EC 551 Telecommunication Systems Engineering

Section 1

Overview of Digital Modulation Techniques

Modulation

Digital modulation

- digital data is translated into an analog signal (baseband)
- ASK, FSK, PSK - main focus in this chapter
- differences in spectral efficiency, power efficiency, robustness

Analog modulation

- shifts center frequency of baseband signal up to the radio carrier

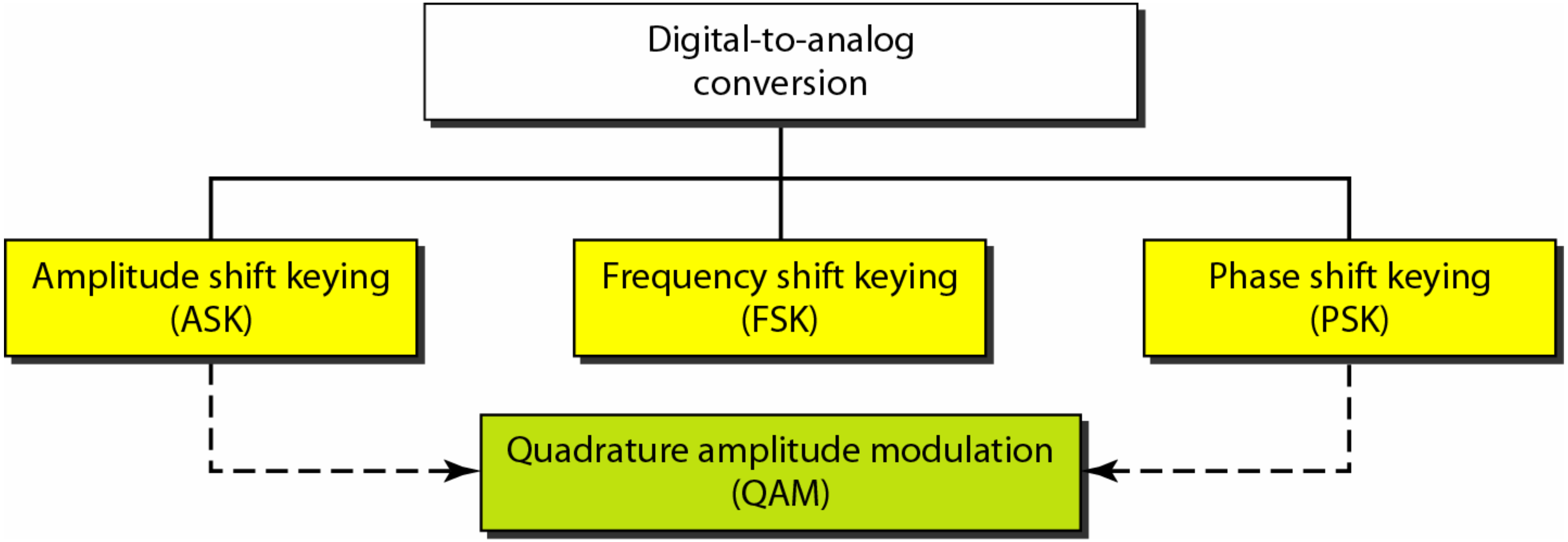
Motivation

- smaller antennas (e.g., $\lambda/4$)
- Frequency Division Multiplexing
- medium characteristics

Basic schemes

- Amplitude Modulation (AM)
- Frequency Modulation (FM)
- Phase Modulation (PM)

Types of digital-to-analog conversion



Note

- Bit rate is the number of bits per second.
- Baud rate is the number of signal elements per second.
- The Baud rate is less than or equal to the bit rate.

Example 1

An signal carries 4 bits per signal element. If 1000 signal elements are sent per second, find the bit rate.

Solution

In this case, r = 4, S = 1000, and N is unknown. We can find the value of N from

$$S = N \times \frac{1}{r} \qquad \text{or} \qquad N = S \times r = 1000 \times 4 = 4000 \text{ bps}$$

Example 2

An signal has a bit rate of 8000 bps and a baud rate of 1000 baud. How many data elements are carried by each signal element? How many signal elements do we need?

Solution

In this example, S = 1000, N = 8000, and r and L are unknown. We find first the value of r and then the value of L.

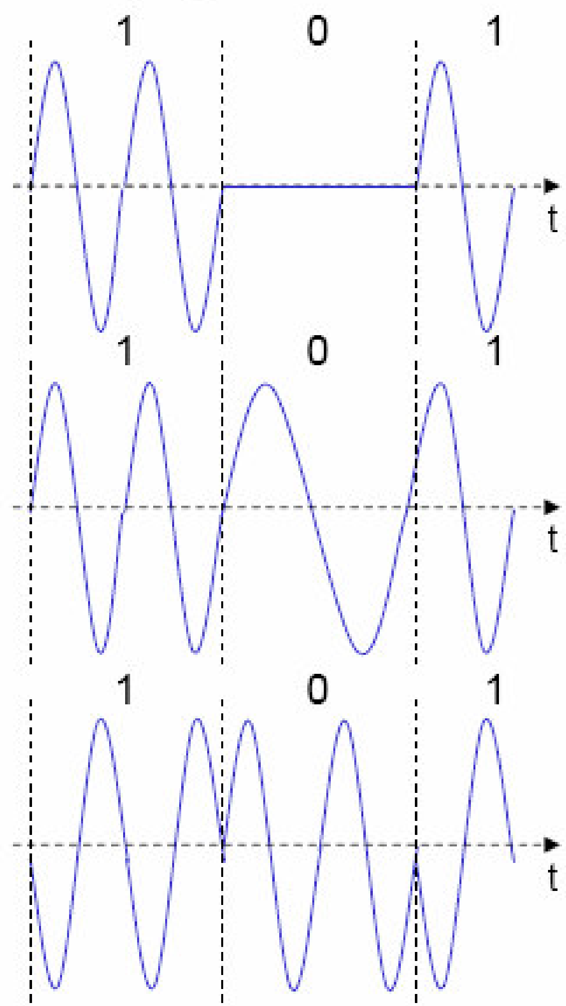
$$S = N \times \frac{1}{r} \rightarrow r = \frac{N}{S} = \frac{8000}{1000} = 8 \text{ bits/baud}$$

$$r = \log_2 L \rightarrow L = 2^r = 2^8 = 256$$

Digital modulation

Modulation of digital signals known as Shift Keying

- Amplitude Shift Keying (ASK):
  - very simple
  - low bandwidth requirements
  - very susceptible to interference
- Frequency Shift Keying (FSK):
  - needs larger bandwidth
- Phase Shift Keying (PSK):
  - more complex
  - robust against interference

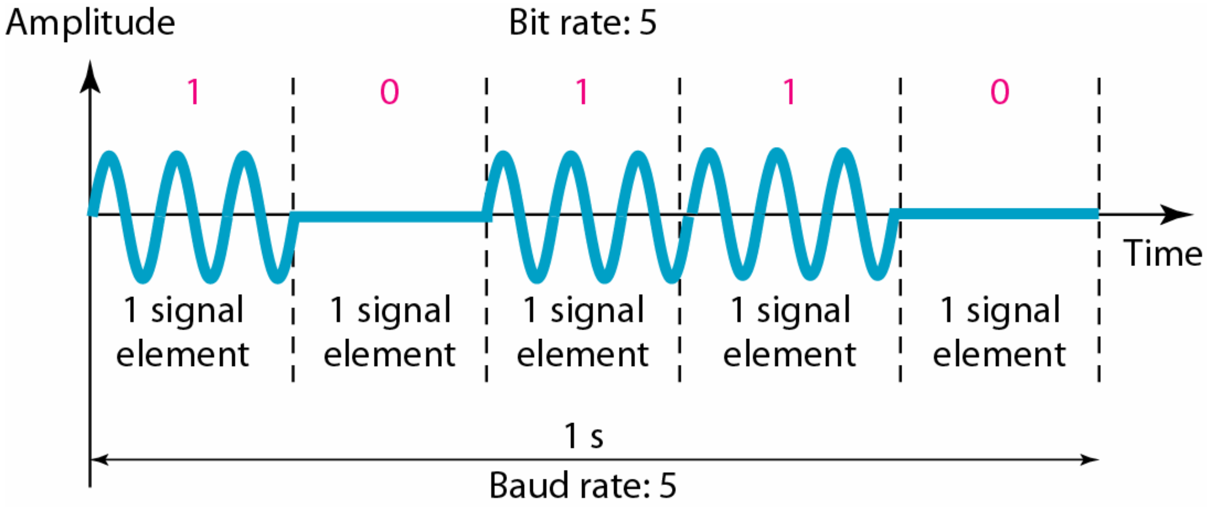


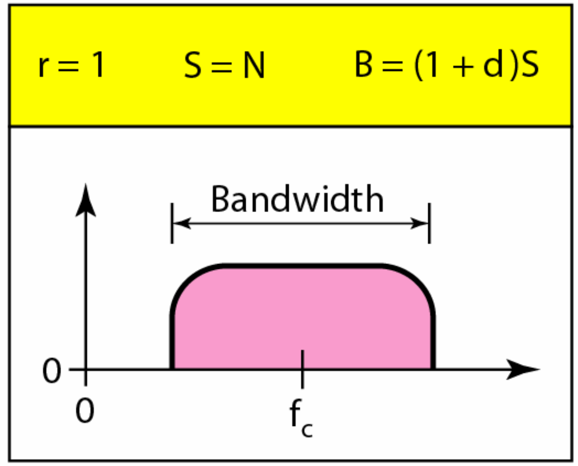
Digital Modulation

Amplitude Shift Keying

(ASK)

Binary amplitude shift keying





d = Rolloff factor

d= 1 (Rectangular pulse)

Example 3

We have an available bandwidth of 100 kHz which spans from 200 to 300 kHz. What are the carrier frequency and the bit rate if we modulated our data by using ASK with d = 1?

Solution

The middle of the bandwidth is located at 250 kHz. This means that our carrier frequency can be at f_c = 250 kHz. We can use the formula for bandwidth to find the bit rate (with d = 1 and r = 1).

$$B = (1 + d) \times S = 2 \times N \times \frac{1}{r} = 2 \times N = 100 \text{ kHz} \quad \rightarrow \quad N = 50 \text{ kbps}$$

Digital Modulation

Frequency Shift Keying

(FSK)

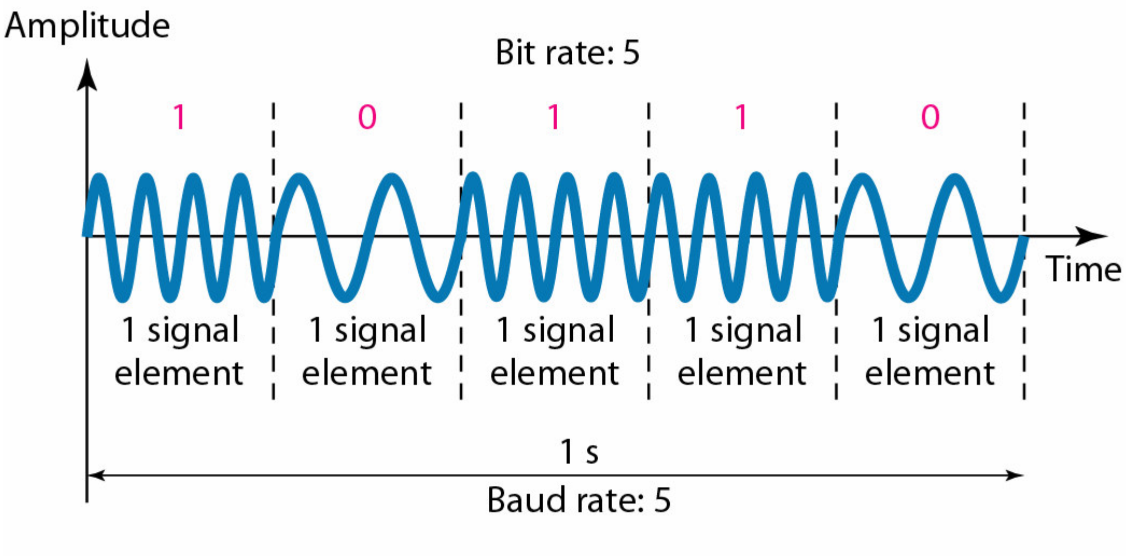
Binary frequency shift keying

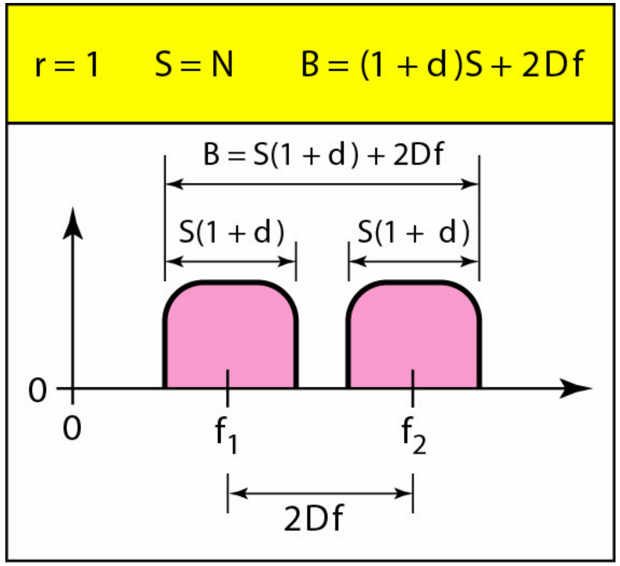
- In Binary FSK system, Symbol 0 and 1 are distinguished from each other By transmitting one of two sinusoidal wave that differ in frequency by a fixed amount.
- $$s_i(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t)$$

$$f_i \times T_b = Intgral$$

Eb : transmitted signal energy per bit

Binary frequency shift keying





Example 4

We have an available bandwidth of 100 kHz which spans from 200 to 300 kHz. What should be the carrier frequency and the bit rate if we modulated our data by using FSK with d = 1?

Solution

This problem is similar to Example 3, but we are modulating by using FSK. The midpoint of the band is at 250 kHz. We choose 2Δf to be 50 kHz; this means

$$B = (1 + d) \times S + 2\Delta f = 100 \rightarrow 2S = 50 \text{ kHz} \quad S = 25 \text{ kbaud} \quad N = 25 \text{ kbps}$$

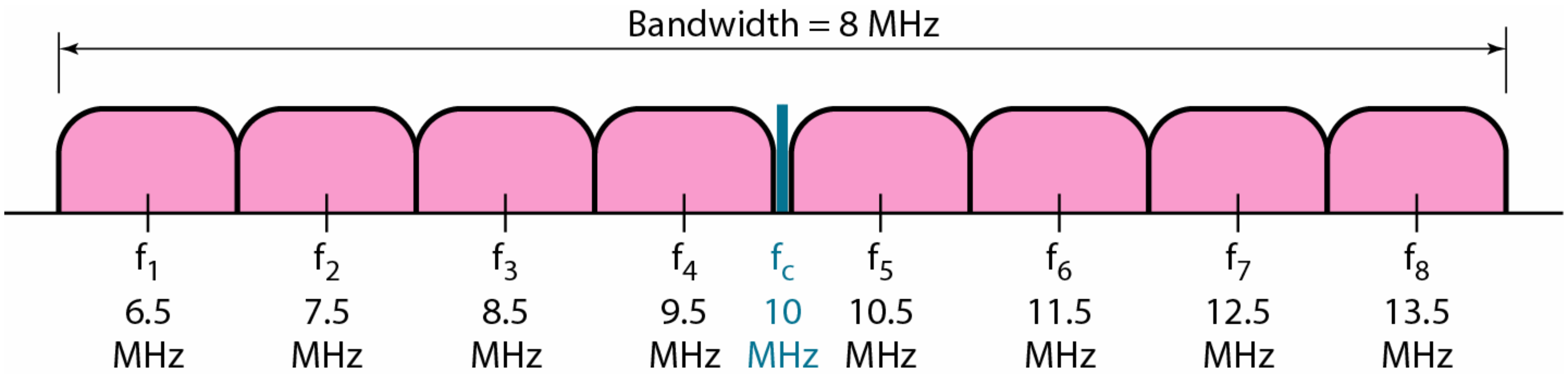
Example 5

We need to send data 3 bits at a time at a bit rate of 3 Mbps. The carrier frequency is 10 MHz. Calculate the number of levels (different frequencies), the baud rate, and the bandwidth.

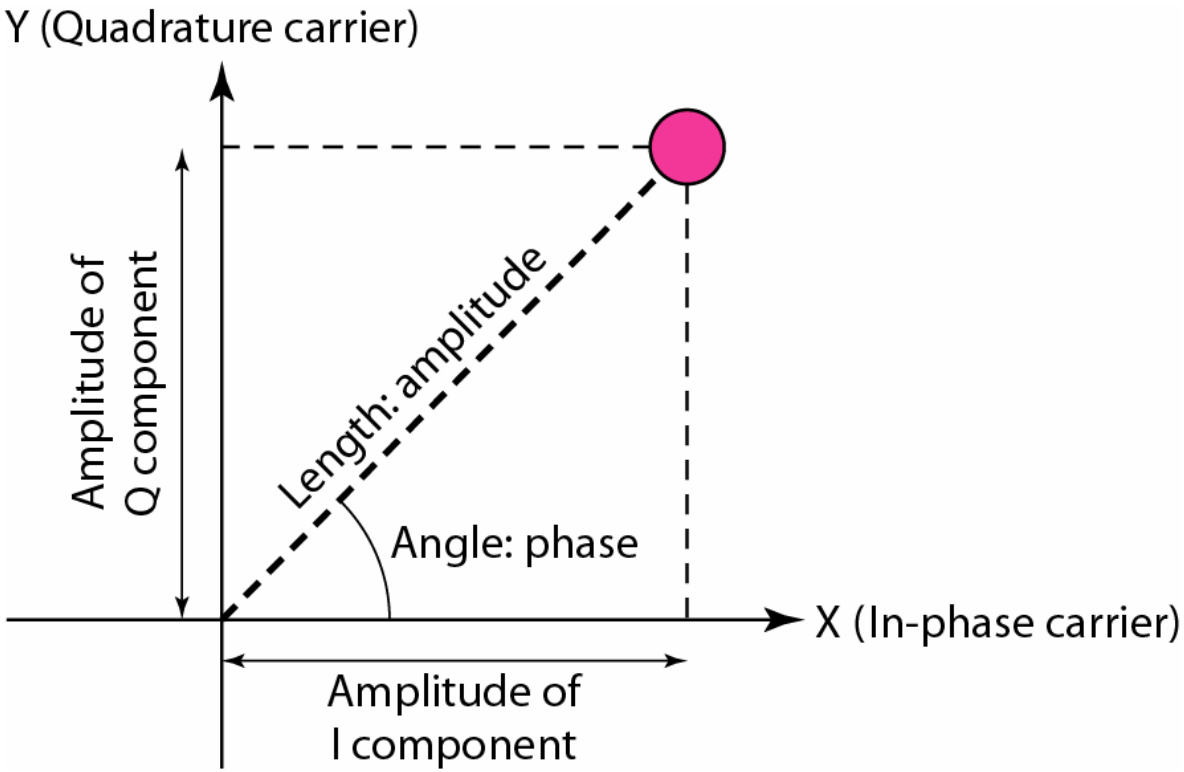
Solution

We can have L = 2^3= 8. The baud rate is S = 3 MHz/3 = 1000 Mbaud. This means that the carrier frequencies must be 1 MHz apart ($2\Delta f$ = 1 MHz). The bandwidth is B = 8 × 1000 = 8000. Figure 1 shows the allocation of frequencies and bandwidth.

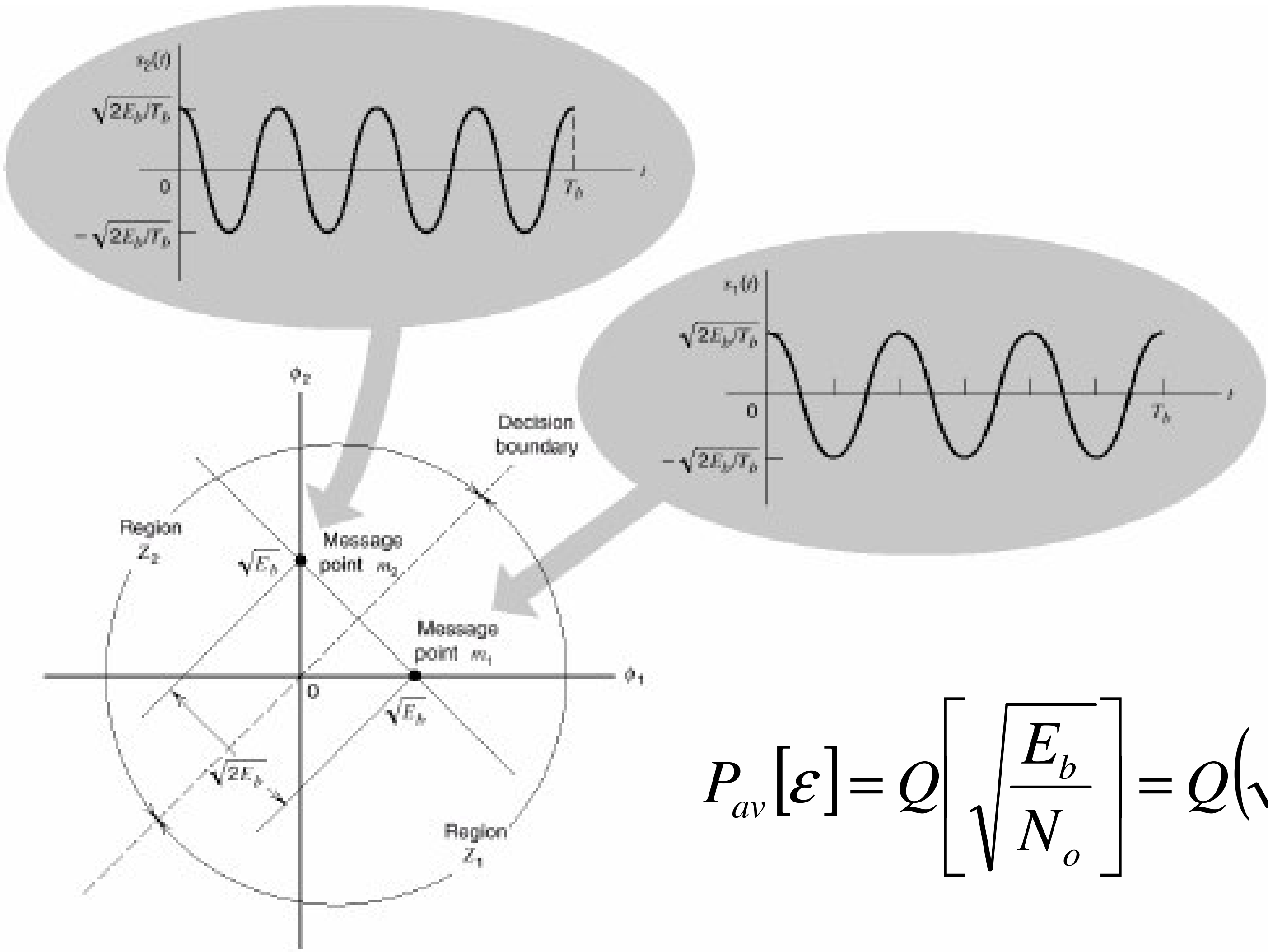
Figure 1 *Bandwidth of MFSK used in Example 5*



Concept of a constellation diagram



Signal Space of FSK



$$P_{av}[\varepsilon] = Q\left[\sqrt{\frac{E_b}{N_o}}\right] = Q\left(\sqrt{SNR}\right)$$

Digital Modulation

Phase Shift Keying

(PSK)

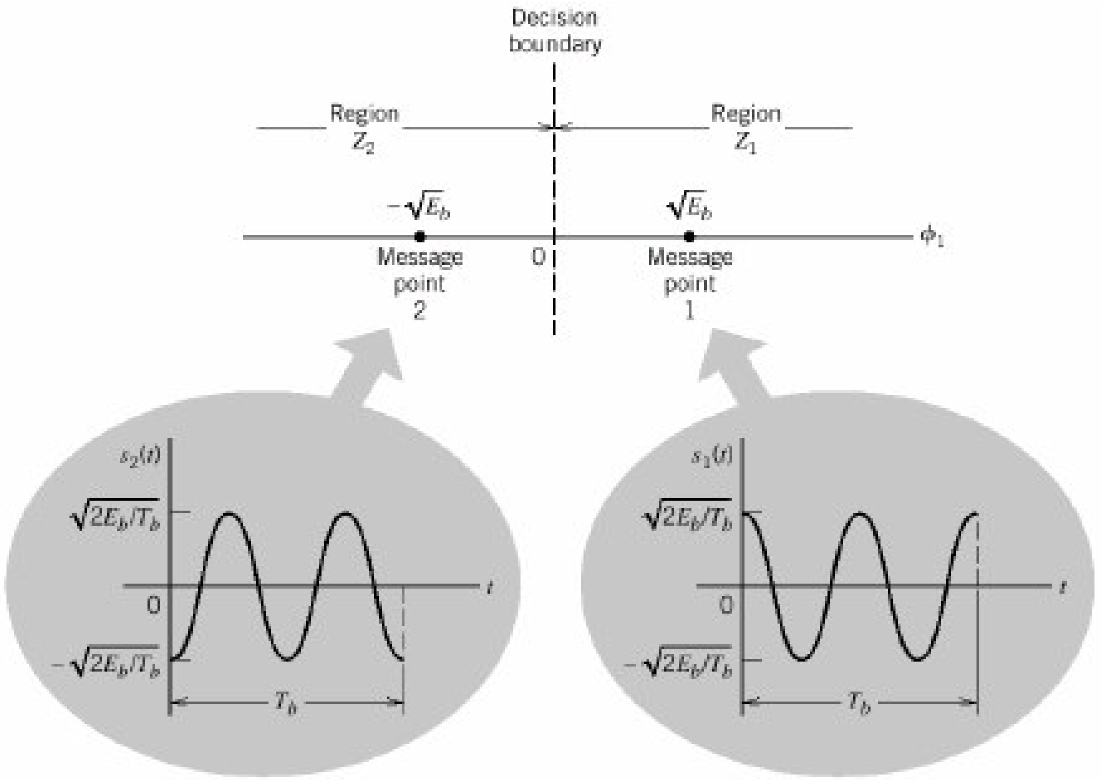
Binary phase shift keying

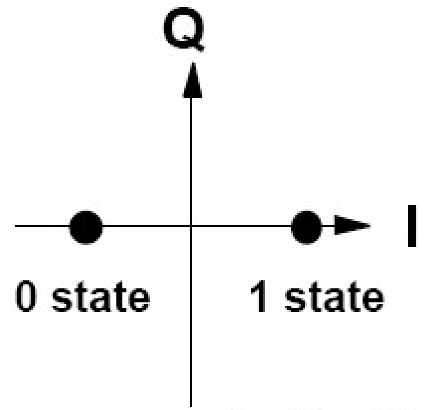
- The pair of signals S1(t) and S2(t) used to represent binary symbols 1 and 0

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t + \pi\right) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

Signal Space of BPSK

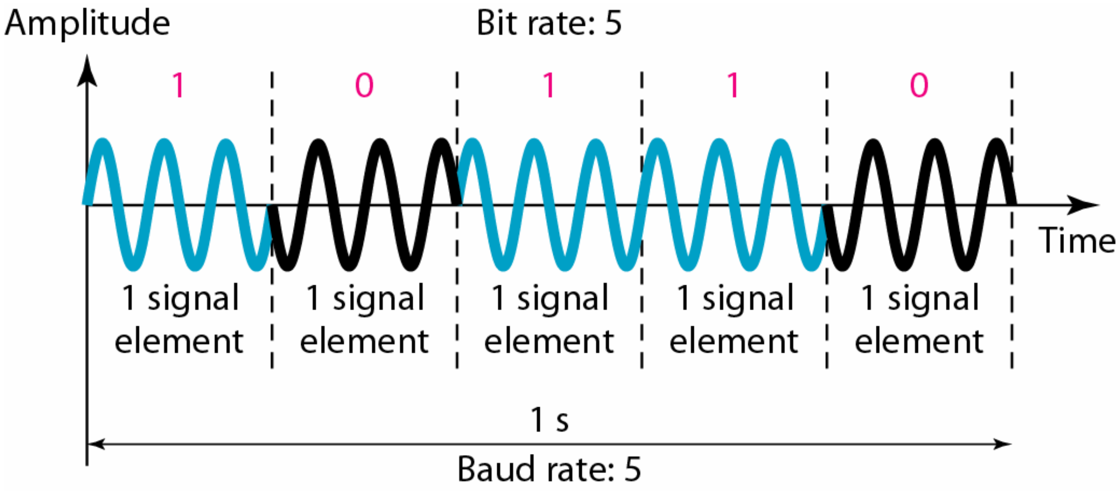


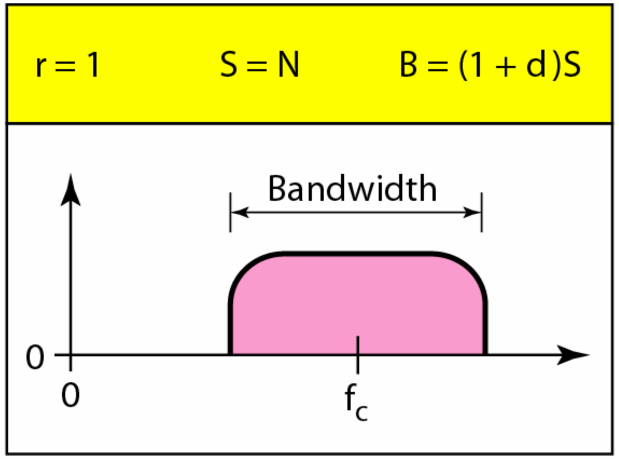


Phases separated by 180° (π radians)

$$P_e = Q\left[\frac{d}{\sqrt{2N_o}}\right] \qquad d = 2\sqrt{E_b} \qquad P_e = Q\left[\sqrt{\frac{2E_b}{N_o}}\right]$$

Binary phase shift keying





M-array PSK

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t - \theta_i)$$

$$\theta_i = \frac{\pi}{M}(2i - 1)$$

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_o}} \sin\frac{\pi}{M}\right)$$

Quadriphase-shift keying (QPSK)

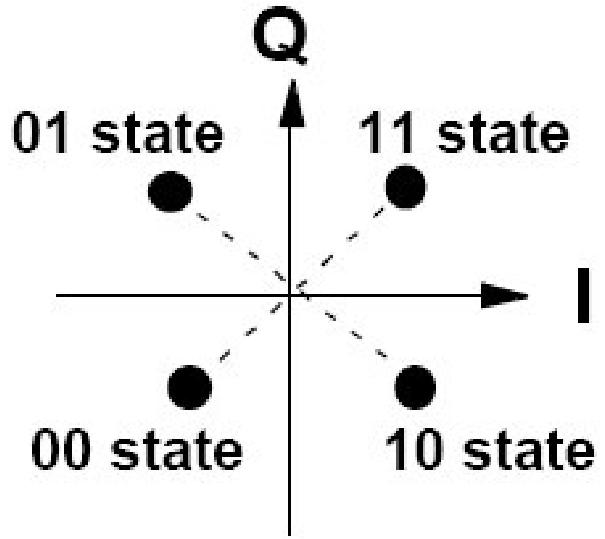
$$s_i(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t - \theta_i)$$

$$T = m \times T_b$$

$$E = m \times E_b$$

$$\theta_i = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

- **Quadrature Phase Shift Keying (QPSK)**
  - Multilevel modulation technique: 2 bits per symbol
  - More spectrally efficient, more complex receiver

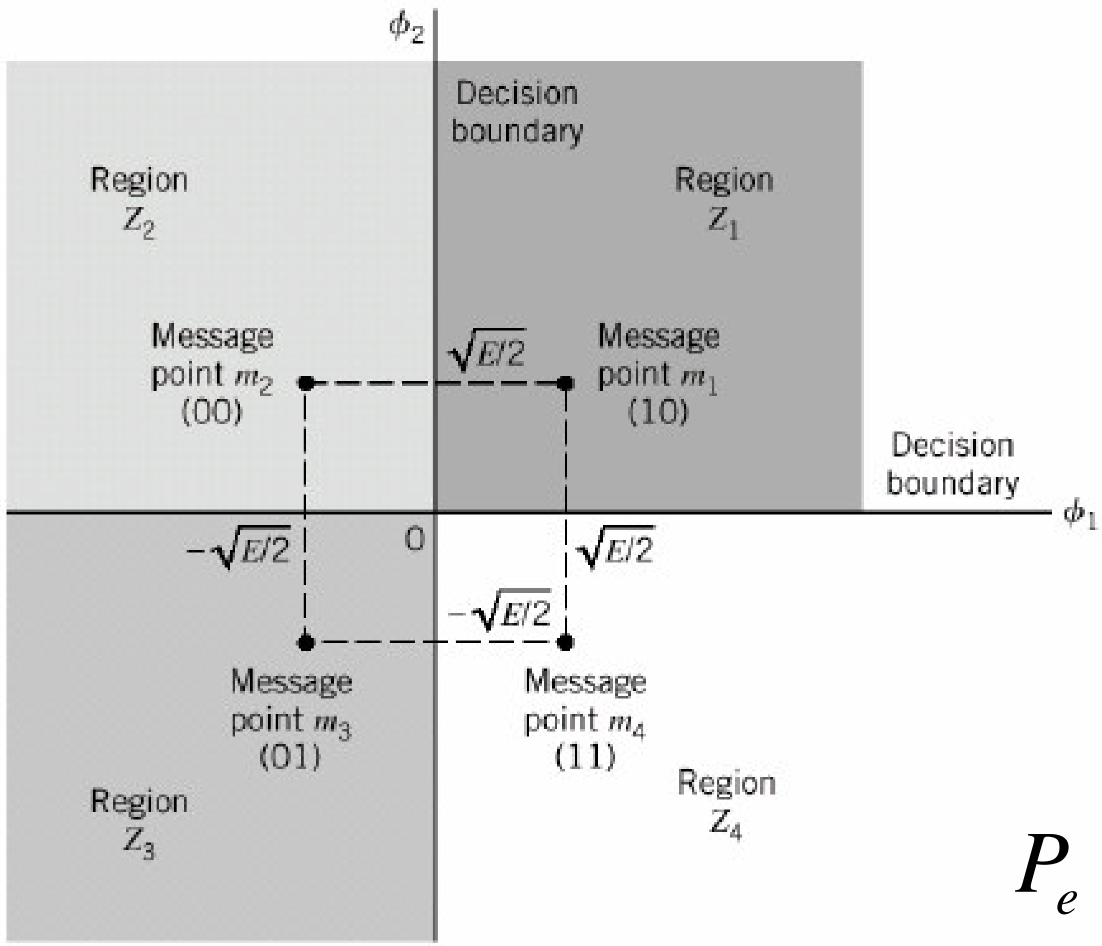


Phase of carrier:
$\pi/4$, $3\pi/4$, $5\pi/4$, $7\pi/4$

Output waveform is sum of modulated ± Cosine and ±Sine wave

2x bandwidth efficiency of BPSK

Quadriphase-shift keying (QPSK)



$$P_e = Q\left(\sqrt{\frac{2E_b}{N_o}}\sin\frac{\pi}{4}\right)$$

Example 6

Find the bandwidth for a signal transmitting at 12 Mbps for QPSK. The value of d = 0.

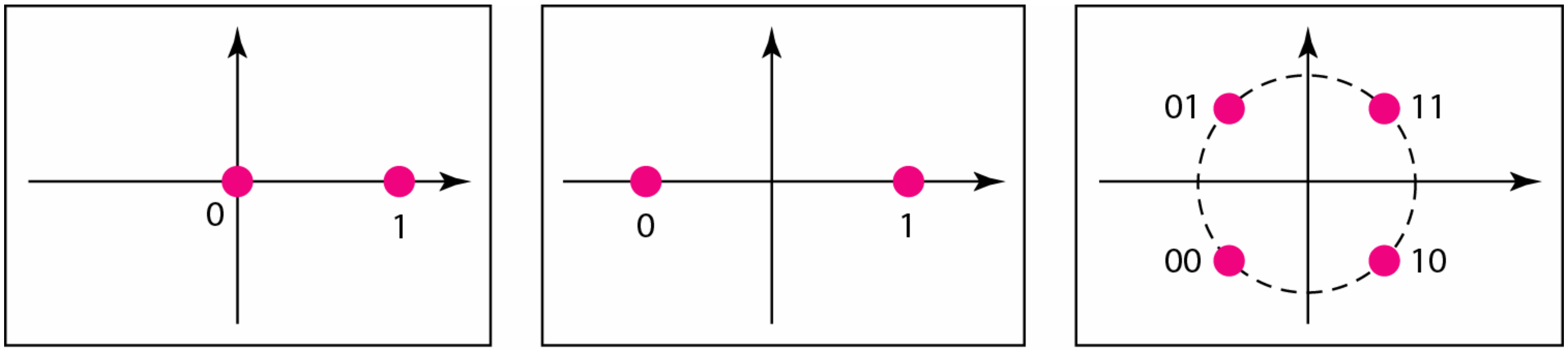
Solution

For QPSK, 2 bits is carried by one signal element. This means that r = 2. So the signal rate (baud rate) is $S = N \times (1/r) = 6$ Mbaud. With a value of d = 0, we have B = S = 6 MHz.

Example 7

Show the constellation diagrams for an ASK (OOK), BPSK, and QPSK signals.

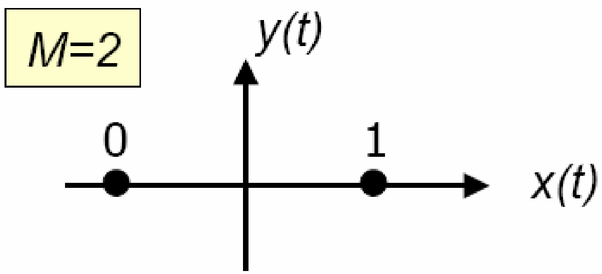
Solution

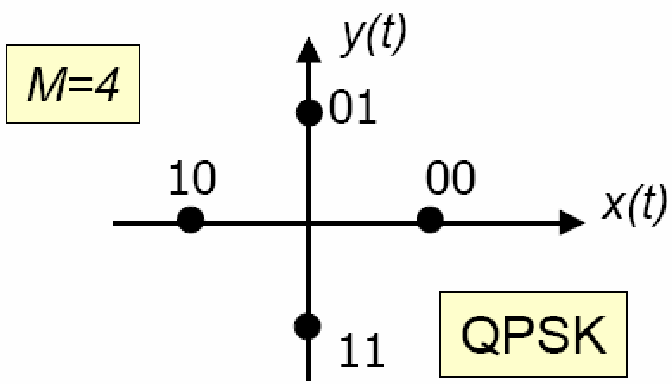


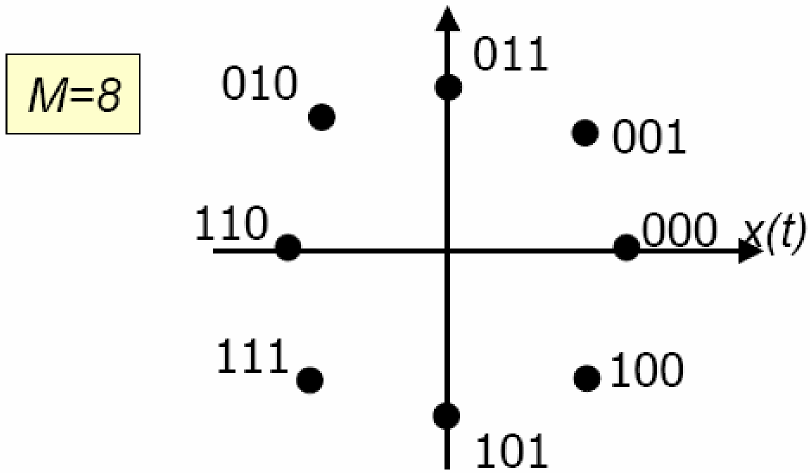
a. ASK (OOK) b. BPSK c. QPSK

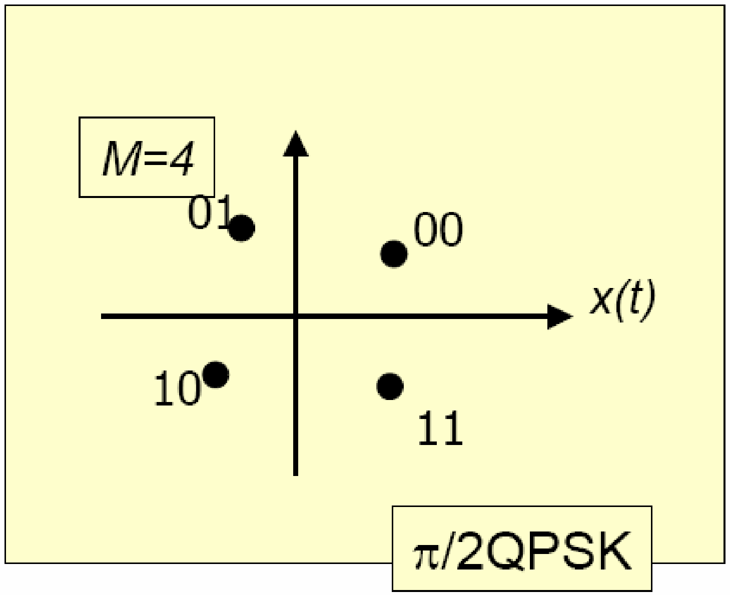
Digital Phase-Modulated signals

Signal Space Diagram of PSK

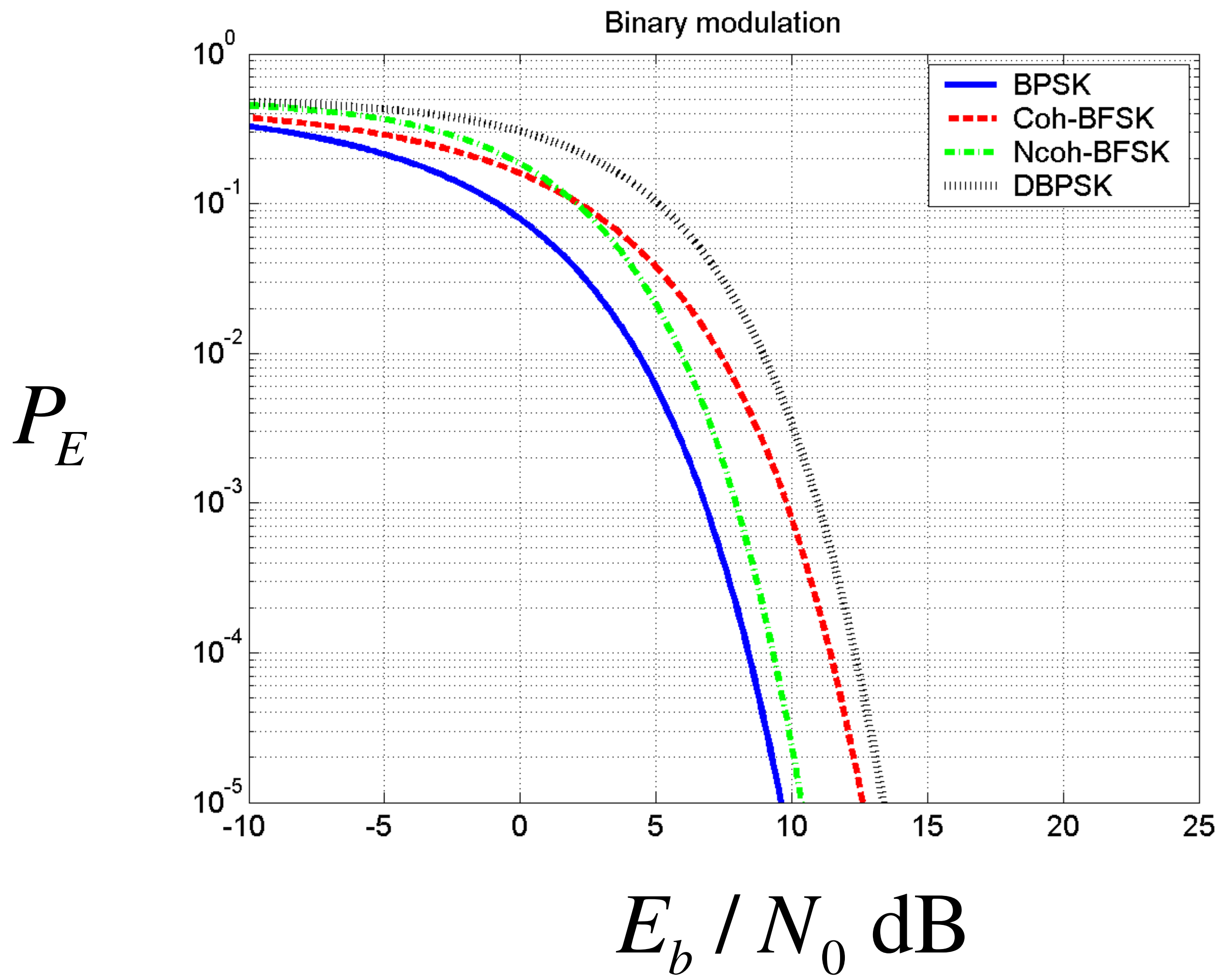








Probability of symbol error for binary modulation

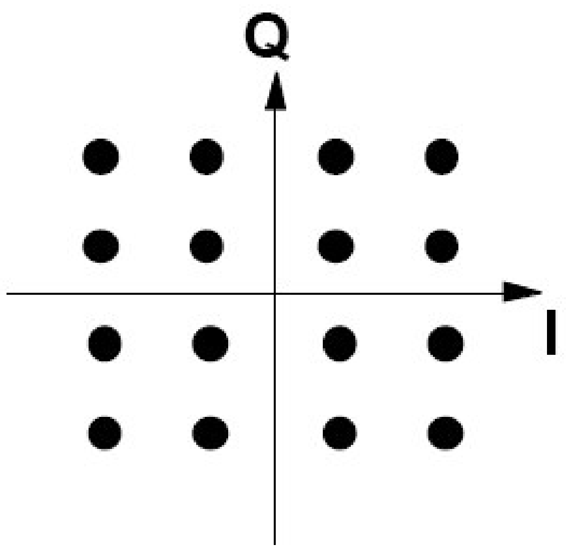


Note

Quadrature amplitude modulation is a combination of ASK and PSK.

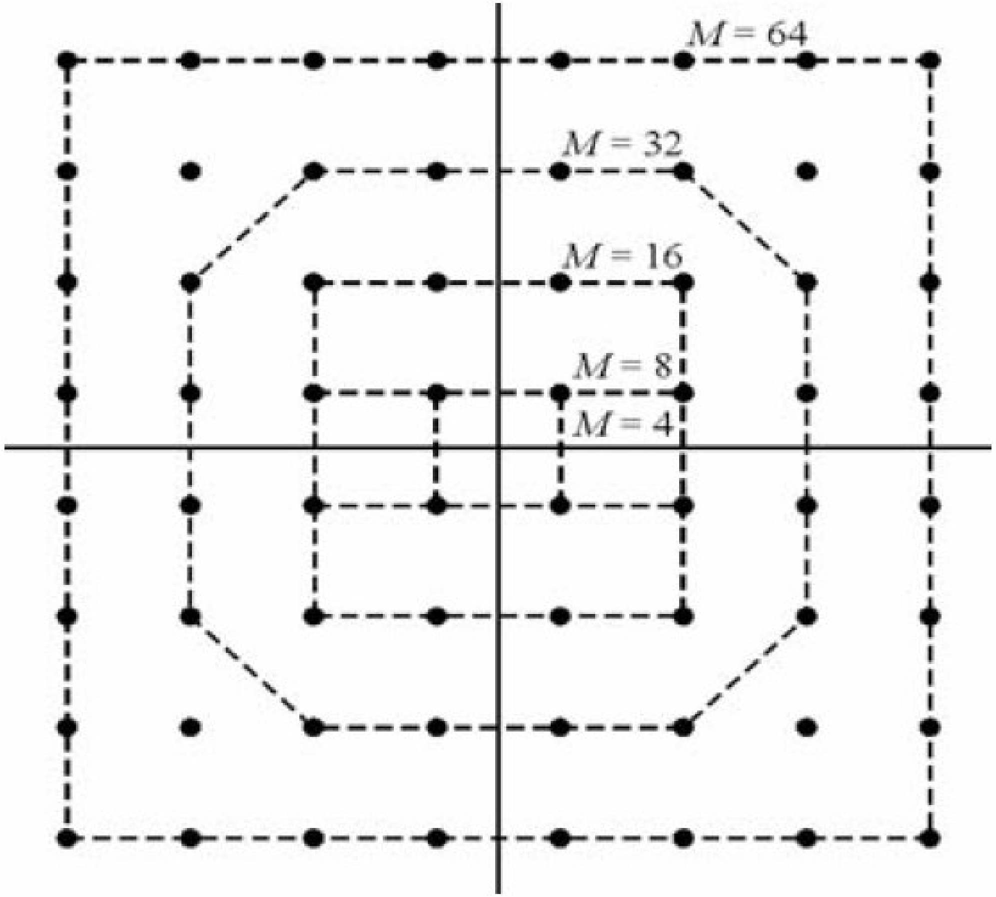
Quadrature Amplitude Modulation

- Quadrature Amplitude Modulation (QAM)
  - Amplitude modulation on both quadrature carriers
  - 2^n discrete levels, n = 2 same as QPSK
- Extensive use in digital microwave radio links

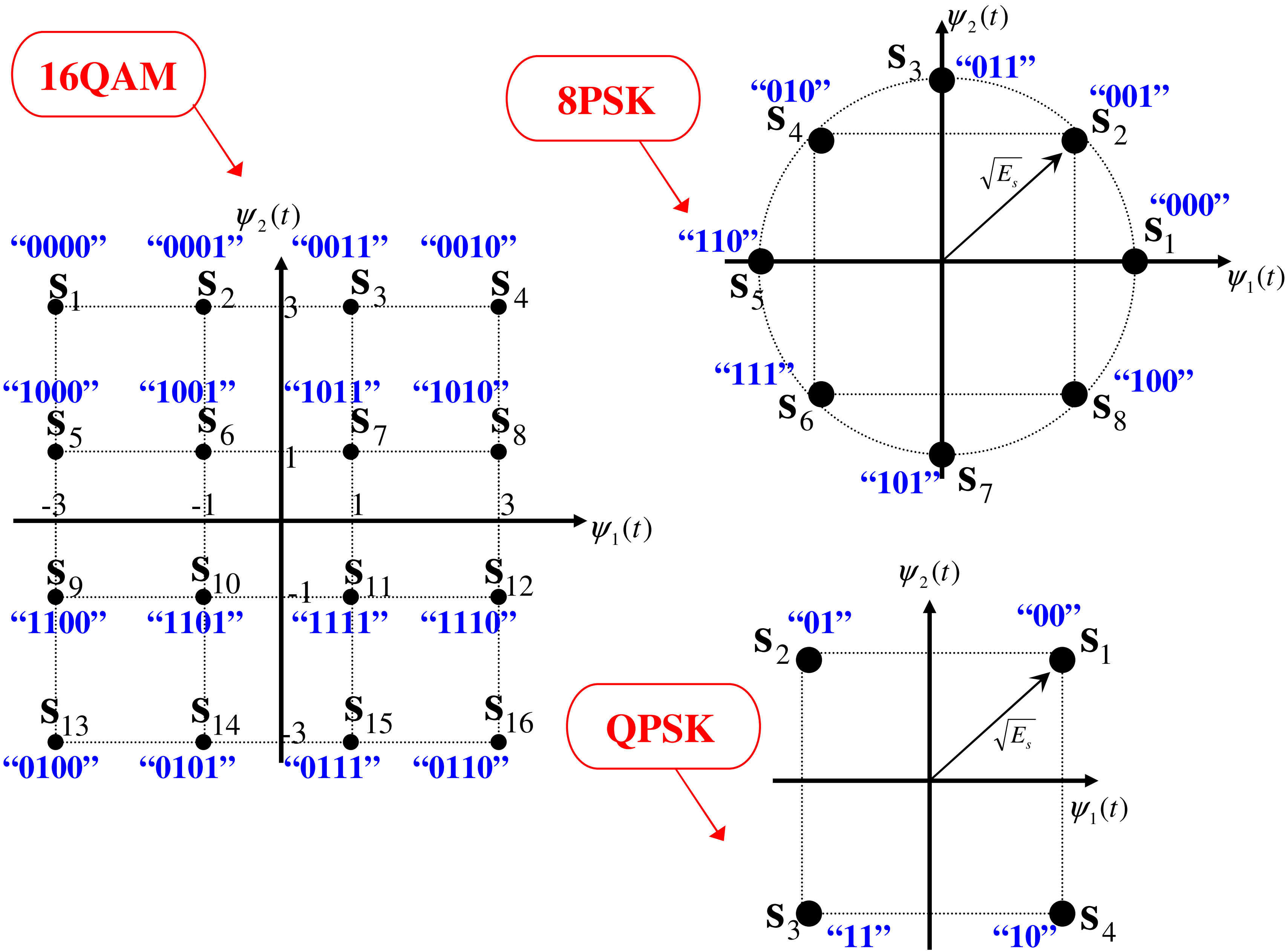


16 Level QAM

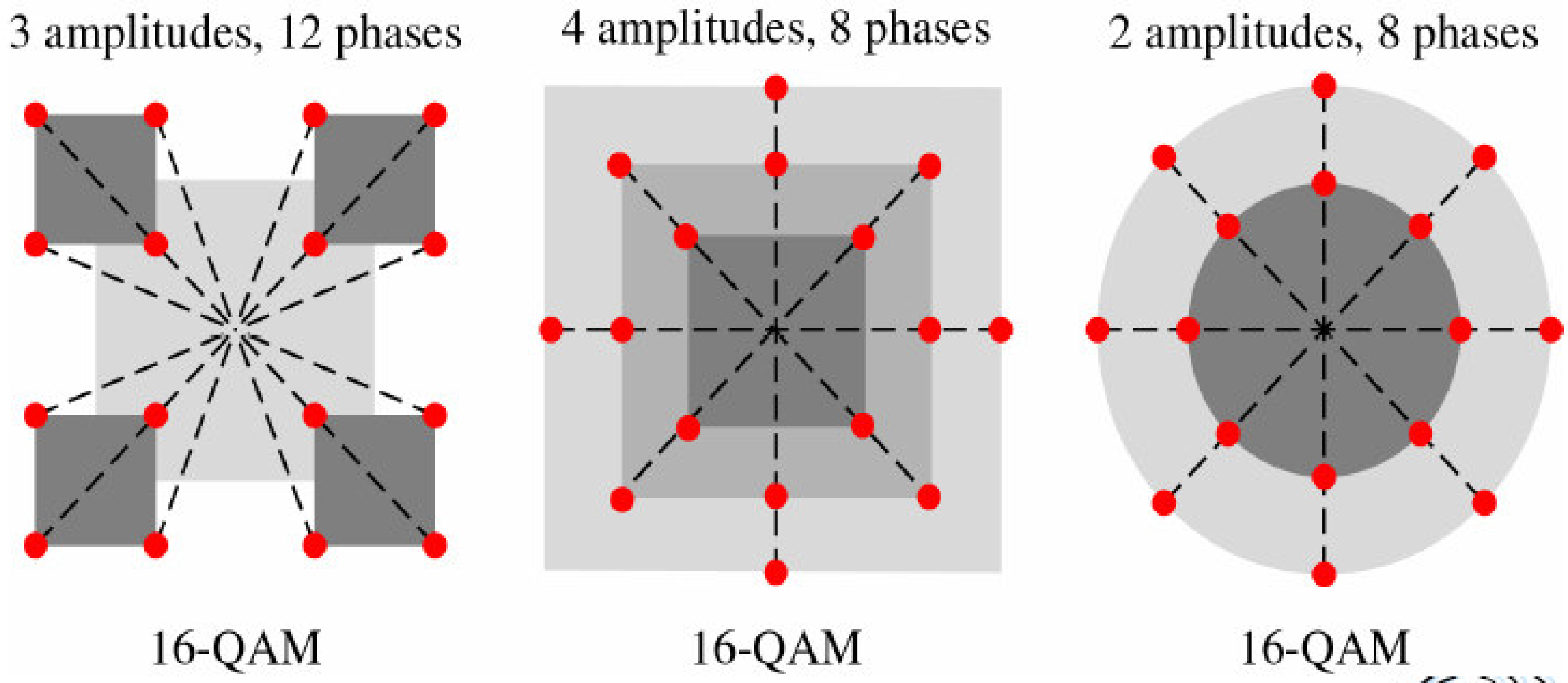
Quadrature Amplitude Modulation



Signal space diagram of QAM


16QAM
ψ2(t)
ψ1(t)
"0000" "0001" "0011" "0010"
S1 S2 S3 S4
"1000" "1001" "1011" "1010"
S5 S6 S7 S8
-3 -1 1 3
3 1 -1 -3
S9 S10 S11 S12
"1100" "1101" "1111" "1110"
S13 S14 S15 S16
"0100" "0101" "0111" "0110"
8PSK
ψ2(t)
ψ1(t)
S1 "000"
S2 "001"
S3 "011"
S4 "010"
S5 "110"
S6 "111"
S7 "101"
S8 "100"
√Es
QPSK
ψ2(t)
ψ1(t)
S1 "00"
S2 "01"
S3 "11"
S4 "10"
√Es

16-QAM


3 amplitudes, 12 phases
16-QAM
4 amplitudes, 8 phases
16-QAM
2 amplitudes, 8 phases
16-QAM

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