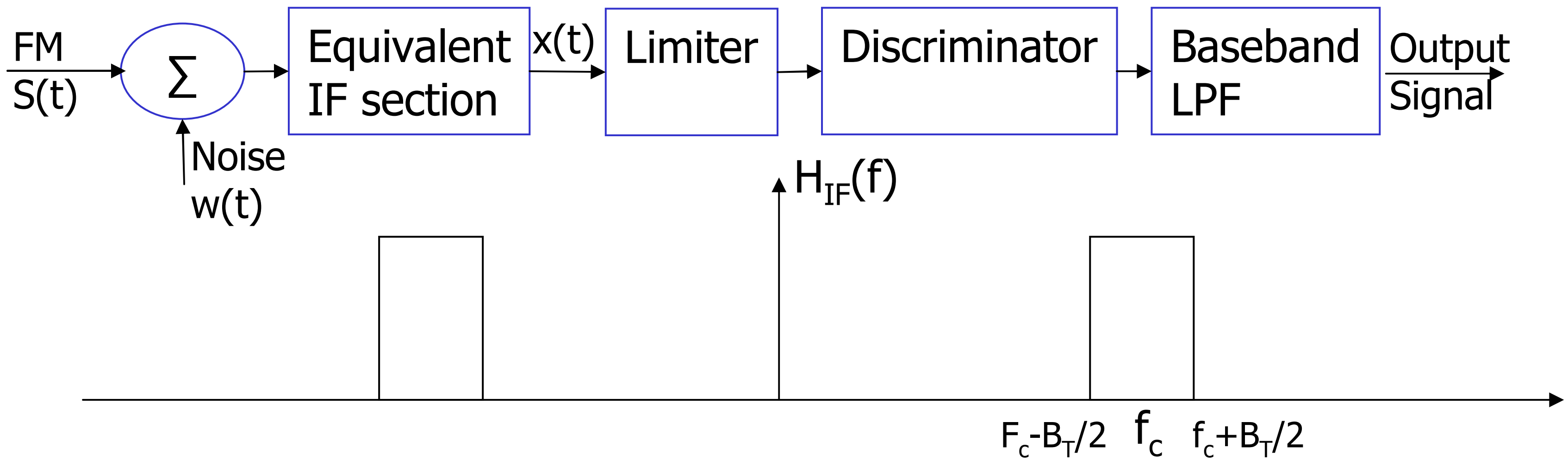
Communication Systems II

Noise in CW modulation

FM Modulation

FM Receiver Model

- W(t) is modeled as white Gaussian noise with zero mean and PSD of $N_0/2$.
- FM signal of center frequency f_c and BW $=B_T$



- IF filter is assumed IBPF with bandwidth B_T
  - We can use the narrow band noise representation in terms of its inphase and quadrature components.

Noise in FM reception

- The NBN at the IF output is defined as

$$n(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

- Equivalently

$$n(t) = r(t)\cos(2\pi f_c t + \psi(t))$$

$$r(t) = \sqrt{n_c^2(t) + n_s^2(t)}$$

$$\psi(t) = \tan^{-1}\frac{n_s(t)}{n_c(t)}$$

- r(t) is Rayleigh distributed and phase is uniformely distributed over 2π

Noise in FM reception

- The FM signal at the IF output is

$$s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t m(t)dt)$$

$$\phi(t) = 2\pi k_f \int_0^t m(t)dt$$

$$s(t) = A_c \cos(2\pi f_c t + \phi(t))$$

- The total signal at the output of IF section is
  - X(t)=s(t)+n(t)

$$x(t) = A_c \cos(2\pi f_c t + \phi(t)) + r(t)\cos(2\pi f_c t + \psi(t))$$

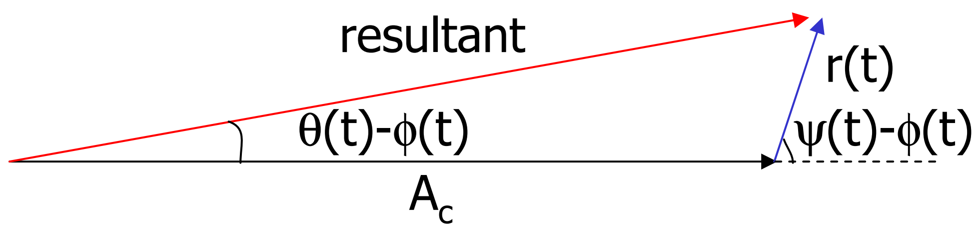
Noise in FM reception

- Represent x(t) by means of a phasor diagram

$$x(t) = A_c \cos(2\pi f_c t + \phi(t)) + r(t)\cos(2\pi f_c t + \psi(t))$$

- The relevant phase θ(t) can be calculated

$$\theta(t) = \phi(t) + \tan^{-1}\left\{\frac{r(t)\sin(\psi(t) - \phi(t))}{A_c + r(t)\cos(\psi(t) - \phi(t))}\right\}$$



Noise in FM reception

- Envelope of x(t) is not of interest to us
- The output of the ideal discriminator will be proportional to θ′(t)/2π

$$\theta(t) = \phi(t) + \tan^{-1}\left\{\frac{r(t)\sin(\psi(t)-\phi(t))}{A_c + r(t)\cos(\psi(t)-\phi(t))}\right\}$$

$$\therefore \theta(\mathrm{t}) \approx \phi(\mathrm{t}) + \frac{\mathrm{r(t)}}{A_c}\sin(\psi(\mathrm{t}) - \phi(\mathrm{t}))$$

$$\theta(\mathrm{t}) \approx 2\pi k_f \int_0^t m(t)dt + \frac{\mathrm{r(t)}}{A_c}\sin(\psi(\mathrm{t}) - \phi(\mathrm{t}))$$

$$\mathrm{v(t)\ discriminator\ output} = \frac{1}{2\pi}\theta'(\mathrm{t})$$

$$\mathrm{v(t)} = k_f m(t) + n_d(t)$$

$$where\ n_d(t) = \frac{1}{2\pi}\frac{d}{dt}\{\frac{\mathrm{r(t)}}{A_c}\sin(\psi(\mathrm{t}) - \phi(\mathrm{t}))\}$$

- Assumptions
  - Carrier to noise ratio is large compared with unity

Noise in FM reception

- The noise component can be simplified further to

$$n_d(t) \approx \frac{1}{2\pi}\frac{d}{dt}\{\frac{\mathrm{r(t)}}{A_c}\sin(\psi(\mathrm{t}))\}$$

however

$$\mathrm{n_s}(t) = r(t)\sin(\psi(\mathrm{t}))$$

$$n_d(t) \approx \frac{1}{2\pi}\frac{d}{dt}\{\frac{\mathrm{n_s}(t)}{A_c}\}$$

- The power in the output message is k_f^2P where p is power in transmitted message
- Required to find the power in the noise component
  - Derivative in time → j2πf in frequency

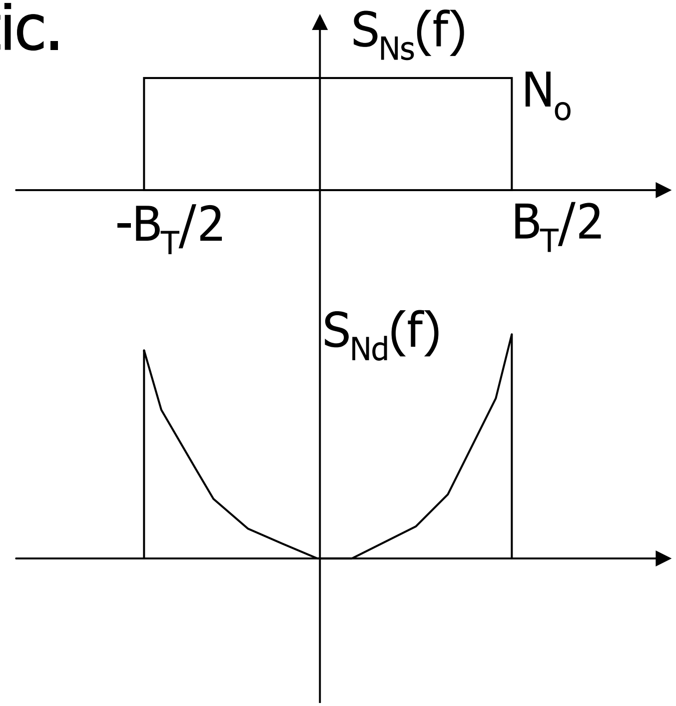
$$\therefore \mathrm{S_{n_d}}(f) = \frac{f^2}{A_c^2}\mathrm{S_{n_s}}(f)$$

Noise in FM reception

- With the Equivalent IF filter has ideal response.
  - It follows that the narrow-band noise n(t) will have a PSD similar in shape to the transfer function of the IF filter
  - Therefore the quadrature component of the NBN will have the ideal low pass characteristic.
  - The corresponding PSD of n_d will be

$$\therefore S_{n_d}(f) = \frac{f^2}{A_c^2} S_{n_s}(f)$$

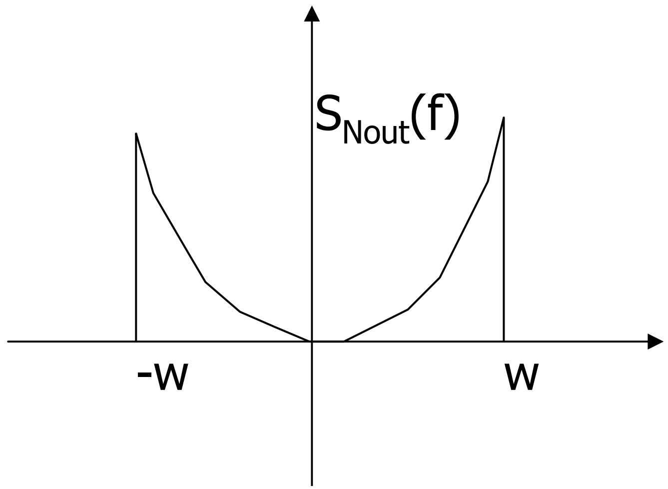
$$S_{n_d}(f) = \begin{cases} \dfrac{N_o f^2}{A_c^2}, & |f| \le \dfrac{B_T}{2} \\ 0, & elsewhere \end{cases}$$



Noise in FM reception

- The discriminator output is followed by a low pass filter with bandwidth equal to the bandwidth of the message W.
  - $W >> B_T/2$
  - Therefore the noise component $n_d(t)$ will have rejection region.
  - The corresponding PSD of n_d after LPF will be

$$S_{n_{out}}(f) = \begin{cases} \dfrac{N_o f^2}{A_c^2}, & |f| \leq W \\ 0, & elsewhere \end{cases}$$



Noise in FM reception

- The average output noise power is determined by integrating the PSD S_{nout} from –W to W

$$average\ \text{power of output noise} = \frac{\text{N}_\text{o}}{\text{A}_\text{c}^2}\int_{-W}^{w} f^2 df$$

$$= \frac{2N_o}{3A_c^2}W^3$$

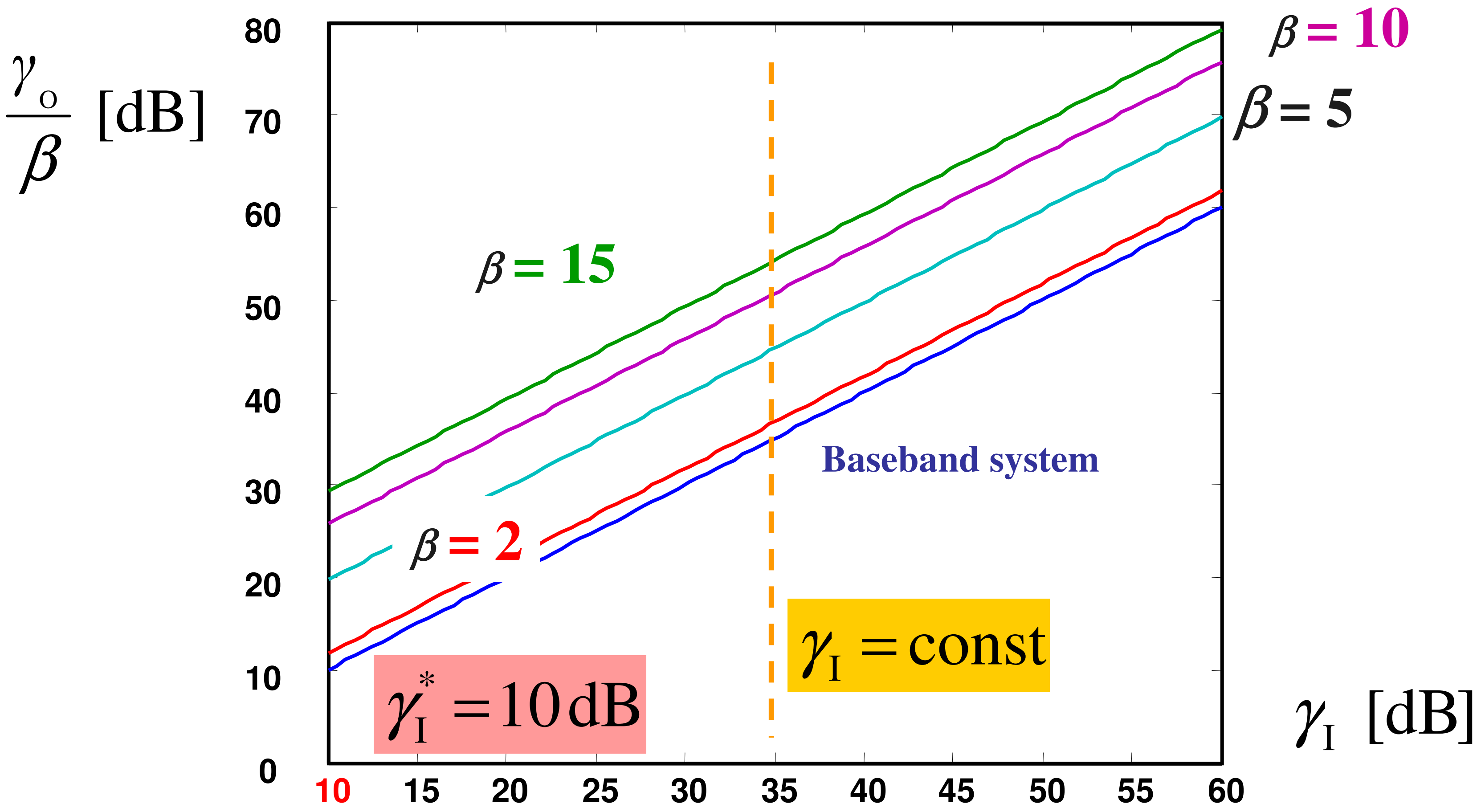
- Note that the average output noise power is inversely proportional to the carrier power $A_c^2/2$

$$SNR_{c,FM} = \frac{A_c^2}{2N_oW}$$

$$SNR_{o,FM} = \frac{3A_c^2k_f^2P}{2N_oW^3}$$

$$Figure\ \text{of Merit} = \frac{3k_f^2P}{W^2}$$

FM Noise Characteristic



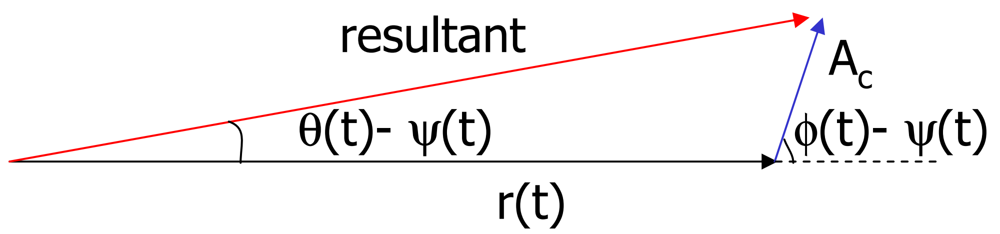
What if the noise is bigger than the signal

- Represent x(t) by means of a phasor diagram

$$x(t) = A_c \cos(2\pi f_c t + \phi(t)) + r(t)\cos(2\pi f_c t + \psi(t))$$

- $r(t) >>> A_c$
- The relevant phase θ(t) can be calculated

$$\theta(t) = \psi(\mathrm{t}) + \tan^{-1}\left\{ \frac{A_c \sin(\phi(t) - \psi(t))}{r(t) + A_c \cos(\phi(t) - \psi(t))} \right\}$$



Noise >>> FM reception

- Envelope of x(t) is not of interest to us
- The output of the ideal discriminator will be proportional to $\theta'(t)/2\pi$

$$\theta(t) = \psi(t) + \tan^{-1}\left\{\frac{A_c \sin(\phi(t) - \psi(t))}{r(t) + A_c \cos(\phi(t) - \psi(t))}\right\}$$

$$\therefore \theta(t) \approx \psi(t) + \frac{A_c}{r(t)} \sin(\phi(t) - \psi(t))$$

- Assumptions
  - Carrier to noise ratio is small compared with unity
- V(t) is complete noise
  - No detection → FM threshold effect
  - SNR>1

$$\theta(t) \approx \psi(t) + \frac{A_c}{r(t)} \sin(2\pi k_f \int_0^t m(t)dt - \psi(t))$$

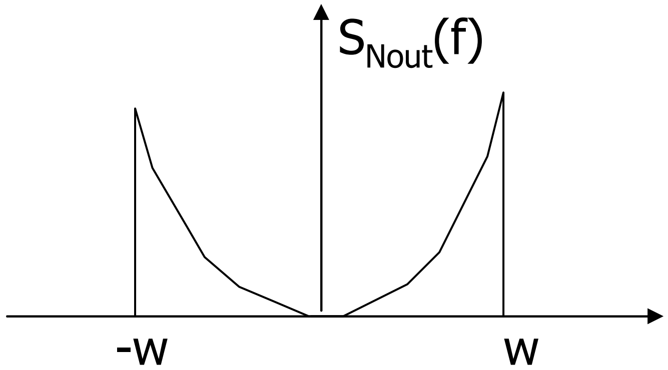
$$v(t)\ \text{discriminator output} = \frac{1}{2\pi}\theta'(t)$$

$$v(t) = \frac{d\psi(t)}{2\pi dt} + n_d(t)$$

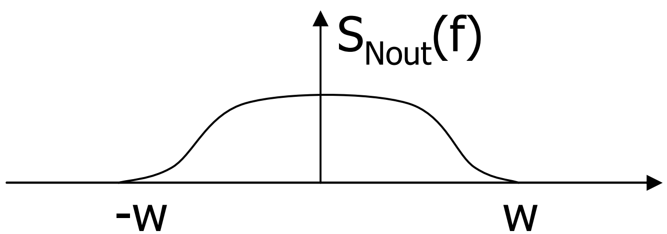
$$where\ n_d(t) = \frac{1}{2\pi}\frac{d}{dt}\{\frac{A_c}{r(t)}\sin(2\pi k_f \int_0^t m(t)dt - \psi(t))\}$$

Pre-emphasis and De-emphasis in FM

- The PSD of the noise at the receiver output has a square law dependence on the operating frequency

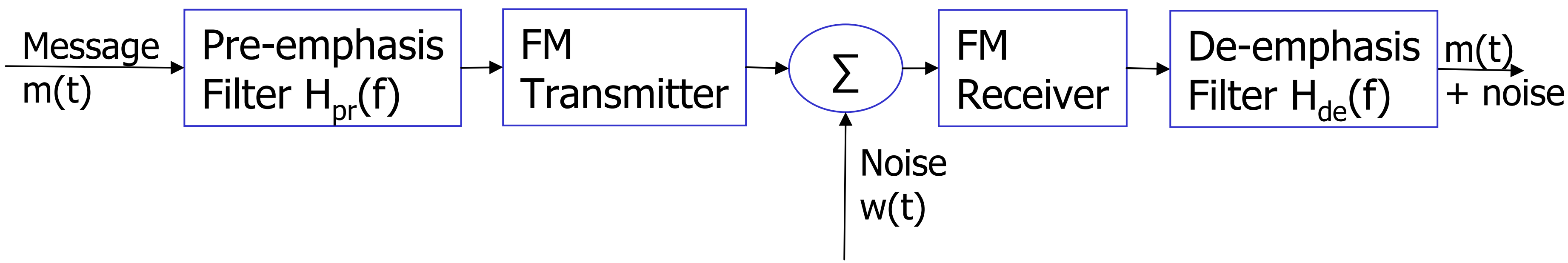


- The PSD of a typical message source (audio, video) has the following general shape
- The PSD falls significantly at high frequencies while the noise PSD increases rapidly with frequency



Pre-emphasis and De-emphasis in FM

- One way to improve the noise performance of the system is to slightly reduce the bandwidth of the post detection low pass filter so to reject large amount of noise power → unacceptable
- Another method is to use a pre-emphasis at the transmitter and a de-emphasis at the receiver as shown in the following figure



Pre-emphasis and De-emphasis in FM

- In this method, emphasize the high frequency components of the message signal prior to modulation in the transmitter.
- The low frequency and the high frequency portions of the message are equalized to occupy the whole frequency band allocated to it.
- At the discriminator output, inverse process is performed to de-emphasize the high frequency components to restore the original signal power distribution of the message.
  - The high frequency portion of the noise will be reduced thus increasing the output signal to noise ratio of the system

Pre-emphasis and De-emphasis in FM

- In order to produce an undistorted version of the original message at the receiver output, the pre-emphasis and de-emphasis must ideally have transfer function that is inverse to each other

$$H_{de}(f) = \frac{1}{H_{pe}(f)}, \qquad -W \le f \le W$$

- This makes the message power independent of this operation.
- For the noise component after the discriminator

$$S_{n_d}(f) = \begin{cases} \dfrac{N_o f^2}{A_c^2}, & |f| \le \dfrac{B_T}{2} \\ 0, & elsewhere \end{cases}$$

Pre-emphasis and De-emphasis in FM

- Thus the modified PSD of the noise at the de-emphasis output is equal to $\left|H_{de}(f)\right|^2 S_{n_d}(f)$

- Since the final LPF has BW= W<<B_T/2

$$Av.\text{ o/p noise power}$$

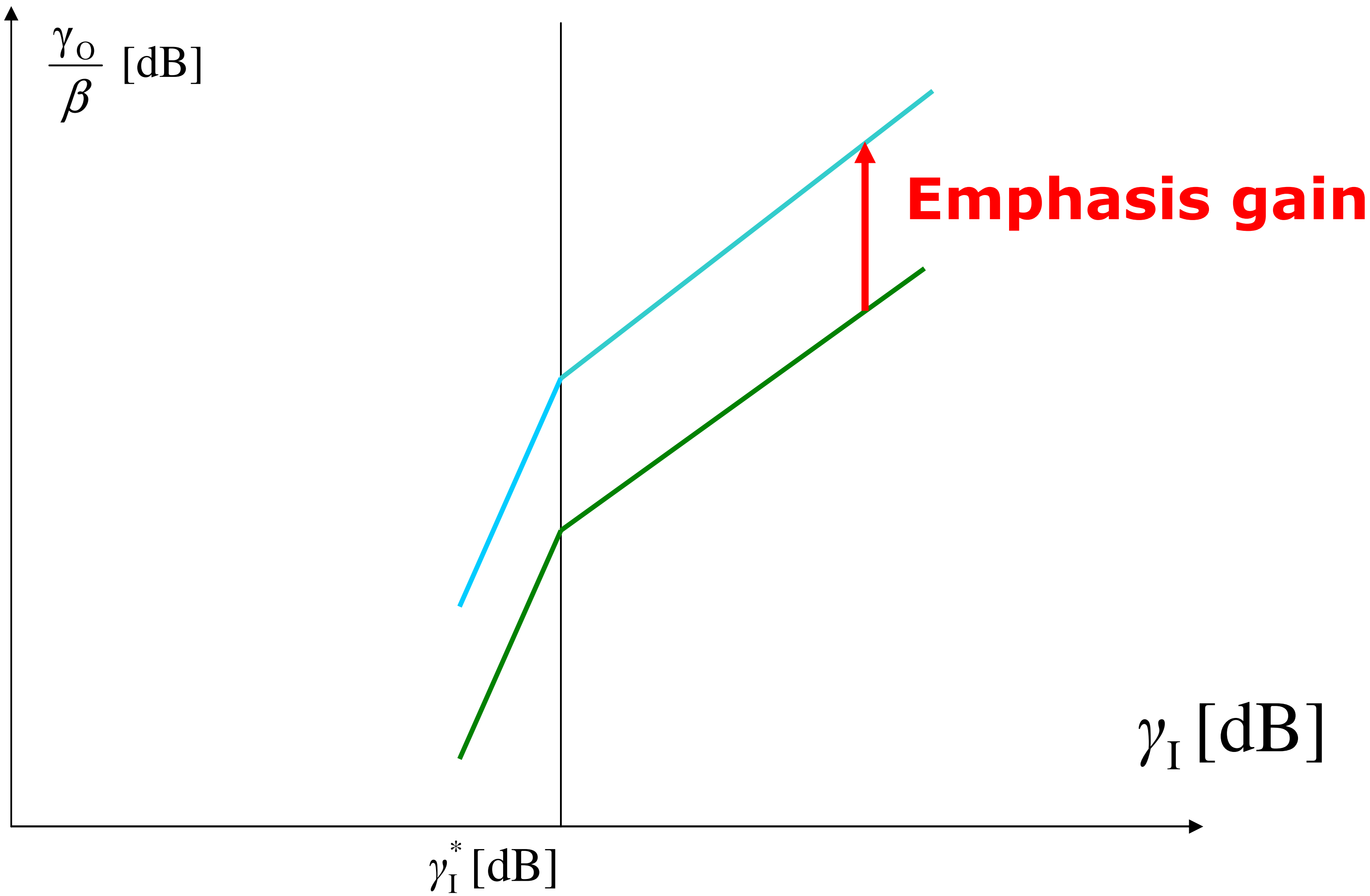
$$\text{with de-emphasis} = \frac{N_o}{A_c^2}\int_{-w}^{w} f^2 \left|H_{de}(f)\right|^2 df$$

- The improvement in o/p SNR due to de-emphasis is

$$D = \frac{\text{Average o/p noise power without emphasis}}{\text{Average o/p noise power with emphasis}}$$

$$D = \frac{2W^3}{3\int_{-w}^{w} f^2 \left|H_{de}(f)\right|^2 df}$$

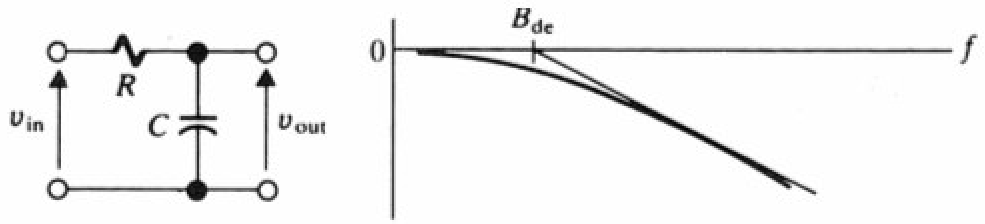
Noise characteristics



FM preemphases and deemphases filters

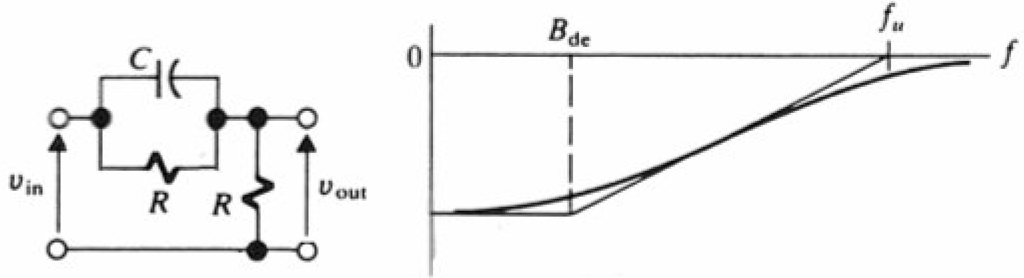
- FM related noise emphases can be suppressed by *pre-distortion* and post detection filters (preemphases and deemphases filters):

receiver filter



$H_{de}(f)$

transmitter filter



$H_{pe}(f)$

$$H_{de}(f) = \left[1 + j(f / B_{de})\right]^{-1} \approx \begin{cases} 1, |f| << B_{de} \\ B_{de} / (jf), |f| >> B_{de} \end{cases}$$

LPF

$$H_{pe}(f) = \left[1 + j(f / B_{de})\right] \approx \begin{cases} j(f / B_{de}), f_u > |f| > B_{de} \\ 1, |f| >> f_u \end{cases}$$

HPF

FM post-detection S/N with deemphases

- Deemphases filter (that is a lowpass filter connected after detector) can suppress noise further. FM post-detection noise PSD and total noise power without deemphases:

$$G_{FM}(f) = \frac{N_0 f^2}{2S_R}\Pi\left(\frac{f}{B_T}\right) \quad N_D = \int_{-W}^{W} G_{FM}(f)df = \frac{N_0 W^3}{3S_R}$$

- With deemphases filter (for simplification assume W/B_{de}>>1):

$$N_D = \int_{-W}^{W} G_{FM}(f)\left|H_{de}(f)\right|^2 df = \frac{N_0 B_{de}^3}{S_R}\left[\frac{W}{B_{de}} - \underbrace{\arctan\frac{W}{B_{de}}}_{W/B_{de} >> \arctan(W/B_{de})}\right] \approx N_0 B_{de}^2 W / S_R$$

where

$$\left|H_{de}(f)\right|^2 = \frac{1}{1+(f/B_{de})^2}$$

$W >> B_{de}$

$$S_D/N_D = \frac{f_\Delta^2 S_x}{N_0 B_{de}^2 W / S_R} = \frac{S_x S_R}{N_0 W}\left(\frac{f_\Delta}{B_{de}}\right)^2 = \underline{S_x \gamma \left(\frac{f_\Delta}{B_{de}}\right)^2}$$

Example

- FM radio

$$f_\Delta = 75 \text{ kHz}, W = 15 \text{ kHz}, D = 5, S_x = 1/2, B_{de} = 2.1\text{kHz}$$

- Without deemphases

$$S_D / N_D = 3D^2 S_x \gamma$$

$$= (3 \times 5^2 \times ½)\gamma = 38\gamma$$

$$\gamma = \frac{S_R}{N_o W}$$

- With deemphases

$$S_D / N_D = (f_\Delta / B_{de})^2 S_x \gamma \approx 640\gamma$$

- Therefore if DSB or SSB system could be exchanged to FM system 640 fold transmission power savings could be achieved.
- Note, however that the required transmission bandwidth is now about 220 kHz /15 kHz = 15 times larger!