Communication Systems II

Noise in CW modulation

SSB, QAM, Envelope Detection

Signal to noise ratio SNR (S/R)

- The SNR can be measured at the output of the receiver as follows

$$(SNR)_o = \frac{Average \text{ power of the message signal at the receiver output}}{\text{Average power of noise at the receiver output}}$$

- The SNR requires that noise and signal are additive and this is satisfied by linear receivers as coherent detector.
- SNR depends on the modulation used and the type of receiver.
- This suggest comparative evaluation w.r.t a reference point.
- Reference point is taken to be the input to the receiver from the channel
- $$(SNR)_c = \frac{Average \text{ power of the message signal at the receiver input}}{\text{Average power of noise at the receiver input}}$$
- We can normalize the performance by dividing the two equations

$$Figure \text{ of merit} = \frac{\text{SNR}_\text{o}}{\text{SNR}_\text{c}}$$

SNR of Base Band signal

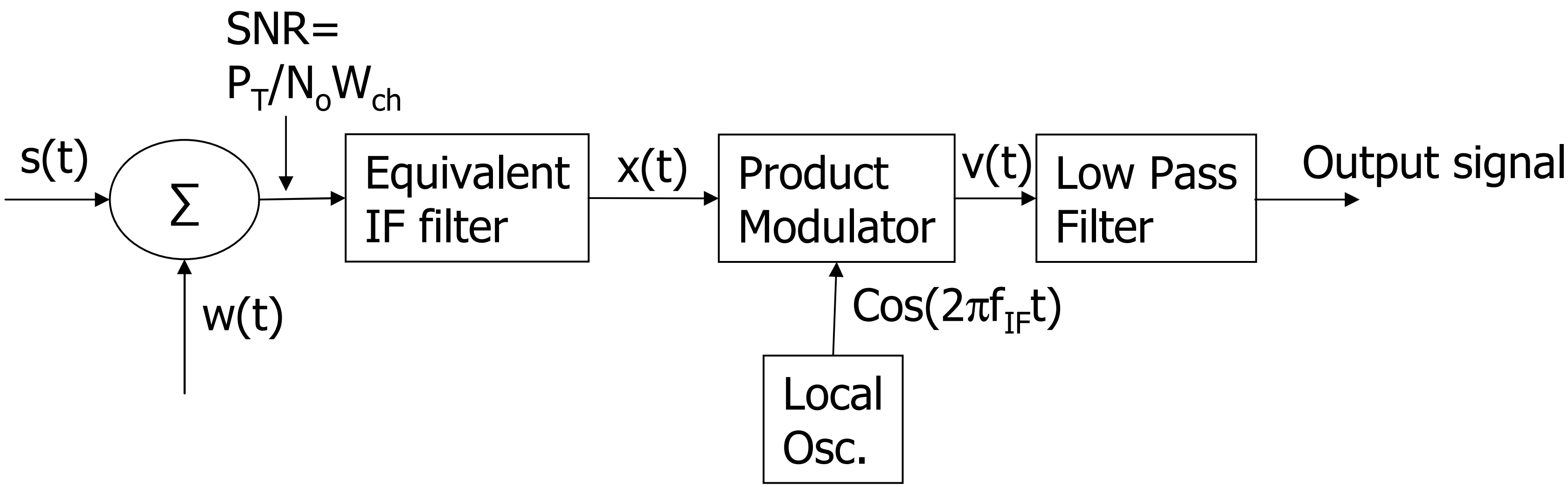
- S(t)=m(t)+w(t)
- After LPF
  - X(t)=m(t)+n(t)

$$SNR_{BB} = \frac{P_T}{\frac{N_o}{2} 2W}$$

P_T Power in the transmitted signal

SNR for Coherent Reception of SSB Modulation

- The IF filter output is multiplied by a local carrier with frequency f_{IF} and then low pass filtering the product



- Consider SSB-LSB

$$s(t) = \frac{A_c}{2} m(t)\cos(2\pi f_c t) + \frac{A_c}{2} \hat{m}(t)\sin(2\pi f_c t)$$

where m(t) is considered a sample function of a stationary process of zero mean and PSD $S_M(f)$ limited to maximum frequency W

SNR for Coherent Reception of SSB Modulation

- m(t) and m^(t) are orthogonal to each other.
  - If m(t) is zero mean → m(t) and m^(t) are uncorrelated
  - Their PSD are additive →
  - m^(t) is obtained by passing m(t) on a LTIS with H(f)=-jsgn(f) → square magnitude is equal to one → m^(t) have the same PSD as m(t)

$$z = m(t) + \hat{m}(t)$$

$$R_z(t) = E[z(t)z(t+\tau)]$$

$$= E[m(t)m(t+\tau)] + E[\hat{m}(t)\hat{m}(t+\tau)] + zeros$$

$$R_z(\tau) = R_m(\tau) + R_{\hat{m}}(\tau)$$

$$S_z(f) = S_m(f) + S_{\hat{m}}(f)$$

SNR for Coherent Reception of SSB Modulation

- Inphase component $\frac{A_c}{2} m(t)\cos(2\pi f_c t)$
- Quadrature component $\frac{A_c}{2} \hat{m}(t)\sin(2\pi f_c t)$
- Average power in the inphase component = average power in quadrature component= $\frac{A_c^2}{8} P_m$
- Average power in modulated signal s(t)= $\frac{A_c^2}{4} P_m$
  - Half that of DSB-SC
- Channel noise power in message bandwidth is $N_o W$
- $$SNR_c = \frac{A_c^2 P_m}{4 N_o W}$$

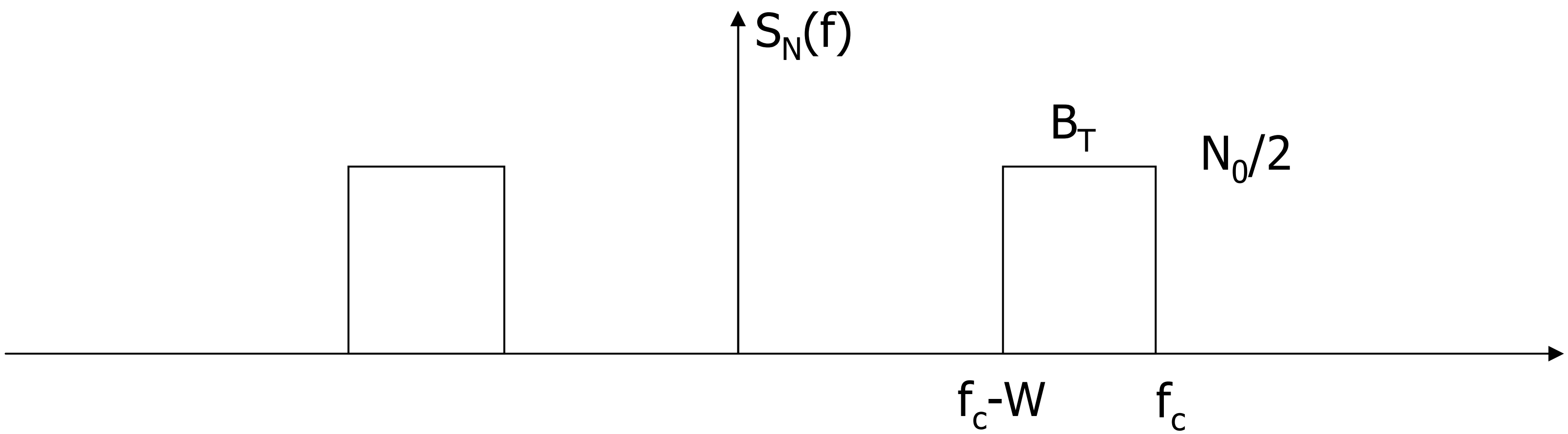
SNR for Coherent Reception of SSB Modulation

- The NBN can be represented as

$$n(t) = n_c(t)\cos(2\pi(f_c - \frac{w}{2})t) - n_s(t)\sin(2\pi(f_c - \frac{w}{2})t)$$

- The output of the coherent detector

$$y(t) = [s(t) + n(t)]\cos(2\pi f_c t)$$



SNR for Coherent Reception of SSB Modulation

- The output of the coherent detector

$$y(t) = \frac{A_c}{2} m(t)\cos^2(2\pi f_c t) + \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)\cos(2\pi f_c t)$$

$$+$$

$$n_c(t)\cos(2\pi(f_c - \frac{w}{2})t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi(f_c - \frac{w}{2})t)\cos(2\pi f_c t)$$

$$= \frac{A_c}{4} m(t) + \frac{A_c}{4} m(t)\cos(2\pi 2 f_c t) + \frac{A_c}{4}\hat{m}(t)\sin(2\pi 2 f_c t)$$

$$+$$

$$\frac{n_c(t)}{2}\cos(2\pi \frac{w}{2} t) + \frac{n_s(t)}{2}\sin(\frac{w}{2} t) + \text{higher frequency terms}$$

SNR for Coherent Reception of SSB Modulation

- The output of the after LPF

$$= \frac{A_c}{4} m(t) + \frac{n_c(t)}{2} \cos(2\pi \frac{w}{2} t) + \frac{n_s(t)}{2} \sin(\frac{w}{2} t)$$

- Average power in the detected message is $\frac{A_c^2}{16} P_m$
- Average power in the received noise is

$$= \frac{N_o W}{8} + \frac{N_o W}{8} = \frac{N_o W}{4}$$

$$SNR_o = \frac{A_c^2 P_m}{16 N_o W / 4} = \frac{A_c^2 P_m}{4 N_o W}$$

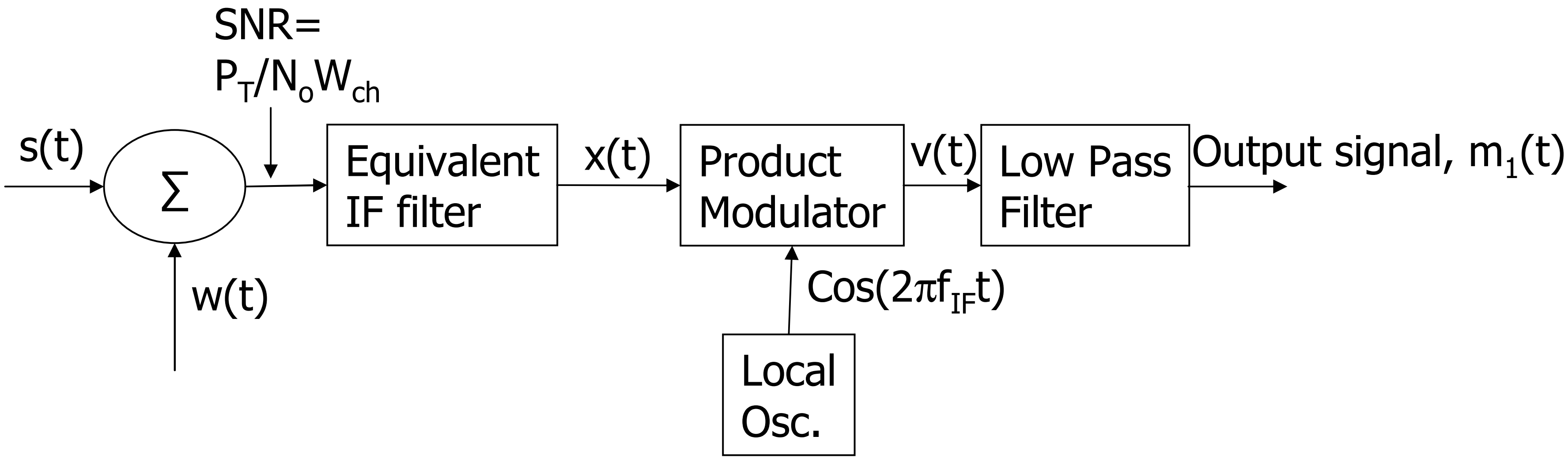
- Figure of Merit=1

Comparison between SSB, DSB-SC

- HW assignment
- Compare between the SNR of the DSB-SC, and the SSB

SNR for Coherent Reception of QAM Modulation

- The IF filter output is multiplied by a local carrier with frequency f_{IF} and then low pass filtering the product



- Consider SSB-LSB

$$s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)$$

where $m_1(t)$ and $m_2(t)$ are considered a sample functions of stationary processes of zero mean and PSD $S_M(f)$ limited to maximum frequency W

SNR for Coherent Reception of QAM Modulation

$$s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)$$

$$R_s(\tau) = \frac{A_c^2}{2} R_{m_1}(\tau)\cos(2\pi f_c \tau) + \frac{A_c^2}{2} R_{m2}(\tau)\cos(2\pi f_c \tau)$$

$$S_s(f) = \frac{A_c^2}{4}[S_{m_1}(f - f_c) + S_{m_1}(f + f_c) + S_{m_2}(f - f_c) + S_{m_2}(f + f_c)]$$

Received Signal

$$\mathrm{X(t) = s(t) + w(t)}$$

Received Signal after IF filter

$$\mathrm{X(t) = s(t) + n(t)}$$

$$= A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t) + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

$$= [A_c m_1(t) + n_c(t)]\cos(2\pi f_c t) + [A_c m_2(t) - n_s(t)]\sin(2\pi f_c t)$$

Received Signal after coherent detector

$$= [A_c m_1(t) + n_c(t)]\cos(2\pi f_c t)\cos(2\pi f_c t) + [A_c m_2(t) - n_s(t)]\sin(2\pi f_c t)\cos(2\pi f_c t)$$

Received Signal after LPF

$$[A_c m_1(t) + n_c(t)]/2$$

$$SNR_o = \frac{A_c^2 P_{m_1}}{2N_o W}$$

Noise in AM receivers using Envelope Detection

- The transmitted signal $s(t) = A_c[1+k_a m(t)]\cos(2\pi f_c t)$
- The average power in s(t) = $A_c^2[1+k_a^2 P_m]/2$
- The average noise power in the message bandwidth= $N_o W$
- The $SNR_{c,AM} = \dfrac{A_c^2[1+k_a^2 P_m]}{2N_o W}$
- The received signal at the envelope detector x(t) consists of the transmitted signal s(t) and NBN n(t)
- $$x(t) = s(t) + n(t)$$
  $$= A_c[1+k_a m(t)]\cos(2\pi f_c t) + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$
  $$= [A_c + A_c k_a m(t) + n_c(t)]\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

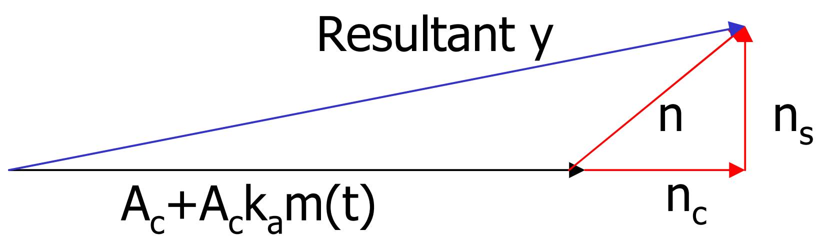
Phasor diagram for AM wave plus NBN

- From the phasor diagram, the received output y(t) is obtained as the envelope of x(t)

$$y(t) = \sqrt{[A_c + A_c k_a m(t) + n_c(t)]^2 + n_s^2(t)}$$

- Y(t) need to be approximated to a message term plus a noise and distortion term.
- If average carrier power is lager compared to noise, then the first term in y(t) will be larger than the second term

$$y(t) \approx [A_c + A_c k_a m(t) + n_c(t)]$$



Phasor diagram for AM wave plus NBN

- DC component can be removed with DC block.
- The output signal to noise ratio

$$SNR_{o,AM} = \frac{A_c^2 k_a^2 P}{2WN_o}$$

- This expression is only valid IFF
  - The noise at the receiver input is small compared to the signal.
  - The K_a is adjusted for a percentage modulation less than or equal to 100%
- The Figure of merit is always less than unity

$$\frac{SNR_{o,AM}}{SNR_{c,AM}} = \frac{A_c^2 k_a^2 P}{2WN_o} \cdot \frac{2WN_o}{A_c^2 [1 + k_a^2 P]} = \frac{k_a^2 P}{[1 + k_a^2 P]}$$

The threshold effect

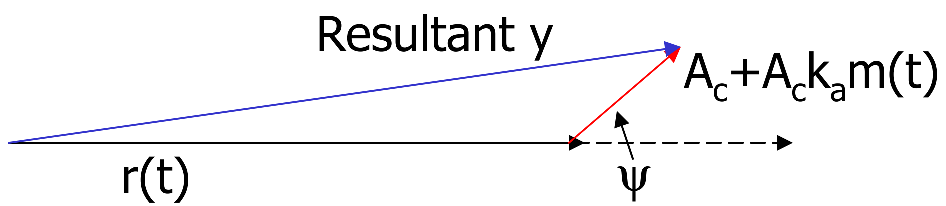
- When the signal to noise ratio is small, the noise component dominates and the performance of the envelope detector completely changes.
- NBN can be represented as shown

$$n(t) = r(t)\cos(2\pi f_c t + \psi(t))$$

Received signal at envelope detector input

$$x(t) = s(t) + n(t)$$

- We can neglect the quadrature component of the signal $y(t) \approx [r(t) + A_c \cos(\psi(t)) + A_c k_a m(t)\cos(\psi(t))]$



The threshold effect

- When the signal to noise ratio is small, The envelope detector output does not have a component proportional to the message m(t), the noise angle is uniformly distributed from 0 to 2π
- The threshold effect is defined as the value of the carrier to noise ratio below which the noise performance of a detector deteriorates much more rapidly than proportionately to the carrier to noise ratio

Example of threshold effect calculations

- If the noise component $r(t) > A_c$ with probability 0.5 then there is a good chance that the envelope detector will operate. On the other hand if the probability is 0.01 then it is considered that the detector will operate satisfactory.
- The envelope of a NBN noise follows rayleigh dist.

$$P(R \geq A_c) = \exp(-\frac{A_c^2}{4WN_o})$$

$$\rho = \frac{\textit{average}\ \text{carrier power}}{\textit{average}\ \text{noise power in message BW}}$$

$$\rho = \frac{A_c^2}{2N_o 2W}$$

$$P(R \geq A_c) = \exp(-\rho)$$

$$\text{for } P(R \geq A_c) = 0.5,\ \rho = \ln 2 = 0.69 = -1.6dB$$

$$\text{for } P(R \geq A_c) = 0.01,\ \rho = \ln 100 = 4.6 = 6.6dB$$