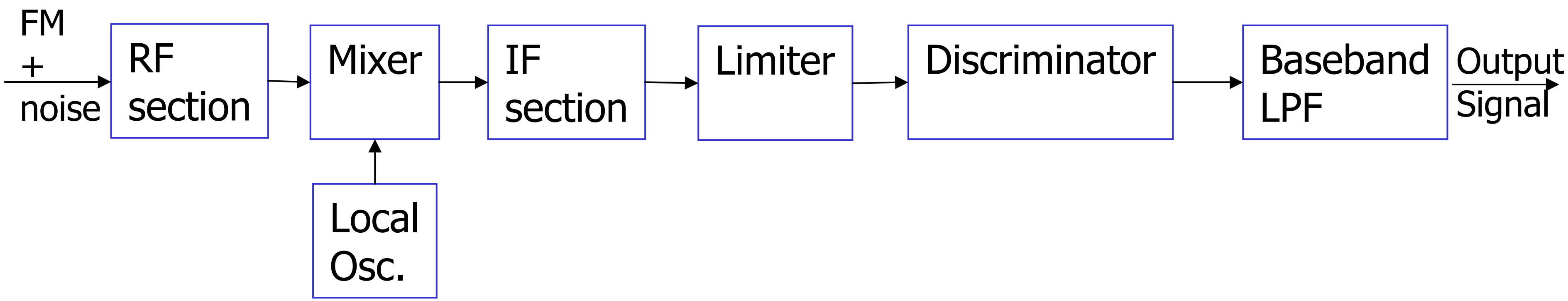
Communication Systems II

Noise in CW modulation

FM Modulation

FM Receivers

- FM receivers are also of superheterodyne type.
- Typical frequencies for commercial FM radio are:
  - RF carrier range: 88-108 MHz
  - Mid band frequency of IF section: 10.7 MHz
  - IF bandwidth :0.2 MHz



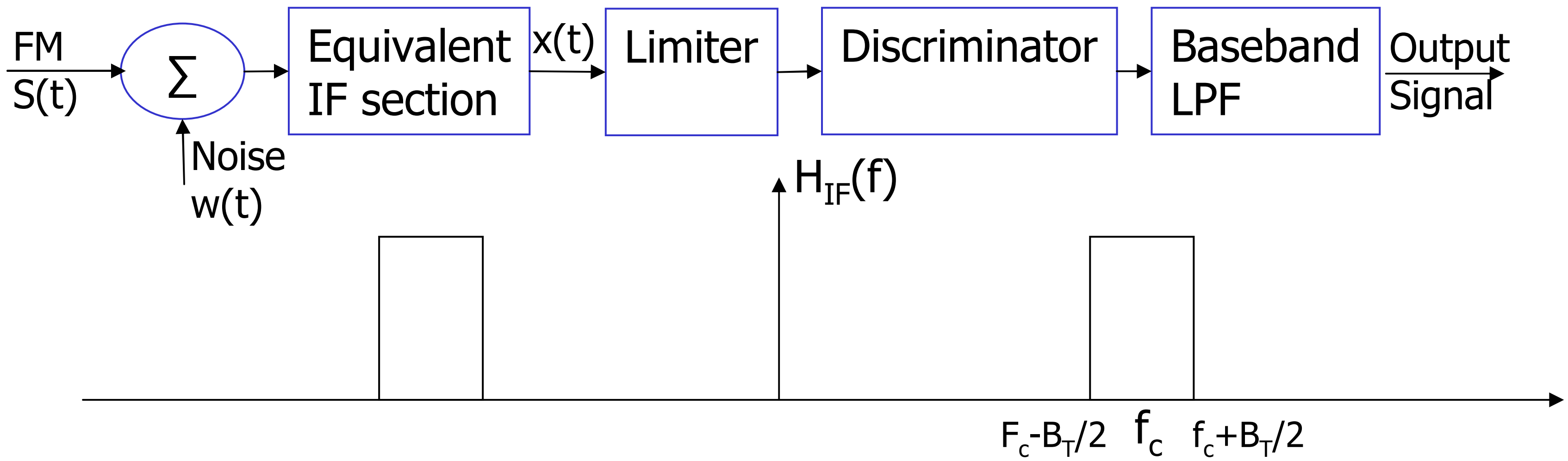
- The amplitude limiter removes amplitude variations by clipping the modulated wave at the IF section output.
- The resulting rectangular wave is rounded off by a bandpass filter which suppresses harmonics of the carrier frequency.

FM Receivers

- The discriminator consists of two components
  - A slope network or differentiator with purely imaginary transfer function that varies linearly with frequency.
    - Produces a hybrid modulation of amplitude and frequency
  - An envelope detector that recovers the amplitude variation and thus reproduces the message signal.
- The post detection filter removes the out of band components of the noise at the discriminator output and thus keeps the output noise as small as possible.

FM Receiver Model

- W(t) is modeled as white Gaussian noise with zero mean and PSD of $N_0/2$.
- FM signal of center frequency f_c and BW $=B_T$



- IF filter is assumed IBPF with bandwidth B_T
  - We can use the narrow band noise representation in terms of its inphase and quadrature components.

Noise in FM reception

- The NBN at the IF output is defined as

$$n(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

- Equivalently

$$n(t) = r(t)\cos(2\pi f_c t + \psi(t))$$

$$r(t) = \sqrt{n_c^2(t) + n_s^2(t)}$$

$$\psi(t) = \tan^{-1}\frac{n_s(t)}{n_c(t)}$$

- r(t) is Rayleigh distributed and phase is uniformely distributed over 2π

Noise in FM reception

- The FM signal at the IF output is

$$s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t m(t)dt)$$

$$\phi(t) = 2\pi k_f \int_0^t m(t)dt$$

$$s(t) = A_c \cos(2\pi f_c t + \phi(t))$$

- The total signal at the output of IF section is
  - X(t)=s(t)+n(t)

$$x(t) = A_c \cos(2\pi f_c t + \phi(t)) + r(t)\cos(2\pi f_c t + \psi(t))$$

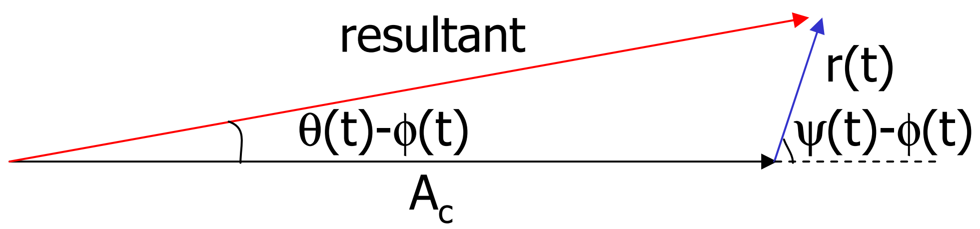
Noise in FM reception

- Represent x(t) by means of a phasor diagram

$$x(t) = A_c \cos(2\pi f_c t + \phi(t)) + r(t)\cos(2\pi f_c t + \psi(t))$$

- The relevant phase θ(t) can be calculated

$$\theta(t) = \phi(t) + \tan^{-1}\left\{\frac{r(t)\sin(\psi(t)-\phi(t))}{A_c + r(t)\cos(\psi(t)-\phi(t))}\right\}$$



Noise in FM reception

- Envelope of x(t) is not of interest to us
- The output of the ideal discriminator will be proportional to $\theta'(t)/2\pi$

$$\theta(t) = \phi(t) + \tan^{-1}\left\{ \frac{r(t)\sin(\psi(t) - \phi(t))}{A_c + r(t)\cos(\psi(t) - \phi(t))} \right\}$$

$$\therefore \theta(\mathrm{t}) \approx \phi(\mathrm{t}) + \frac{\mathrm{r(t)}}{A_c}\sin(\psi(\mathrm{t}) - \phi(\mathrm{t}))$$

$$\theta(\mathrm{t}) \approx 2\pi k_f \int_0^t m(t)dt + \frac{\mathrm{r(t)}}{A_c}\sin(\psi(\mathrm{t}) - \phi(\mathrm{t}))$$

- Assumptions
  - Carrier to noise ratio is large compared with unity

$$\mathrm{v(t)\ discriminator\ output} = \frac{1}{2\pi}\theta'(\mathrm{t})$$

$$\mathrm{v(t)} = k_f m(t) + n_d(t)$$

$$where\ n_d(t) = \frac{1}{2\pi}\frac{d}{dt}\{\frac{\mathrm{r(t)}}{A_c}\sin(\psi(\mathrm{t}) - \phi(\mathrm{t}))\}$$

Noise in FM reception

- The noise component can be simplified further to

$$n_d(t) \approx \frac{1}{2\pi}\frac{d}{dt}\{\frac{\mathrm{r(t)}}{A_c}\sin(\psi(\mathrm{t}))\}$$

however

$$\mathrm{n_s}(t) = r(t)\sin(\psi(\mathrm{t}))$$

$$n_d(t) \approx \frac{1}{2\pi}\frac{d}{dt}\{\frac{\mathrm{n_s}(t)}{A_c}\}$$

- The power in the output message is k_f^2P where p is power in transmitted message
- Required to find the power in the noise component
  - Derivative in time → j2πf in frequency

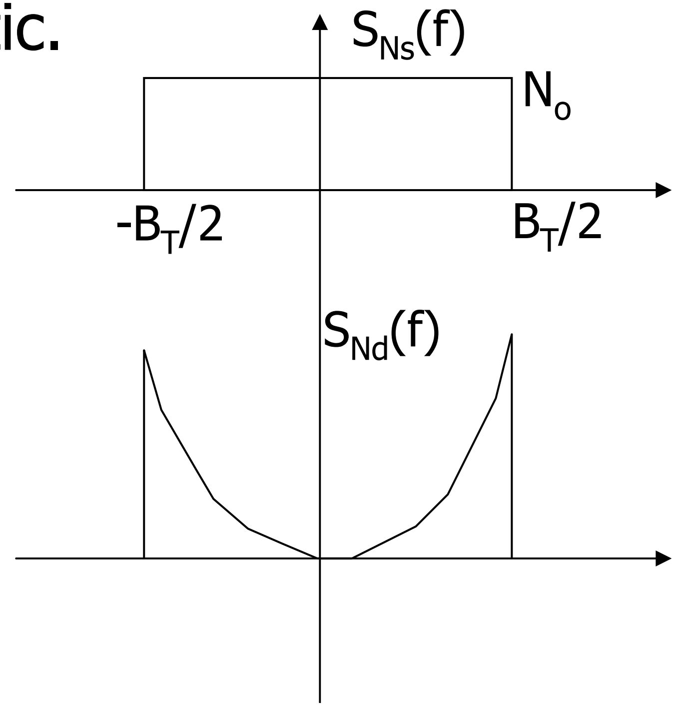
$$\therefore \mathrm{S}_{\mathrm{n_d}}(f) = \frac{f^2}{A_c^2}\mathrm{S}_{\mathrm{n_s}}(f)$$

Noise in FM reception

- With the Equivalent IF filter has ideal response.
  - It follows that the narrow-band noise n(t) will have a PSD similar in shape to the transfer function of the IF filter
  - Therefore the quadrature component of the NBN will have the ideal low pass characteristic.
  - The corresponding PSD of n_d will be

$$\therefore S_{n_d}(f) = \frac{f^2}{A_c^2} S_{n_s}(f)$$

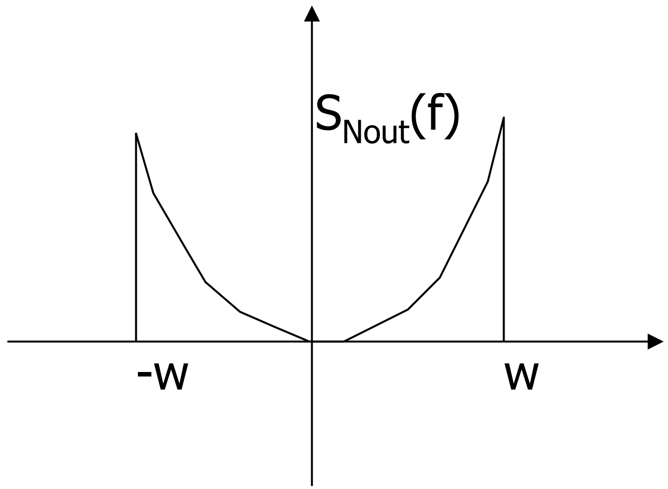
$$S_{n_d}(f) = \begin{cases} \frac{N_o f^2}{A_c^2}, & |f| \le \frac{B_T}{2} \\ 0, & elsewhere \end{cases}$$



Noise in FM reception

- The discriminator output is followed by a low pass filter with bandwidth equal to the bandwidth of the message W.
  - $W >> B_T/2$
  - Therefore the noise component $n_d(t)$ will have rejection region.
  - The corresponding PSD of n_d after LPF will be

$$S_{n_{out}}(f) = \begin{cases} \dfrac{N_o f^2}{A_c^2}, & |f| \leq W \\ 0, & elsewhere \end{cases}$$



Noise in FM reception

- The average output noise power is determined by integrating the PSD S_{nout} from –W to W

$$average\ \text{power of output noise} = \frac{\mathrm{N_o}}{\mathrm{A_c^2}} \int_{-W}^{w} f^2 df$$

$$= \frac{2N_o}{3A_c^2} W^3$$

- Note that the average output noise power is inversely proportional to the carrier power $A_c^2/2$

$$SNR_{c,FM} = \frac{A_c^2}{2N_o W}$$

$$SNR_{o,FM} = \frac{3A_c^2 k_f^2 P}{2N_o W^3}$$

$$Figure\ \text{of Merit} = \frac{3k_f^2 P}{W^2}$$

Single tone modulation example

- Consider the case of single tone modulation with maximum frequency deviation Δf.

$$s(t) = A_c \cos(2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t))$$

$$m(t) = \frac{\Delta f}{k_f} \cos(2\pi f_m t)$$

$$P = \frac{\Delta f^2}{2k_f^2}$$

$$SNR_{o,FM} = \frac{3A_c^2 \Delta f^2}{4N_o W^3} = \frac{3A_c^2}{4N_o W}\beta^2$$

$$FoM = \frac{3}{2}\beta^2$$

Narrow Band FM

- NBFM→ β<1/3

$$s_{NBFM}(t) = A_c \cos(2\pi f_c t) - A_c \sin(2\pi f_c t)[k_f \int_0^t m(t)dt]$$

$$P = \frac{A_c^2}{2}$$

$$R_{s_{NBFM}}(\tau) = \frac{A_c^2}{2}\cos(2\pi f_c \tau) + \frac{A_c^2}{2}\cos(2\pi f_c \tau) R_{\phi_m}(\tau)$$

$$\phi_m = \begin{cases} k_p m(t), PM \\ k_f \int_0^t m(t)dt, FM \end{cases}$$

$$S_m(f) = \begin{cases} k_p^2 S_m(f) \\ \dfrac{k_f^2 S_m(f)}{(2\pi f)^2} \end{cases}$$

- Received signal

$$x(t) = A_c \cos(2\pi f_c t) - A_c \sin(2\pi f_c t)[k_f \int_0^t m(t)dt] + w(t)$$

$$SNR_{c,NBFM} = \frac{A_c^2}{2N_0 W_{ch}}$$

after IF section

$$SNR_{NBFM} = \frac{A_c^2}{2N_0 2W}$$

$$P = \frac{A_c^2}{2}$$

$$SNR_{o,FM} = \frac{3A_c^2 k_f^2 P}{2N_o W^3}$$

$$\textit{Figure of Merit} = \frac{3k_f^2 P}{W^2} = \frac{3}{2}\beta^2, \beta << 1$$