Communication Systems II

Noise

Narrow-Band Noise

Envelope of NBN

Noise

- Unwanted signal that tend to disturb the transmission and processing of signals in communication systems.
- Thermal Noise → random motion of electrons in a conductor.
- Shot noise → arises in electronic devices, sudden change in voltage or current.

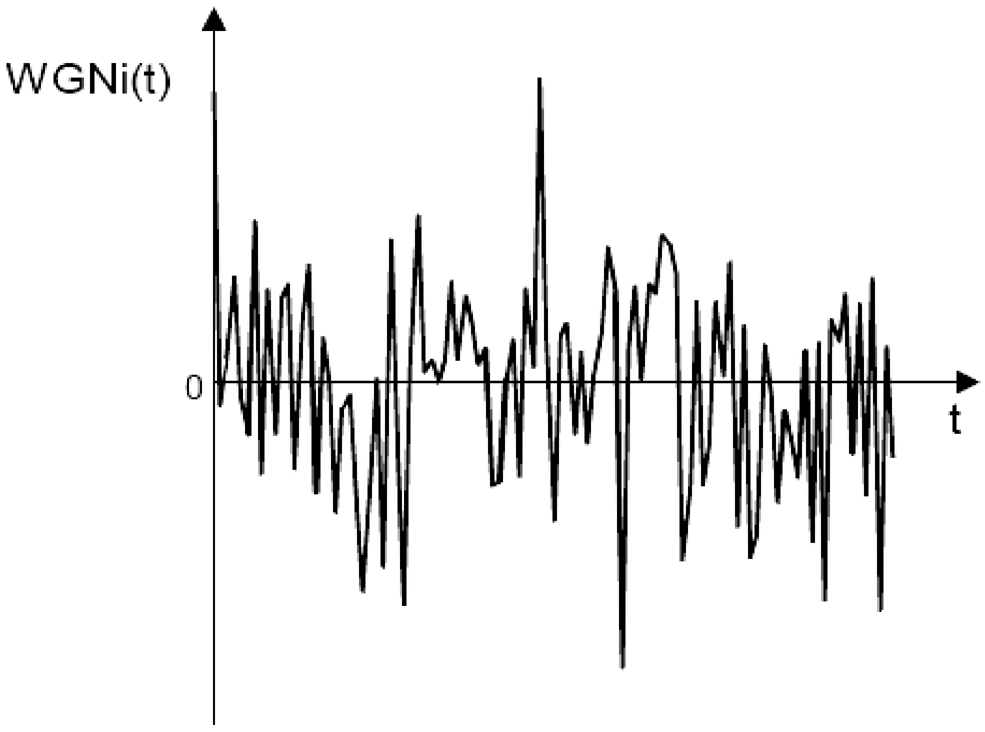
White Noise

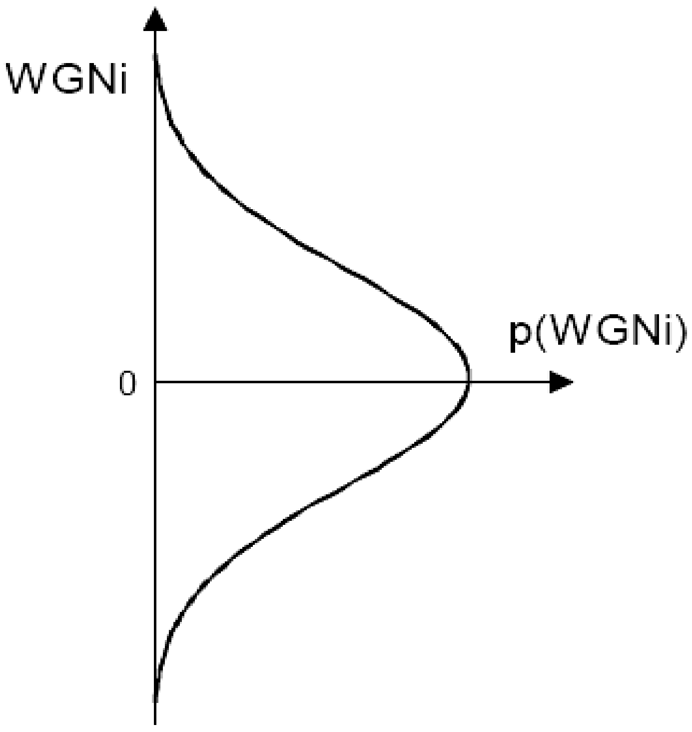
- White → occupies all frequencies→ PSD is independent on the operating frequency
- Dimensions of N_o is watt per Hertz, N_o=KT

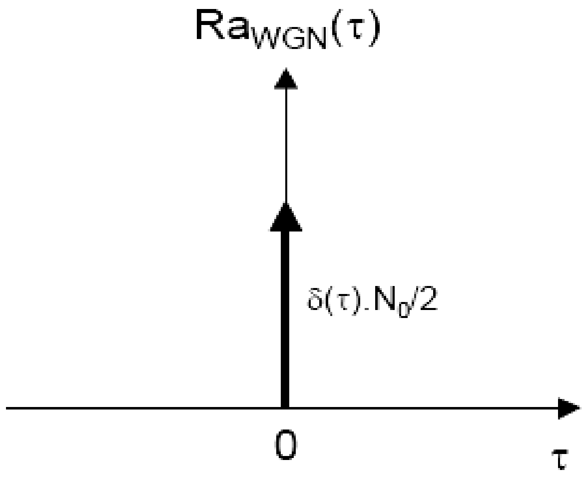
$$S_N(f) = \frac{N_o}{2}$$

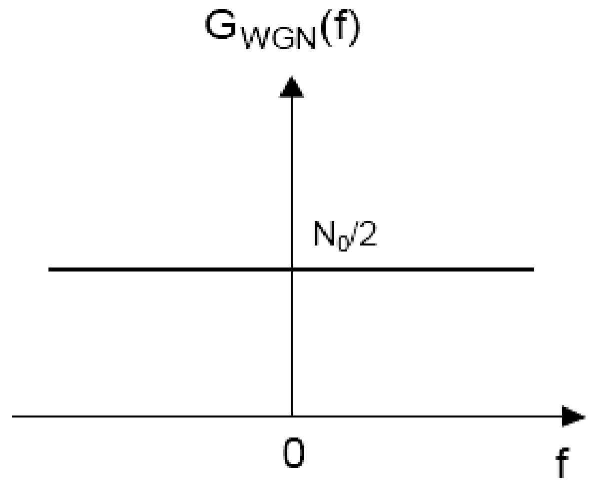
$$R_N(\tau) = \frac{N_o}{2}\delta(\tau)$$

- Any two different samples of white noise no matter how close they are will be uncorrelated.
- If white noise is Gaussian then they will also be independent


WGNi(t)
0
t


WGNi
0
p(WGNi)


$Ra_{WGN}(\tau)$
$\delta(\tau).N_0/2$
0
τ


$G_{WGN}(f)$
$N_0/2$
0
f

Ideal low pass filtered white noise

- A white Gaussian noise with zero mean and variance $N_0/2$ is applied to an ideal low pass filter of bandwidth B and amplitude response of one.
- PSD of output Y(t) is

$$S_Y(f) = \begin{cases} \frac{N_o}{2} & -B \leq f \leq B \\ 0 & otherwise \end{cases}$$

- The autocorrelation is $R_Y(\tau) = N_0 B \sin c(2B\tau)$
- Autocorrelation maximum at τ equal zero equal $N_o B$ and passes through zero at $\tau = n/2B$ for n= integer values and variance $N_o B$
- If noise is samples at rate 2B then they are uncorrelated and being Gaussian then statistically independent

RC low pass filtered white noise

- The H(f) of RC filter is $H(f) = \frac{1}{1 + j2\pi fRC}$

- The PSD of the o/p is $S_Y(f) = \frac{1}{1 + (2\pi fRC)^2}$

- The autocorrelation of the output is $R_Y(\tau) = \frac{N_o}{4RC} \exp(-\frac{|\tau|}{RC})$

- If noise is samples at rate 0.217/RC then they are uncorrelated and being Gaussian then statistically independent

Ex sine wave plus white noise

$$X(t) = A\cos(2\pi f_c t + \theta) + N(t)$$

θ is Uniformely distributed, N(t) is WGN

$$\mathrm{R_X}(\tau) = E[X(t)X(t+\tau)] = \frac{A^2}{2}\cos(2\pi f_c \tau) + \frac{N_0}{2}\delta(\tau)$$

$$S_X(f) = \frac{A^2}{4}[\delta(f - f_c) + \delta(f + f_c)] + \frac{N_0}{2}$$

Noise Equivalent Bandwidth

- Output average power of ILPF→ N_oB
- Output average power of RCLPF→ $N_o/4RC$

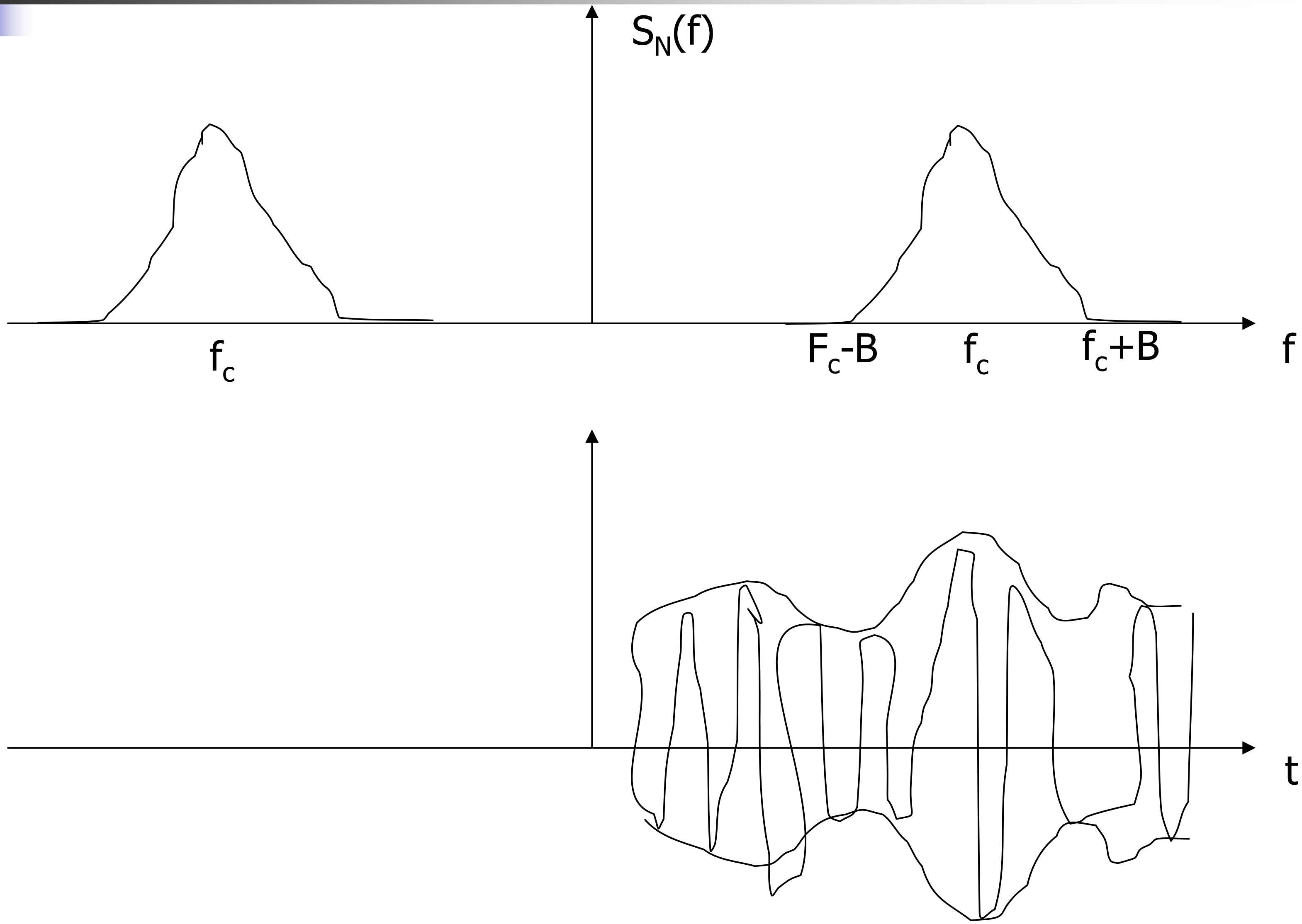
- Output average power of any filter → $N_oBH^2(0)$
- Equivalent Bandwidth =

$$B = \frac{\int_0^\infty |H(f)|^2 df}{H^2(0)}$$

Narrow Band Noise

- Filters at the receiver have enough bandwidth to pass the desired signal but not to big to pass excess noise.
- Narrow band→ center frequency is much bigger that the bandwidth.
- Noise at the output of such filters are called NBN.
- NBN has spectral concentrated about some mid-band frequency $\pm f_c$
- The sample function of such NBN n(t) appears as a sine wave of frequency f_c which modulates slowly in amplitude and phase.

$$S_N(f) = \frac{N_o}{2} \left| H(f) \right|^2$$



$S_N(f)$
f_c
F_c-B
f_c
f_c+B
f
t

Representation of NBN

- n(t) can be represented by its pre-envelope and complex envelope as follows

$$n_+(t) = n(t) + j\hat{n}(t)$$

$$\tilde{n}(t) = n_+(t)\exp(-j2\pi f_c t)$$

$$\tilde{n}(t) = n_c(t) + jn_s(t)$$

In phase comp. of NBN $$n_c(t) = n(t)\cos(2\pi f_c t) + \hat{n}(t)\sin(2\pi f_c t)$$

Quadrature comp. of NBN $$n_s(t) = \hat{n}(t)\cos(2\pi f_c t) - n(t)\sin(2\pi f_c t)$$

$$\therefore$$

$$\boxed{n(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)}$$

Properties of Inphase and Quadrature components of NBN

1. If n(t) is zero mean then $n_c(t)$ and $n_s(t)$ are also zero mean
2. If n(t) is G. RP then $n_c(t)$ and $n_s(t)$ are jointly G. RP
3. If n(t) is WSS then $n_c(t)$ and $n_s(t)$ are jointly WSS

$$R_{n_c}(\tau) = R_{n_s}(\tau) = R_N(\tau)\cos(2\pi f_c \tau) + \hat{R}_N(\tau)\sin(2\pi f_c \tau)$$

$$crosscorrelation$$

$$R_{n_c n_s}(\tau) = -R_{n_s n_c}(\tau) = R_N(\tau)\sin(2\pi f_c \tau) - \hat{R}_N(\tau)\cos(2\pi f_c \tau)$$

Properties of inphase and quadrature components of NBN

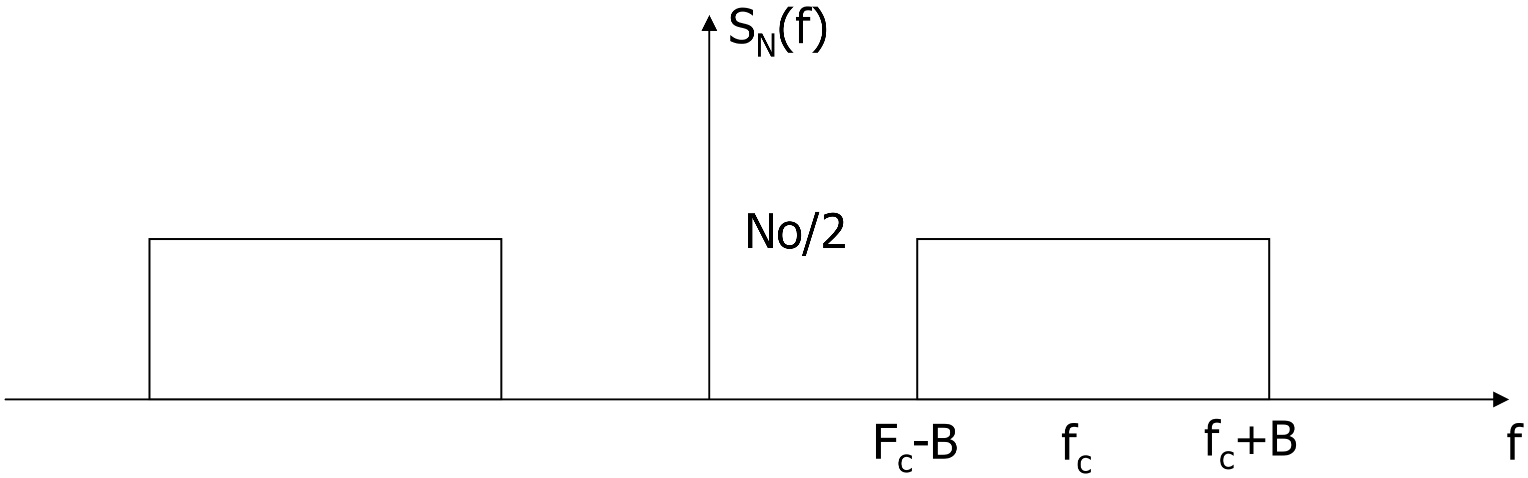
4. PSD of inphase and quadrature components are the same and are related to the PSD of the original NBN PSD as follows

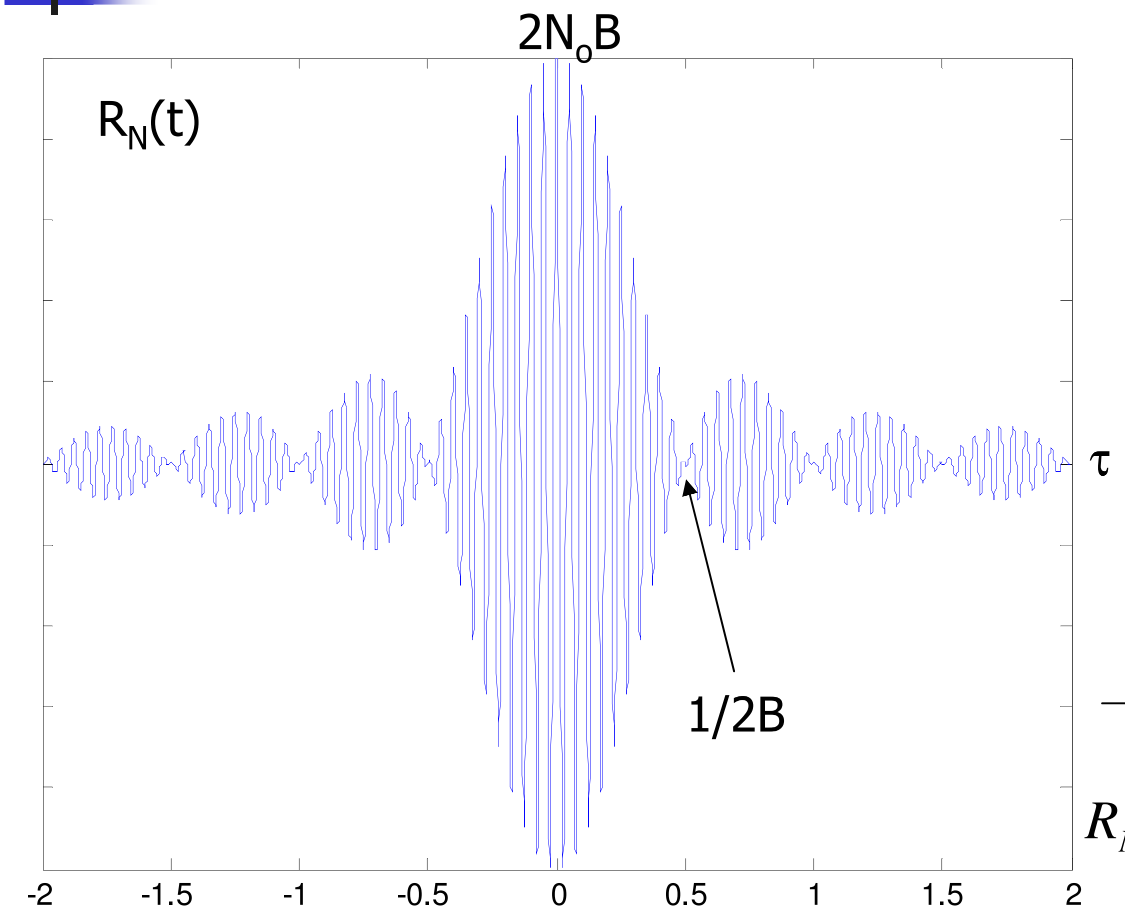
$$S_{n_c}(f) = S_{n_s}(f) = \begin{cases} S_n(f - f_c) + S_n(f + f_c), & -B \le f \le B \\ 0 & otherwise \end{cases}$$

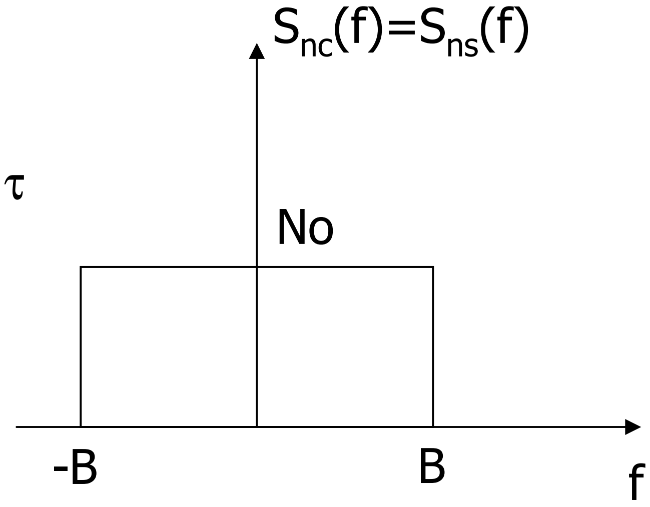
5. If n(t) is zero mean then $n_c(t)$ and $n_s(t)$ will have the same variance as n(t) itself
6. If n(t) is zero mean Gaussian with symmetric PSD around f_c, then $n_c(t)$ and $n_s(t)$ are statistically independent

Ideal BPF white noise

- Consider a WGN of zero mean and PSD $N_o/2$ which passes by an IBPF of unit amplitude response and BW 2B.
- The PSD of the filtered noise n(t) has the same shape of the BPF







$$R_{N_c}(\tau) = R_{Ns}(\tau) = 2N_o B \sin c(2B\tau)$$

$$R_N(\tau) = 2N_o B \sin c(2B\tau)\cos(2\pi f_c \tau)$$

Representation of NBN w.r.t Envelope and phase components

- n(t) can be represented as

$$n(t) = r(t)\cos(2\pi f_c t + \theta(t))$$

where

$$r(t) = \sqrt{n_c^{\,2}(t) + n_s^{\,2}(t)} \qquad\qquad f_R(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{\sigma^2}\right) \quad \mathrm{r} \geq 0$$

and

$$\theta(t) = \tan^{-1}\left(\frac{n_s(t)}{n_c(t)}\right)$$

- r(t) is called the envelope of n(t) and θ(t) is called the phase of n(t)
- r(t) will have a Rayleigh distribution and θ(t) will have a uniform distribution

Envelope of sine wave plus NBN

- Suppose we add NBN to a sinusoidal signal

$$x(t) = A\cos(2\pi f_c t) + n(t)$$

$$x(t) = A\cos(2\pi f_c t) + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

$$x(t) = [A + n_c(t)]\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

- If n(t) is a zero mean σ^2 variance G. RP
  - $n_c(t)$ and $n_s(t)$ are G RP and S.I
  - $n_c(t)$ and $n_s(t)$ are zero mean
  - $n_c(t)$ and $n_s(t)$ are σ^2 variance

Envelope of sine wave plus NBN

- The envelope of x(t) will be Rician distribution

$$f_R(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2 + A^2}{2\sigma^2}\right) I_o\left(\frac{Ar}{\sigma^2}\right)$$

where

I_o is the modified Bessel function of first kind of zero order

- If A =0, it becomes Rayleigh distribution
- If A>>, it becomes approximately Gaussian