Communication Systems II

Noise in CW modulation

Noise in CW modulation

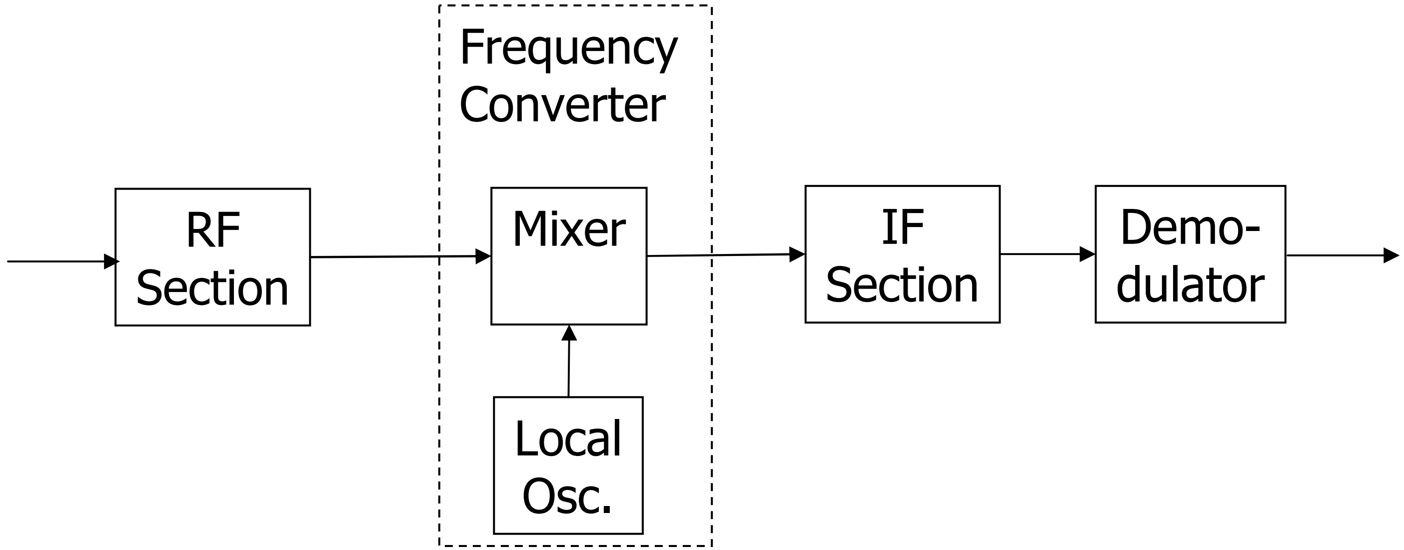
- Analysis the effect of noise on the performance of the receiver.
- Analysis of different modulation-demodulation schemes
- Need a criterion to measure the performance
- **Output signal to noise ratio**
  - **Ratio of the average power of the message aignal to the average power in the noise, both measured at the receiver output**
  - **Adequate as long as noise and signal are additive at the receiver output**

Assumptions in analysis of noise in CW Modulation

- Noise is
  - Stationary
  - White
  - Gaussian
  - Additive
- We consider
  - DSB-SC, SSB, using coherent demodulation
  - DSB-TC using envelope detector
  - FM

AM Receivers

- The usual AM radio receiver so called superheterodyne type is shown in following figure



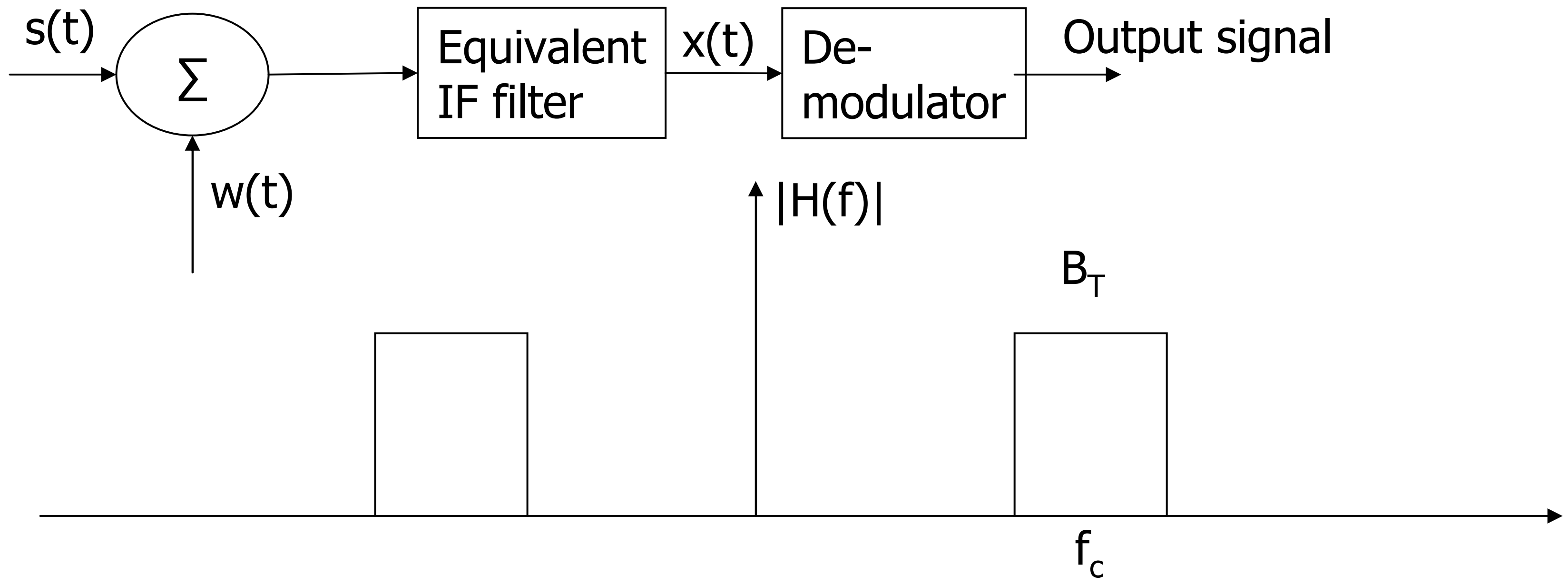
- Typical frequency parameters
  - RF carrier range: 0.535-1.605 MHz
  - Mid band frequency of IF section : 0.455 MHz
  - IF bandwidth: 10 MHz

Superheterodyne AM Receiver

- RF section is tuned to the carrier frequency of interest and amplifies the received signal
- The mixer and local osc. Provides the frequency conversion from the carrier frequency to the IF frequency $F_{IF} = F_{RF} - F_{LO}$
- The output of the IF is applied to a demodulator to recover the transmitted message.

AM Receiver model

- The previous model can be modeled as an equivalent IF filter and demodulator.



- At the filter input, we have a signal consists of the received modulated signal s(t) and noise w(t) modeled as AWGN with zero mean and PSD of $N_o/2$
- We assume that the IF filter has an ideal BPF response as shown

AM Receiver model

- The composite signal x(t) at the IF filter output is

$$x(t) = s(t) + n(t)$$

- Where n(t) is a band-limited noise with zero mean and PSD

$$S_n(f) = \begin{cases} \frac{N_o}{2} & f_c - \frac{B_T}{2} \le |f| \le f_c - \frac{B_T}{2} \\ 0 & otherwise \end{cases}$$

- n(t) is a narrow band pass noise that can be modeled with $n_c(t)$ and $n_s(t)$

Signal to noise ratio SNR (S/R)

- The SNR can be measured at the output of the receiver as follows

$$(SNR)_o = \frac{Average\ \text{power of the message signal at the receiver output}}{\text{Average power of noise at the receiver output}}$$

- The SNR requires that noise and signal are additive and this is satisfied by linear receivers as coherent detector.
- SNR depends on the modulation used and the type of receiver.
- This suggest comparative evaluation w.r.t a reference point.
- Reference point is taken to be the input to the receiver from the channel
- $$(SNR)_c = \frac{Average\ \text{power of the message signal at the receiver input}}{\text{Average power of noise at the receiver input}}$$
- We can normalize the performance by dividing the two equations

$$Figure\ \text{of merit} = \frac{\text{SNR}_\text{o}}{\text{SNR}_\text{c}}$$

SNR of Base Band signal

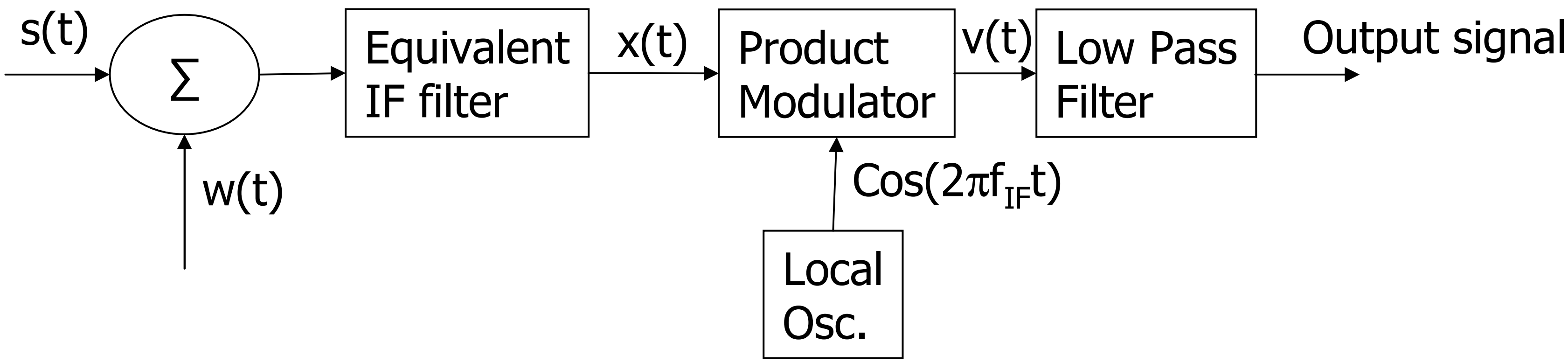
- S(t)=m(t)+w(t)
- After LPF
  - X(t)=m(t)+n(t)

$$SNR_{BB} = \frac{P_T}{\frac{N_o}{2} 2W}$$

P_T Power in the transmitted signal

SNR for Coherent Reception of DSB-SC Modulation

- The IF filter output is multiplied by a local carrier with frequency f_{IF} and then low pass filtering the product



- Consider DSB-SC $s(t) = A_c \cos(2\pi f_c t) m(t)$ where m(t) is considered a sample function of a stationary process of zero mean and PSD $S_M(f)$ limited to maximum frequency W as shown in the next slide

SNR of Coherent Demodulation of DSB-SC

- For a stationary RP, the area under the PSD is equal to the average power of the RP
  - $$P = \int_{-w}^{w} S_M(f)df$$

$$R_S(\tau) = E[S(t)S(t+\tau)]$$

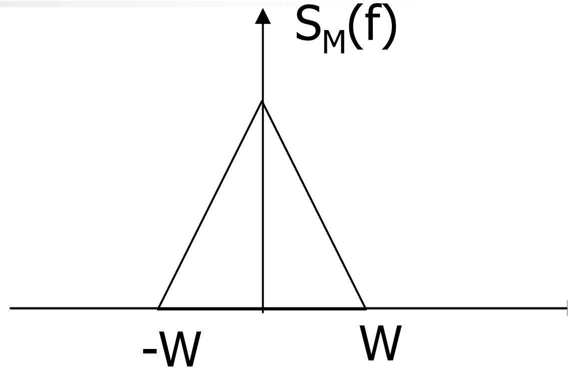
$$= A_c^2 E[\cos(2\pi f t)\cos(2\pi f(t+\tau))m(t)m(t+\tau)]$$

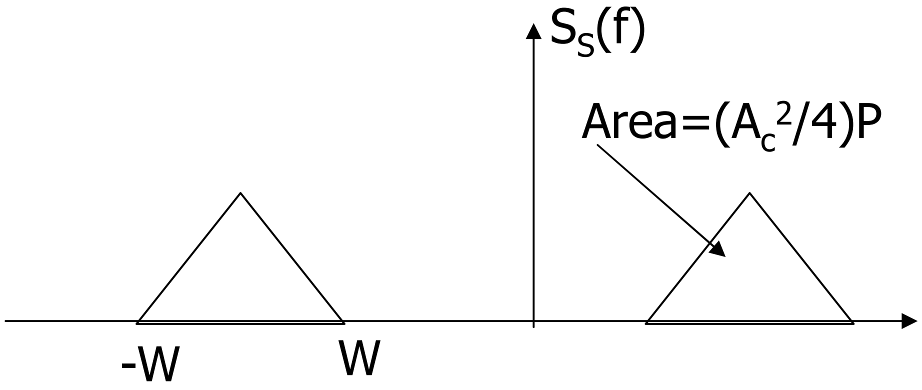
$$= \frac{A_c^2}{2}\cos(2\pi f\tau)R_m(\tau)$$

$$\therefore$$

$$S_s(f) = \frac{A_c^2}{4}[S_M(f-f_c)+S_M(f+f_c)]$$

  - Transmitted BW=2W
  - Average power in transmitted signal is $(A_c^2/2)P$





SNR of Coherent Demodulation of DSB-SC

- Noise PSD of $N_o/2$, the average noise power in the message BW W is equal to WN_o $(N_o/2)(2W)$.
- The SNR_c:

$$SNR_c = \frac{A_c^2 P}{2N_o W}$$

- Next we want to determine the SNR_o using the NBN representation of the filtered noise n(t)

SNR of Coherent Demodulation of DSB-SC

- The total signal at the coherent input may be expressed as: $x(t) = s(t) + n(t)$

$$= A_c \cos(2\pi f_c t) m(t) + n_c \cos(2\pi f_c t) - n_s \sin(2\pi f_c t)$$

- The output of the product modulator

$$v(t) = x(t)\cos(2\pi f_c t)$$

$$= A_c m(t)\cos(2\pi f_c t)\cos(2\pi f_c t) + n_c(t)\cos(2\pi f_c t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)\cos(2\pi f_c t)$$

$$= \frac{A_c m(t)}{2} + \frac{A_c m(t)\cos(2\pi 2 f_c t)}{2} + \frac{n_c(t)}{2} + \frac{n_c(t)\cos(2\pi 2 f_c t)}{2} - \frac{n_s(t)\sin(2\pi 2 f_c t)}{2}$$

- The output after the LPF $= \dfrac{A_c m(t)}{2} + \dfrac{n_c(t)}{2}$

SNR of Coherent Demodulation of DSB-SC

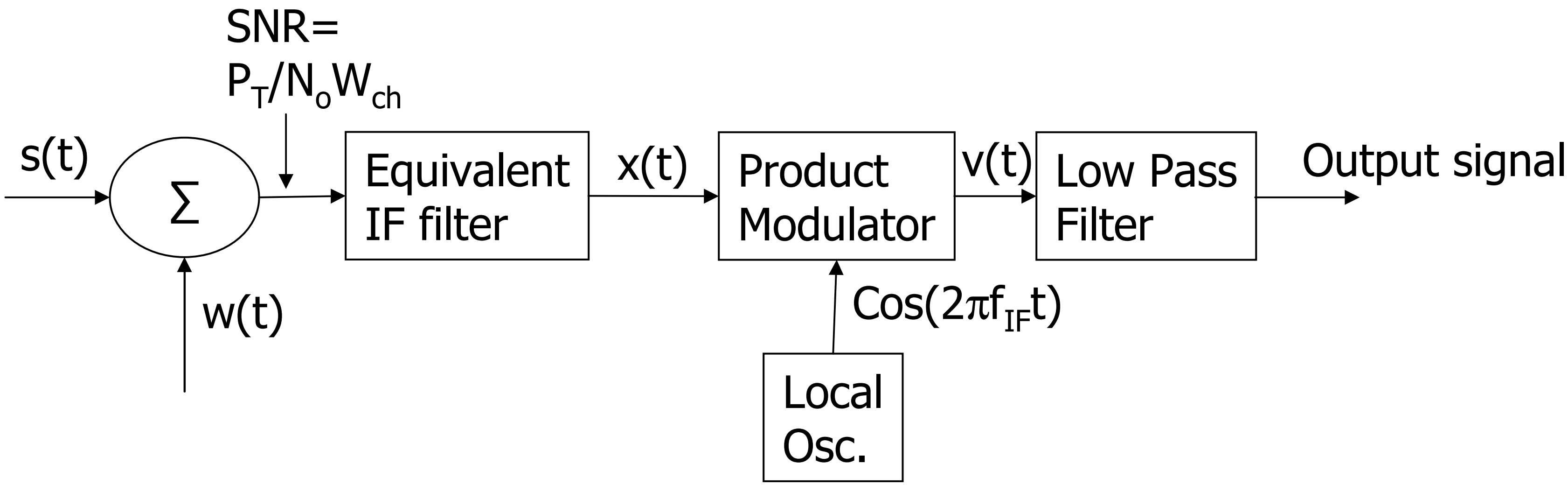
- The output after the LPF $= \frac{A_c m(t)}{2} + \frac{n_c(t)}{2}$
- The output indicates that
  - The message and inphase noise components are additive
  - The quadrature component of the noise is removed
  - The message component at the output is $A_c m(t)/2$
  - The message power at the receiver output is $A_c^2 P/4$
  - The noise component at the reciver output is $n_c(t)/2$
  - PSD of $n_c(t)$ is $S_N(f-f_c)+S_N(f+f_c)= N_o$ for BW=2W
  - The average power of noise component is $2WN_o/4$

$$SNR_o = \frac{A_c^2 P}{2WN_0} = \frac{P_T}{WN_0} = SNR_{BB}$$

$$\frac{SNR_o}{SNR_c} = 1$$

SNR for Coherent Reception of DSB-TC Modulation

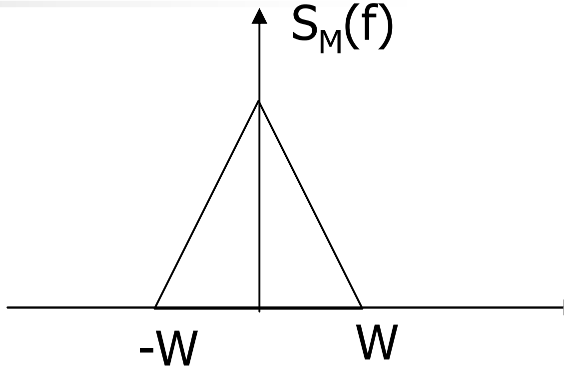
- The IF filter output is multiplied by a local carrier with frequency f_{IF} and then low pass filtering the product



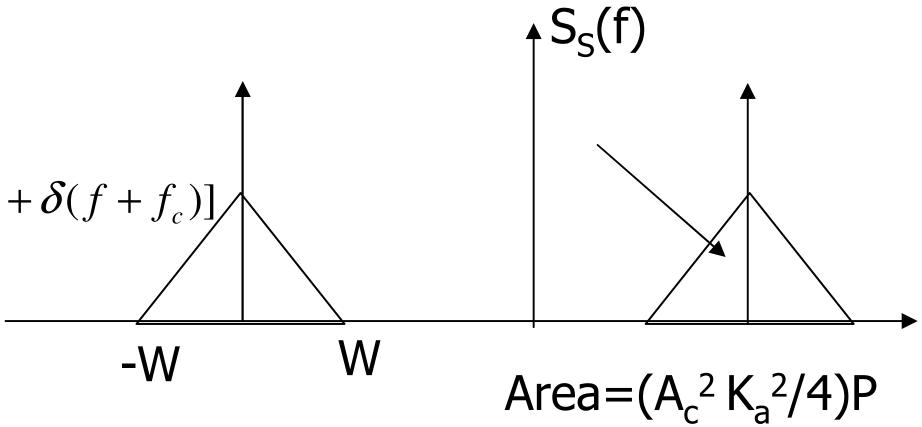
- Consider DSB-TC $s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$ where m(t) is considered a sample function of a stationary process of zero mean and PSD $S_M(f)$ limited to maximum frequency W as shown in the next slide

SNR of Coherent Demodulation of DSB-SC

- For a stationary RP, the area under the PSD is equal to the average power of the RP
  - $$P = \int_{-w}^{w} S_M(f)\,df$$



$$R_S(\tau) = E[S(t)S(t+\tau)]$$
$$= A_c^2 E[\cos(2\pi f t)\cos(2\pi f(t+\tau))(1+k_a m(t))(1+k_a m(t+\tau))]$$
$$= \frac{A_c^2}{2}\cos(2\pi f\tau)[1+k_a^2 R_m(\tau)]$$
$$\therefore$$
$$S_s(f) = \frac{A_c^2 k_a^2}{4}[S_M(f-f_c)+S_M(f+f_c)] + \frac{A_c^2}{4}[\delta(f-f_c)+\delta(f+f_c)]$$



- Transmitted BW=2W
- Average power in transmitted signal is $(A_c^2 K_a^2/2)P$

SNR of Coherent Demodulation of DSB-SC

- Noise PSD of $N_o/2$, the average noise power in the message BW W is equal to WN_o.
- The SNR_c:

$$SNR_c = \frac{A_c^2 K_a^2 P}{2N_o W}$$

- Next we want to determine the SNR_o using the NBN representation of the filtered noise n(t)

SNR of Coherent Demodulation of DSB-SC

- The total signal at the coherent input may be expressed as: $x(t) = s(t) + n(t)$

$$= A_c[1 + k_a m(t)]\cos(2\pi f_c t) + n_c\cos(2\pi f_c t) - n_s\sin(2\pi f_c t)$$

- The output of the product modulator

$$v(t) = x(t)\cos(2\pi f_c t)$$

$$= A_c[1 + k_a m(t)]\cos(2\pi f_c t)\cos(2\pi f_c t) + n_c(t)\cos(2\pi f_c t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)\cos(2\pi f_c t)$$

$$= \frac{A_c}{2} + \frac{A_c k_a m(t)}{2} + \frac{A_c[1 + k_a m(t)]\cos(2\pi 2 f_c t)}{2} + \frac{n_c(t)}{2} + \frac{n_c(t)\cos(2\pi 2 f_c t)}{2} - \frac{n_s(t)\sin(2\pi 2 f_c t)}{2}$$

- The output after the LPF $= \frac{A_c}{2} + \frac{A_c m(t)}{2} + \frac{n_c(t)}{2}$

SNR of Coherent Demodulation of DSB-SC

- The output after the LPF and dc block $= \frac{A_c k_a m(t)}{2} + \frac{n_c(t)}{2}$
- The output indicates that
  - The message and inphase noise components are additive
  - The quadrature component of the noise is removed
  - The message component at the output is $A_c k_a m(t)/2$
  - The message power at the receiver output is $(A_c^2/4)(k_a^2 P)$
  - The noise component at the receiver output is $n_c(t)/2$
  - PSD of $n_c(t)$ is $S_N(f-f_c)+S_N(f+f_c)= N_o$ for BW=2W
  - The average power of noise component is $2WN_o/4$

$$SNR_o = \frac{A_c^2 (K_a^2 P)}{2WN_0}, \mathrm{P_T} = \frac{A_c^2 (1+K_a^2 P)}{2}$$

$$SNR_o = \frac{(K_a^2 P)}{WN_0} . \frac{P_T}{(1+K_a^2 P)} = SNR_{BB} \frac{(K_a^2 P)}{(1+K_a^2 P)}$$

$$\frac{SNR_o}{SNR_c} = \frac{A_c^2 (K_a^2 P)}{A_c^2 K_a^2 P} = 1$$