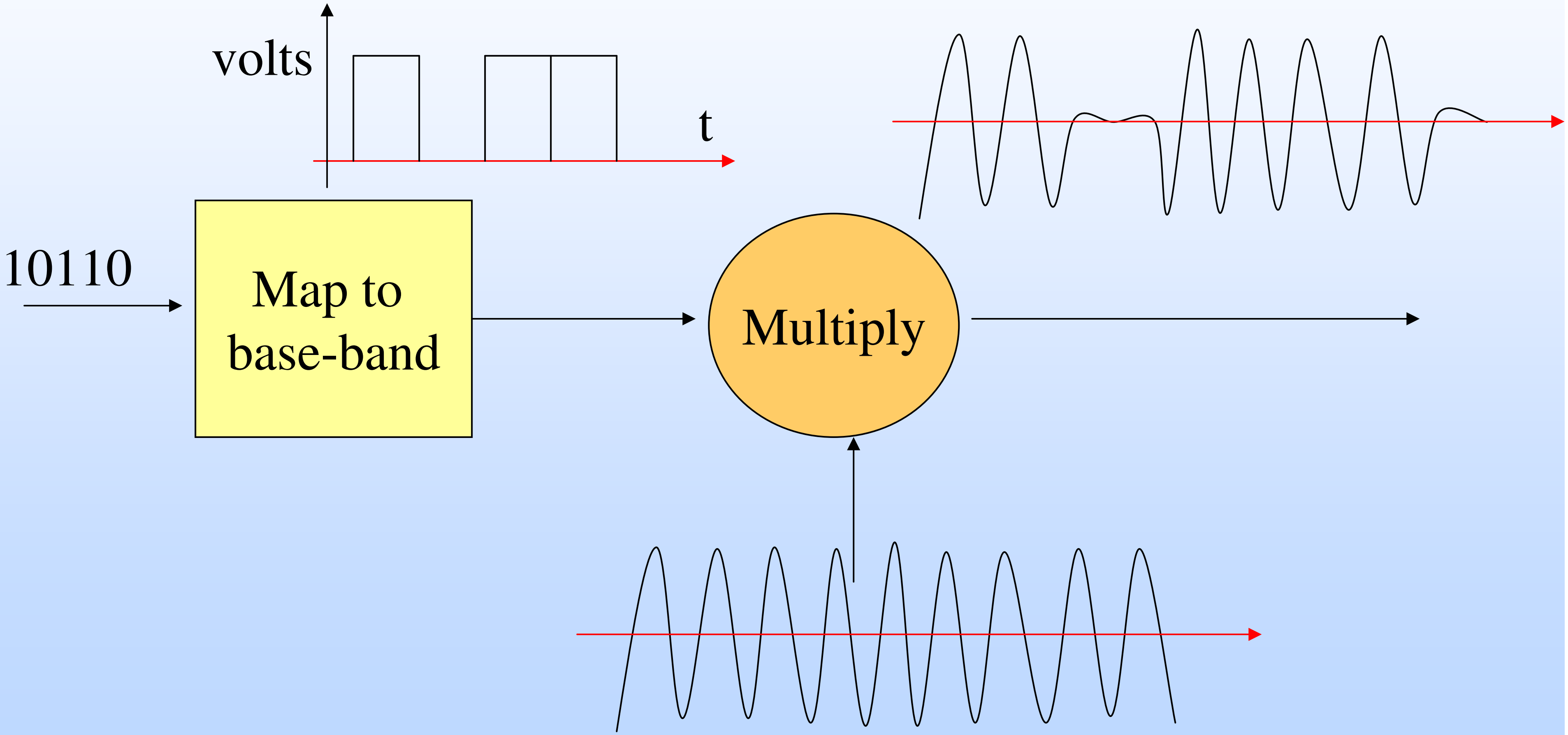
  (a) amplitude shift keying (ASK)
  (b) frequency shift keying (FSK)
  (c) phase shift keying (PSK).

- There are many versions of each of these.
- Possible to use a combination of more than one form.
- Consider simplest binary forms first.

Binary frequency shift keying (B-FSK)



Binary amplitude shift keying (B-ASK)



volts

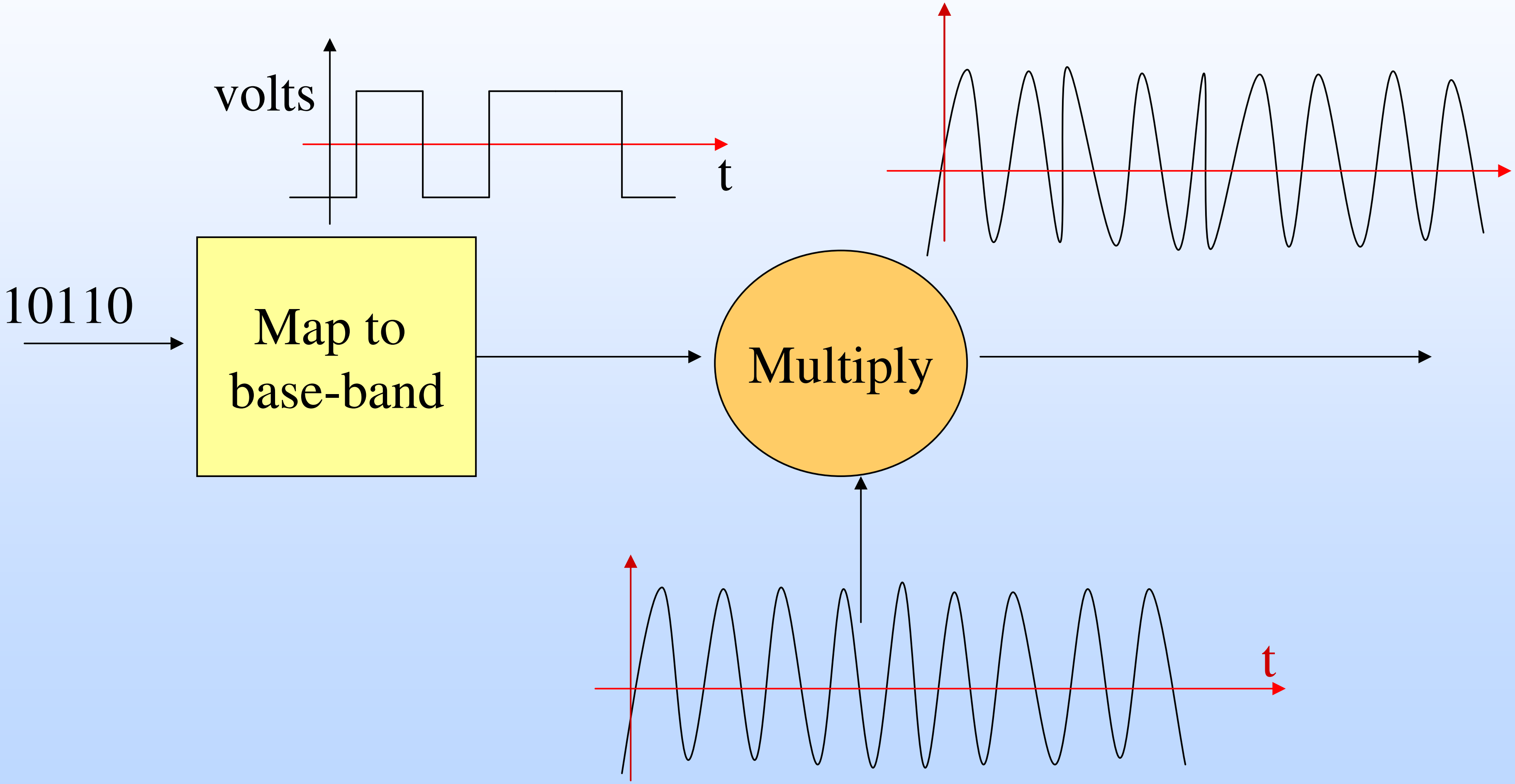
t

10110

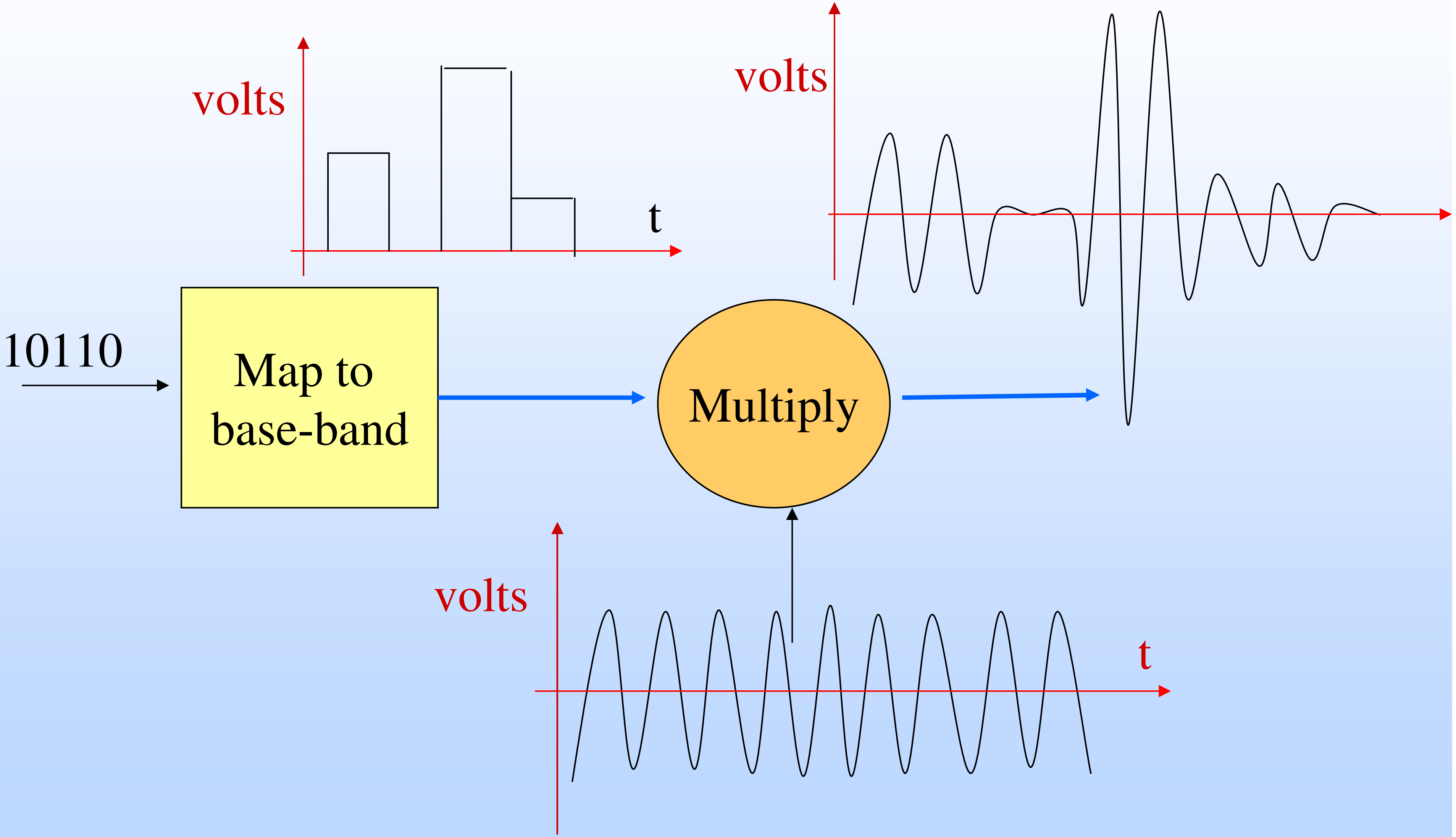
Map to base-band

Multiply

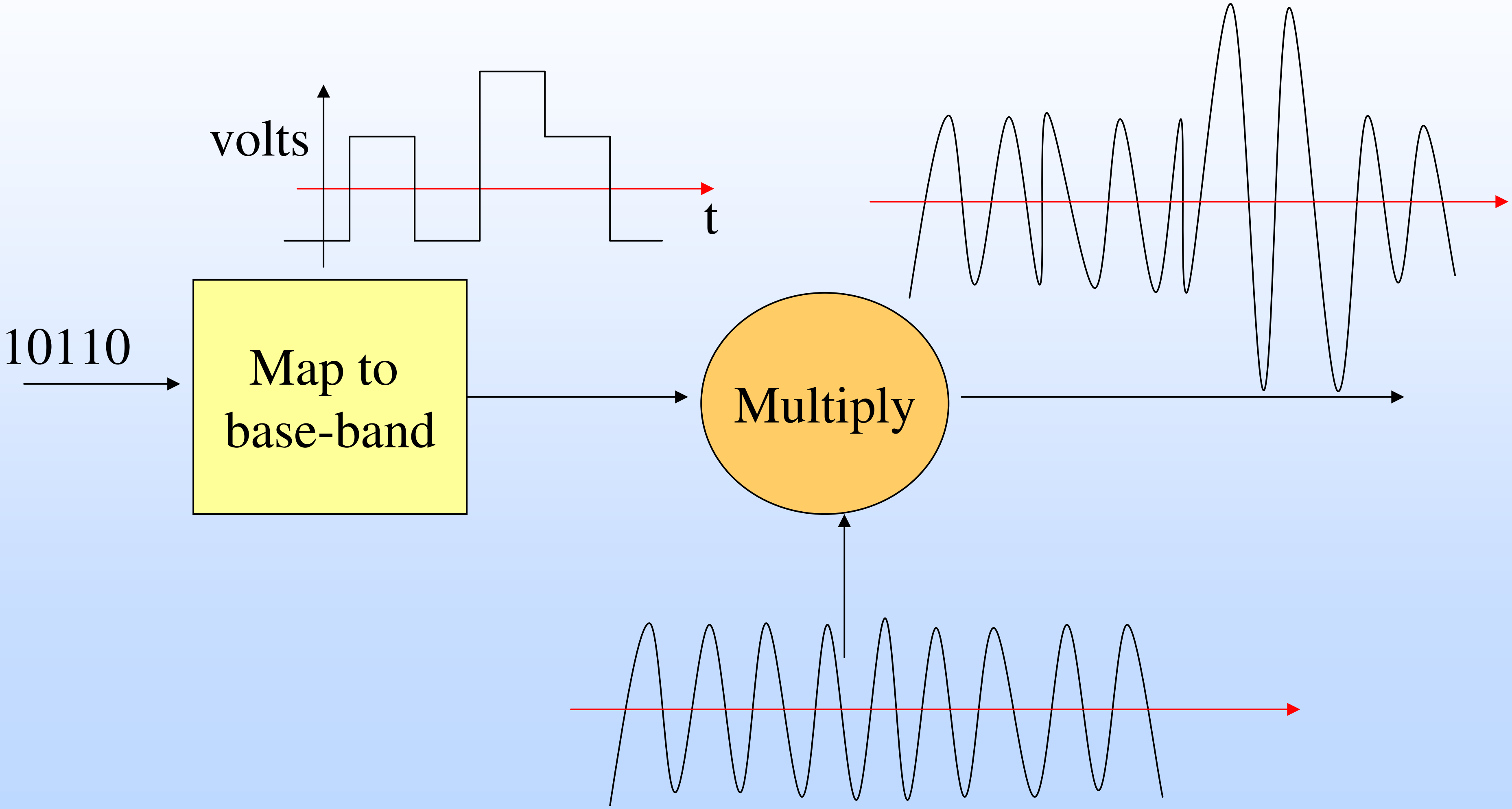
Binary phase shift keying (B-PSK)



4-ary amplitude shift keying (ASK)



Combined multi-level ASK & PSK



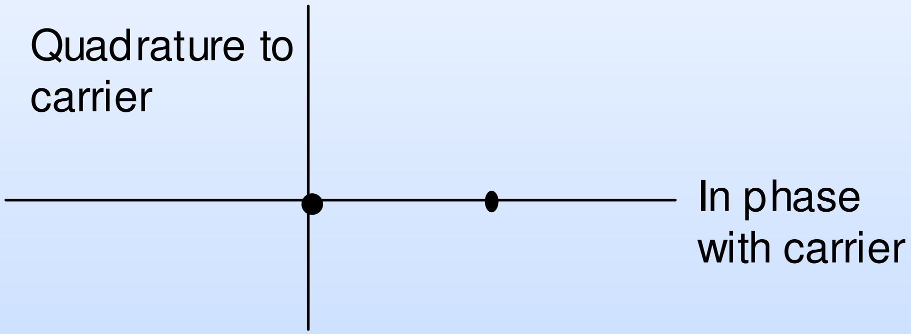
10110

Map to base-band

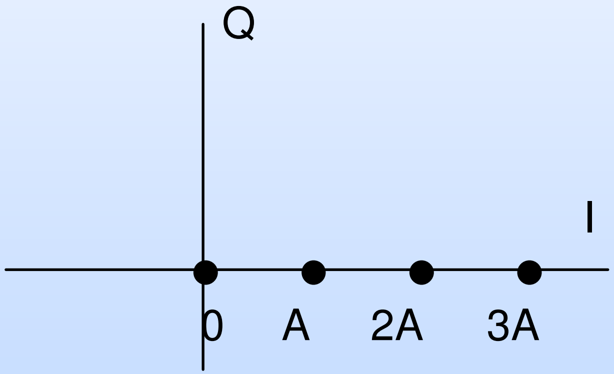
Multiply

Constellation diagrams

Show “in phase” and “quadrature” components as a graph as illustrated below for two examples:



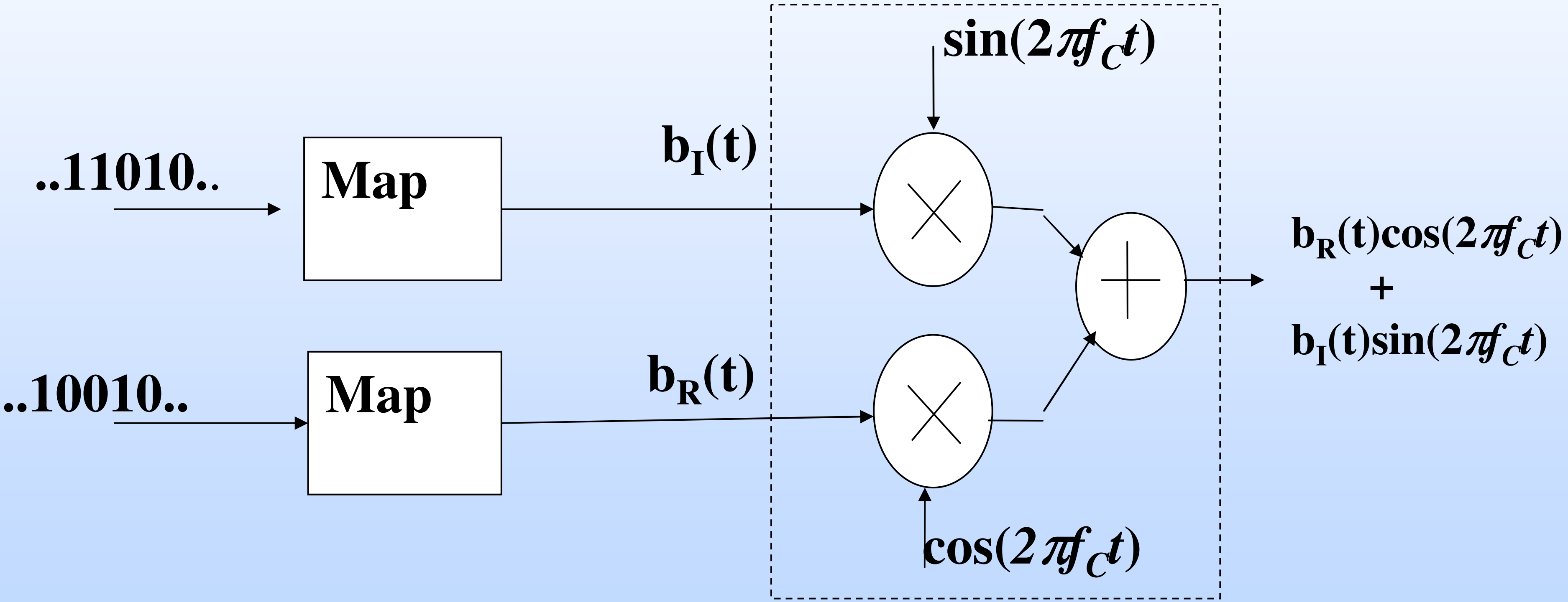
Binary ASK with symbols 0 & Acos(..)



4-ary ASK with symbols 0, Acos(..), 2Acos(..), 3Acos(..)

Complex baseband & vector-modulator/demodulator

Vector modulator:



..11010..

Map

$b_I(t)$

$\sin(2\pi f_C t)$

..10010..

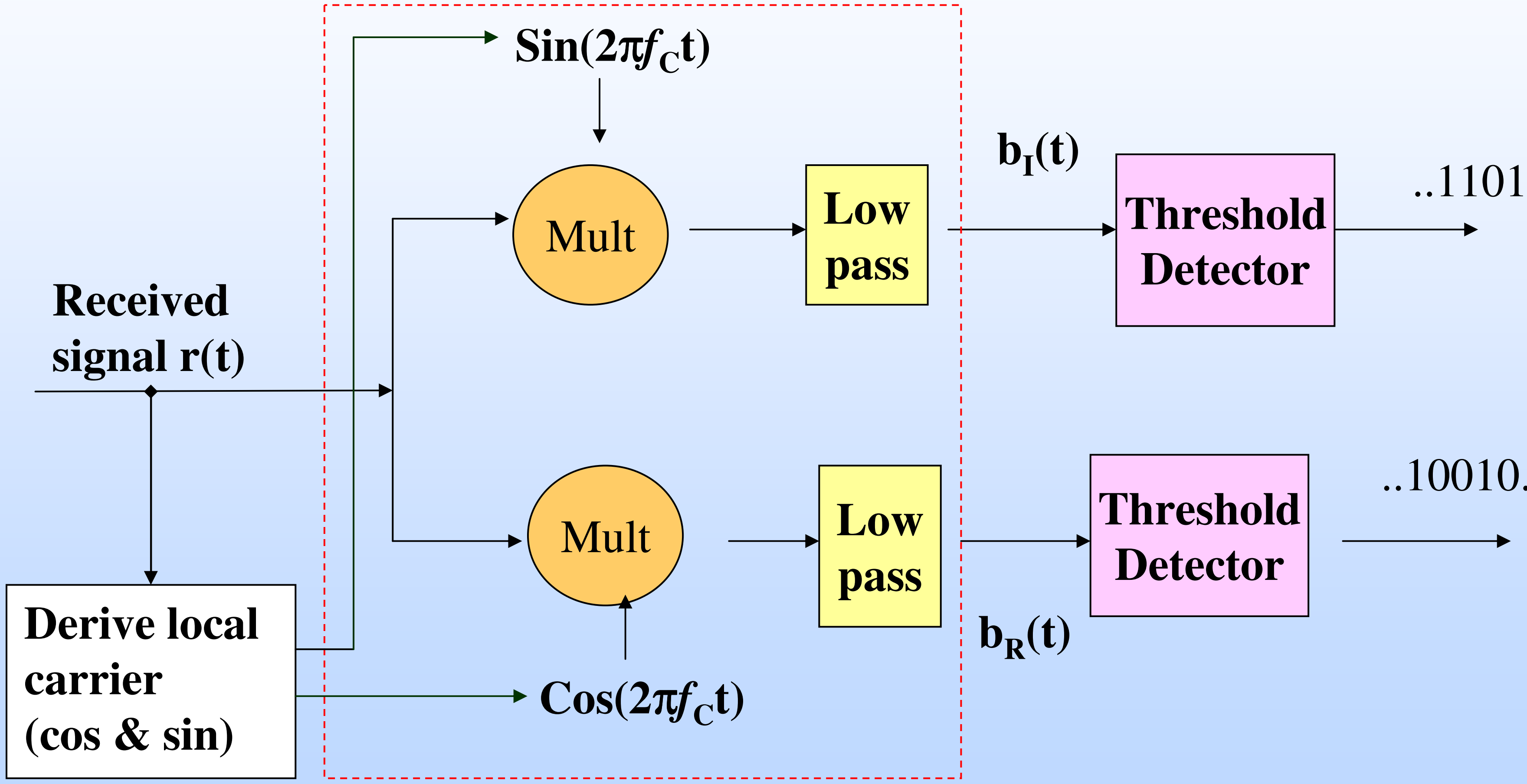
Map

$b_R(t)$

$\cos(2\pi f_C t)$

$b_R(t)\cos(2\pi f_C t)$
$+$
$b_I(t)\sin(2\pi f_C t)$

Vector demodulator



Sin($2\pi f_C$t)

Mult

Low pass

b_I(t)

Threshold Detector

..11010..

Received signal r(t)

Mult

Low pass

Threshold Detector

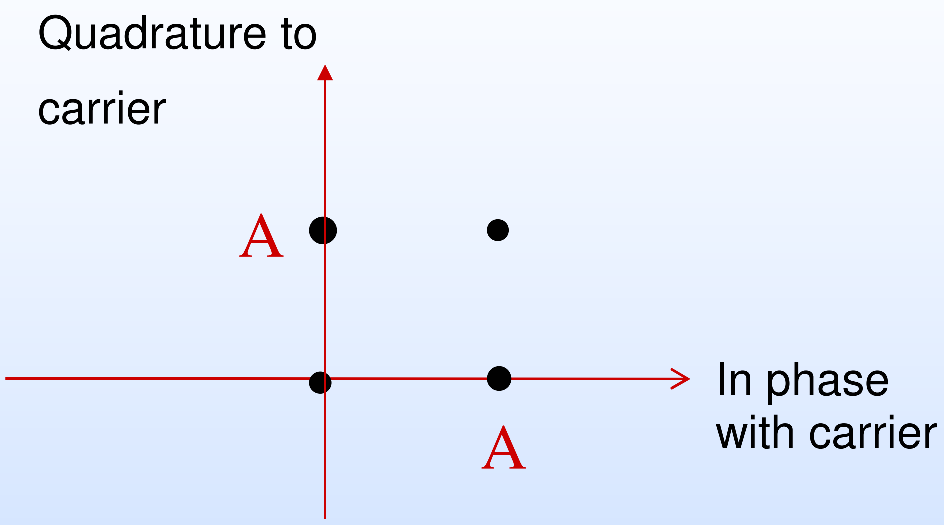
..10010..

b_R(t)

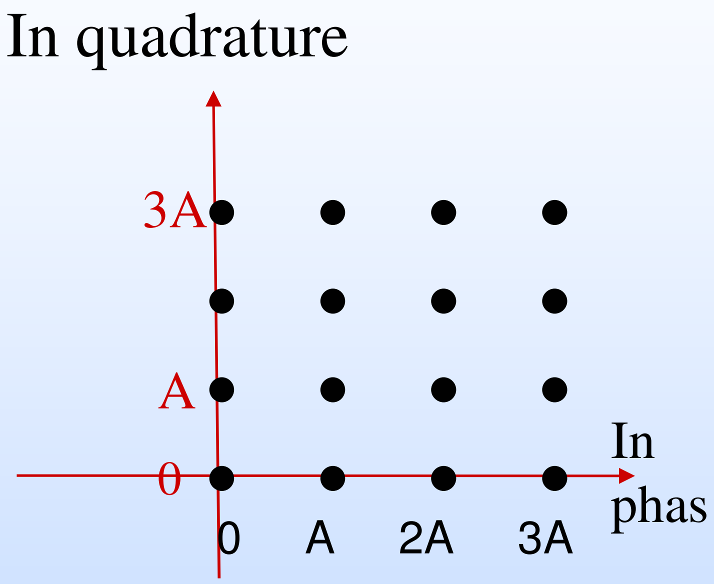
Derive local carrier (cos & sin)

Cos($2\pi f_C$t)

Constellation diags for ASK with complex baseband



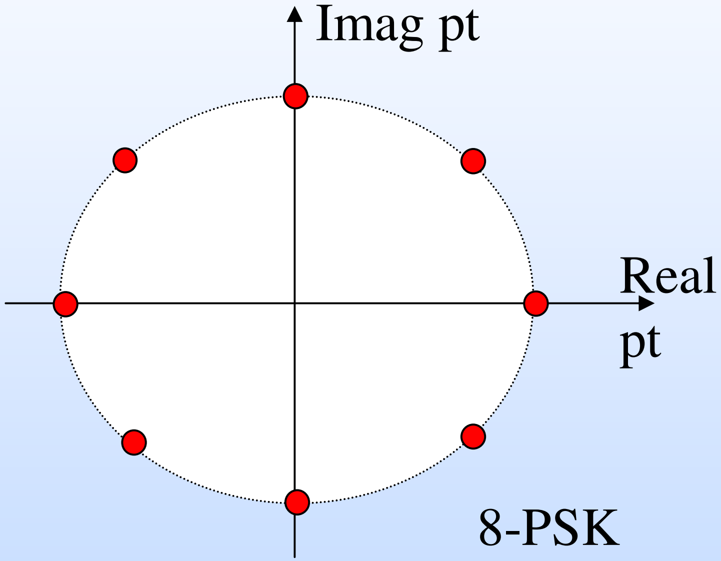
Binary ASK
for $b_R(t)$ & $b_I(t)$

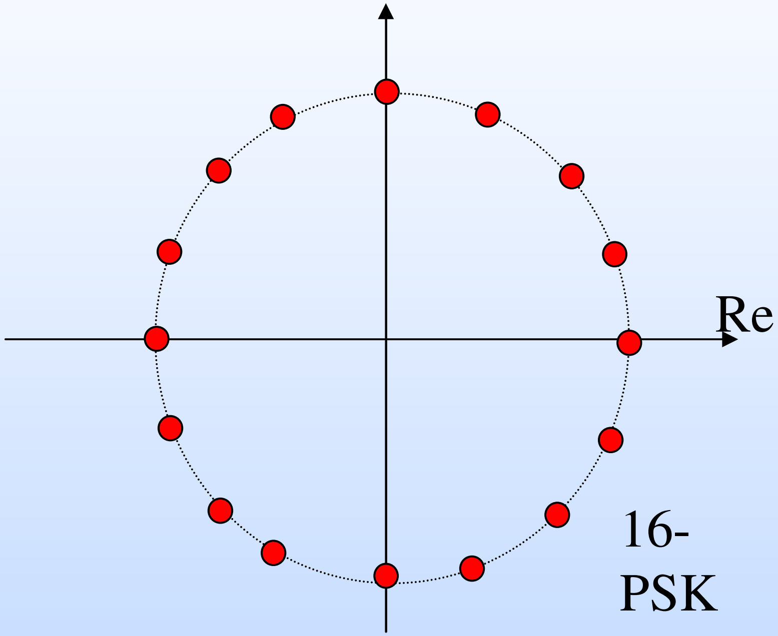


4-ary ASK
for $b_R(t)$ & $b_I(t)$

QPSK is 4-PSK. What about 8-PSK & 16-PSK?

Can have 8-PSK (3 bits/symbol) & 16-PSK (4 bits/symbol).
Constellation diagrams for shown below.

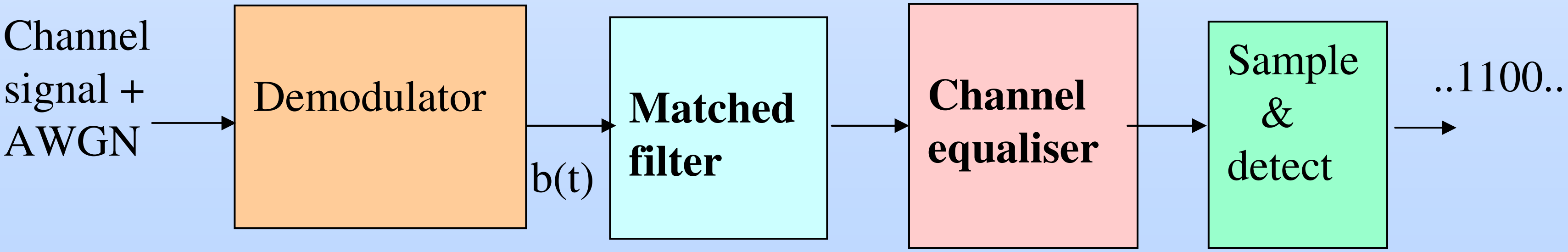




Differential forms of QPSK & M-PSK often used where changes in phase signify the data. Principle similar to DPSK .

‘Single carrier’ receiver

- **Receiver must demodulate to obtain base-band b(t) .**
- **Pulse shapes distorted & affected by noise.**
- **Sample & detect for rectangular pulses discussed in last lecture.**
- **May work for low bit-rates over channels with little distortion or noise**
- **Performance can be improved by introduction of**
  - a **matched filter** optimally tuned to shape of transmitted pulses to minimize effect of noise (AWGN).
  - a **channel equalizer** to cancel out distortion introduced by channel.



Channel signal + AWGN

Demodulator

b(t)

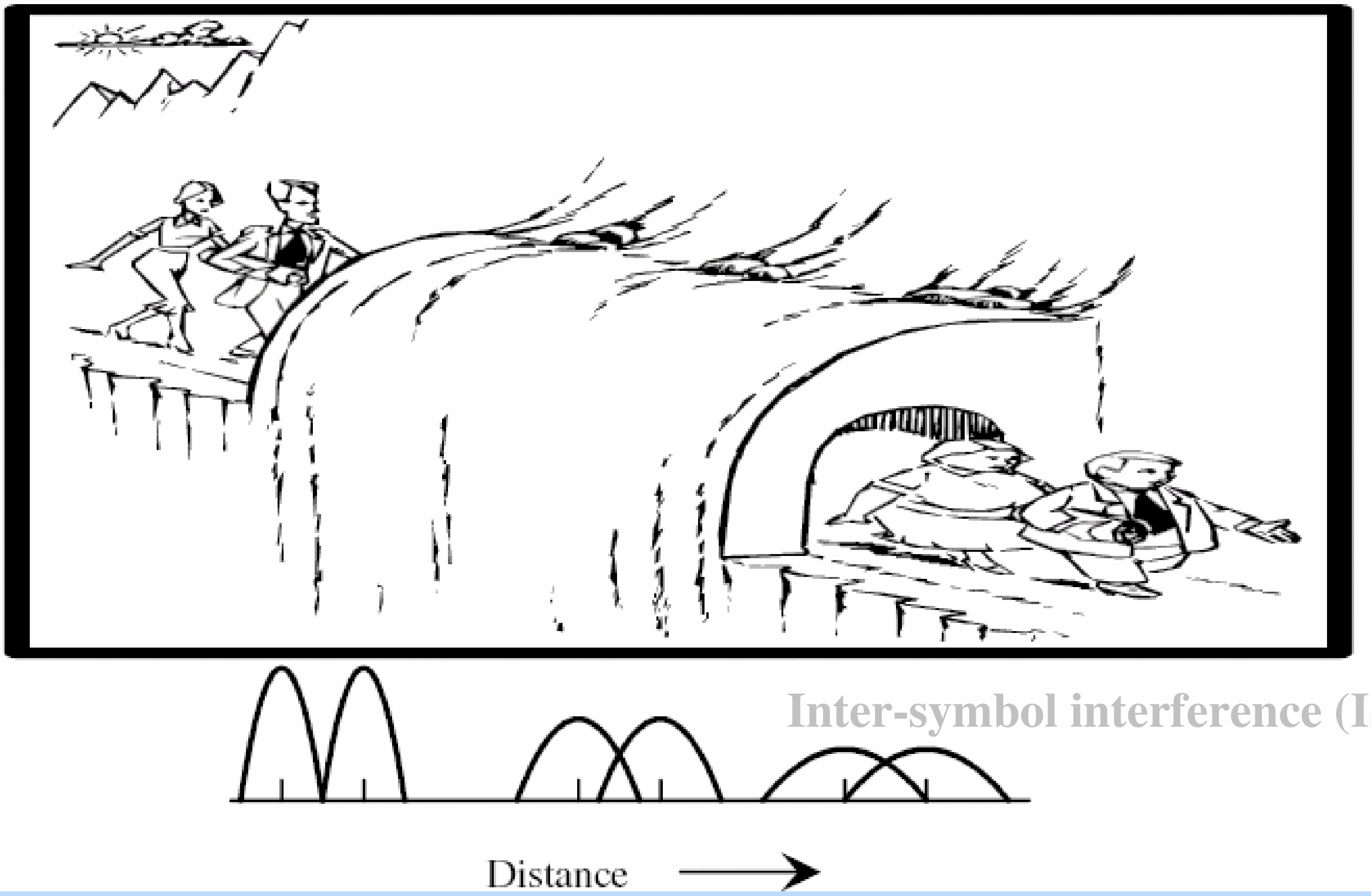
Matched filter

Channel equaliser

Sample & detect

..1100..

Recall: Attenuation, Dispersion Effects: ISI!



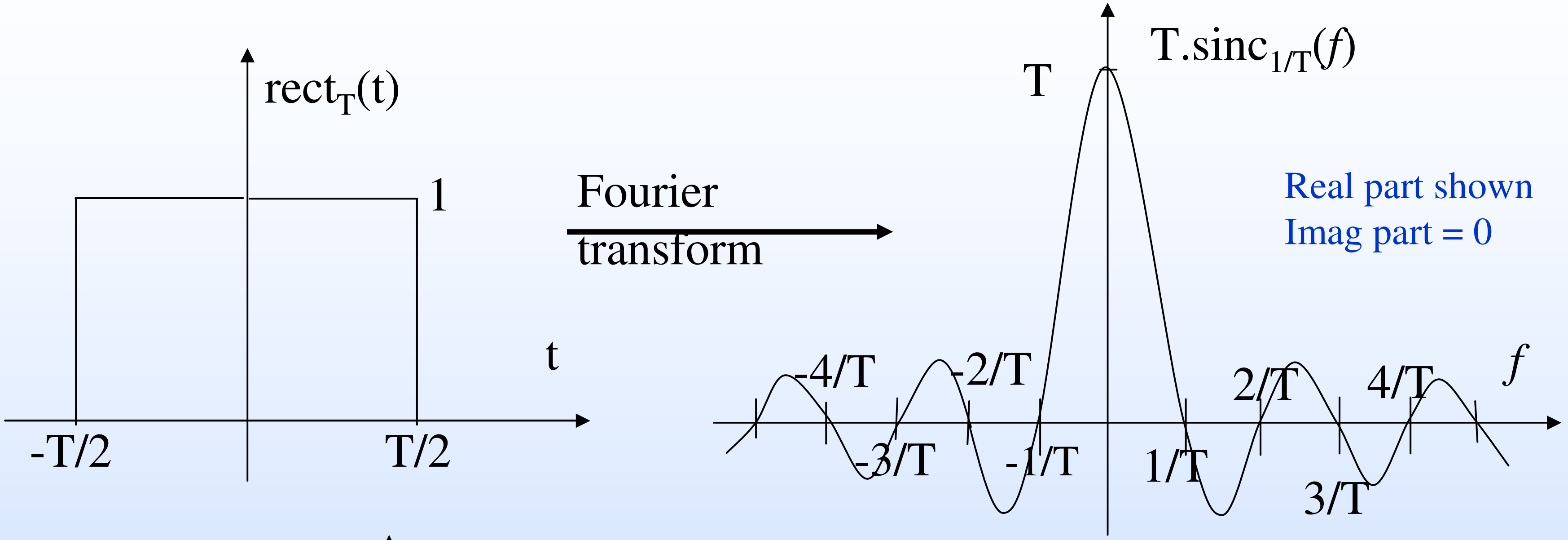
Inter-symbol interference (ISI)

Source: Prof. Raj Jain, WUSTL

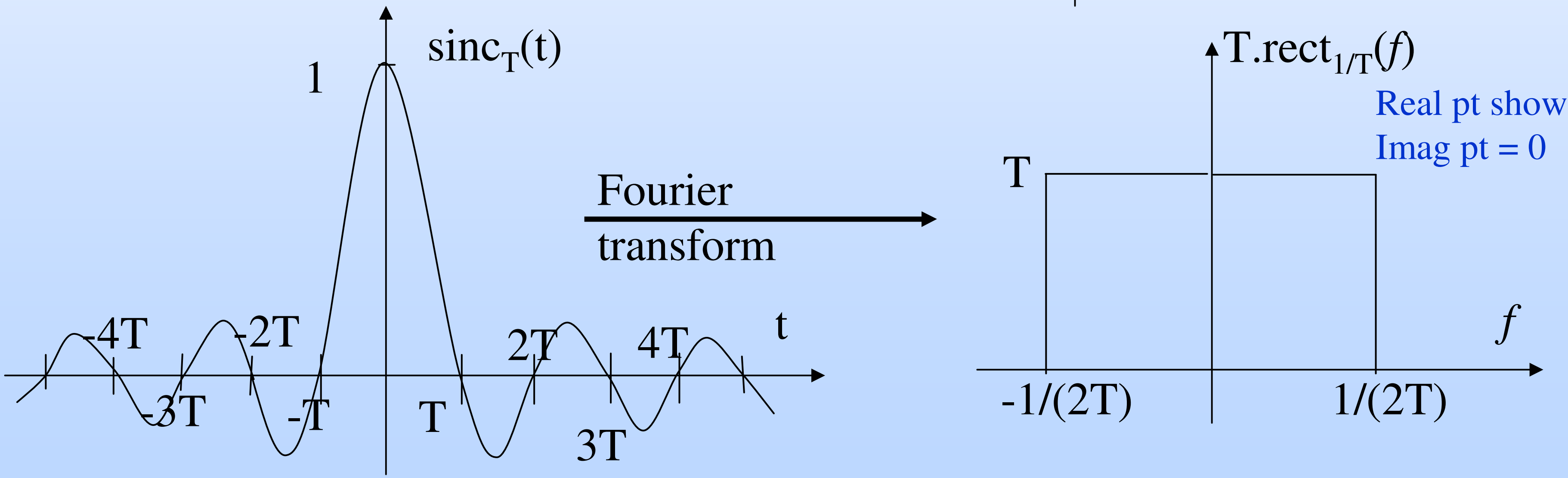
Channel equaliser

- Channel equaliser’ is an ‘adaptive filter’
- Programmed to correct any differences between pulses seen at output of matched filter & ideal RC pulses required by detector.
- Aims to cancel out effect of the channel,
- In particular the effects of frequency selective fading.
- Received amplitude reduced at some frequencies & reinforced at others.
- Equalizer must do opposite of this.
- Must adapt to changes in fading channel characteristics.
- A demanding filtering task, and it cannot always be successful.
- If there is a very deep fade, it will just not be possible to reverse it.
- Trying to do so will just emphasize noise at frequency of deep fade.
- Single carrier sine-wave modulation still widely used.

Spectra of rect & sinc pulses

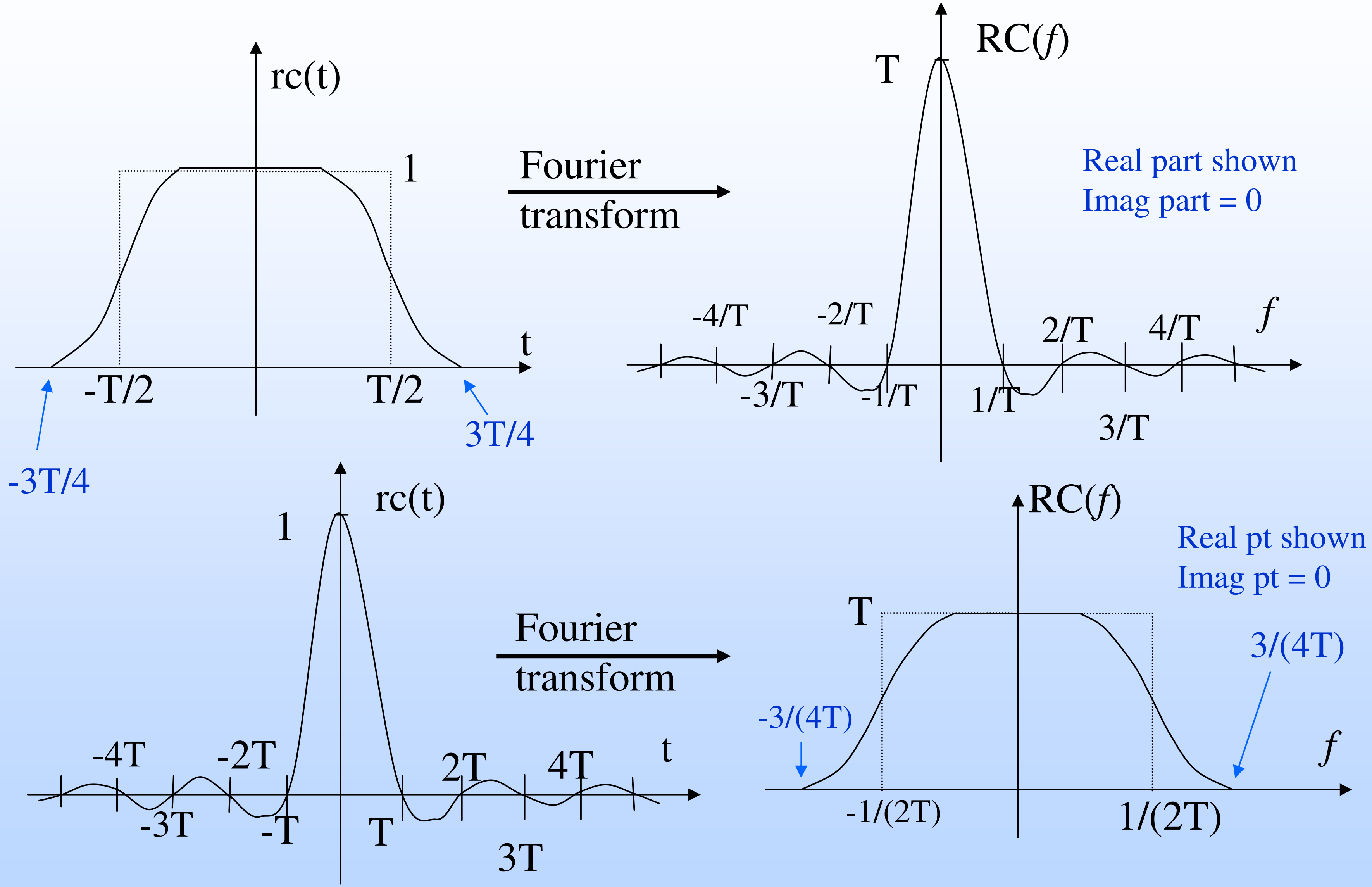


Real part shown
Imag part = 0



Real pt shown
Imag pt = 0

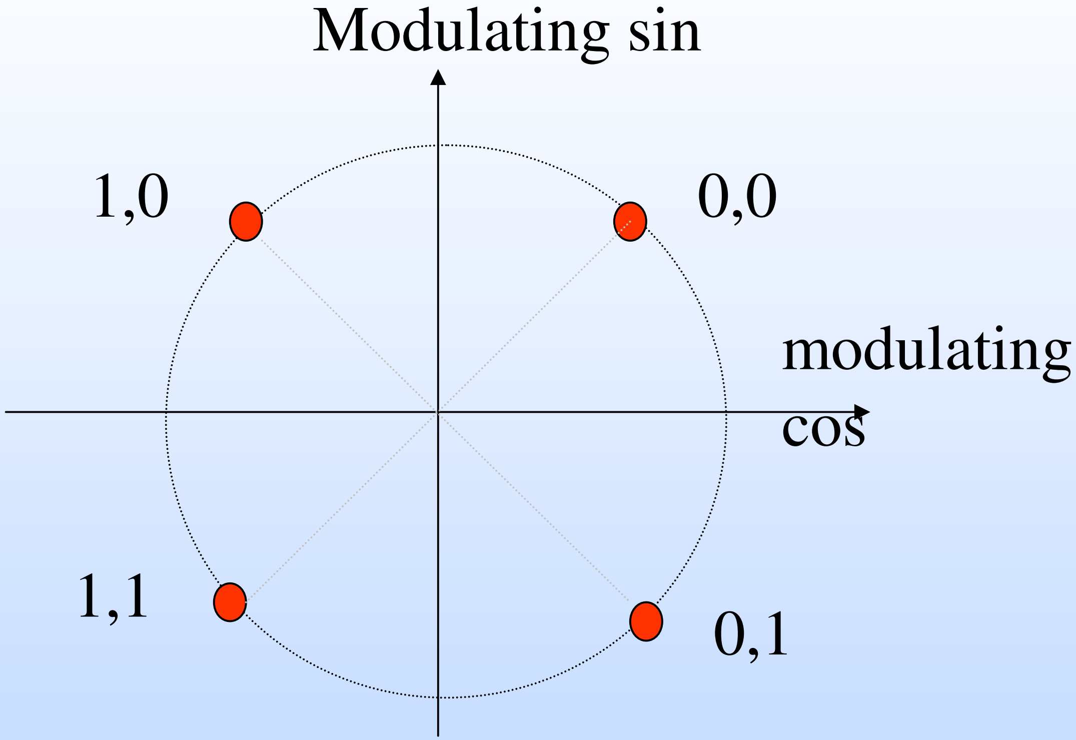
Spectra of 50% RC pulses & spectra



Modulation of sub-carriers

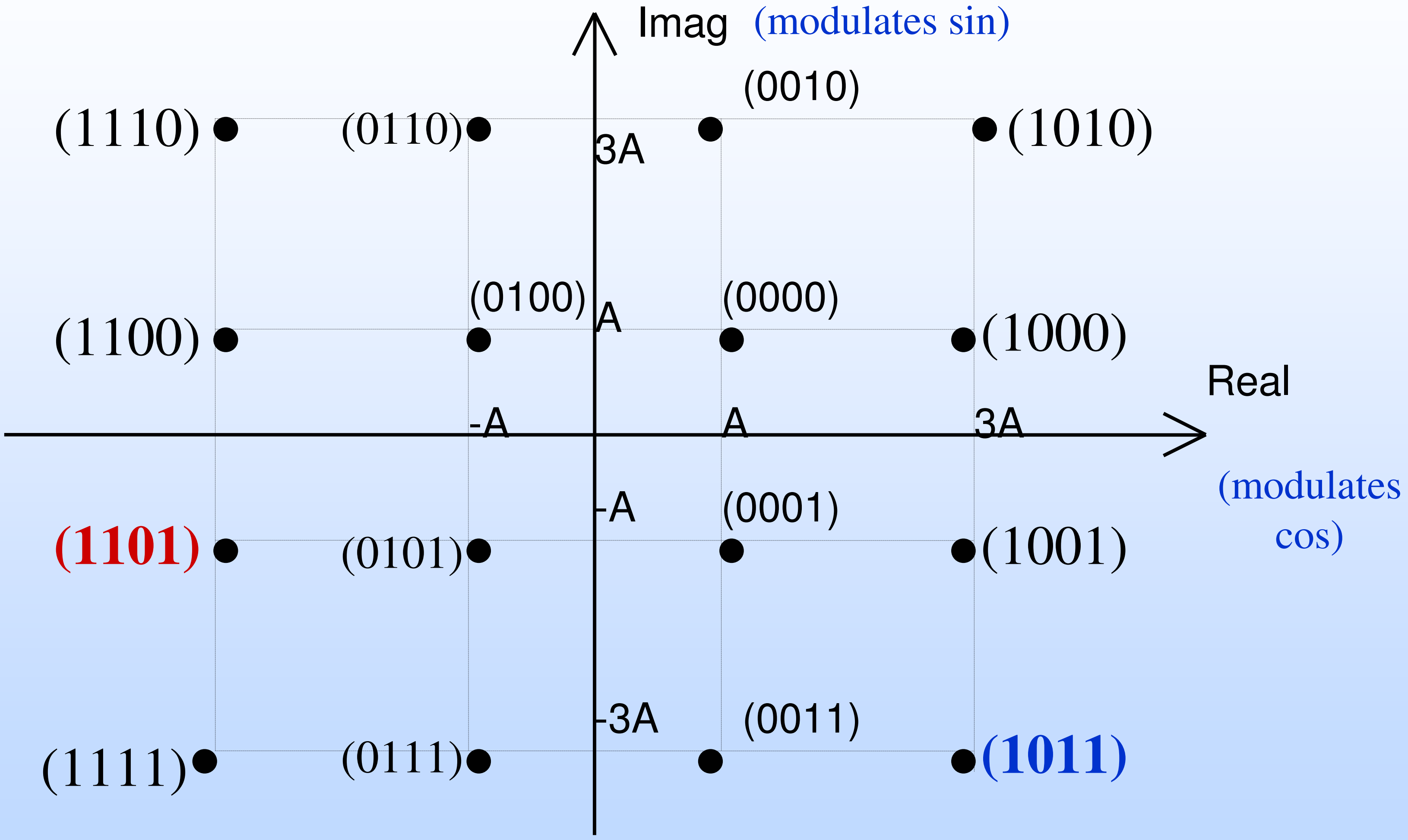
- **With IEEE802.11, each OFDM sub-carrier modulated by choice of:**
  - binary-PSK, (1 bit per pulse)
  - QPSK, (2 bits per pulse)
  - 16-QAM (4 bits per pulse)
  - 64-QAM (6 bits per pulse)
- **16-QAM & 64-QAM are multi-level schemes.**
- **Implement by vector-modulator according to ‘constellations’.**
- **Illustrate for QPSK & 16-QAM**
- **‘Gray coding’ for 16-QAM makes nearest dots differ in just 1 bit.**
- **Differential PSK, QPSK & QAM used where the difference between the current & previous pulse specifies the bit pattern.**

Constellation for QPSK



Bit1	Bit2	b_R	b_I
0	0	A	A
0	1	A	-A
1	0	-A	A
1	1	-A	-A

‘16_QAM’ constellation

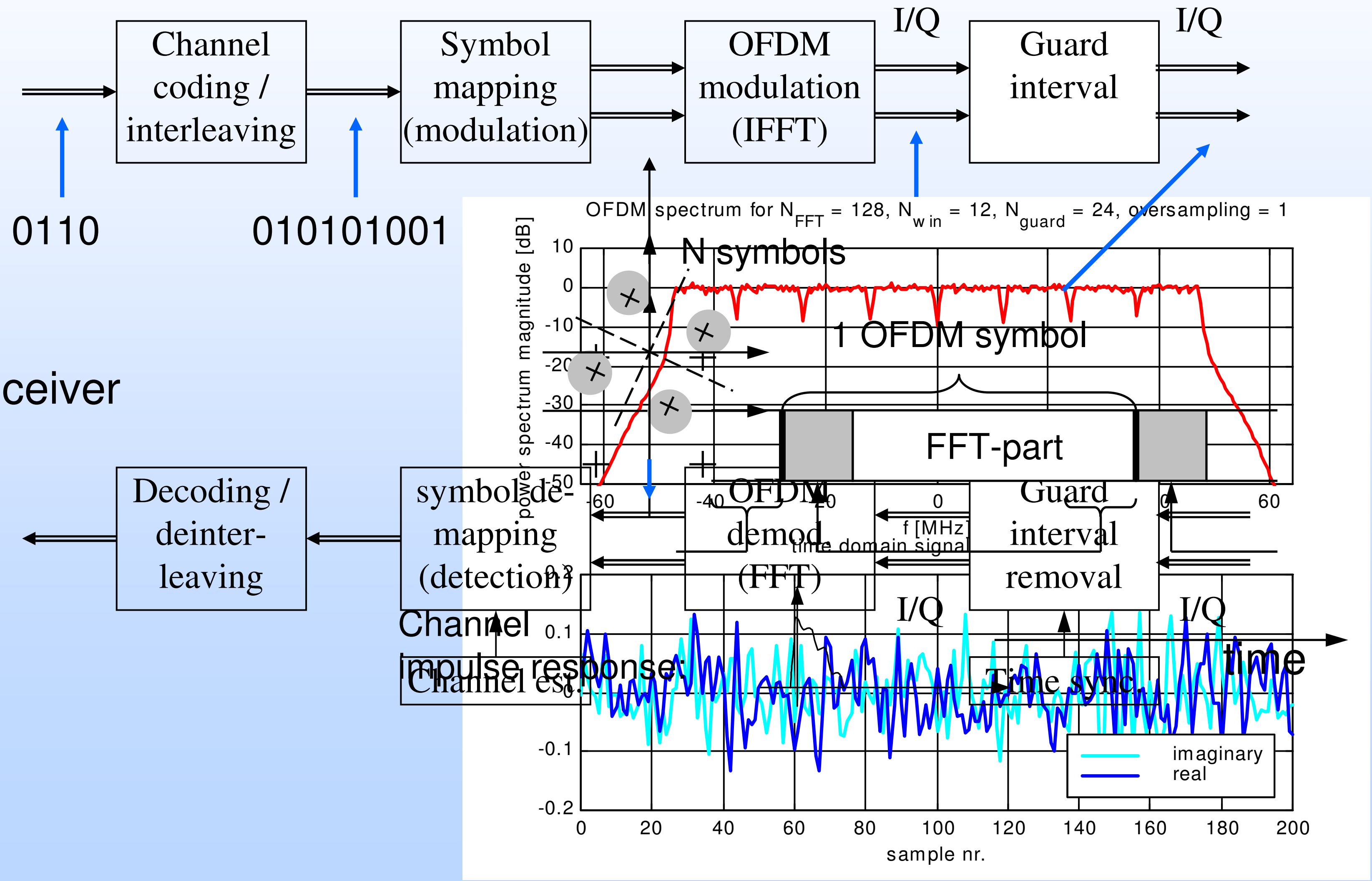


Multicarrier vs Equalizers

- **Equalizers use signal processing in receiver to eliminate ISI.**
- **Linear equalizers can completely eliminate ISI (ZF), but this may enhance noise. MMSE better tradeoff.**
- **Equalizer design involves tradeoffs in complexity, overhead, and performance (ISI vs. noise).**
  - Number of filter taps, linear versus nonlinear, complexity and overhead of training and tracking
- **Multicarrier is an alternative to equalization**
  - Divides signal bandwidth to create flat-fading subchannels.

OFDM Block Diagram

Transmitter



Channel coding / interleaving

Symbol mapping (modulation)

OFDM modulation (IFFT)

I/Q

Guard interval

I/Q

0110

010101001

Receiver

Decoding / deinter-leaving

symbol de-mapping (detection)

OFDM demod (FFT)

Guard interval removal

I/Q

I/Q

Channel est.

Time sync

N symbols

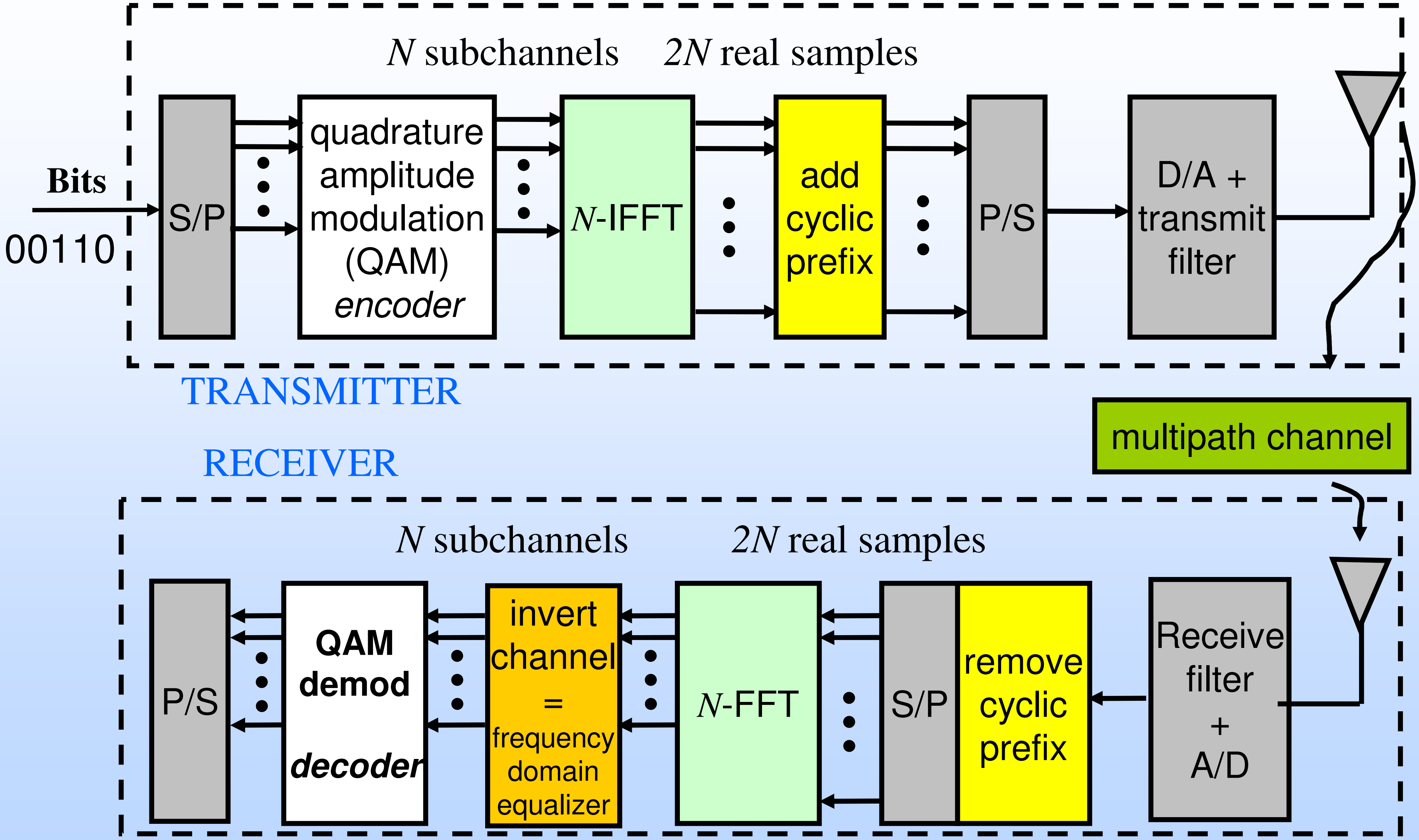
1 OFDM symbol

FFT-part

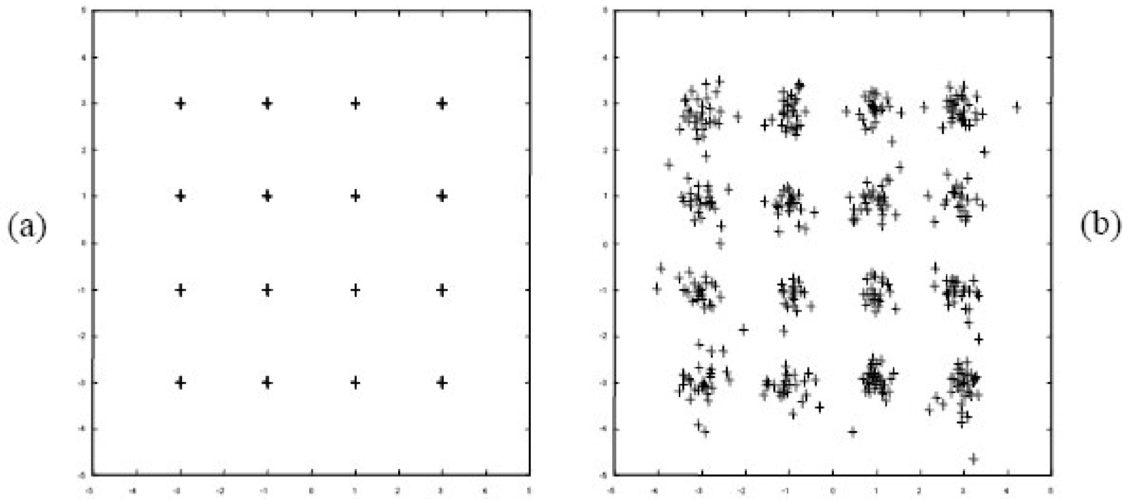
Channel impulse response:

time

Summary: An OFDM Modem



Example of ISI/ICI



16-QAM constellations for a 48-subcarrier OFDM signal in a 2-ray multipath channel with

(a) multipath delay < guard time
(b) multipath delay = 1.03*guard time