EC 551
Telecommunication System Engineering

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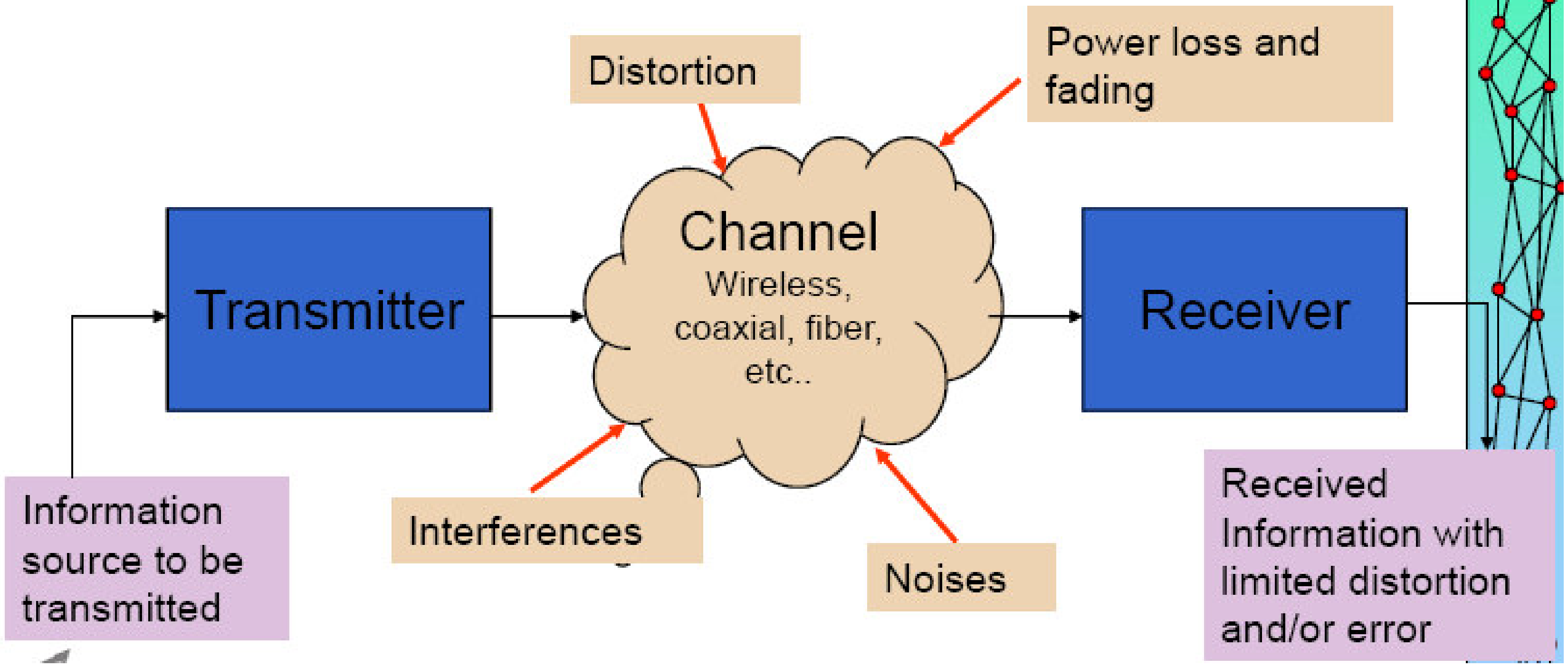
Syllabus

- Tentatively

Week 1	Overview
Week 2	**Wireless Channel characteristics**
Week 3	**Large scale Wireless Channel**
Week 4	**Small scale Wireless Channel**
Week 5	OFDM and modulation techniques
Week 6	Coding techniques in wireless systems
Week 7	WiMax Physical Layer
Week 8	WiMax MAC Layer
Week 9	WLAN Physical/MAC Layer
Week 10	Cellular Communication Concept
Week 11	FDMA, TDMA, CDMA and Duplexing
Week 12	GSM System
Week 13	GPRS System
Week 14	UMTS
Week 15	VOIP

- The following figure shows the basic block diagram of a communication system.



Distortion

Power loss and fading

Transmitter

Channel

Wireless, coaxial, fiber, etc..

Receiver

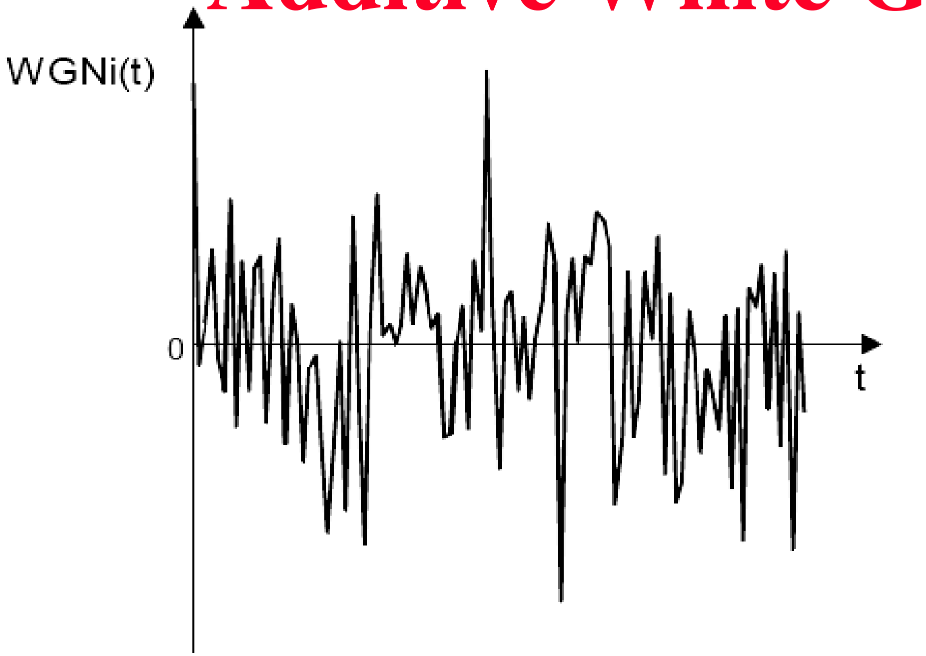
Information source to be transmitted

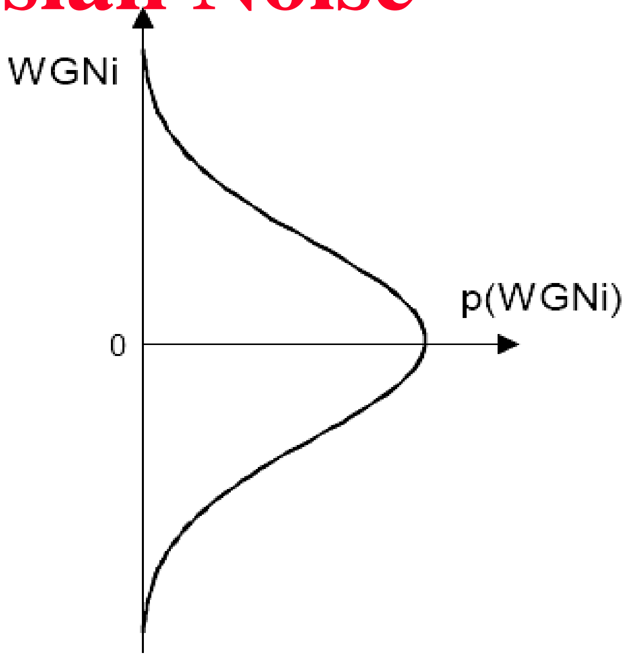
Interferences

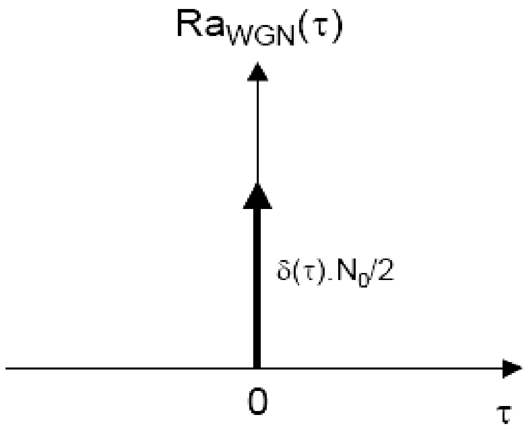
Noises

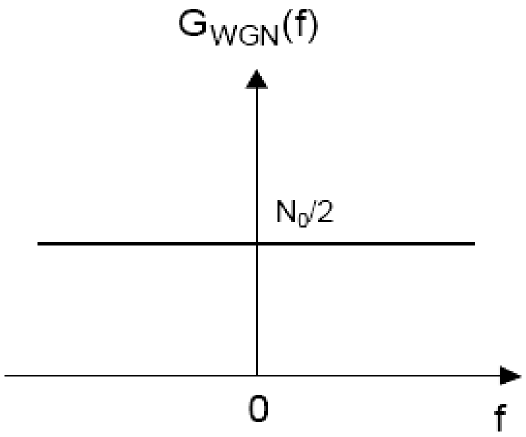
Received Information with limited distortion and/or error

Additive White Gaussian Noise

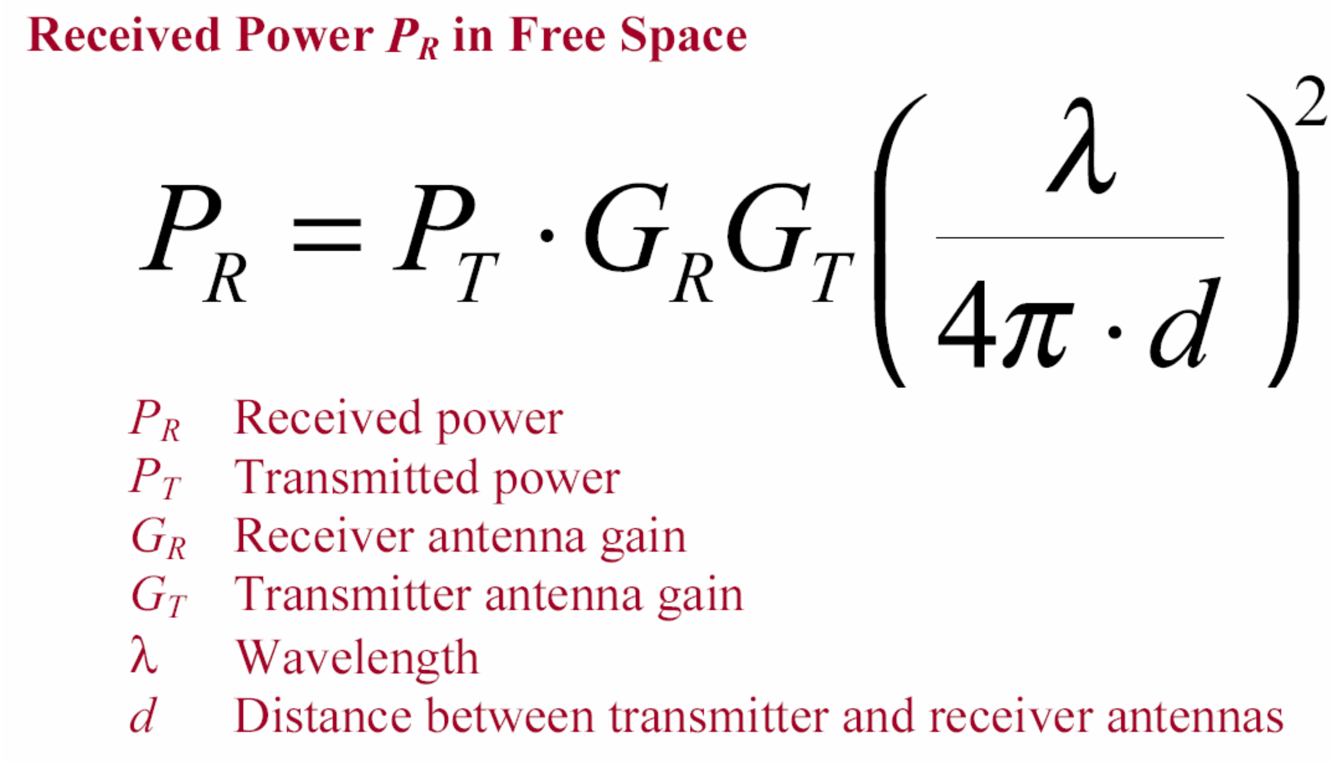








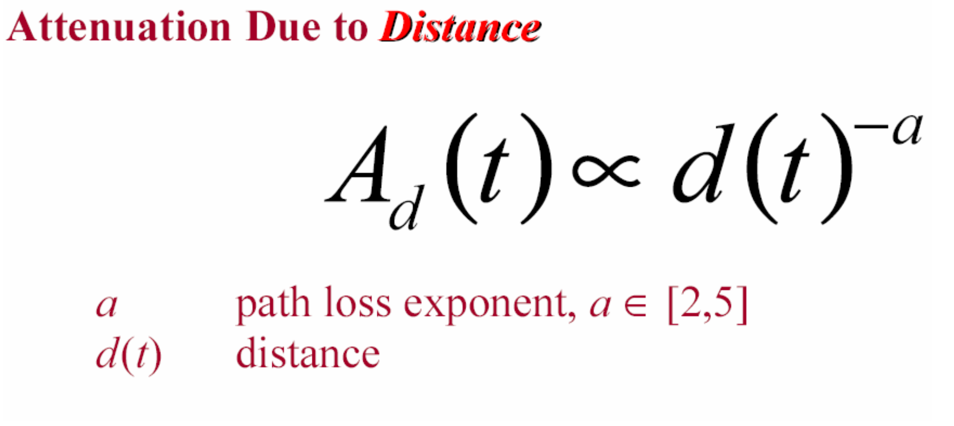
Radio propagation



Received Power P_R in Free Space

$$P_R = P_T \cdot G_R G_T \left(\frac{\lambda}{4\pi \cdot d} \right)^2$$

P_R	Received power
P_T	Transmitted power
G_R	Receiver antenna gain
G_T	Transmitter antenna gain
λ	Wavelength
d	Distance between transmitter and receiver antennas

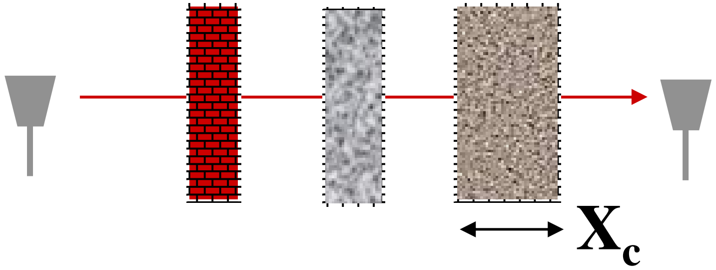


Attenuation Due to *Distance*

$$A_d(t) \propto d(t)^{-a}$$

a	path loss exponent, $a \in [2,5]$
$d(t)$	distance

Shadowing

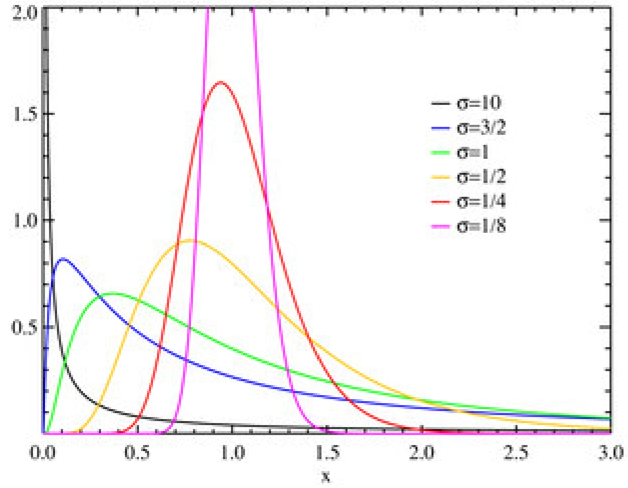


- Models attenuation from obstructions
- Random due to random # and type of obstructions
- Typically follows a log-normal distribution
  - dB value of power is normally distributed
  - $\mu=0$ (mean captured in path loss), $4<\sigma^2<12$ (empirical)

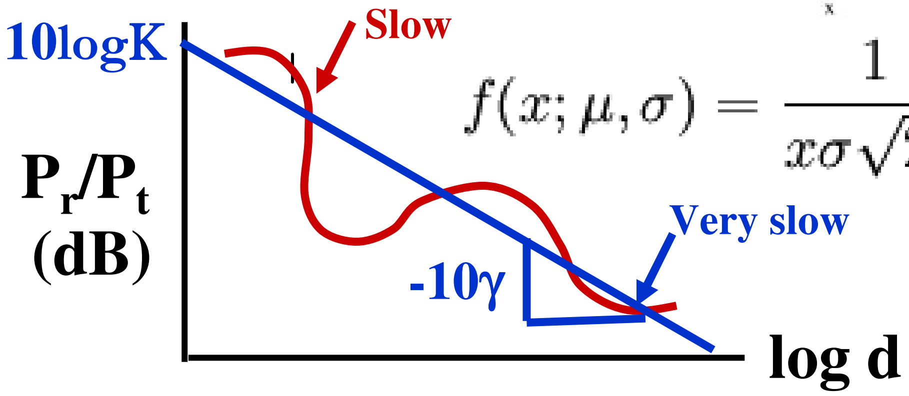
Combined Path Loss and Shadowing

- Linear Model: ψ lognormal

$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma}\psi$$



- dB Model

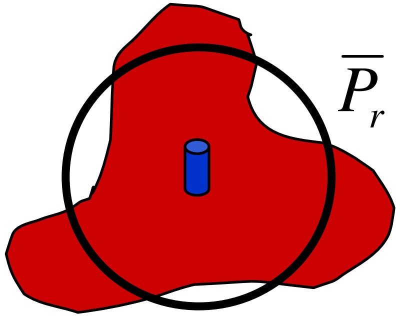


$$f(x;\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}}e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}$$

$$\frac{P_r}{P_t}(dB) = 10\log_{10} K - 10\gamma\log_{10}\left(\frac{d_0}{d}\right) + \psi_{dB},$$

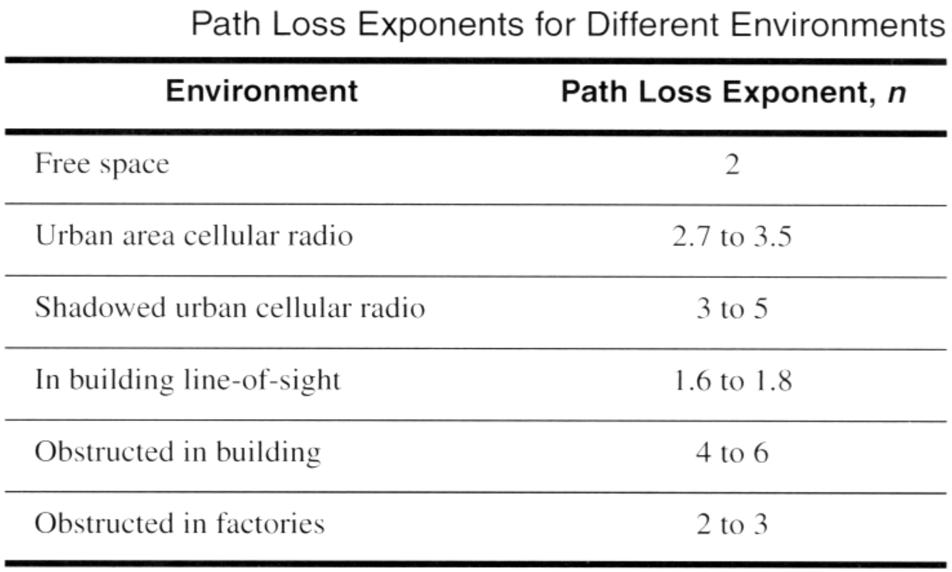
$$\psi_{dB} \sim N(0,\sigma_\psi^2)$$

Outage Probability and Cell Coverage Area



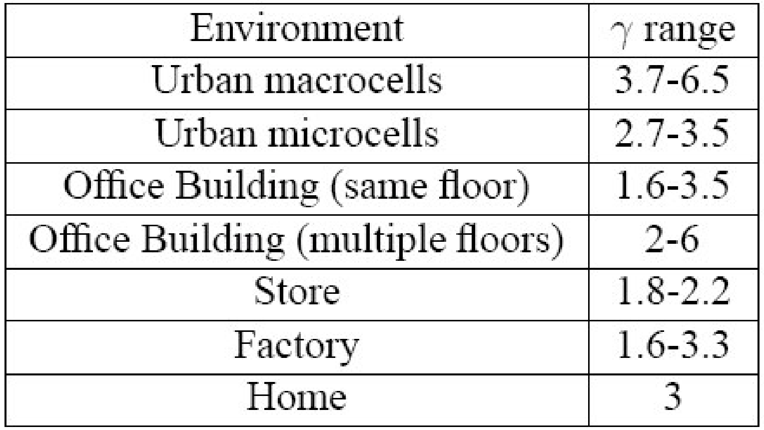
- Path loss: circular cells
- Path loss+shadowing: amoeba cells
  - Tradeoff between coverage and interference
- Outage probability
  - Probability received power below given minimum
- Cell coverage area
  - # of cell locations at desired power
  - Increases as shadowing variance decreases
  - Large # indicates interference to other cells

Typical large-scale path loss



Path Loss Exponents for Different Environments

Environment	Path Loss Exponent, *n*
Free space	2
Urban area cellular radio	2.7 to 3.5
Shadowed urban cellular radio	3 to 5
In building line-of-sight	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factories	2 to 3



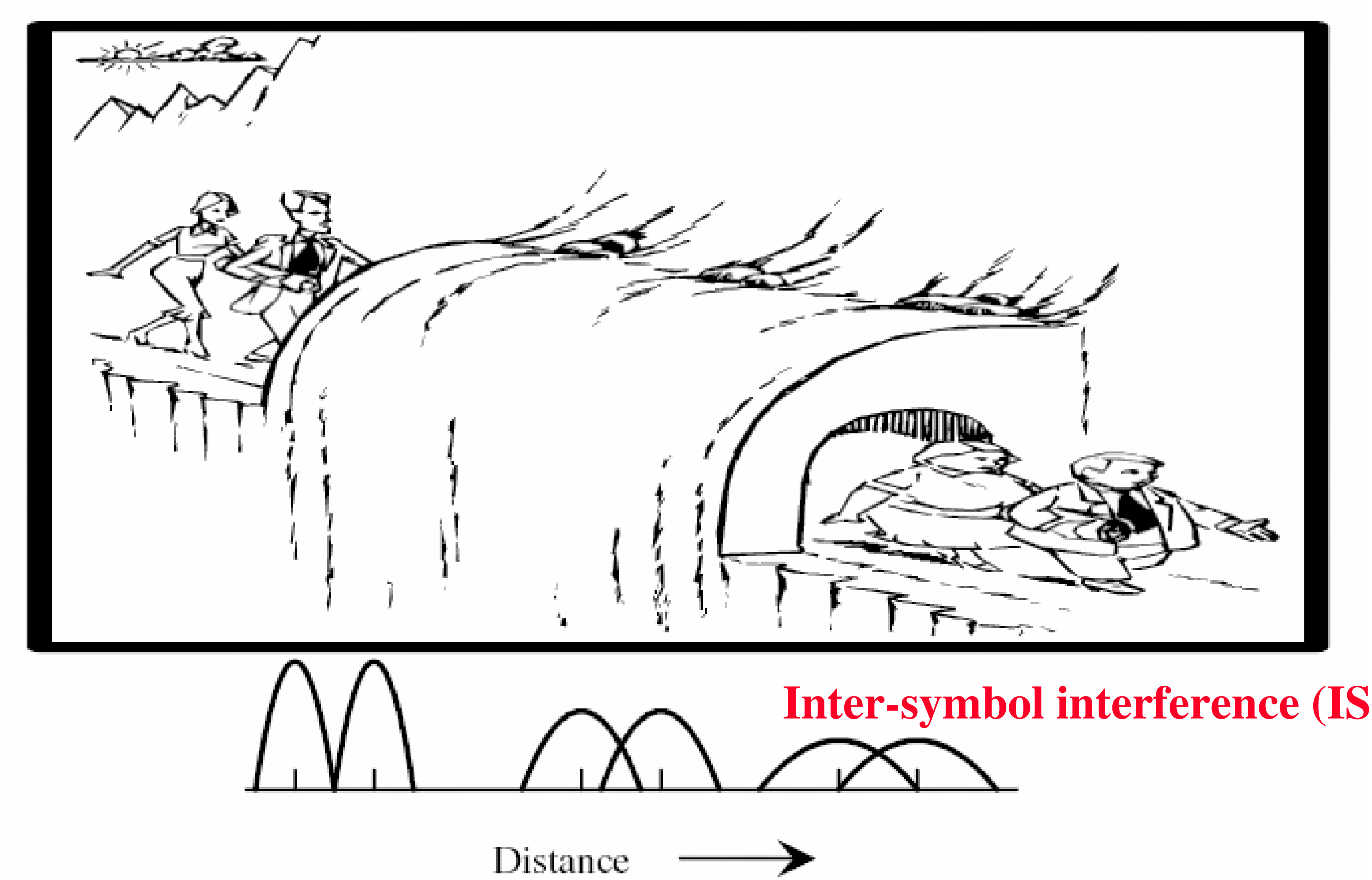
Environment	γ range
Urban macrocells	3.7-6.5
Urban microcells	2.7-3.5
Office Building (same floor)	1.6-3.5
Office Building (multiple floors)	2-6
Store	1.8-2.2
Factory	1.6-3.3
Home	3

<u>Cell design impact:</u> If the radius of a cell is reduced by half when the propagation path loss exponent is 4, the transmit power level of a base station is reduced by 12dB (=10 log 16 dB).

Costs: More base stations, frequent handoffs

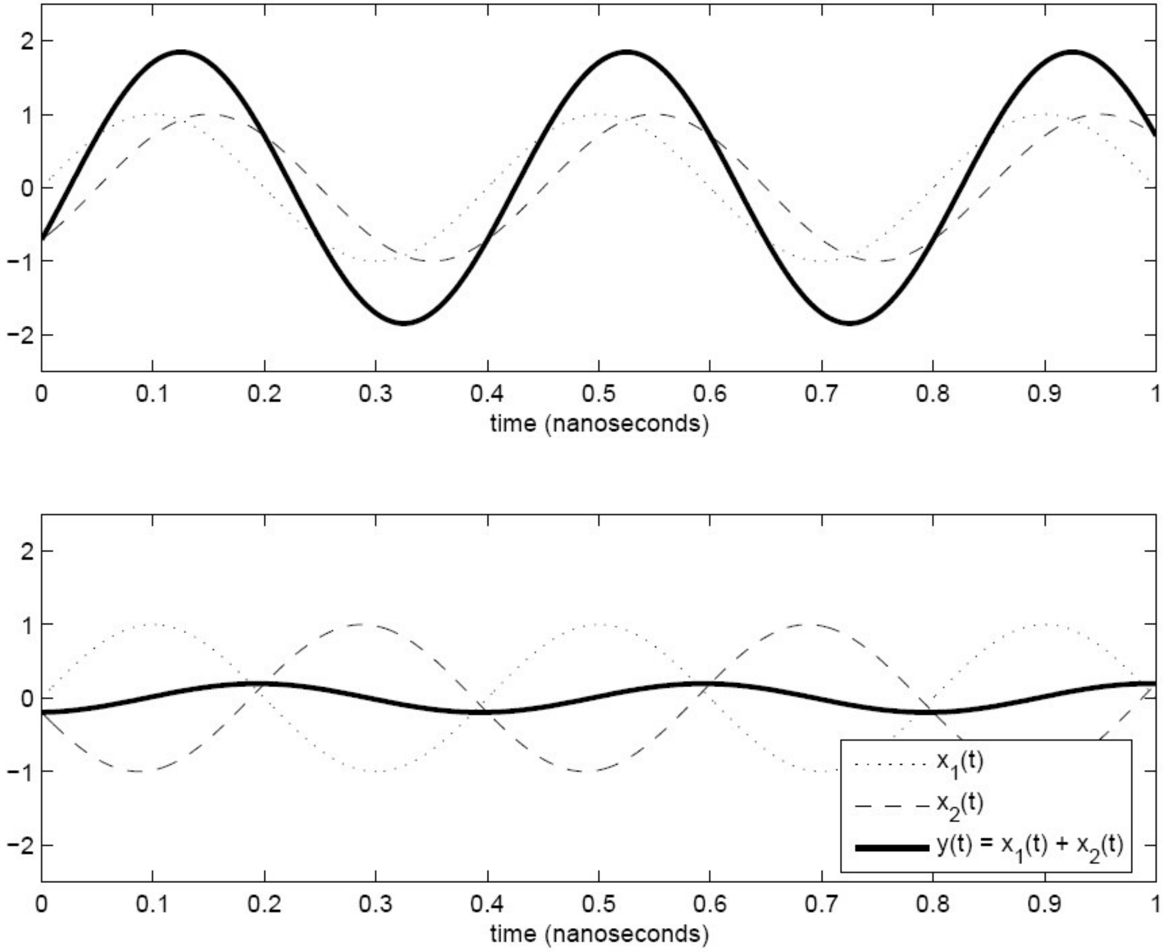
Source: Rappaport and A. Goldsmith books

Attenuation, Dispersion Effects: ISI!



Source: Prof. Raj Jain, WUSTL

MultiPath Interference: Constructive & Destructive



The difference between constructive interference (top) and destructive interference (bottom) at $f_c = 2.5$ GHz is less than 0.1 nanoseconds in phase, which corresponds to about 3 cm.

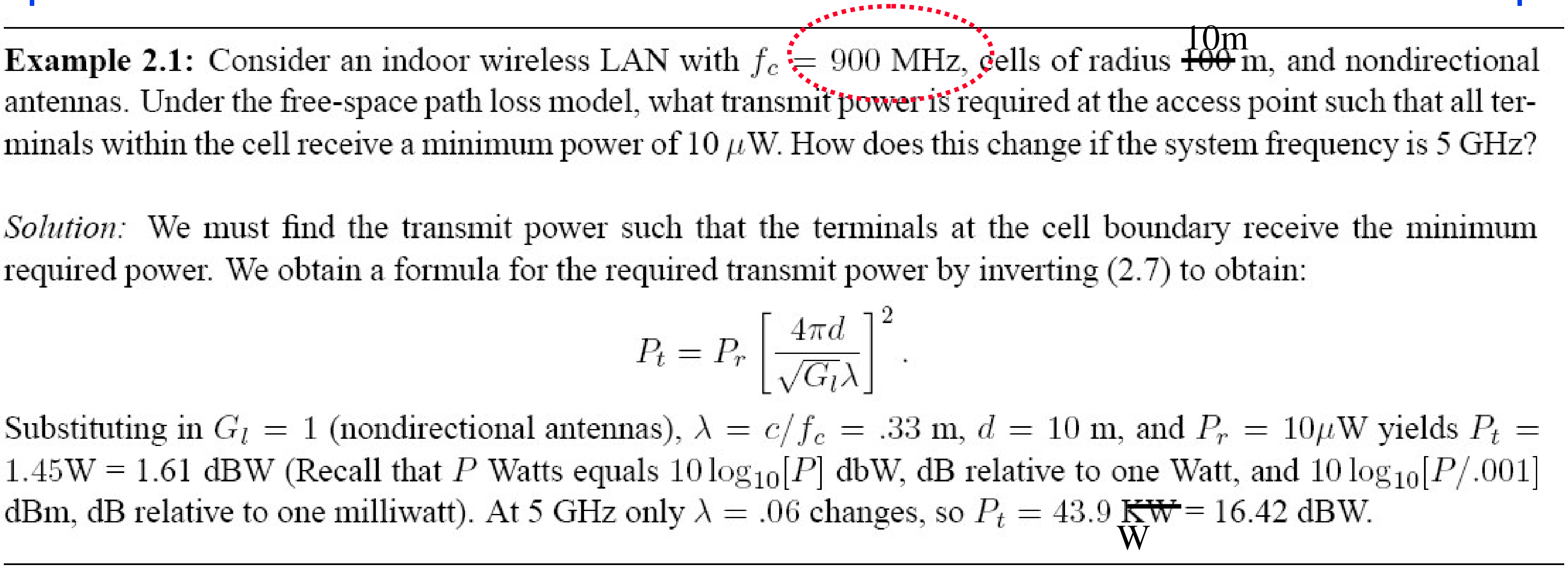
Game plan

- We wish to understand how physical parameters such as
  - carrier frequency
  - mobile speed
  - bandwidth
  - delay spread
  - angular spread

  impact how a wireless channel behaves from the cell planning and communication system point of view.
- We start with deterministic physical model and progress towards statistical models, which are more useful for design and performance evaluation.

Large-scale Fading:
Path Loss, Shadowing

Path Loss (Example 1): Carrier Frequency



Example 2.1: Consider an indoor wireless LAN with $f_c = 900$ MHz, cells of radius ~~100~~ 10m m, and nondirectional antennas. Under the free-space path loss model, what transmit power is required at the access point such that all terminals within the cell receive a minimum power of 10 μW. How does this change if the system frequency is 5 GHz?

Solution: We must find the transmit power such that the terminals at the cell boundary receive the minimum required power. We obtain a formula for the required transmit power by inverting (2.7) to obtain:

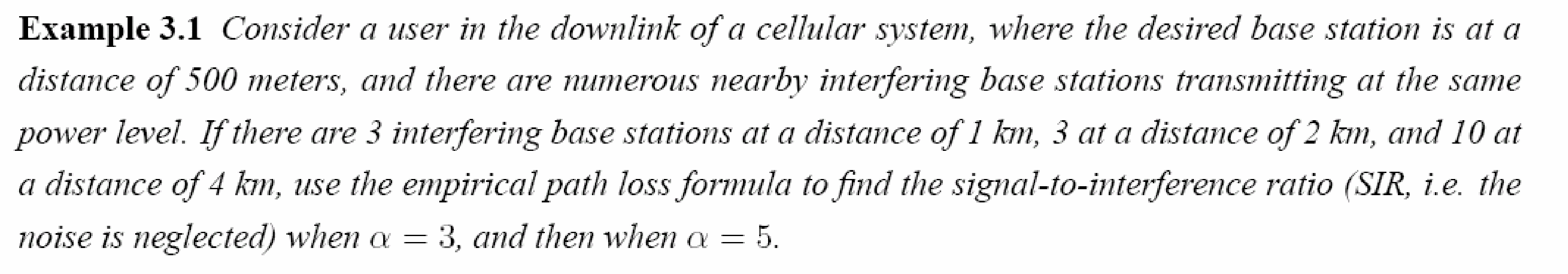
$$P_t = P_r\left[\frac{4\pi d}{\sqrt{G_l}\lambda}\right]^2.$$

Substituting in $G_l = 1$ (nondirectional antennas), $\lambda = c/f_c = .33$ m, $d = 10$ m, and $P_r = 10\mu$W yields $P_t = 1.45$W = 1.61 dBW (Recall that P Watts equals $10\log_{10}[P]$ dbW, dB relative to one Watt, and $10\log_{10}[P/.001]$ dBm, dB relative to one milliwatt). At 5 GHz only $\lambda = .06$ changes, so $P_t = 43.9$ ~~KW~~ W = 16.42 dBW.

- Note: effect of frequency f: 900 Mhz vs 5 Ghz.
  - Either the receiver must have greater sensitivity or the sender must pour 44W of power, even for 10m cell radius!

Source: A. Goldsmith book

Path Loss (Example 2), Interference & Cell Sizing



Example 3.1 *Consider a user in the downlink of a cellular system, where the desired base station is at a distance of 500 meters, and there are numerous nearby interfering base stations transmitting at the same power level. If there are 3 interfering base stations at a distance of 1 km, 3 at a distance of 2 km, and 10 at a distance of 4 km, use the empirical path loss formula to find the signal-to-interference ratio (SIR, i.e. the noise is neglected) when* $\alpha = 3$*, and then when* $\alpha = 5$.

- Desired signal power: $$P_{r,d} = P_t P_o d_o^3 (0.5)^{-3},$$
- Interference power: $$P_{r,I} = P_t P_o d_o^3 \left[3(1)^{-3} + 3(2)^{-3} + 10(4)^{-3}\right].$$
- SIR: $$SIR(\alpha = 3) = \frac{P_{r,d}}{P_{r,I}} = 28.25 = 14.5dB,$$ $$SIR(\alpha = 5) = 99.3 = 20dB,$$
- SIR is much better with higher path loss exponent ($\alpha = 5$)!
- Higher path loss, smaller cells => lower interference, higher SIR

Source: J. Andrews et al book

Path Loss: ***Range*** vs ***Bandwidth*** Tradeoff

- **1.** High frequency RF electronics have traditionally been harder to design and manufacture, and hence more expensive. [less so nowadays]
- **2.** Pathloss increases ~ $O(f_c^2)$
  - A signal at 3.5 GHz (one of WiMAX's candidate frequencies) will be received with about 20 times less power than at 800 MHz (a popular cellular frequency).
  - Effective path loss exponent also increases at higher frequencies, due to increased absorption and attenuation of high frequency signals

- Tradeoff:
  - Bandwidth at higher carrier frequencies is more plentiful and less expensive.
  - Does *not* support large transmission ranges.
  - (also increases problems for mobility/Doppler effects etc)

- WIMAX Choice:
  - Pick any two out of three: *high data rate, high range, low cost.*

Empirical Models

- Okumura model
  - Empirically based (site/freq specific)
  - Awkward (uses graphs)
- Hata model
  - Analytical approximation to Okumura model
- Cost 136 Model:
  - Extends Hata model to higher frequency (2 GHz)
- Walfish/Bertoni:
  - Cost 136 extension to include diffraction from rooftops

Commonly used in cellular system simulations

Empirical Path Loss: Okamura, Hata, COST231

- Empirical models include effects of path loss, shadowing and multipath.
  - Multipath effects are averaged over several wavelengths: local mean attenuation (LMA)
  - Empirical path loss for a given environment is the average of LMA at a distance d over all measurements
- **Okamura**: based upon Tokyo measurements. 1-100 km, 150-1500MHz, base station heights (30-100m), median attenuation over free-space-loss, 10-14dB standard deviation.

$$P_L(d) \text{ dB} = L(f_c, d) + A_{mu}(f_c, d) - G(h_t) - G(h_r) - G_{AREA}$$

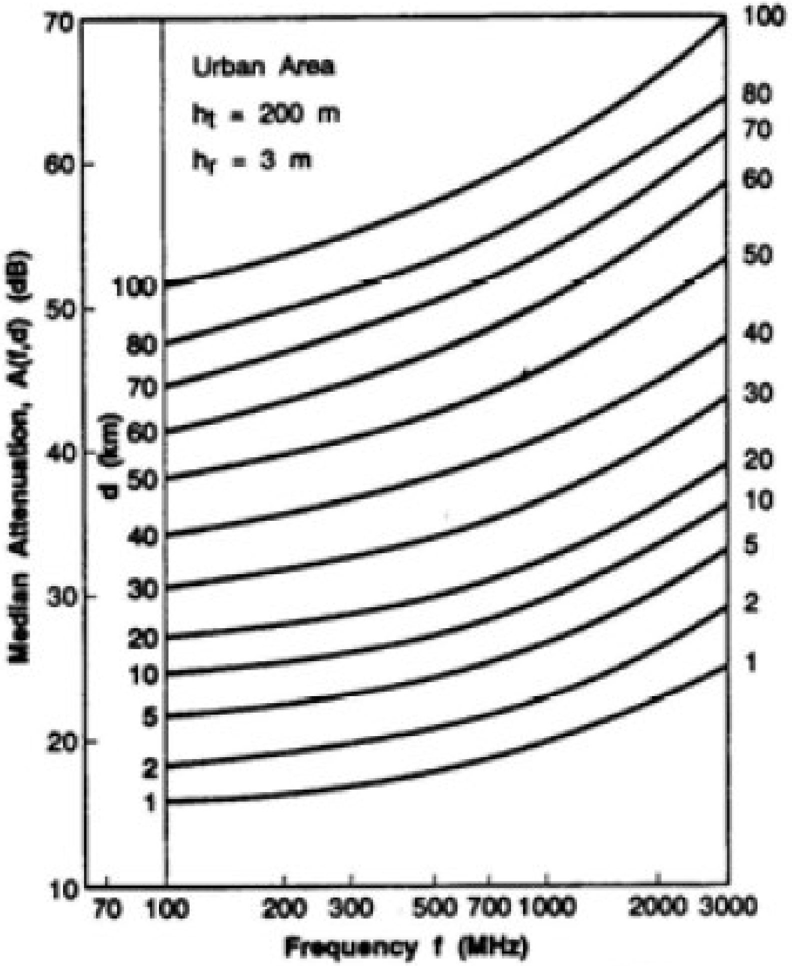
- **Hata**: closed form version of Okamura

$$P_{L,urban}(d) \text{ dB} = 69.55 + 26.16\log_{10}(f_c) - 13.82\log_{10}(h_t) - a(h_r) + (44.9 - 6.55\log_{10}(h_t))\log_{10}(d). \quad (2.31)$$

- **COST 231:** Extensions to 2 GHz

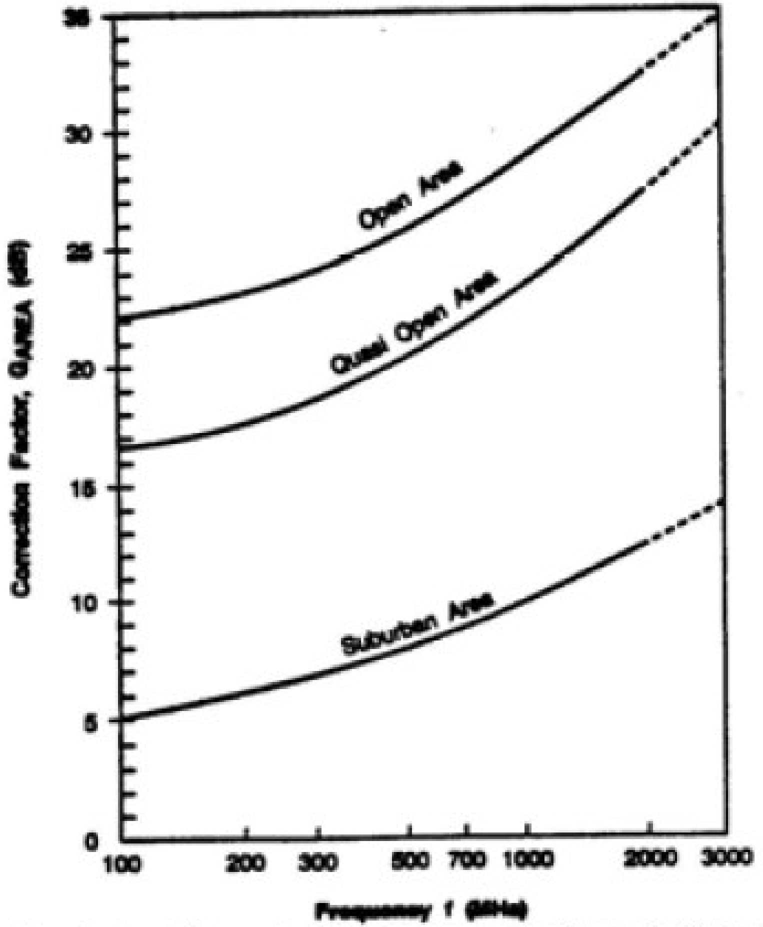
$$P_{L,urban}(d) \text{dB} = 46.3 + 33.9\log_{10}(f_c) - 13.82\log_{10}(h_t) - a(h_r) + (44.9 - 6.55\log_{10}(h_t))\log_{10}(d) + C_M, \quad (2.34)$$

Source: A. Goldsmith book



Median attenuation relative to free space $(A_{mu}(f,d))$, over a

] © IEEE].



Correction factor, G_{AREA}, for different types of terrain [from [

Antenna height correction factors:

$$G(h_{te}) = 20\log_{10}(h_{te}/200), \qquad 30\text{ m} < h_{te} < 1000\text{ m}$$

$$G(h_{re}) = 10\log_{10}(h_{re}/3), \qquad h_{re} < 3\text{m}$$

$$G(h_{re}) = 20\log_{10}(h_{re}/3), \qquad 3\text{ m} < h_{re} < 10\text{ m}$$

Hata Model 2/3

The path loss (in dB) for urban areas is given in the Hata model as

$$L_{50}(urban)=69.55+26.16\log_{10} f_c-13.82\log_{10} h_{te}-a(h_{re})$$
$$+(44.9-6.55\log_{10} h_{te})\log_{10} d$$

For various environments we apply a correction factor for the mobile antenna height. For a small to medium size city

$$a(h_{re})=(1.1\log_{10} f_c-0,7)h_{re}-(1.56\log_{10} f_c-0.8)$$

For a large city the correction factors take the form

$$a(h_{re})=8.29(\log_{10} 1.54h_{re})^2-1.1, \qquad f_c<300\text{ MHz}$$
$$a(h_{re})=3.2(\log_{10} 11.75h_{re})^2-4.97, \qquad f_c>300\text{ MHz}$$

Hata Model 3/3

For a suburban area the original expression is modified as

$$L_{50}(suburban) = L_{50}(urban) - 2\left[\log(f_c/28)\right]^2 - 5.4$$

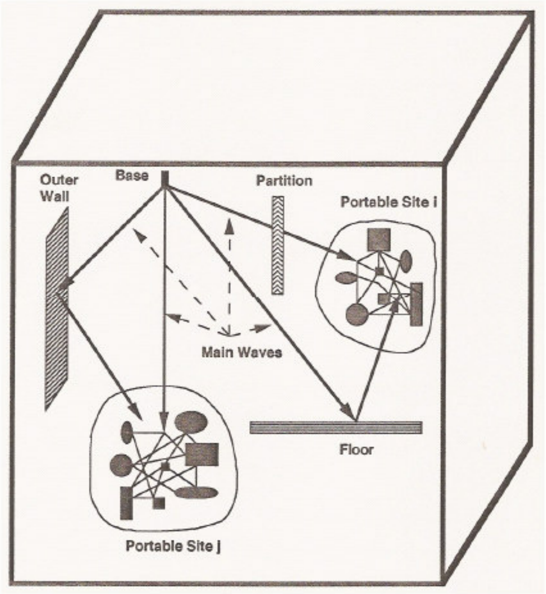
Finally for open rural areas we have

$$L_{50}(suburban) = L_{50}(urban) - 4.78\left[\log(f_c)\right]^2 + 18.33\log_{10}(f_c) - 40.94$$

Note that the Hata model is a formula and does not have the path specific graphical corrections available in the Okumura model.

Indoor Models

- 900 MHz: 10-20dB attenuation for 1-floor, 6-10dB/floor for next few floors (and frequency dependent)
- Partition loss each time depending upton material (see table)
- Outdoor-to-indoor: building penetration loss (8-20 dB), decreases by 1.4dB/floor for higher floors. (reduced clutter)
- Windows: 6dB less loss than walls (if not lead lined)



Partition Type	Partition Loss in dB
Cloth Partition	1.4
Double Plasterboard Wall	3.4
Foil Insulation	3.9
Concrete wall	13
Aluminum Siding	20.4
All Metal	26

Shadowing: Measured large-scale path loss

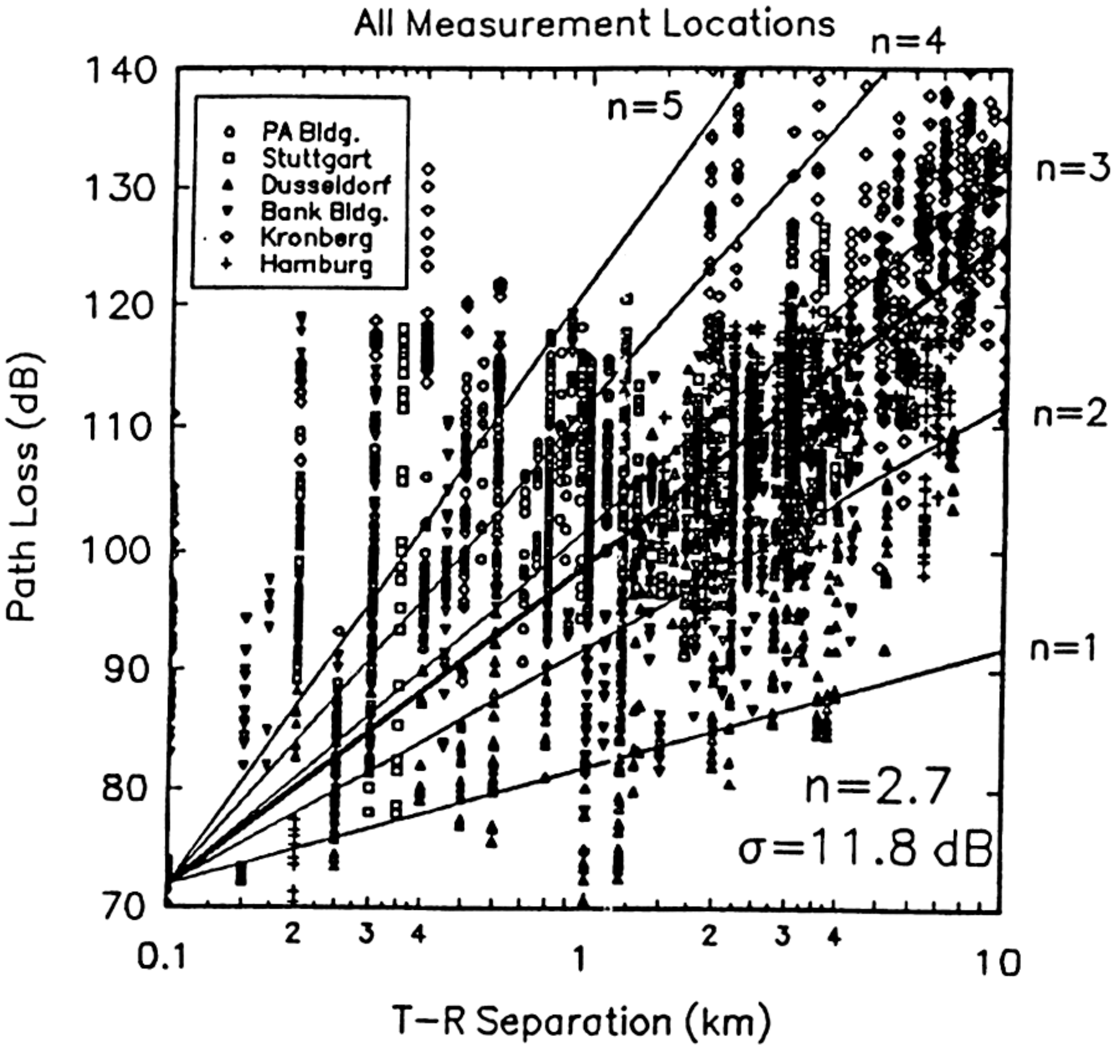
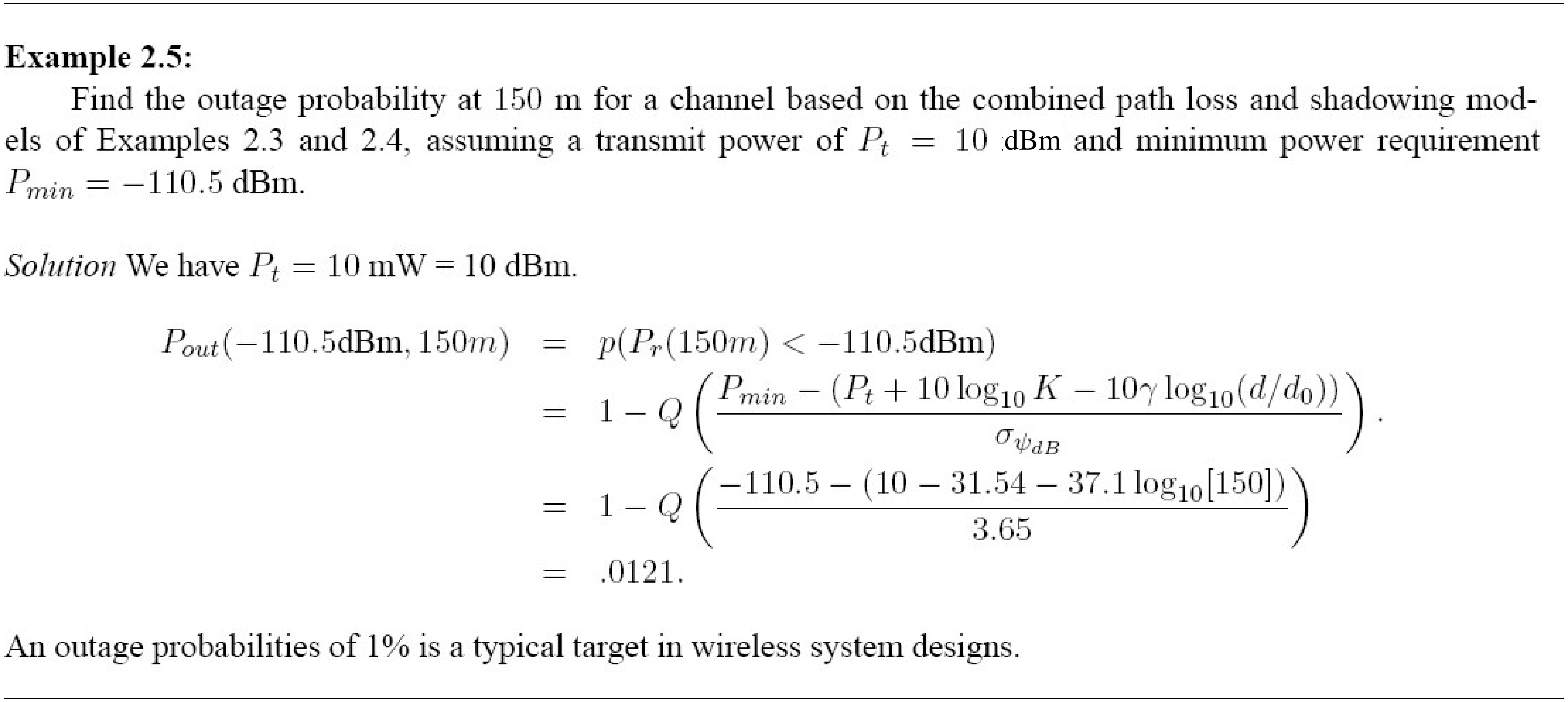


Figure 4.17 Scatter plot of measured data and corresponding MMSE path loss model for many cities in Germany. For this data, $n = 2.7$ and $\sigma = 11.8$ dB [from [Sei91] © IEEE].

Outage Probability w/ Shadowing

$$p(P_r(d) \leq P_{min}) = 1 - Q\left(\frac{P_{min} - (P_t + 10\log_{10} K - 10\gamma\log_{10}(d/d_0))}{\sigma_{\psi_{dB}}}\right),$$



Example 2.5:

Find the outage probability at 150 m for a channel based on the combined path loss and shadowing models of Examples 2.3 and 2.4, assuming a transmit power of $P_t = 10$ dBm and minimum power requirement $P_{min} = -110.5$ dBm.

Solution We have $P_t = 10$ mW = 10 dBm.

$$\begin{aligned} P_{out}(-110.5\text{dBm}, 150m) &= p(P_r(150m) < -110.5\text{dBm}) \\ &= 1 - Q\left(\frac{P_{min} - (P_t + 10\log_{10} K - 10\gamma\log_{10}(d/d_0))}{\sigma_{\psi_{dB}}}\right). \\ &= 1 - Q\left(\frac{-110.5 - (10 - 31.54 - 37.1\log_{10}[150])}{3.65}\right) \\ &= .0121. \end{aligned}$$

An outage probabilities of 1% is a typical target in wireless system designs.

- Need to improve receiver sensitivity (i.e. reduce Pmin) for better coverage.

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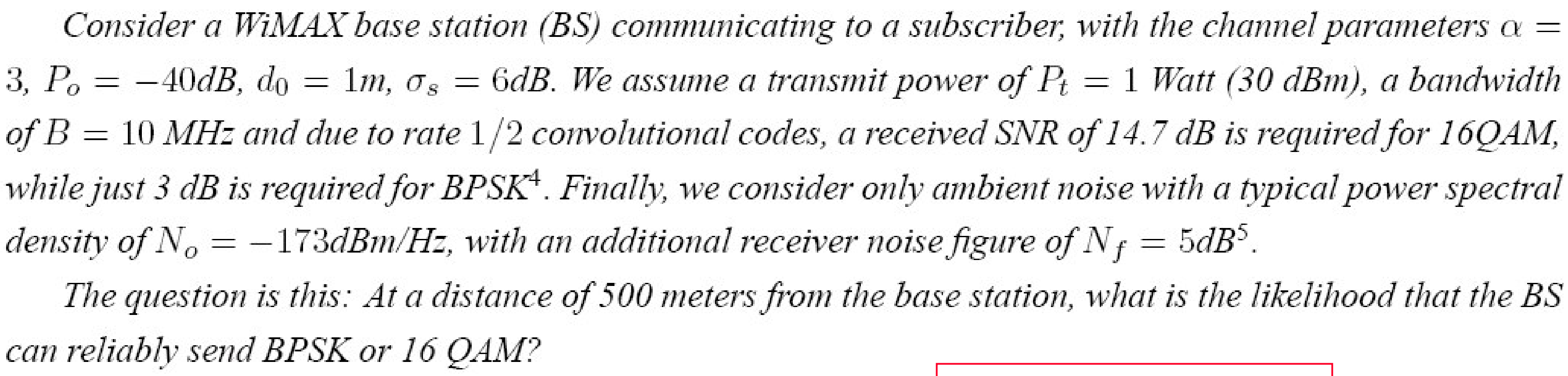
Shadowing: Modulation Design



Consider a WiMAX base station (BS) communicating to a subscriber, with the channel parameters $\alpha = 3$, $P_o = -40dB$, $d_0 = 1m$, $\sigma_s = 6dB$. *We assume a transmit power of* $P_t = 1$ *Watt (30 dBm), a bandwidth of* $B = 10$ *MHz and due to rate* $1/2$ *convolutional codes, a received SNR of 14.7 dB is required for 16QAM, while just 3 dB is required for BPSK*[4]. *Finally, we consider only ambient noise with a typical power spectral density of* $N_o = -173dBm/Hz$*, with an additional receiver noise figure of* $N_f = 5dB$[5].

The question is this: At a distance of 500 meters from the base station, what is the likelihood that the BS can reliably send BPSK or 16 QAM?

- Simple path loss/shadowing model: $$P_r = P_t P_o \chi \left(\frac{d_o}{d}\right)^{\alpha}$$
- Find Pr: $$\begin{aligned} P_r(dB) &= 10\log_{10} P_t + 10\log_{10} P_o - 10\log_{10} d^{\alpha} + 10\log_{10}\chi \\ &= 30dBm - 40dB - 81dB + \chi(dB) = -91dBm + \chi(dB) \end{aligned}$$
- Find Noise power:

$$\begin{aligned} I_{tot}(dB) &= N_o + N_f + 10\log_{10} B \\ &= -173 + 5dB + 70 = -98dBm \end{aligned}$$

Shadowing: Modulation Design (Contd)

- SINR: $\gamma = -91dBm + \chi(dB) + 98dBm = 7dB + \chi(dB).$
- *Without* shadowing ($\chi = 0$), BPSK works 100%, 16QAM fails all the time.
- *With* shadowing ($\sigma_s = 6$dB):

BPSK:

$$\begin{aligned} P[\gamma \geq 3dB] &= P[\frac{\chi+7}{\sigma} \geq \frac{3}{\sigma}] \\ &= P[\frac{\chi}{6} \geq -\frac{4}{6}] \\ &= Q(-\frac{4}{6}) = 0.75 \end{aligned}$$

16 QAM

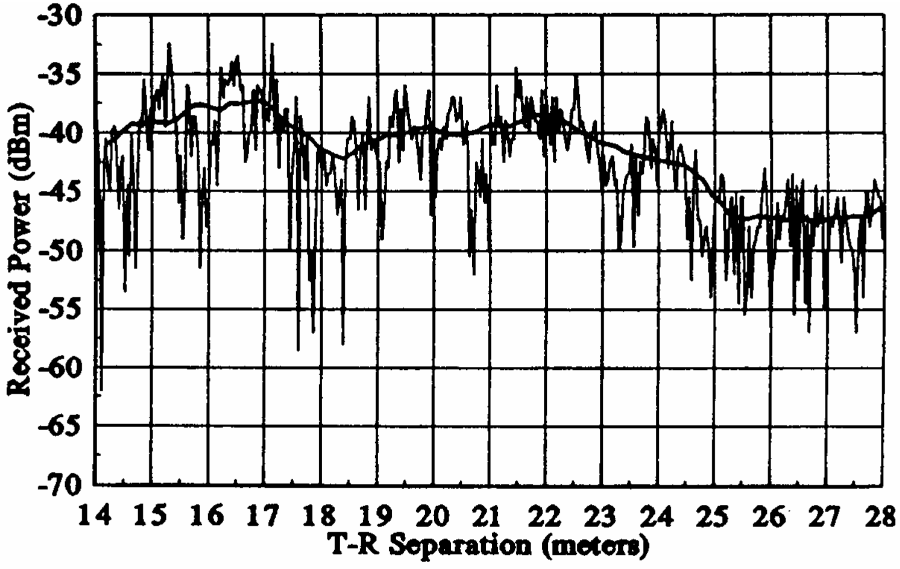
$$\begin{aligned} P[\gamma \geq 14.7dB] &= P[\frac{\chi+7}{\sigma} \geq \frac{14.7}{\sigma}] \\ &= Q(\frac{7.7}{6}) = .007 \end{aligned}$$

- 75% of users can use BPSK modulation and hence get a PHY data rate of 10 MHz · 1 bit/symbol ·1/2 = 5 Mbps
- Less than 1% of users can reliably use 16QAM (4 bits/symbol) for a more desirable data rate of 20 Mbps.
- Interestingly for BPSK, w/o shadowing, we had 100%; and 16QAM: 0%!

Small-Scale Fading: Rayleigh/Ricean Models, Multipath & Doppler

Small-scale Multipath fading: System Design

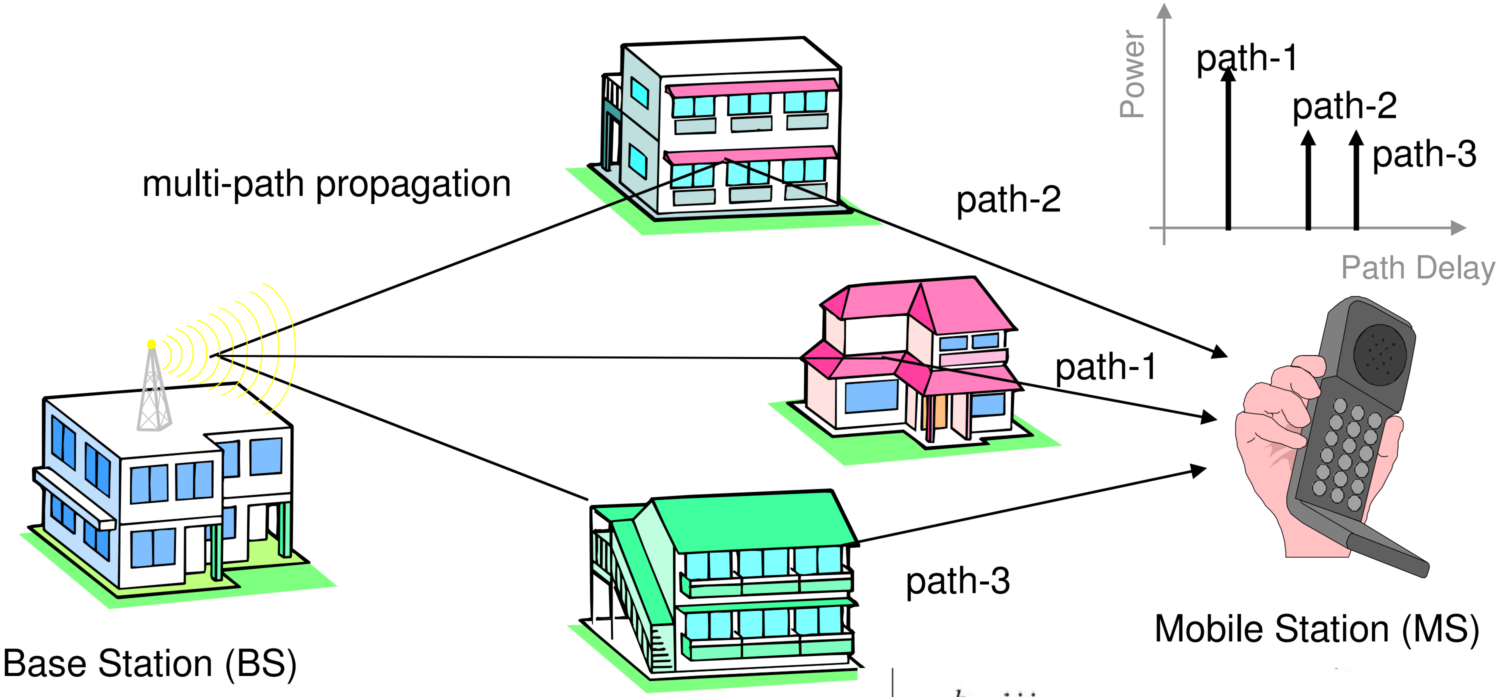
- Wireless communication typically happens at very high carrier frequency. (eg. f_c = 900 MHz or 1.9 GHz for cellular)
- Multipath fading due to constructive and destructive interference of the transmitted waves.
- Channel varies when mobile moves a distance of the order of the carrier wavelength. This is about 0.3 m for 900 Mhz cellular.
- For vehicular speeds, this translates to channel variation of the order of 100 Hz.
- *Primary driver* behind wireless communication system design.



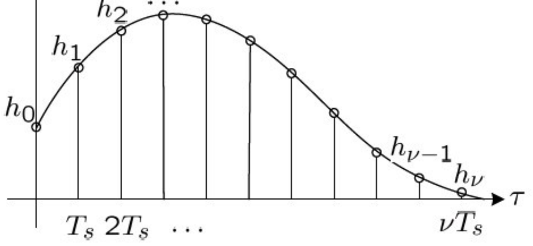
Source #1: Single-Tap Channel: Rayleigh Dist'n

- Path loss, shadowing => average signal power loss
  - Fading around this average.
  - Subtract out average => fading modeled as a zero-mean random process
- Narrowband Fading channel: Each symbol is long in time
  - The channel h(t) is assumed to be uncorrelated across symbols => single "tap" in time domain.
- Fading w/ many scatterers: Central Limit Theorem
  - In-phase (cosine) and quadrature (sine) components of the snapshot r(0), denoted as $r_I(0)$ and $r_Q(0)$ are independent Gaussian random variables.
  - Envelope Amplitude: $|r| = \sqrt{r_I^2 + r_Q^2}$ is Rayleigh.
  - Received Power: $|r|^2 = r_I^2 + r_Q^2$ is exponentially distributed.

Source #2: Multipaths: Power-Delay Profile



Channel Impulse Response:
Channel amplitude |h| <u>correlated</u> at delays τ.
Each “tap” value @ kTs Rayleigh distributed
(actually the sum of several sub-paths)



Eg: Power Delay Profile (WLAN/indoor)

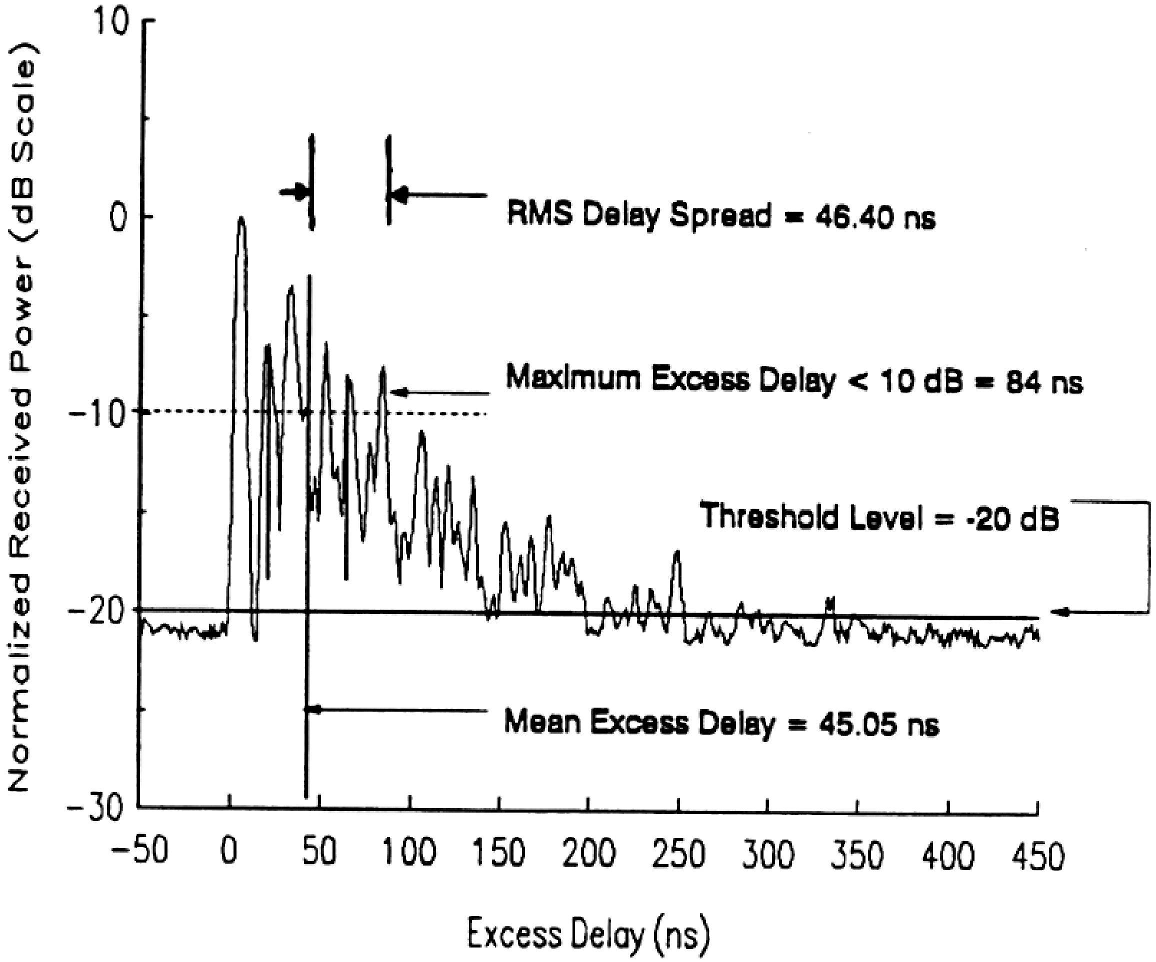
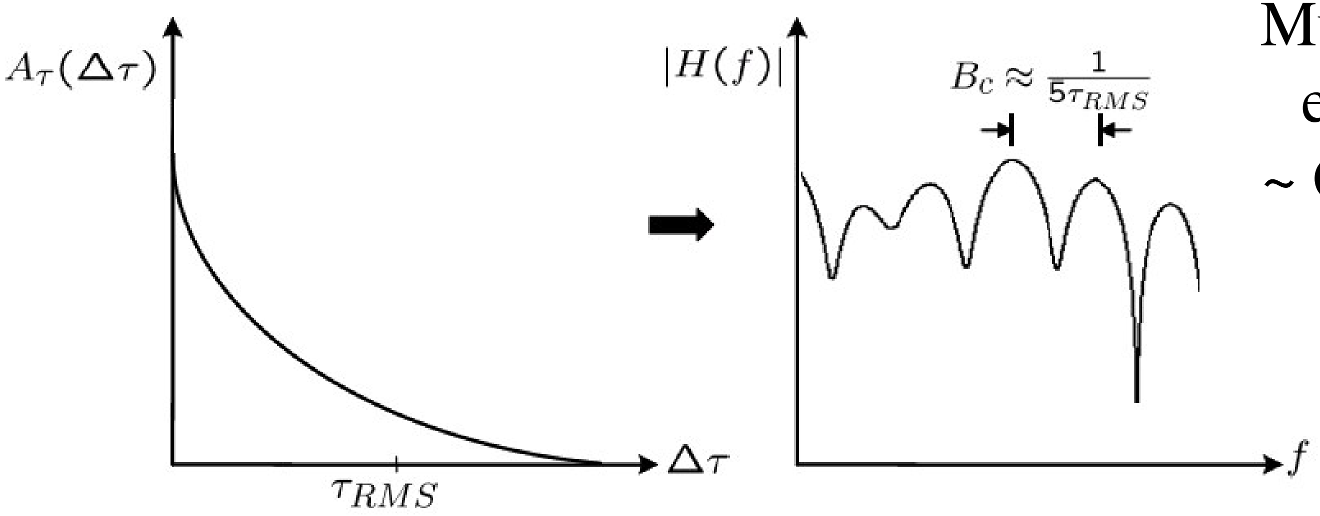


Figure 5.10 Example of an indoor power delay profile; rms delay spread, mean excess delay, maximum excess delay (10 dB), and threshold level are shown.

Multipath: Time-Dispersion => Frequency Selectivity

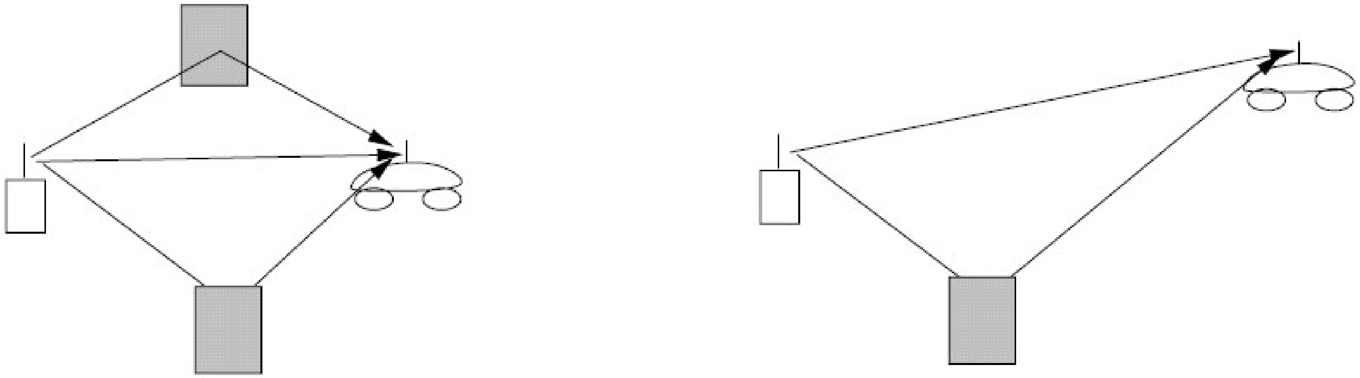
- The impulse response of the channel is correlated in the time-domain (sum of "echoes")
  - Manifests as a power-delay profile.
- Equivalent to "selectivity" or "deep fades" in the frequency domain
- **Delay spread:** τ ~ *50ns (indoor) – 1µs (outdoor/cellular).*
- **Coherence Bandwidth**: Bc = *500kHz (outdoor/cellular) – 20MHz (indoor)*
- Implications: High data rate: symbol smears onto the adjacent ones (ISI).

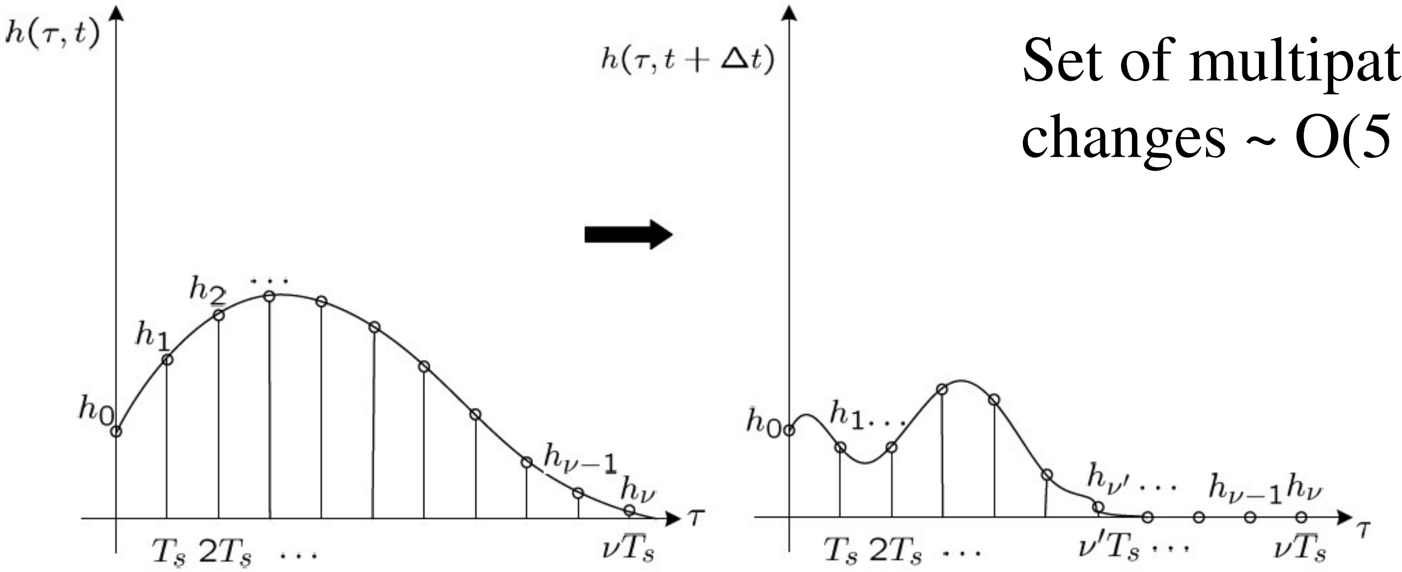


Multipath effects ~ O(1µs)

the shape of the multipath intensity profile $A_\tau(\Delta\tau)$ determines the correlation pattern of the channel frequency response (bottom)

Source #3: Doppler: Non-Stationary Impulse Response.





Set of multipaths changes ~ O(5 ms)

Doppler: Dispersion (Frequency) => Time-Selectivity

- The doppler power spectrum shows dispersion/flatness ~ doppler spread (100-200 Hz for vehicular speeds)
  - Equivalent to “selectivity” or “deep fades” in the time domain correlation envelope.
  - Each envelope point in time-domain is drawn from Rayleigh distribution. But because of Doppler, it is not IID, but correlated for a time period ~ Tc (correlation time).
- **Doppler Spread:** Ds ~ 100 Hz (vehicular speeds @ 1GHz)
- **Coherence Time**: Tc = 2.5-5ms.
- Implications: A deep fade on a tone can persist for 2.5-5 ms! Closed-loop estimation is valid only for 2.5-5 ms.

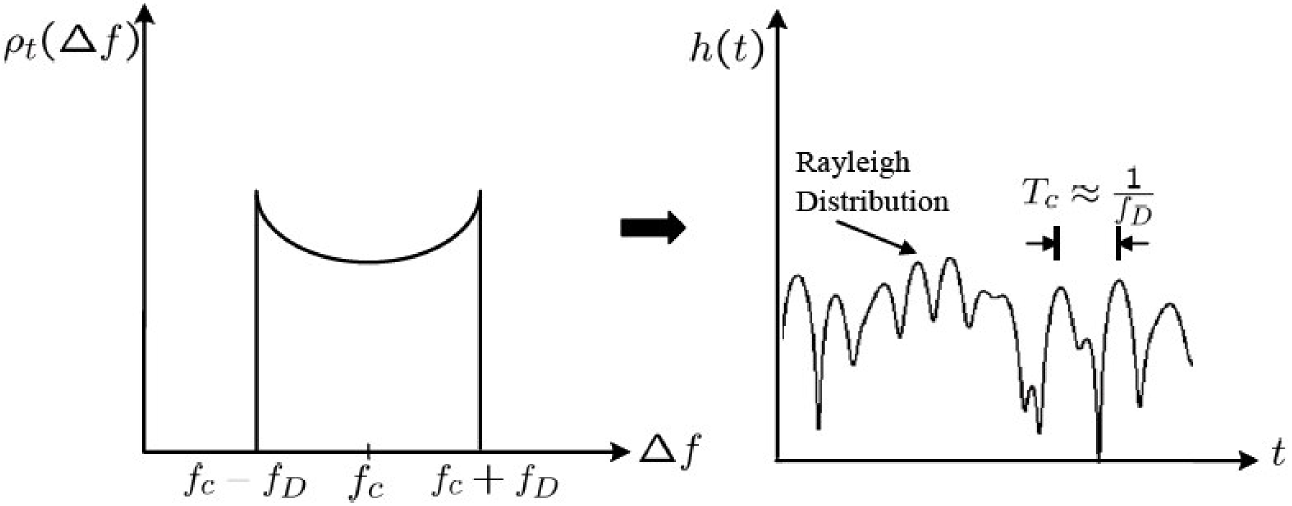


Figure 3.18: The shape of the Doppler power spectrum $\rho_t(\Delta f)$ determines the correlation envelope of the channel in time (top).

Fading Summary: Time-Varying Channel Impulse Response

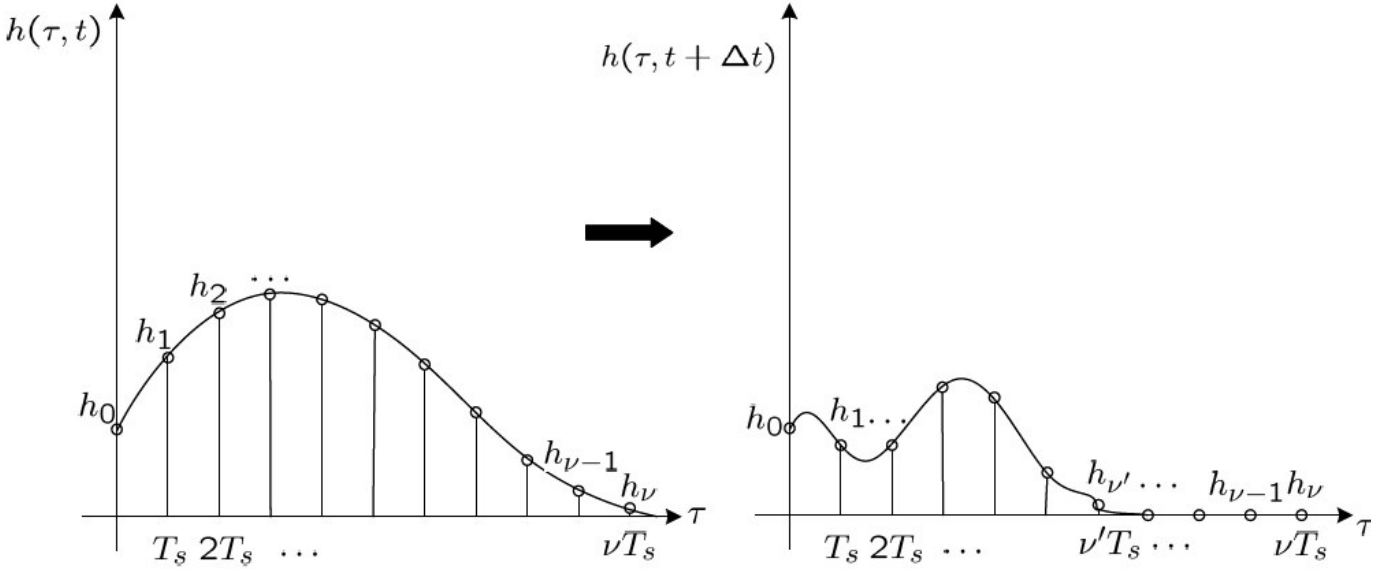


Figure 3.12: The delay τ corresponds to how *long* the channel impulse response lasts. The channel is time varying, so the channel impulse response is also a function of time, i.e. $h(\tau, t)$, and can be quite different at time $t + \Delta t$ than it was at time t.

- ❑ **#1:** At each tap, channel gain |h| is a Rayleigh distributed *r.v.*. The random *process* is not IID.
- ❑ **#2:** Response spreads out in the time-domain (τ), leading to inter-symbol interference and deep fades in the frequency domain: "***frequency-selectivity***" caused by multi-path fading
- ❑ **#3:** Response completely vanish (deep fade) for certain values of t: "***Time-selectivity***" caused by doppler effects (frequency-domain dispersion/spreading)

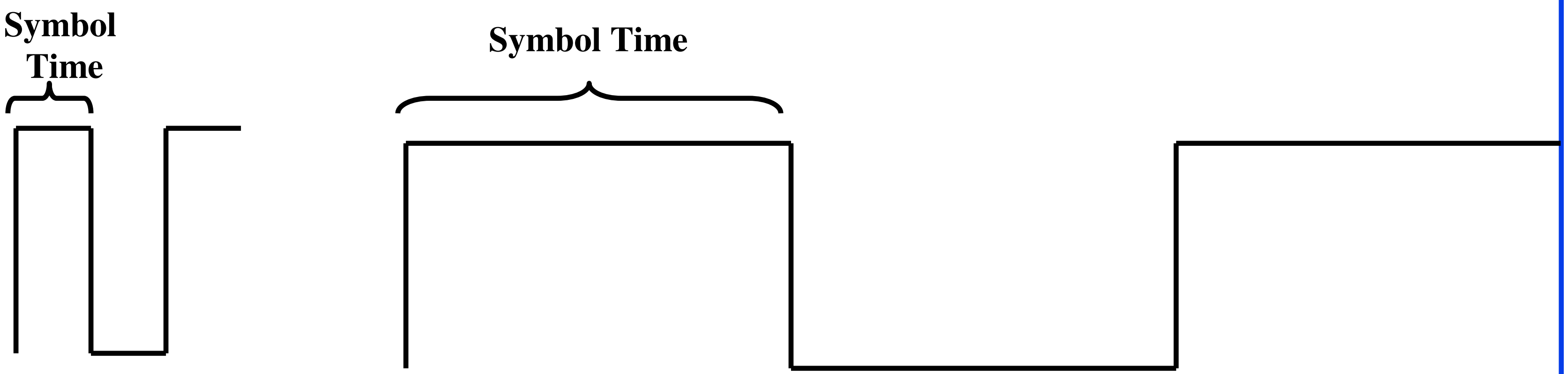
Fading: Jargon

- **Flat fading**: no multipath ISI effects.
  - Eg: narrowband, indoors
- **Frequency-selective fading:** multipath ISI effects.
  - Eg: broadband, outdoor.

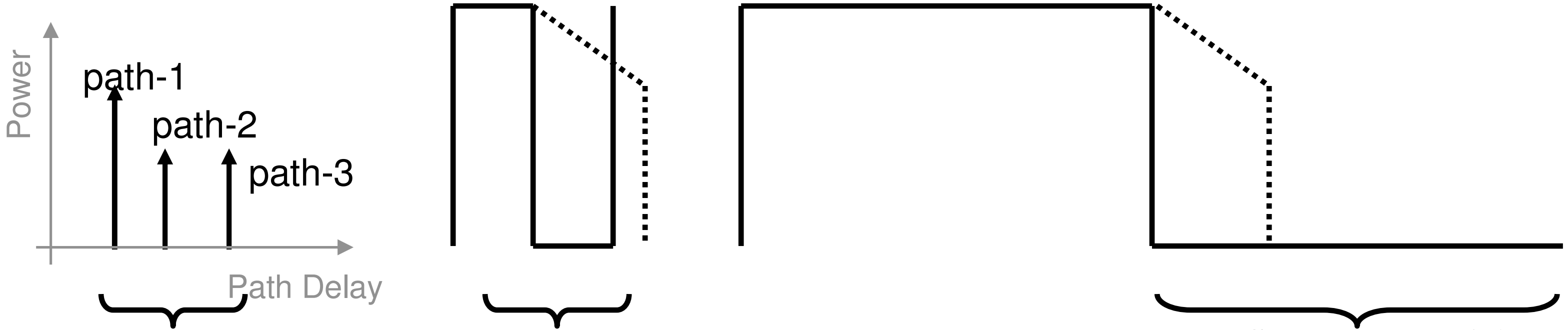
- **Slow fading:** no doppler effects.
  - Eg: indoor Wifi home networking
- **Fast Fading:** doppler effects, time-selective channel
  - Eg: cellular, vehicular

- Broadband cellular + vehicular => Fast + frequency-selective

Power Delay Profile => Inter-Symbol interference



- Higher bandwidth => higher symbol rate, and smaller time per-symbol
- Lower symbol rate, more time, energy per-symbol
- If the delay spread is longer than the symbol-duration, symbols will “smear” onto adjacent symbols and cause symbol errors



Delay spread
~ 1 µs

Symbol Error!
If symbol rate
~ Mbps

No Symbol Error! (~kbps)
(energy is collected
over the full symbol period
for detection)

Multipath Fading Example

Example 3.5:

Consider a wideband channel with multipath intensity profile

$$A_c(\tau) = \begin{cases} e^{-\tau/.00001} & 0 \leq \tau \leq 20 \ \mu\text{sec.} \\ 0 & \text{else} \end{cases} .$$

Find the mean and rms delay spreads of the channel and find the maximum symbol rate such that a linearly-modulated signal transmitted through this channel does not experience ISI.

Solution: The average delay spread is

$$\mu_{T_m} = \frac{\int_0^{20*10^{-6}} \tau e^{-\tau/.00001} d\tau}{\int_0^{20*10^{-6}} e^{-\tau/.00001} d\tau} = 6.87 \ \mu\text{sec.}$$

The rms delay spread is

$$\sigma_{T_m} = \sqrt{\frac{\int_0^{20*10^{-6}} (\tau - \mu_{T_m})^2 e^{-\tau} d\tau}{\int_0^{20*10^{-6}} e^{-\tau} d\tau}} = 5.25 \ \mu\text{sec.}$$

We see in this example that the mean delay spread is roughly equal to its rms value. To avoid ISI we require linear modulation to have a symbol period T_s that is large relative to σ_{T_m}. Taking this to mean that $T_s > 10\sigma_{T_m}$ yields a symbol period of $T_s = 52.5\,\mu$sec or a symbol rate of $R_s = 1/T_s = 19.04$ Kilosymbols per second. This is a highly constrained symbol rate for many wireless systems. Specifically, for binary modulations where the symbol rate equals the data rate (bits per second, or bps), high-quality voice requires on the order of 32 Kbps and high-speed data requires on the order of 10-100 Mbps.

Key Wireless Channel Parameters

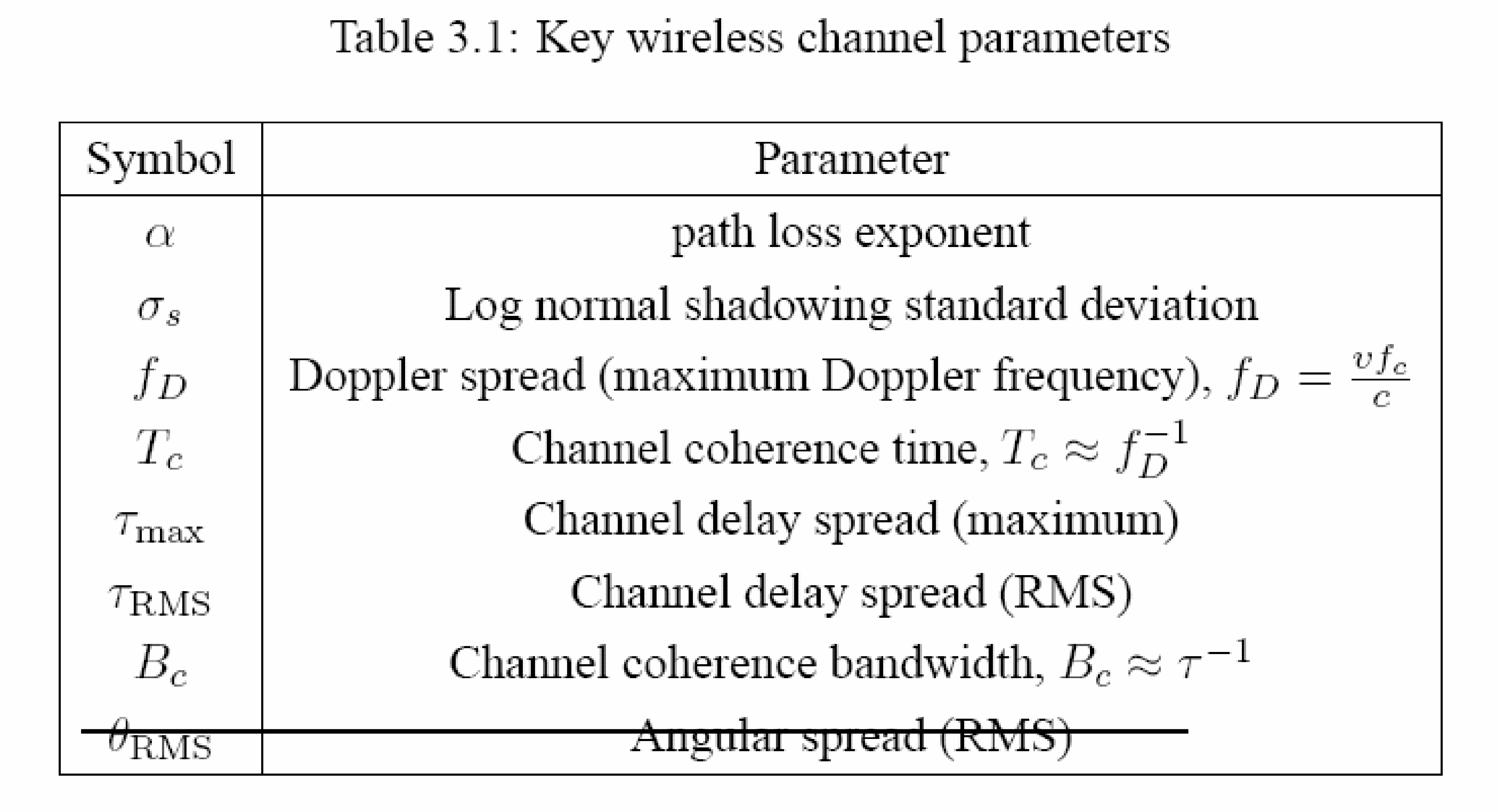
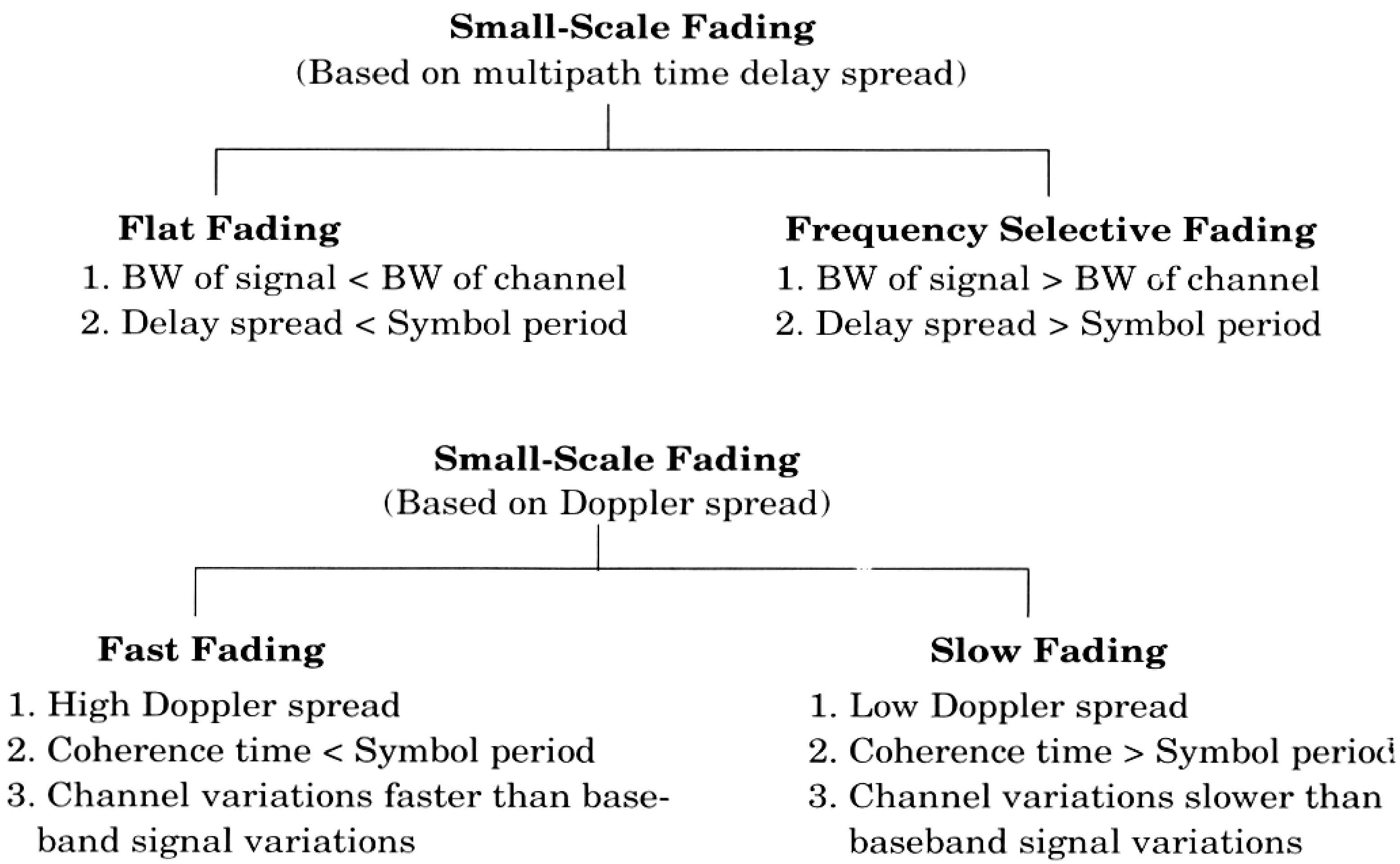


Table 3.1: Key wireless channel parameters

Symbol	Parameter
α	path loss exponent
σ_s	Log normal shadowing standard deviation
f_D	Doppler spread (maximum Doppler frequency), $f_D = \frac{v f_c}{c}$
T_c	Channel coherence time, $T_c \approx f_D^{-1}$
$\tau_{\max}$	Channel delay spread (maximum)
τ_{RMS}	Channel delay spread (RMS)
B_c	Channel coherence bandwidth, $B_c \approx \tau^{-1}$
~~θ_{RMS}~~	~~Angular spread (RMS)~~

Small-Scale Fading Summary



Small-Scale Fading
(Based on multipath time delay spread)

Flat Fading
1. BW of signal < BW of channel
2. Delay spread < Symbol period

Frequency Selective Fading
1. BW of signal > BW of channel
2. Delay spread > Symbol period

Small-Scale Fading
(Based on Doppler spread)

Fast Fading
1. High Doppler spread
2. Coherence time < Symbol period
3. Channel variations faster than baseband signal variations

Slow Fading
1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

Figure 5.11 Types of small-scale fading.

Fading: Design Impacts (Eg: Wimax)

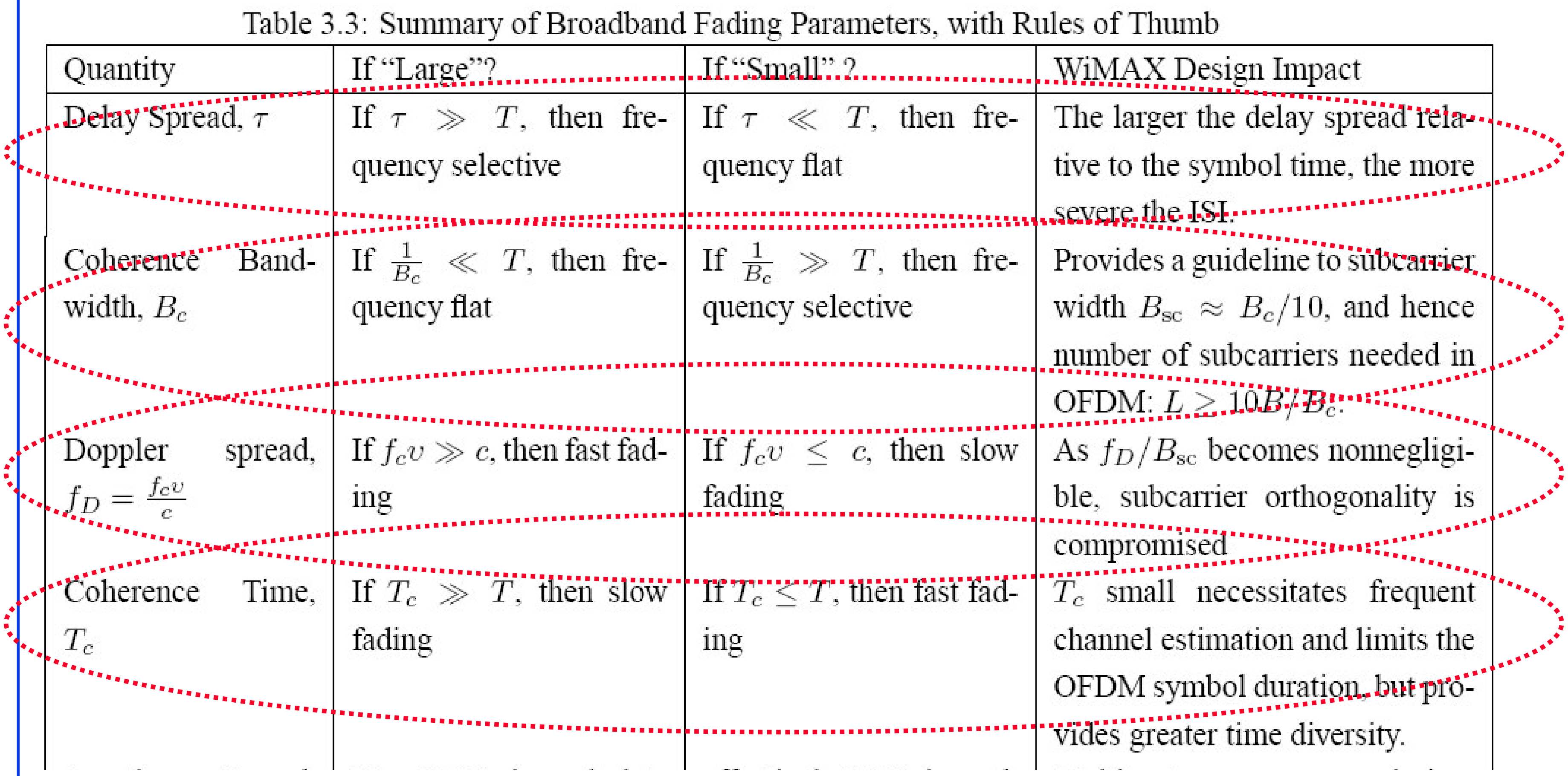


Table 3.3: Summary of Broadband Fading Parameters, with Rules of Thumb

Quantity	If "Large"?	If "Small"?	WiMAX Design Impact
Delay Spread, τ	If $\tau \gg T$, then frequency selective	If $\tau \ll T$, then frequency flat	The larger the delay spread relative to the symbol time, the more severe the ISI.
Coherence Bandwidth, B_c	If $\frac{1}{B_c} \ll T$, then frequency flat	If $\frac{1}{B_c} \gg T$, then frequency selective	Provides a guideline to subcarrier width $B_{sc} \approx B_c/10$, and hence number of subcarriers needed in OFDM: $L \geq 10B/B_c$.
Doppler spread, $f_D = \frac{f_c v}{c}$	If $f_c v \gg c$, then fast fading	If $f_c v \leq c$, then slow fading	As f_D/B_{sc} becomes nonnegligible, subcarrier orthogonality is compromised
Coherence Time, T_c	If $T_c \gg T$, then slow fading	If $T_c \leq T$, then fast fading	T_c small necessitates frequent channel estimation and limits the OFDM symbol duration, but provides greater time diversity.

Summary

- We have understood both qualitatively and quantitatively the concepts of path loss, shadowing, fading (multi-path, doppler), and some of their design impacts.

- We have understood how time and frequency selectivity of wireless channels depend on key physical parameters.

- We have come up with linear, LTI and statistical channel models useful for analysis and design.