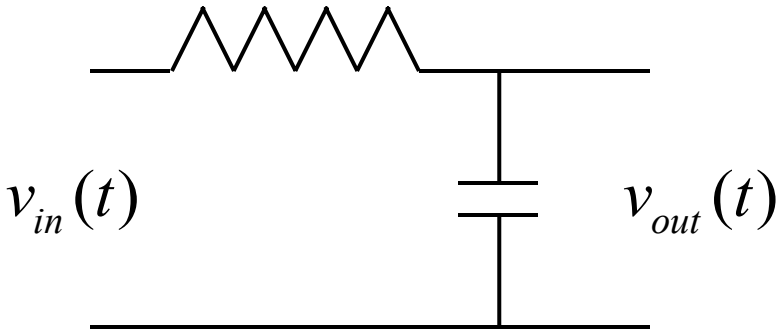
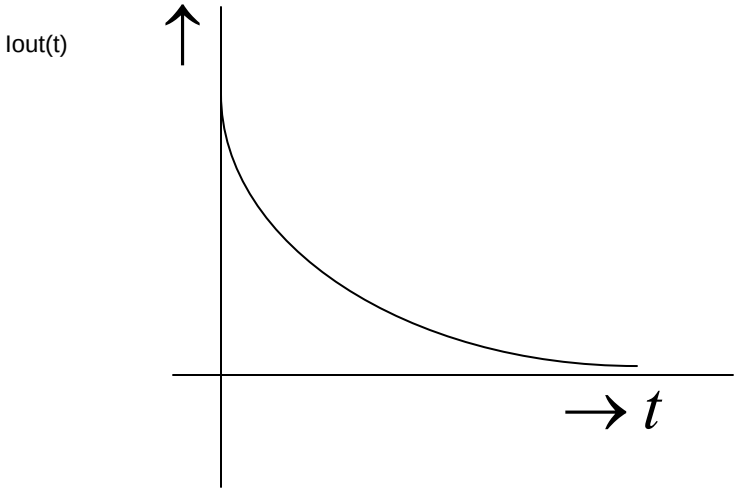
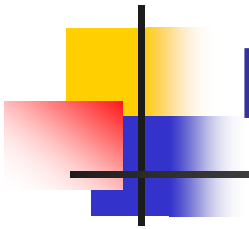
Deterministic Model

- Experiment produces same outcome each time





An actual RC circuit
… does not produce exactly the same outcome each time
… due to temperature variations and thermal noise



Probability Model

- Random experiment
  - *Repeating the experiment produces a different (a priori unknown) outcome each time*
    - Tossing a coin
    - Throwing dice
    - Setting a thermostat
    - Turning on a function generator
    - Powering up a flip-flop circuit
    - Dialing a phone number

Experiment: select ball from urn

- 3 balls, labeled 0, 1, and 2
  - *Blindly select one ball*
  - *Record label*
  - *Replace ball*
  - *Repeat*

a priori unknown!

… 0 2 2 1 0 2 1 1 0 2 0 0 1 2 1 2 0 1 2 0 ? …

a series of outcomes

outcome is element of the sample space $S = \{0, 1, 2\}$

Relative frequency of outcome

0 2 2 1 0 2 1 1 0 2 0 0 1 2 1 2 0 1 2 0 ? …

$N_k(n)$: total # times outcome *k* has occurred after *n* trials

$$N_0(20) = 7 \qquad N_1(20) = 6 \qquad N_2(20) = 7$$

$f_k(n) = \dfrac{N_k(n)}{n}$: Relative frequency (of occurrence) of outcome *k*

statistical regularity: $\lim_{n\to\infty} f_k(n) = p_k$

constant

probability of the outcome

Changing the experiment (condition)

- 3 balls, labeled 0, 0, and 2
  - *Blindly select one ball*
  - *Record label*
  - *Replace ball*
  - *Repeat*

a priori unknown!

a series of outcomes

… 0 2 2 0 0 2 0 0 0 2 0 0 0 2 0 2 0 0 2 0 ? …

each outcome is an element of the sample space $S = \{0, 2\}$

Relative frequency of outcome

0 2 2 0 0 2 0 0 0 2 0 0 0 2 0 2 0 0 2 0 ? …

$N_0(20) = 13$ $N_2(20) = 7$

changed how?

$f_k(n) = \frac{N_k(n)}{n}$: Relative frequency (of occurrence) of outcome *k*

statistical regularity: $\lim_{n\to\infty} f_k(n) = p_k$ probability of the outcome

constant

changed how?

Conditions under which a random experiment is performed determine probabilities of the outcomes of the experiment

Changing the experiment

- 3 balls, labeled 0, 1, and 2
  - *Blindly select one ball*
  - *Record label*
  - *Replace ball*
  - *Repeat*
- Add a ball, labeled 0 ⟹ sample space $S = \{0, 1, 2\}$

This changes the limits of RFO to $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right\}$

Conditions under which a random experiment is performed determine probabilities of the outcomes of the experiment

Properties of RFO

random experiment with K possible outcomes

$$S = \{1, 2, \ldots, K\}$$

$$0 \le N_k(n) \le n \quad for\ k = 1, 2, \ldots, K$$

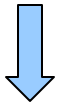
$$0 \le f_k(n) \le 1 \quad for\ k = 1, 2, \ldots, K$$

$$\sum_{k=1}^{K} N_k(n) = n$$

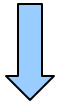
$$\sum_{k=1}^{K} f_k(n) = 1$$

Events

- Occurrence of events associated with the outcomes of an experiment
  - *e.g. An even-numbered ball is selected*

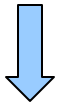


ball is labeled 0 or 2

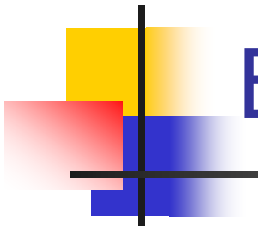


relative frequency of this event?

$$N_E(n) = N_0(n) + N_2(n)$$



$$f_E(n) = f_0(n) + f_2(n)$$



Events

- Event *C*: "*A* or *B* occurs," where *A* and *B* are events that cannot occur simultaneously

$$N_C(n) = N_A(n) + N_B(n)$$

$$f_C(n) = f_A(n) + f_B(n)$$

Basic properties of relative frequency

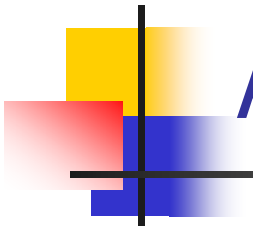
$$0 \le f_k(n) \le 1 \quad for\ k = 1, 2, ..., K$$

$$\sum_{k=1}^{K} f_k(n) = 1$$

$$f_C(n) = f_A(n) + f_B(n)$$

based on experiment and observation — practical

can be used to derive many further results — predictive



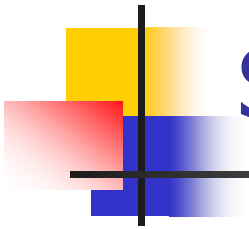
Axioms

- A random experiment has been defined, and a set *S* of all possible outcomes has been identified
- A class of subsets of *S* – called events – has been specified
- Each event *A* has been assigned a number, *P*[*A*], in such a way that the following axioms are satisfied:

1. $0 \le P[A] \le 1$
2. $P[S] = 1$
3. $P[A \text{ or } B] = P[A] + P[B]$

   when *A* and *B* are events that cannot occur simultaneously.

do these look familiar?



Specifying random experiments

- A random experiment is an experiment in which the outcome varies in an unpredictable fashion when the experiment is repeated under the same conditions.
- *A random experiment is specified by stating an experimental procedure and a set of one or more measurements or observations*

Random experiments

{ experimental procedure
measurement/observation

- E_1: select a ball from an urn containing balls numbered 1 to 50. Note the number of the ball.
- E_2: Select a ball from an urn containing balls numbered 1 to 4. Suppose that balls 1 and 2 are black and that balls 3 and 4 are white. Note the number and color of the ball you select.
- E_3: Toss a coin three times and note the sequence of heads and tails.
- E_4: Toss a coin three times and note the number of heads.
- E_5: Count the number of voice packets containing only silence produced from a group of *N* speakers in a 10ms period.
- E_6: A block of information is transmitted repeatedly over a noisy channel until an error-free block arrives at the receiver. Count the number of transmissions required.
- E_7: Pick a number at random between zero and one.
- E_8: Measure the time between two message arrivals at a message center.