Data and Computer Communications

Error Correction

Syllabus

- Tentatively

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10.3 Correction

Retransmission

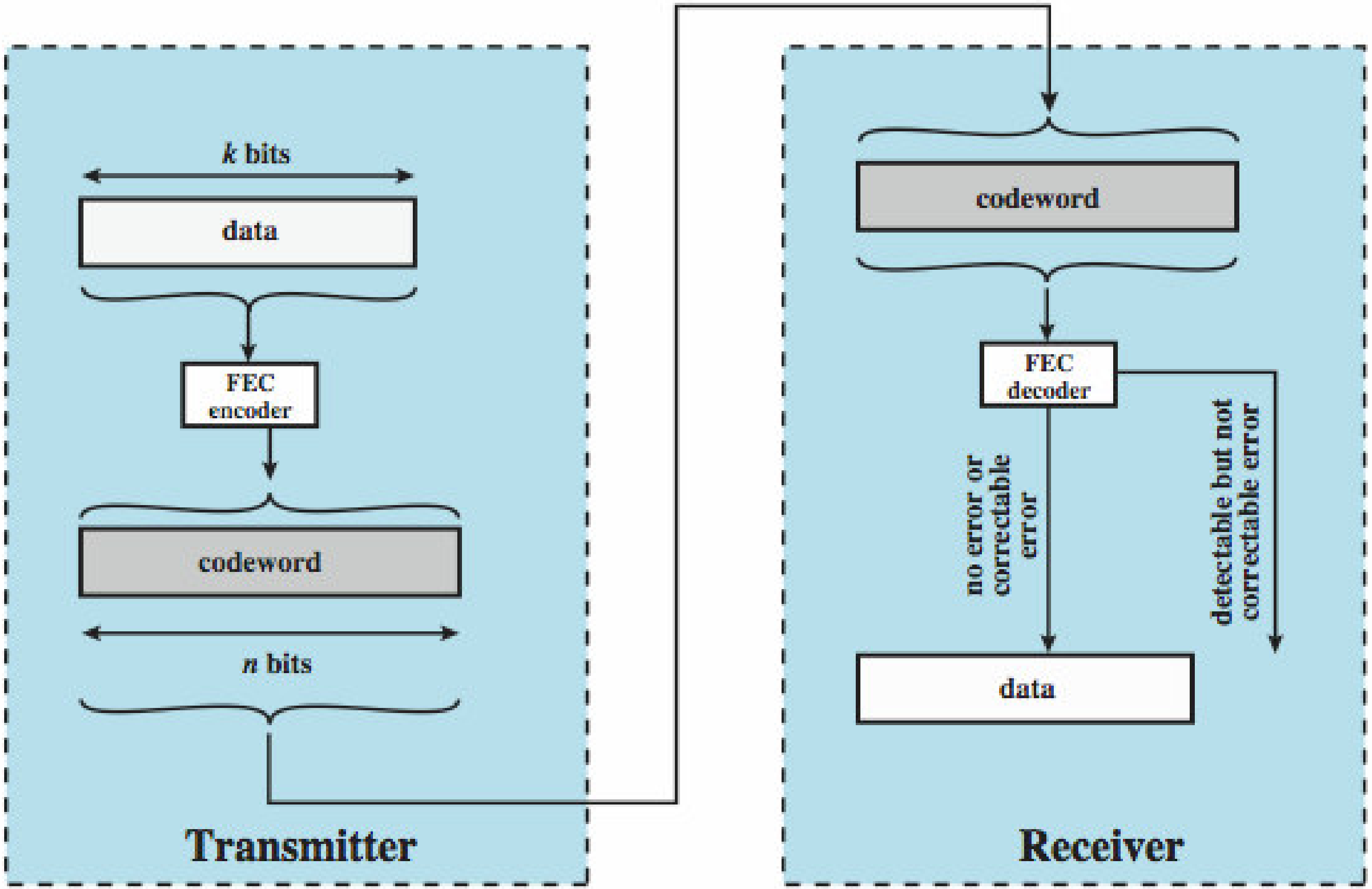
Forward Error Correction

Burst Error Correction

Error Correction

- correction of detected errors usually requires data block to be retransmitted
- not appropriate for wireless applications
  - bit error rate is high causing lots of retransmissions
  - when propagation delay long (satellite) compared with frame transmission time, resulting in retransmission of frame in error plus many subsequent frames
- instead need to correct errors on basis of bits received
- error correction provides this

Error Correction Process



How Error Correction Works

- adds redundancy to transmitted message
- can deduce original despite some errors
- eg. block error correction code
  - map *k* bit input onto an *n* bit codeword
  - each distinctly different
  - if get error assume codeword sent was closest to that received
- means have reduced effective data rate

Table 10.2 Data and redundancy bits

Number of data bits m	Number of redundancy bits r	Total bits m + r
1	2	3
2	3	5
3	3	6
4	3	7
5	4	9
6	4	10
7	4	11

10.14 *Positions of redundancy bits in Hamming code*

11	10	9	8	7	6	5	4	3	2	1
d	**d**	**d**	r_8	**d**	**d**	**d**	r_4	**d**	r_2	r_1

Redundancy bits calculation

r_1 will take care of these bits.

11		9		7		5		3		1
d	d	d	r_8	d	d	d	r_4	d	r_2	r_1

r_2 will take care of these bits.

11	10			7	6			3	2	
d	d	d	r_8	d	d	d	r_4	d	r_2	r_1

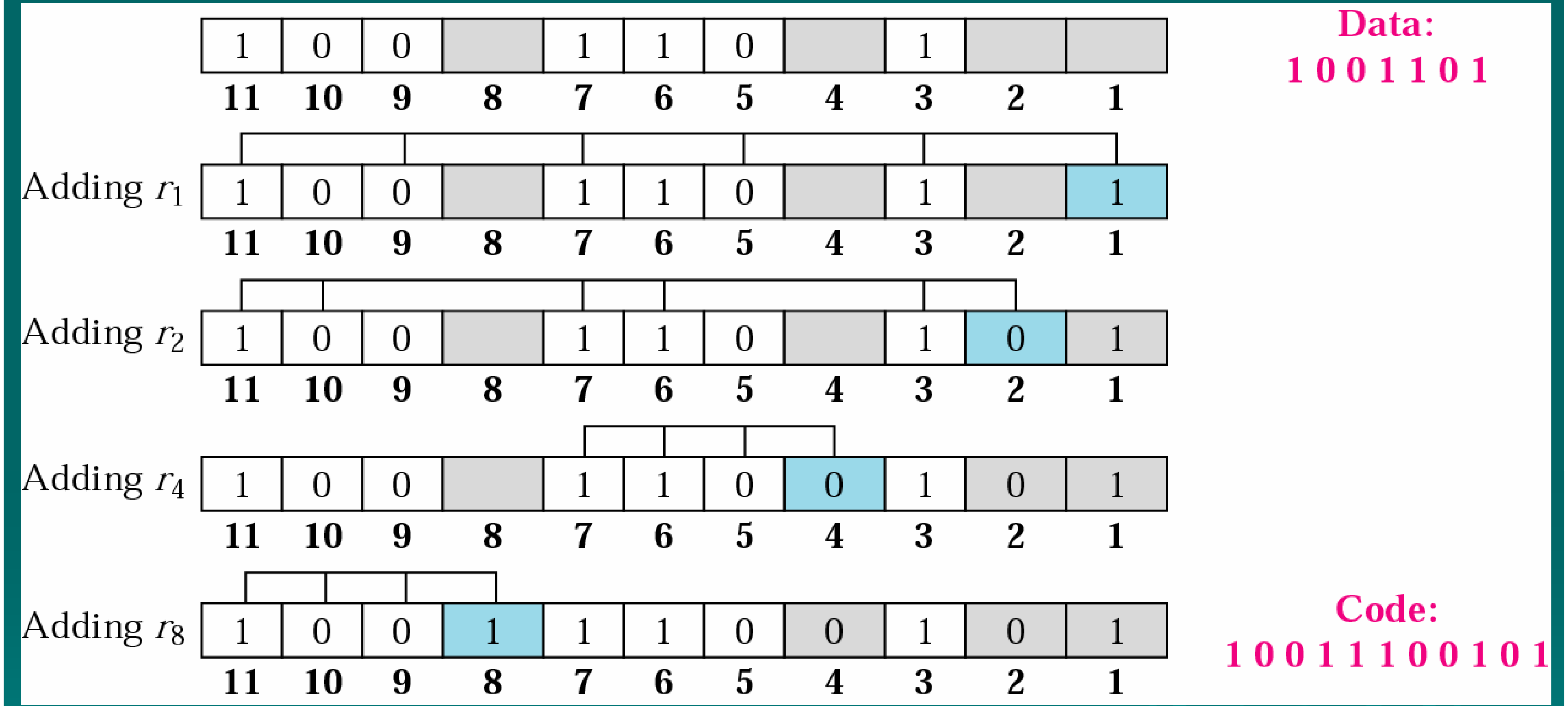
r_4 will take care of these bits.

				7	6	5	4			
d	d	d	r_8	d	d	d	r_4	d	r_2	r_1

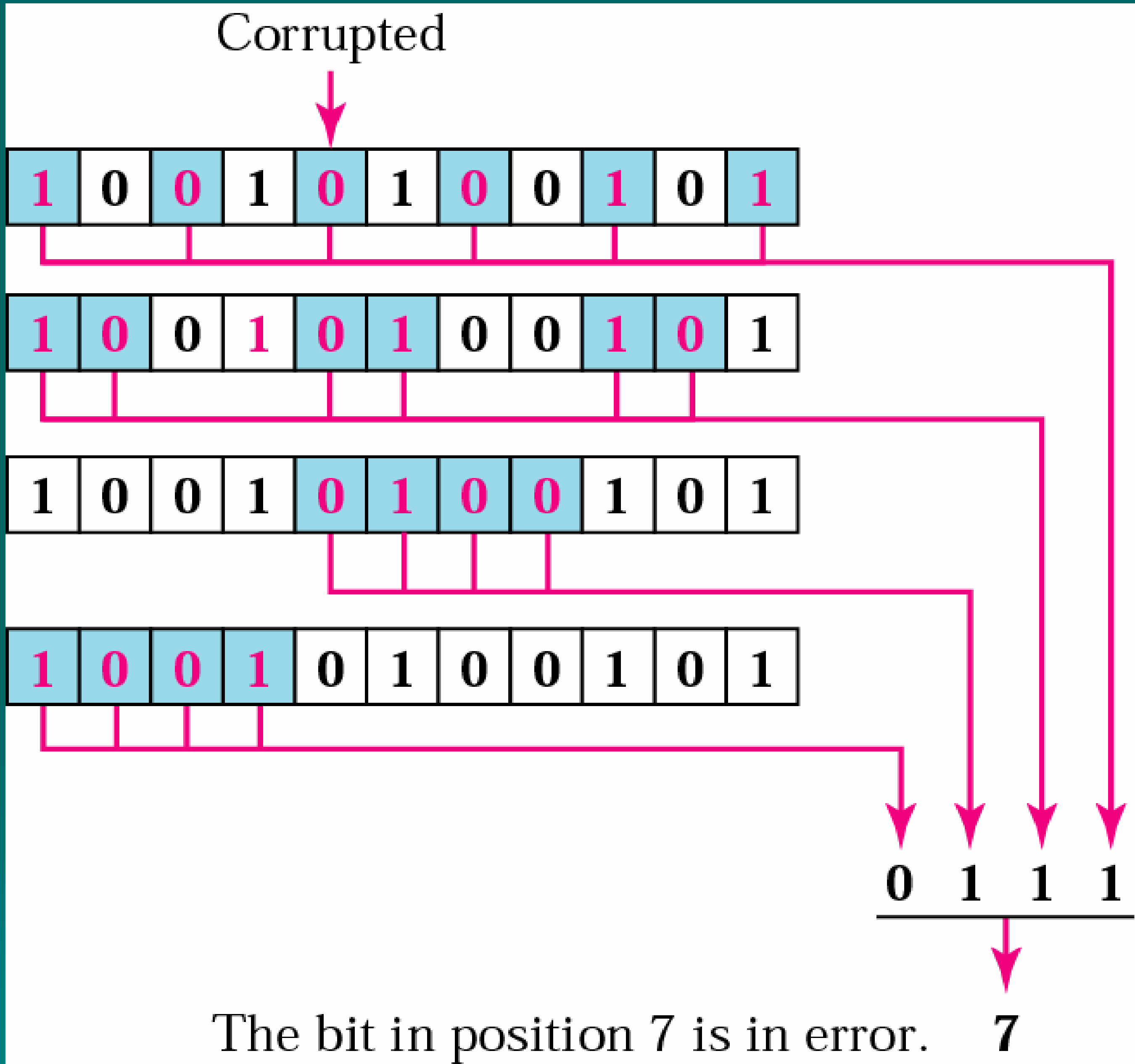
r_8 will take care of these bits.

11	10	9	8							
d	d	d	r_8	d	d	d	r_4	d	r_2	r_1

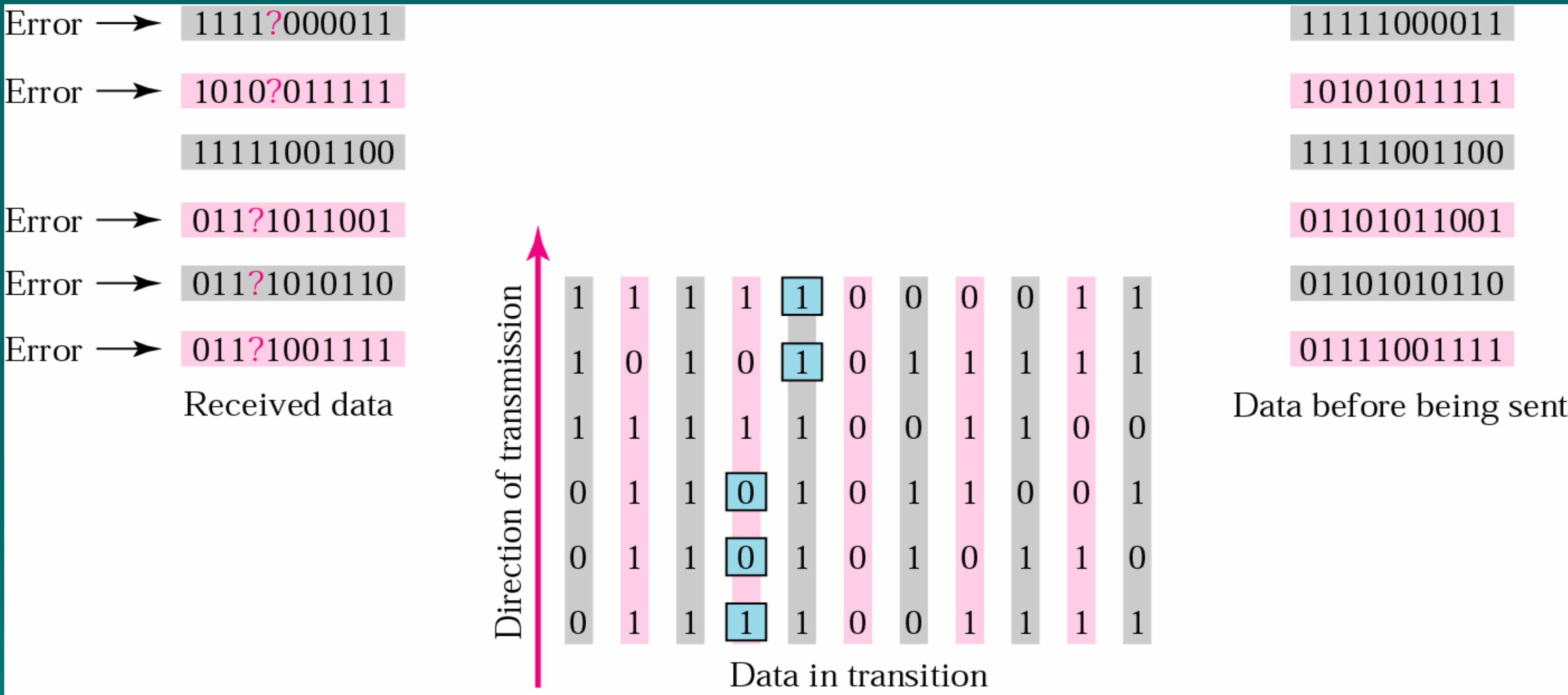
10.16 *Example of redundancy bit calculation*



10.17 *Error detection using Hamming code*



10.18 *Burst error correction example*



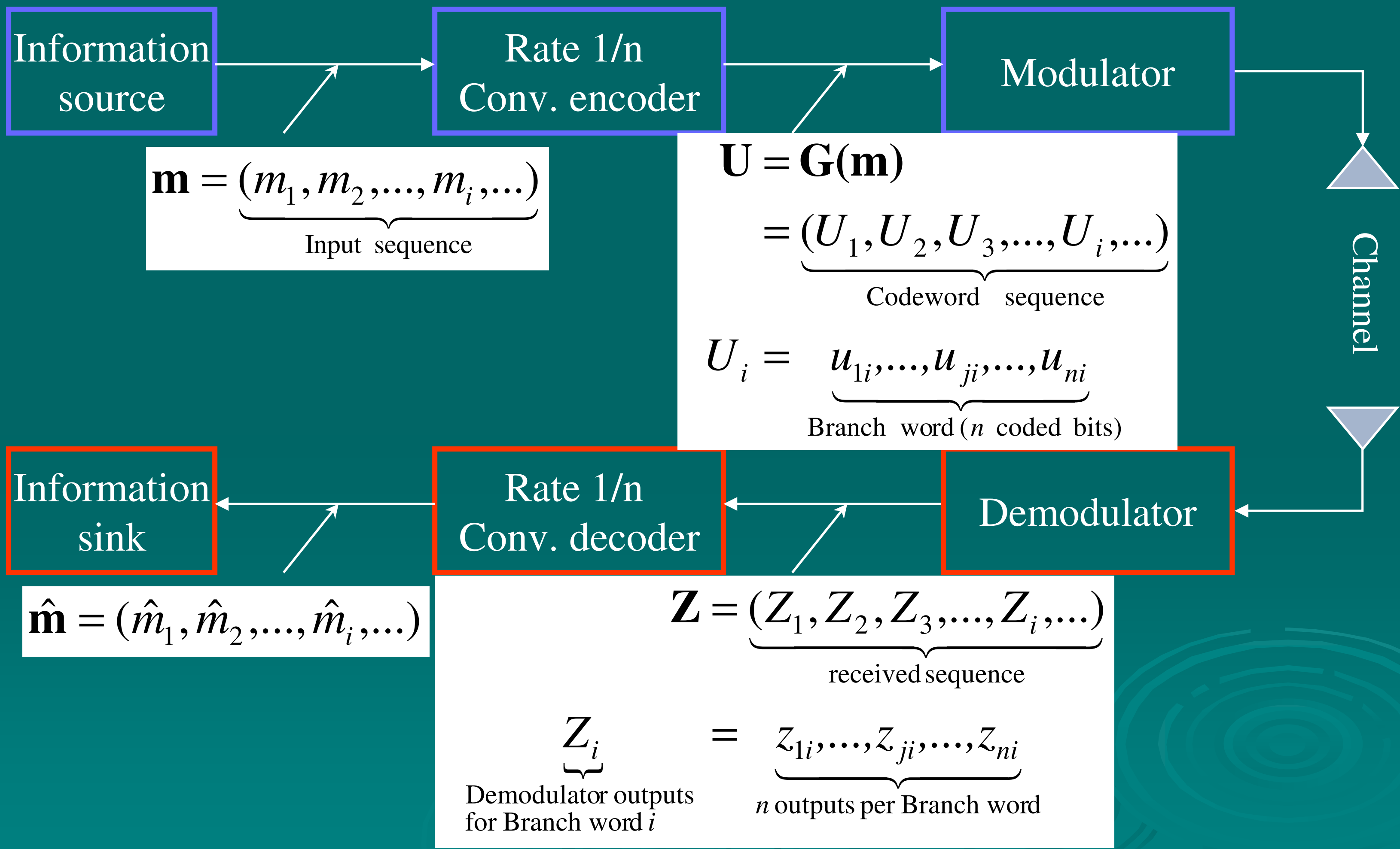
Convolutional codes

- Convolutional codes offer an approach to error control coding substantially different from that of block codes.
  - A convolutional encoder:
    - encodes the entire data stream, into a single codeword.
    - does not need to segment the data stream into blocks of fixed size (*Convolutional codes are often forced to block structure by periodic truncation*).
    - is a machine with memory.
- This fundamental difference in approach imparts a different nature to the design and evaluation of the code.
  - Block codes are based on algebraic/combinatorial techniques.
  - Convolutional codes are based on construction techniques.

Convolutional codes-cont'd

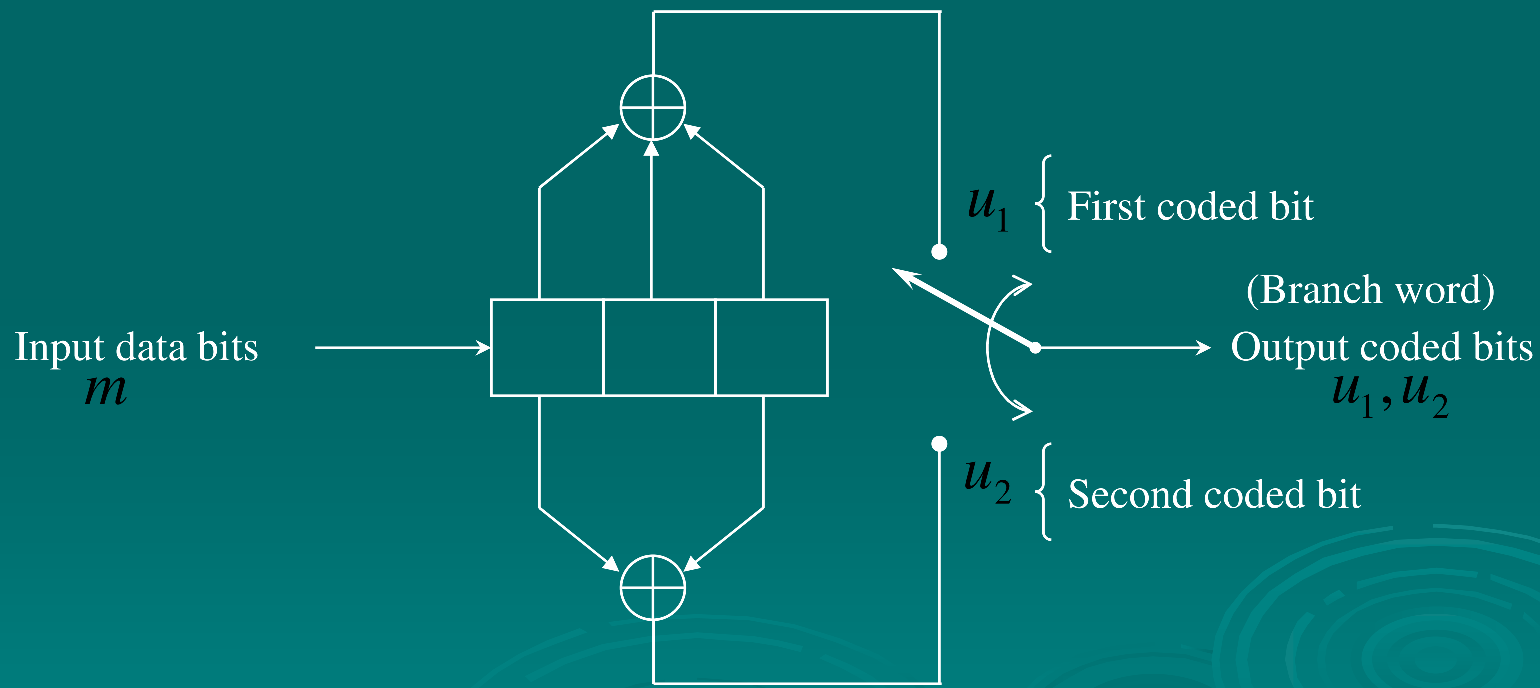
- A Convolutional code is specified by three parameters (n,k,K) or $(k/n,K)$ where
  - $R_c = k/n$ is the coding rate, determining the number of data bits per coded bit.
    - In practice, usually $k=1$ is chosen and we assume that from now on.
  - K is the constraint length of the encoder a where the encoder has $K-1$ memory elements.
    - There is different definitions in literatures for constraint length.

Block diagram of the DCS



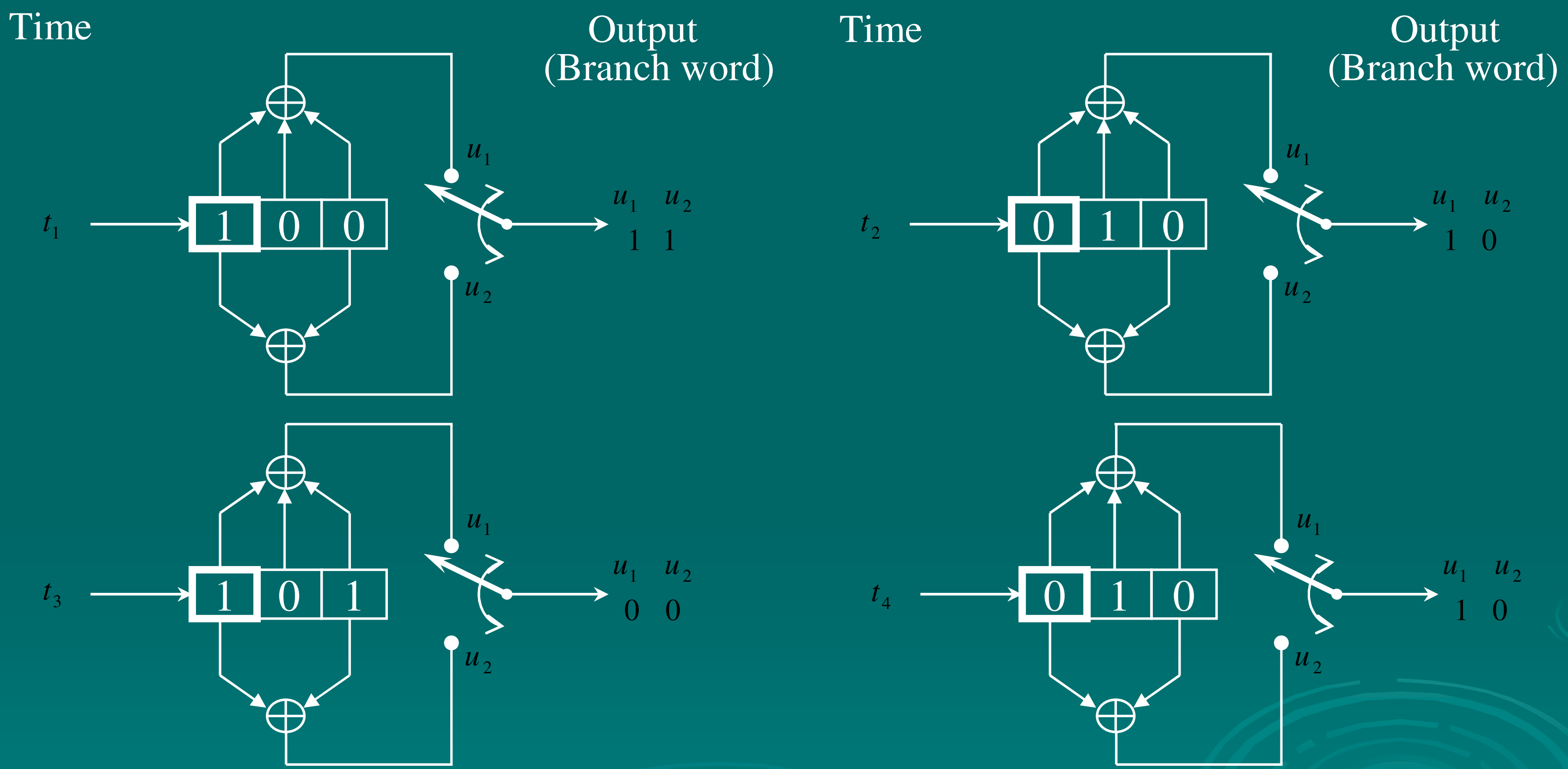
A Rate ½ Convolutional encoder

- Convolutional encoder (rate ½, K=3)
  - 3 shift-registers where the first one takes the incoming data bit and the rest, form the memory of the encoder.

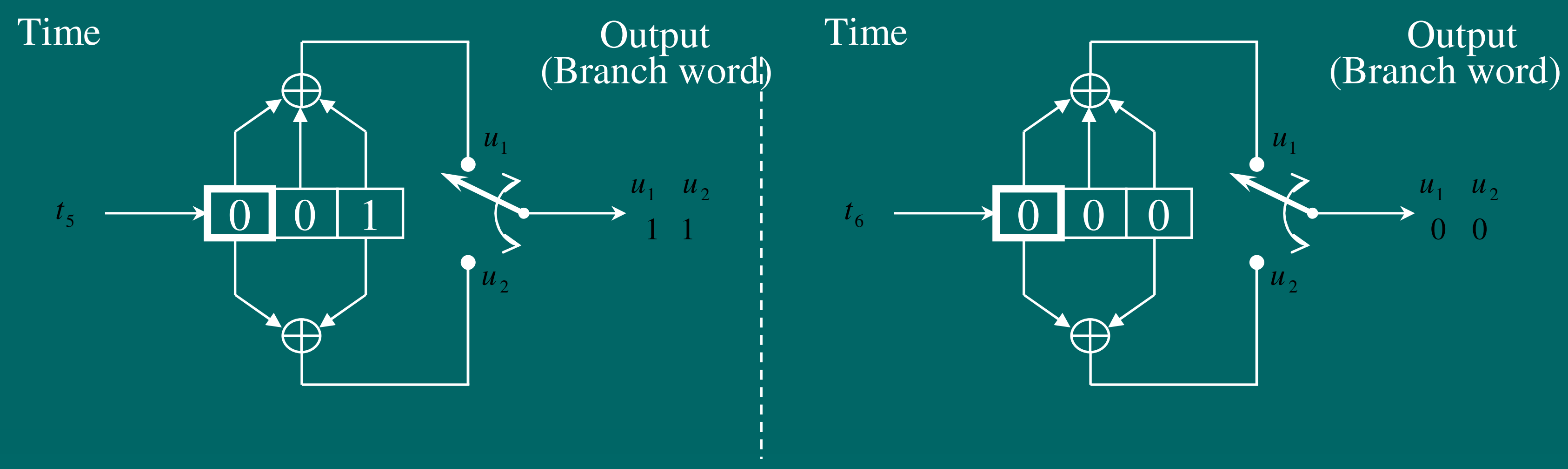


A Rate ½ Convolutional encoder

Message sequence: $\mathbf{m} = (101)$



A Rate ½ Convolutional encoder



$\mathbf{m} = (101)$ → Encoder → $\mathbf{U} = (11 \quad 10 \quad 00 \quad 10 \quad 11)$

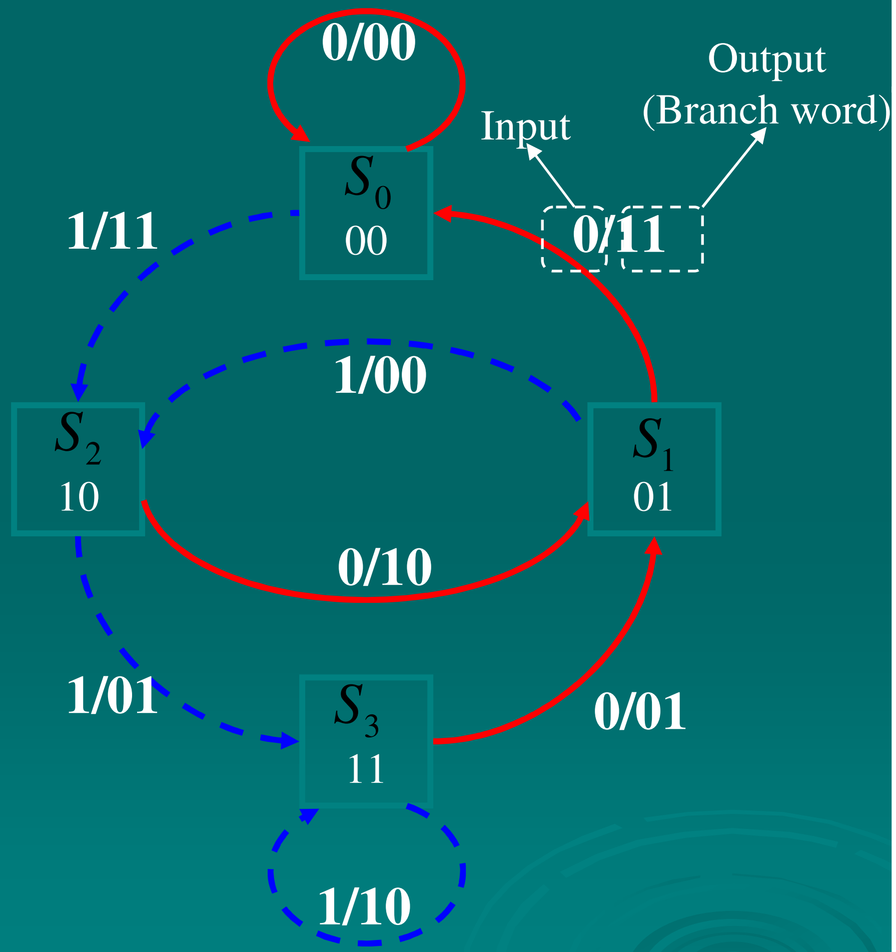
State diagram

- A finite-state machine only encounters a finite number of states.
- State of a machine: the smallest amount of information that, together with a current input to the machine, can predict the output of the machine.
- In a Convolutional encoder, the state is represented by the content of the memory.
- Hence, there are 2^{K-1} states.

State diagram – cont'd

- A state diagram is a way to represent the encoder.
- A state diagram contains all the states and all possible transitions between them.
- Only two transitions initiating from a state
- Only two transitions ending up in a state

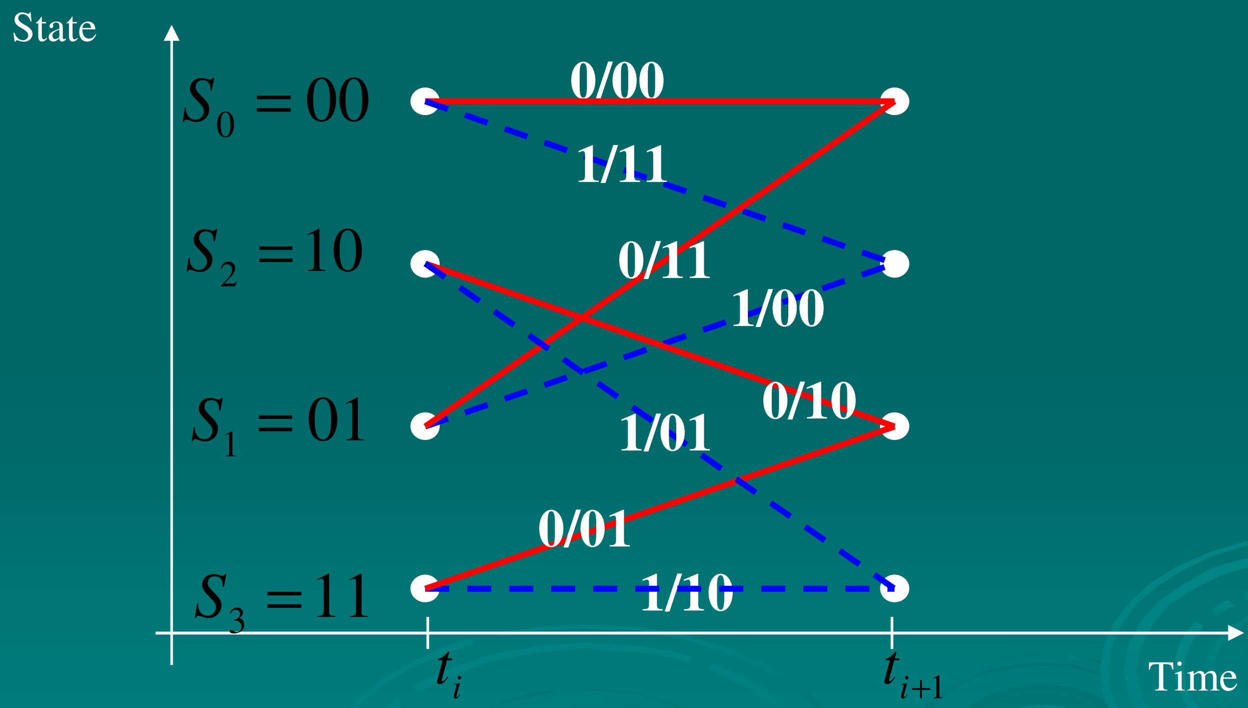
State diagram – cont'd



Current state	input	Next state	output
S_0 00	0	S_0	00
	1	S_2	11
S_1 01	0	S_0	11
	1	S_2	00
S_2 10	0	S_1	10
	1	S_3	01
S_3 11	0	S_1	01
	1	S_3	10

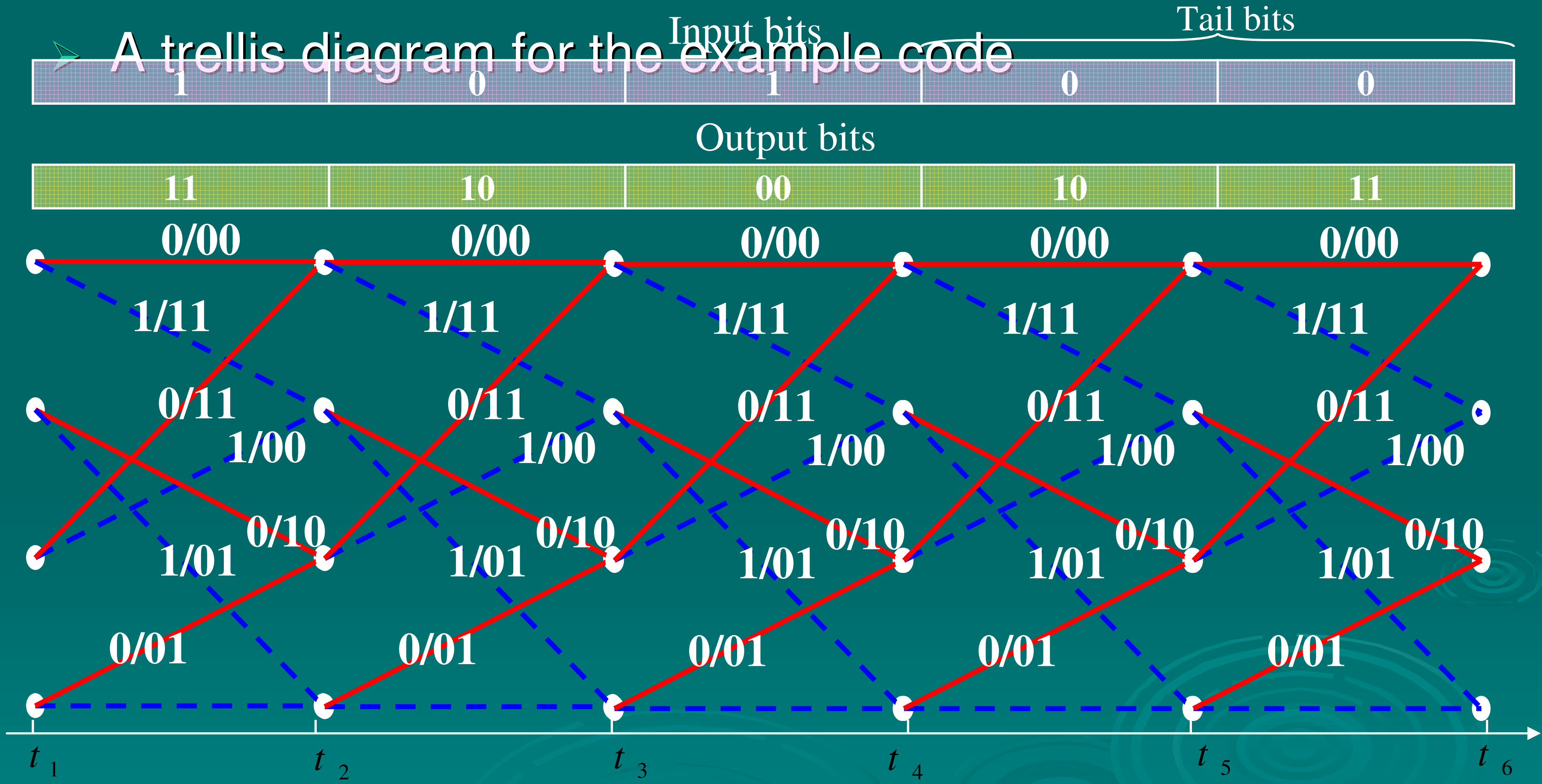
Trellis – cont'd

- Trellis diagram is an extension of the state diagram that shows the passage of time.
  - Example of a section of trellis for the rate ½ code

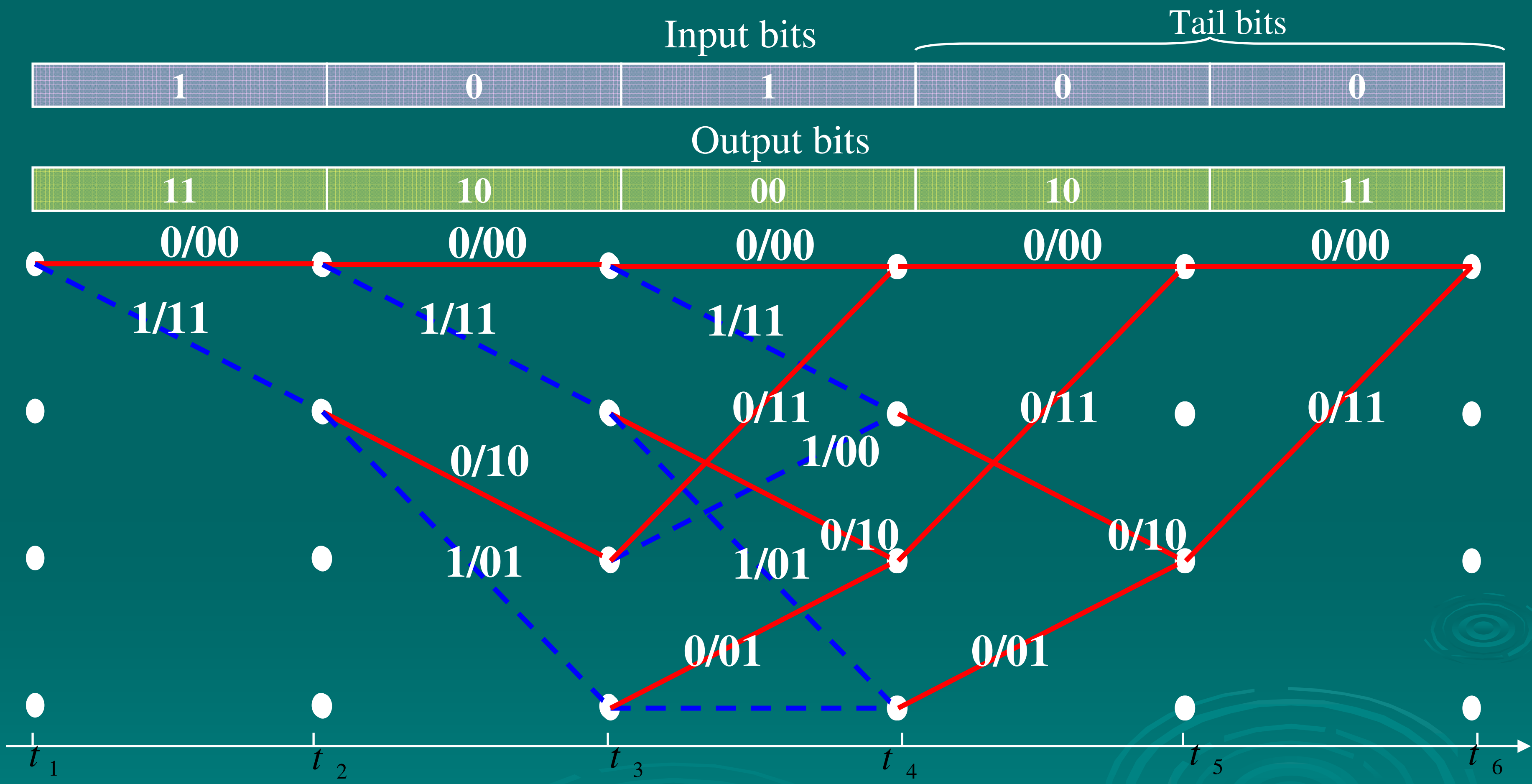


Trellis –cont’d

- A trellis diagram for the example code

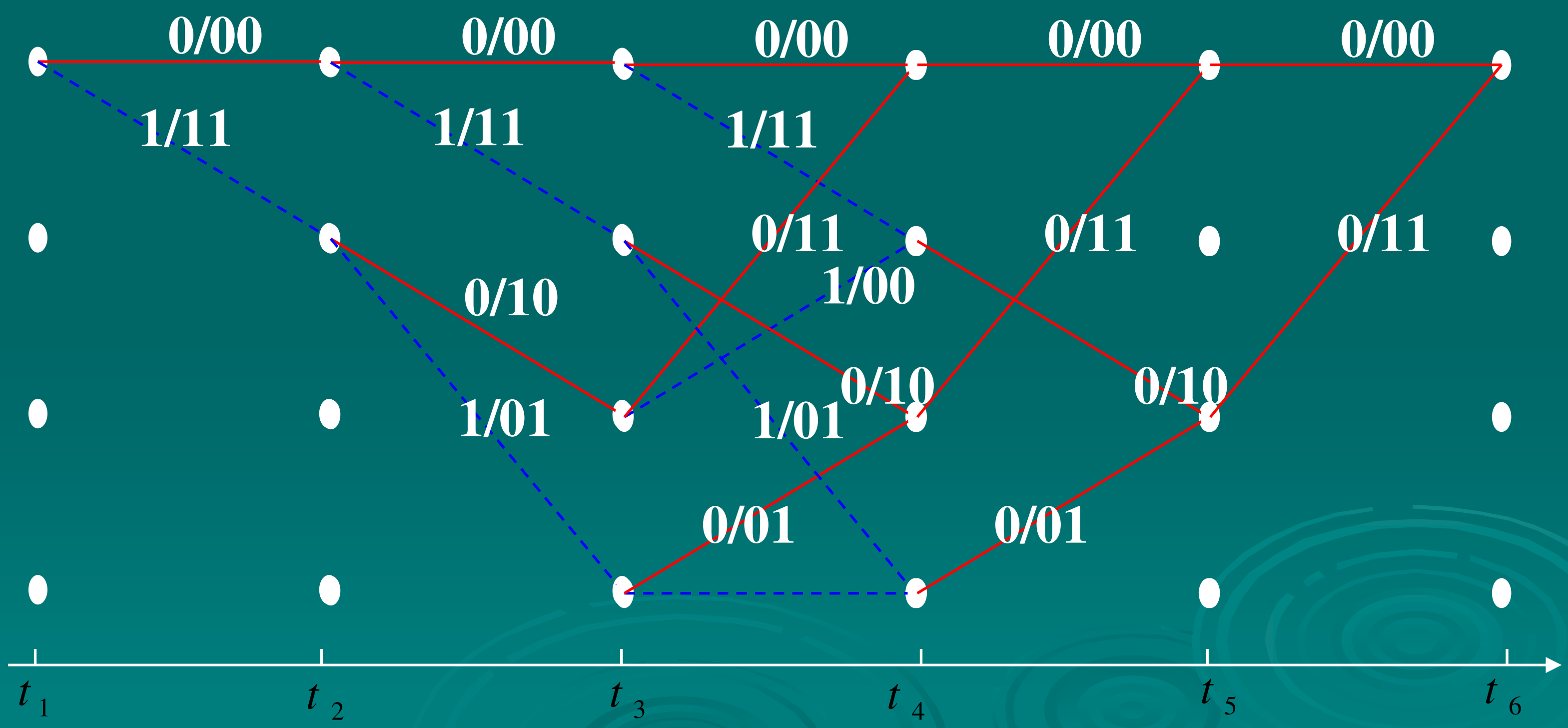


Trellis – cont'd



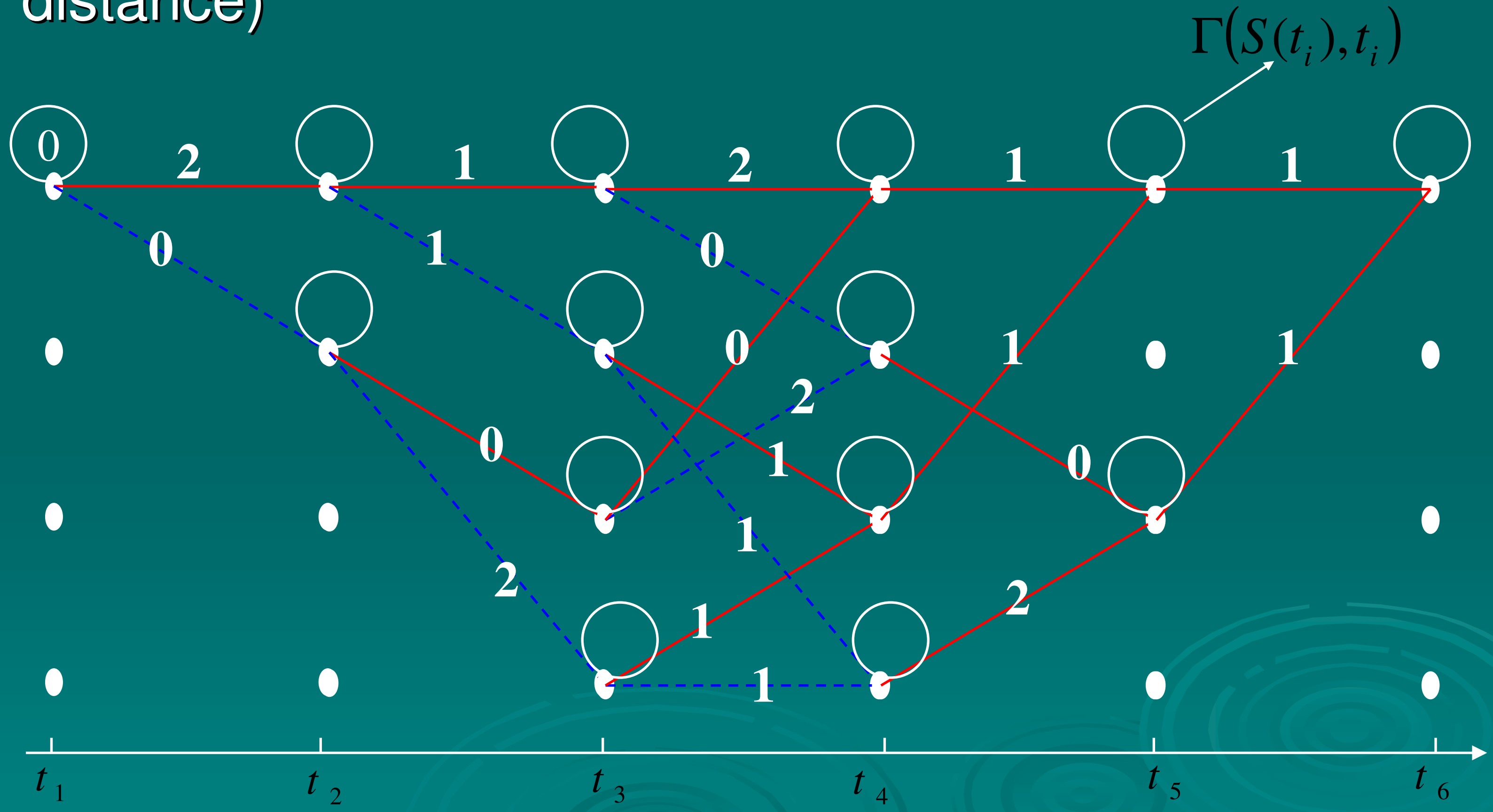
Example of Hard decision Viterbi decoding

$\mathbf{m} = (101)$
$\mathbf{U} = (11 \quad 10 \quad 00 \quad 10 \quad 11)$
$\mathbf{Z} = (11 \quad 10 \quad 11 \quad 10 \quad 01)$



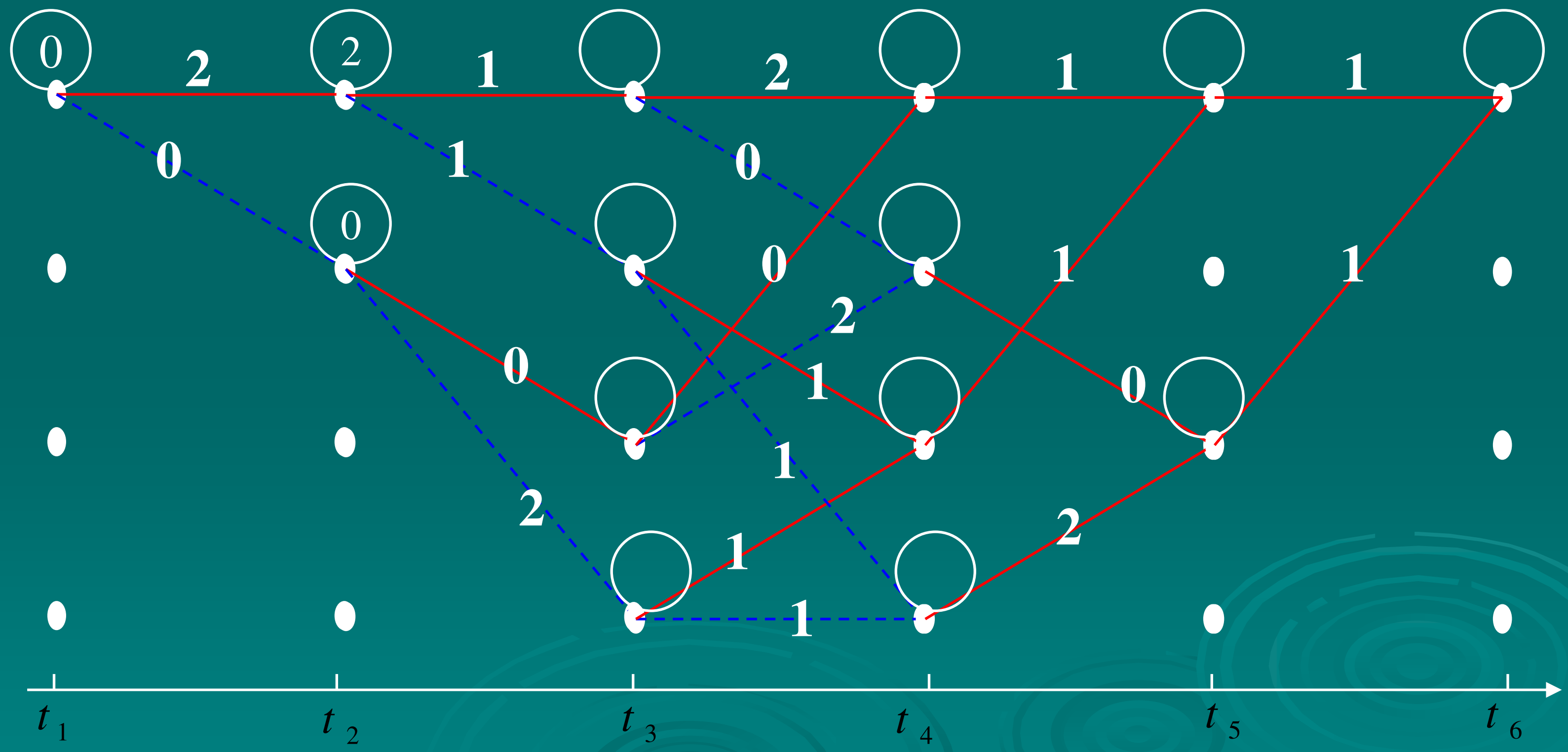
Example of Hard decision Viterbi decoding-cont'd

- Label al the branches with the branch metric (Hamming distance)



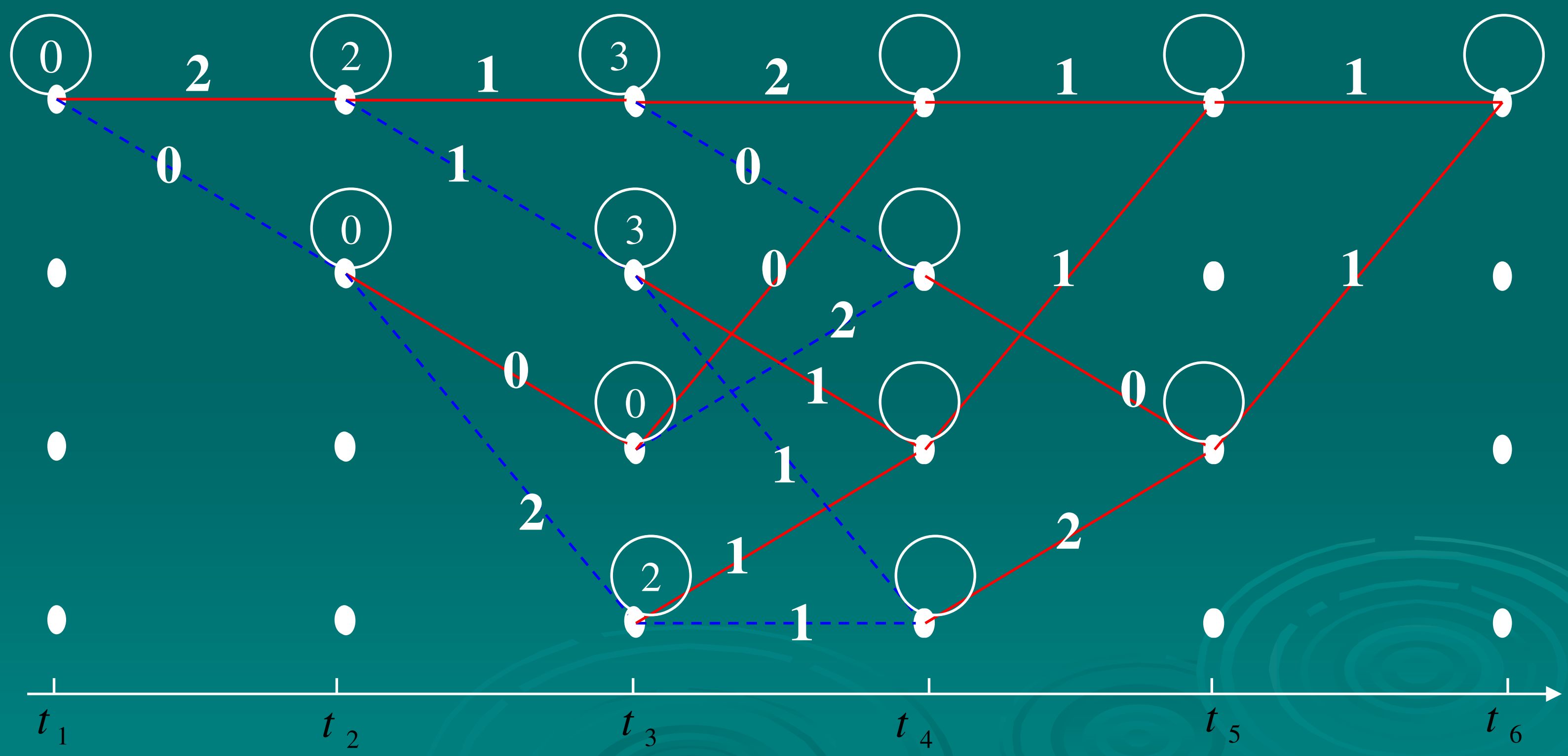
Example of Hard decision Viterbi decoding-cont'd

- i=2



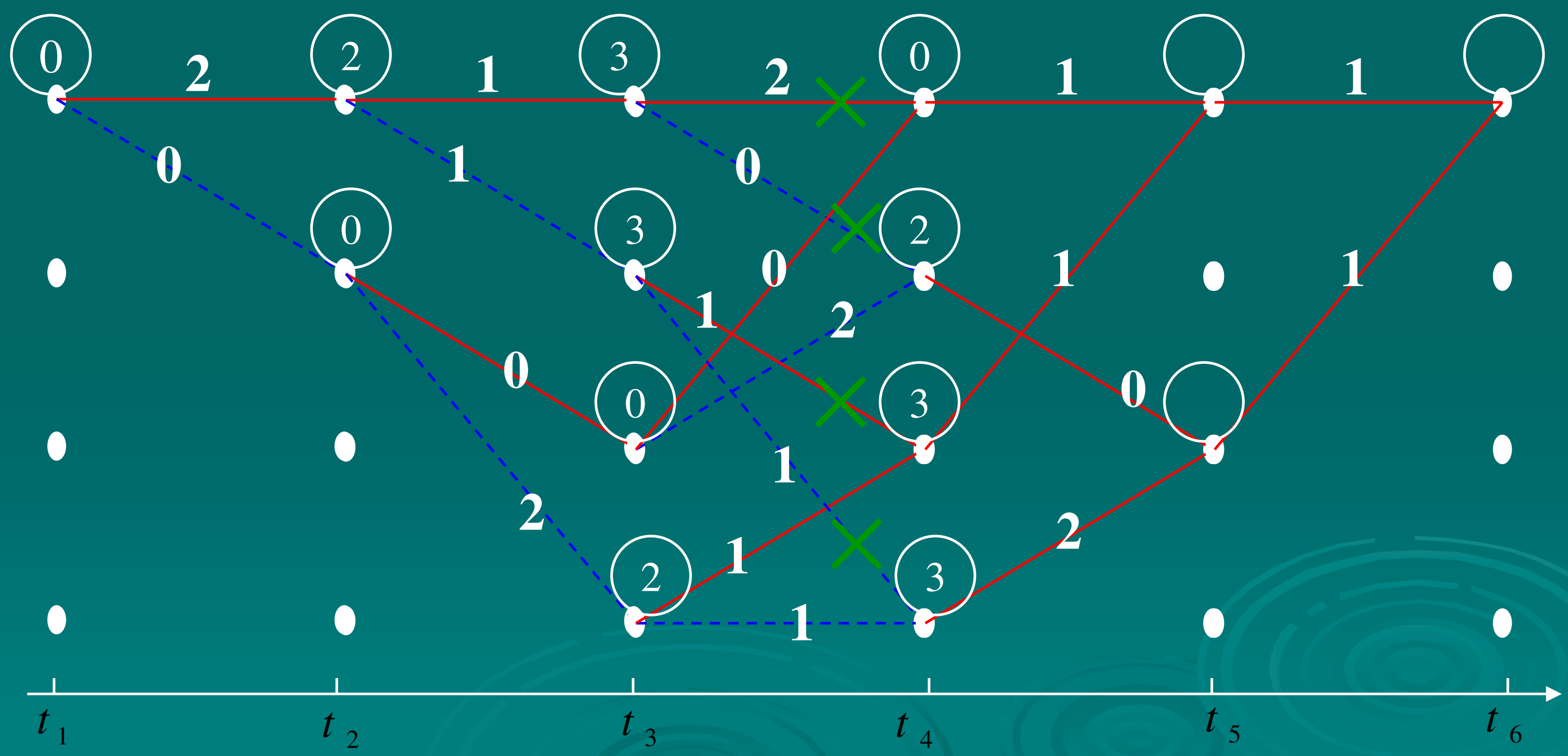
Example of Hard decision Viterbi decoding-cont'd

- i=3



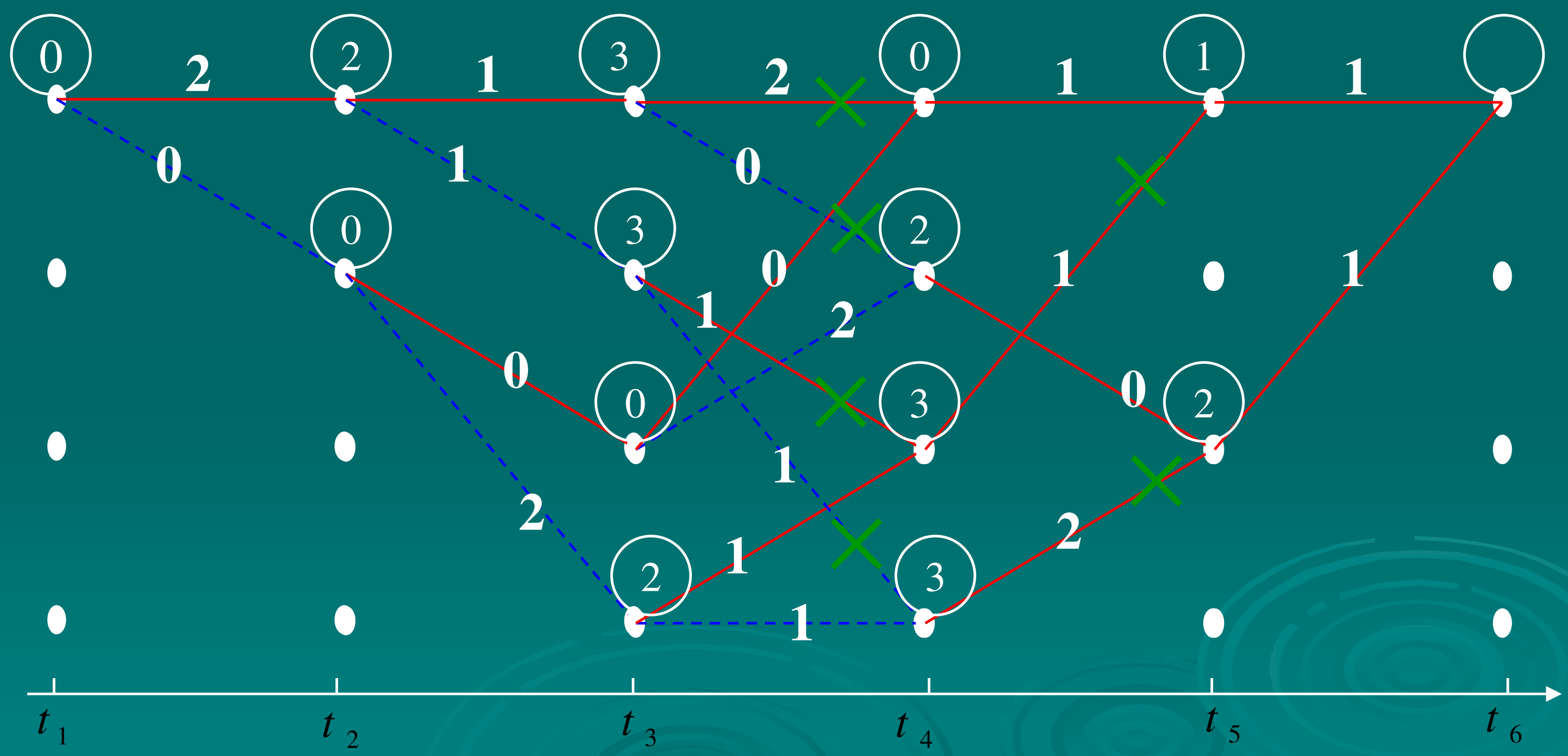
Example of Hard decision Viterbi decoding-cont'd

- i=4



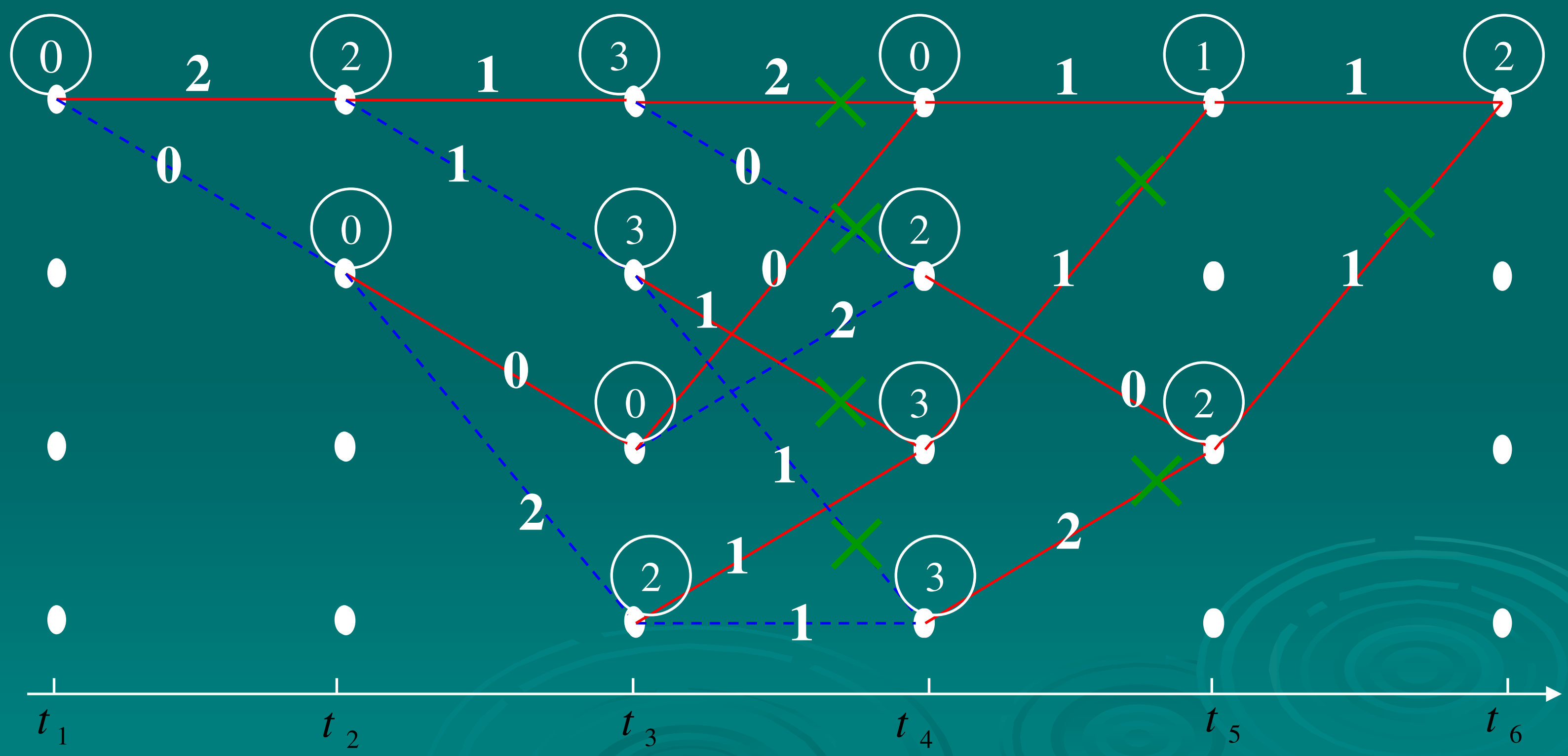
Example of Hard decision Viterbi decoding-cont'd

- i=5



Example of Hard decision Viterbi decoding-cont'd

- i=6



Example of Hard decision Viterbi decoding-cont'd

- Trace back and then:

$$\hat{\mathbf{m}} = (100)$$
$$\hat{\mathbf{U}} = (11 \quad 10 \quad 11 \quad 00 \quad 00)$$