EC744 Wireless Communication
Fall 2008

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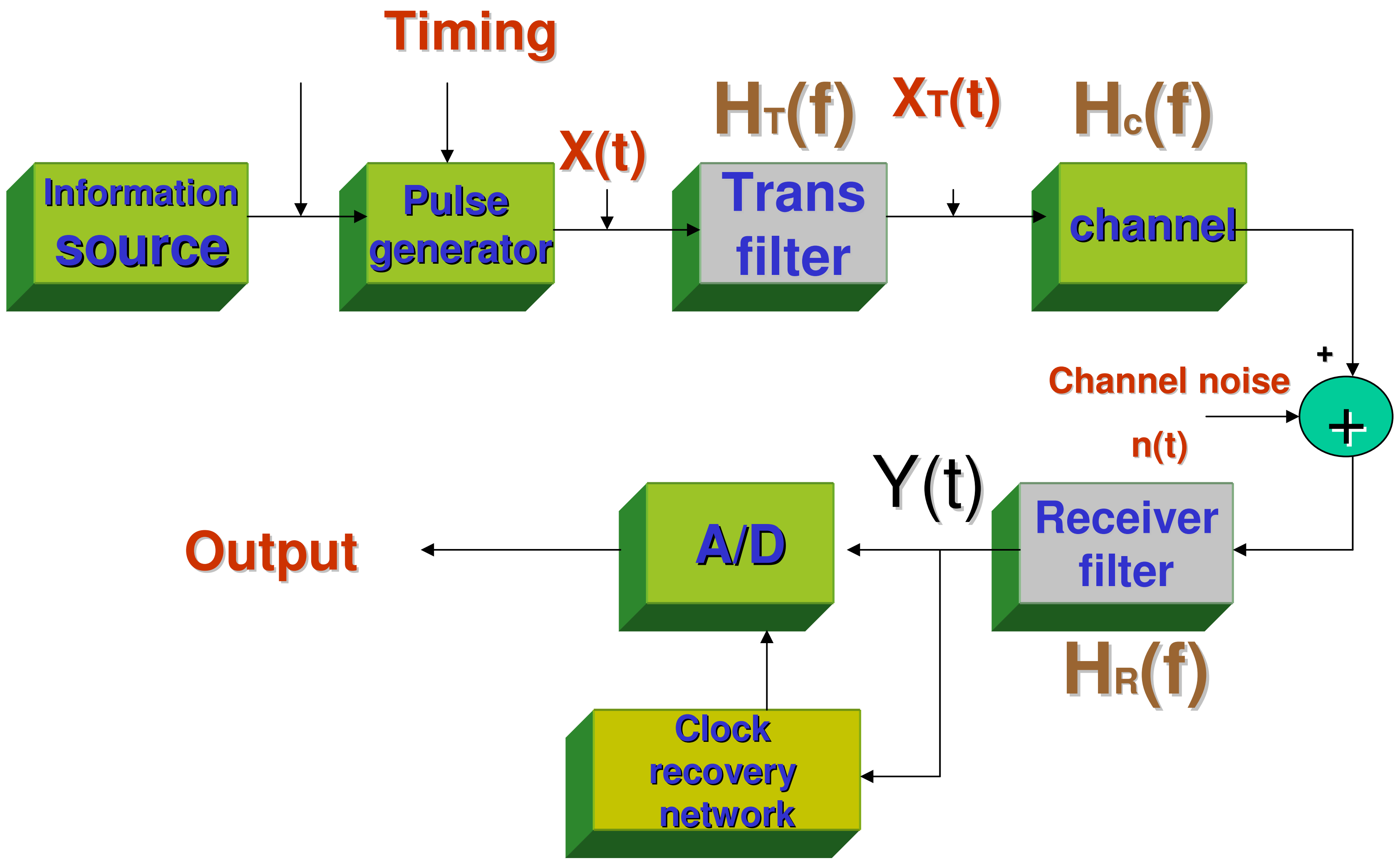
Digital Communication Fundamentals

Syllabus

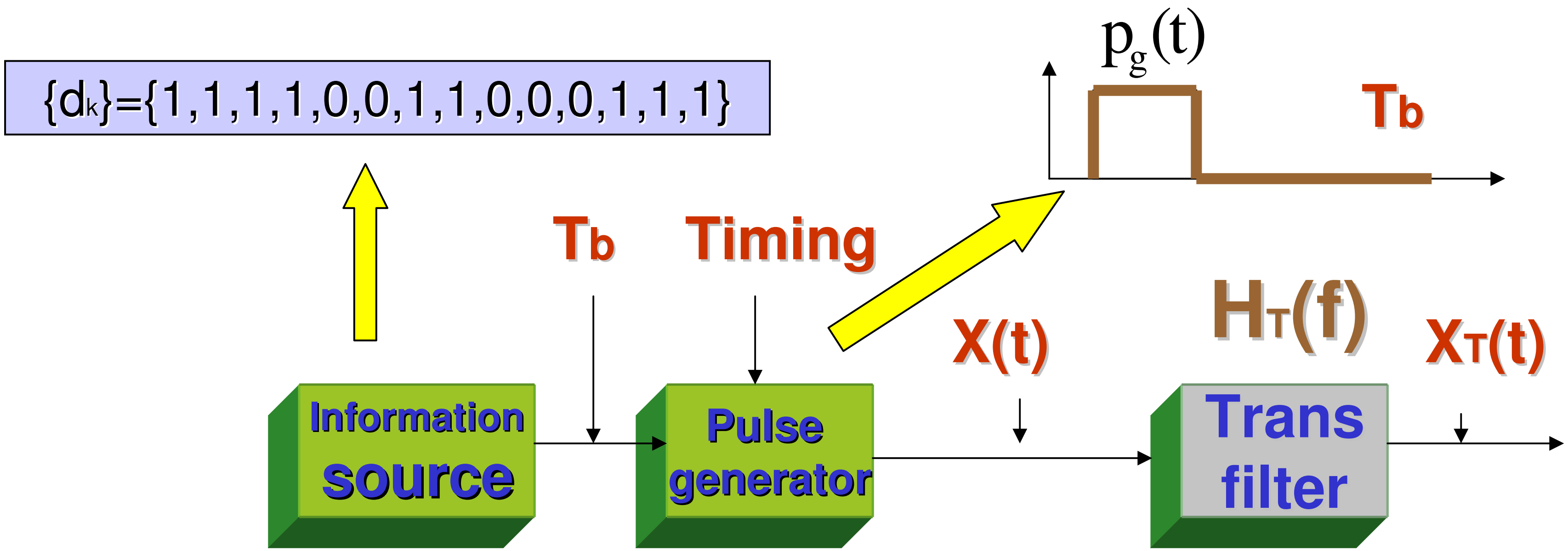
- Tentatively

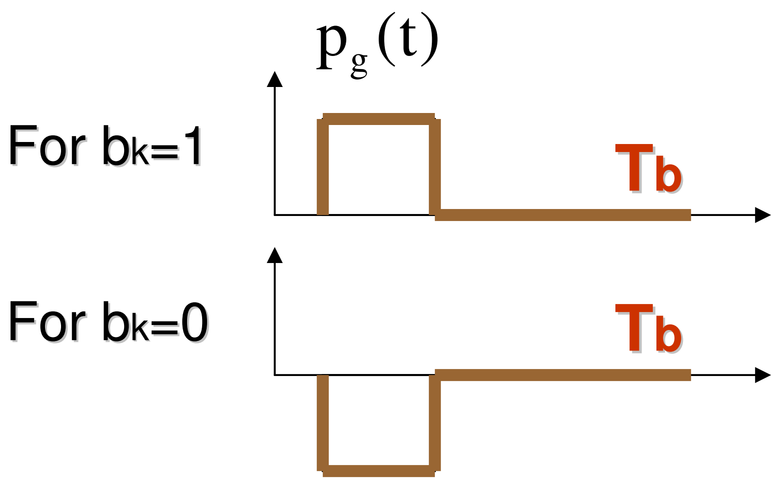
Week 1	Overview wireless communications, Probabilities
Week 2	Digital Communication fundamentals: channel characteristics (AWGN, fading)
Week 3	Modulation techniques
Week 4	Demodulation techniques (coherent and non-coherent)
Week 5	Source coding techniques
Week 6	Channel coding techniques
Week 7	Mid Term exam (take home), Diversity techniques
Week 8	Equalization techniques
Week 9	Spread spectrum, MIMO and OFDM
Week 10	Wireless networking: 802.11, 802.16, UWB
Week 11	Hot topics
Week 12	Presentations
Week 13	Presentations
Week 14	Presentations
Week 15	Final Exam

Block diagram of an Binary/M-ary signaling scheme



Block diagram Description

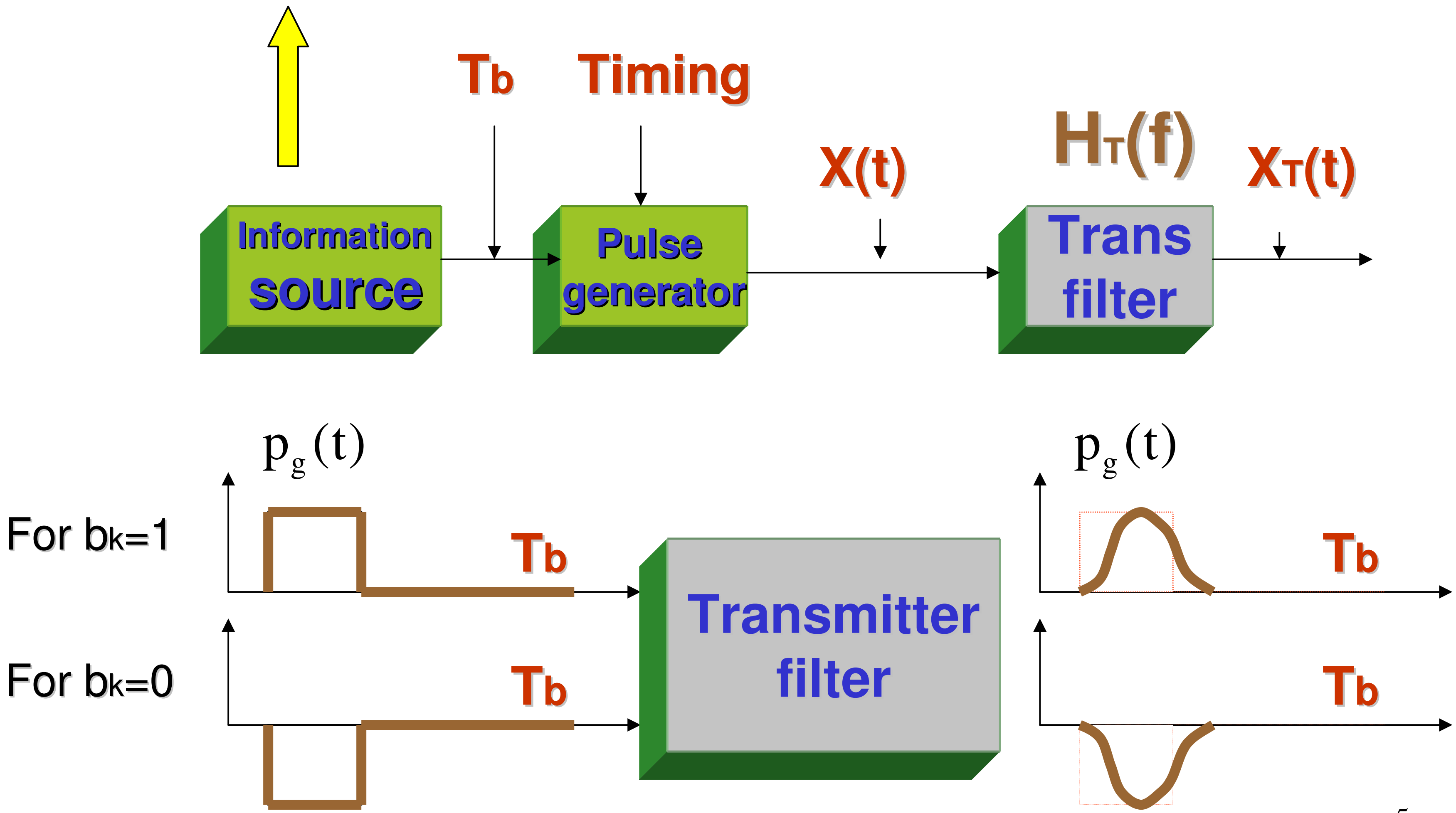




$$a_k = \begin{cases} +a & if \ d_k = "1"; \\ -a & if \ d_k = "0"; \end{cases}$$

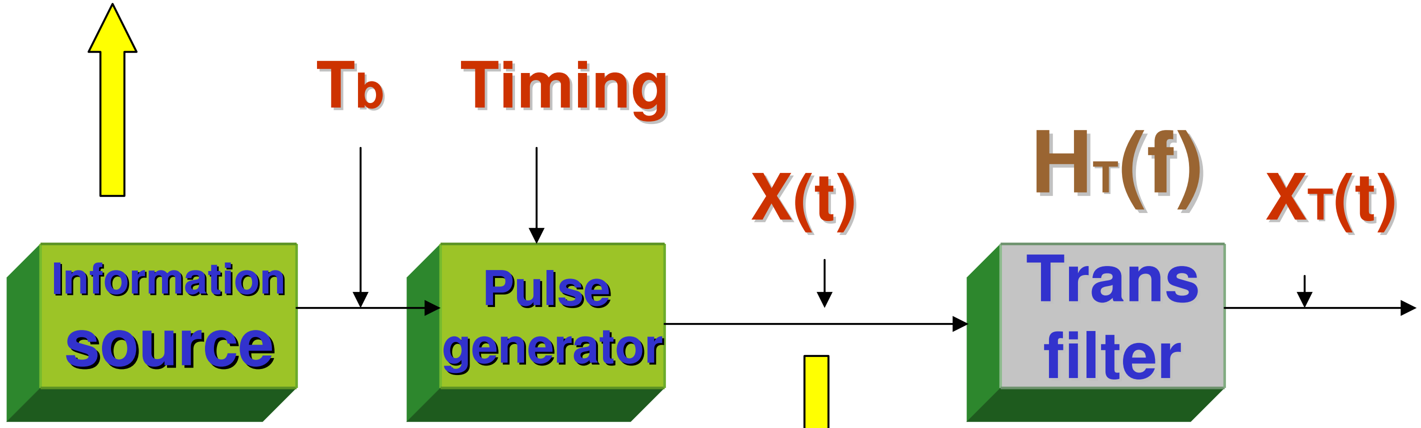
Block diagram Description (Continue - 1)

$\{d_k\}=\{1,1,1,1,0,0,1,1,0,0,0,1,1,1\}$



Block diagram Description (Continue - 2)

$\{d_k\}=\{1,1,1,1,0,0,1,1,0,0,0,1,1,1\}$



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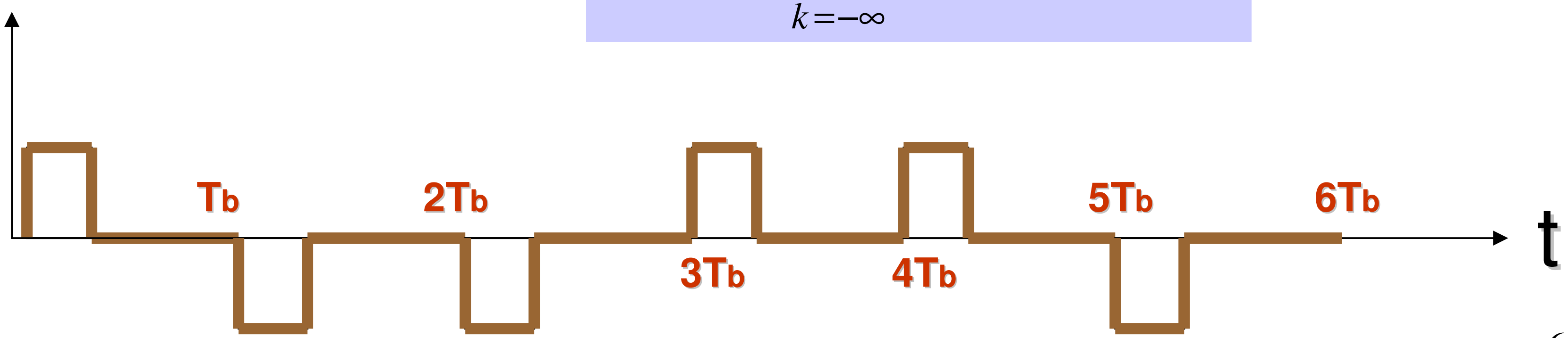
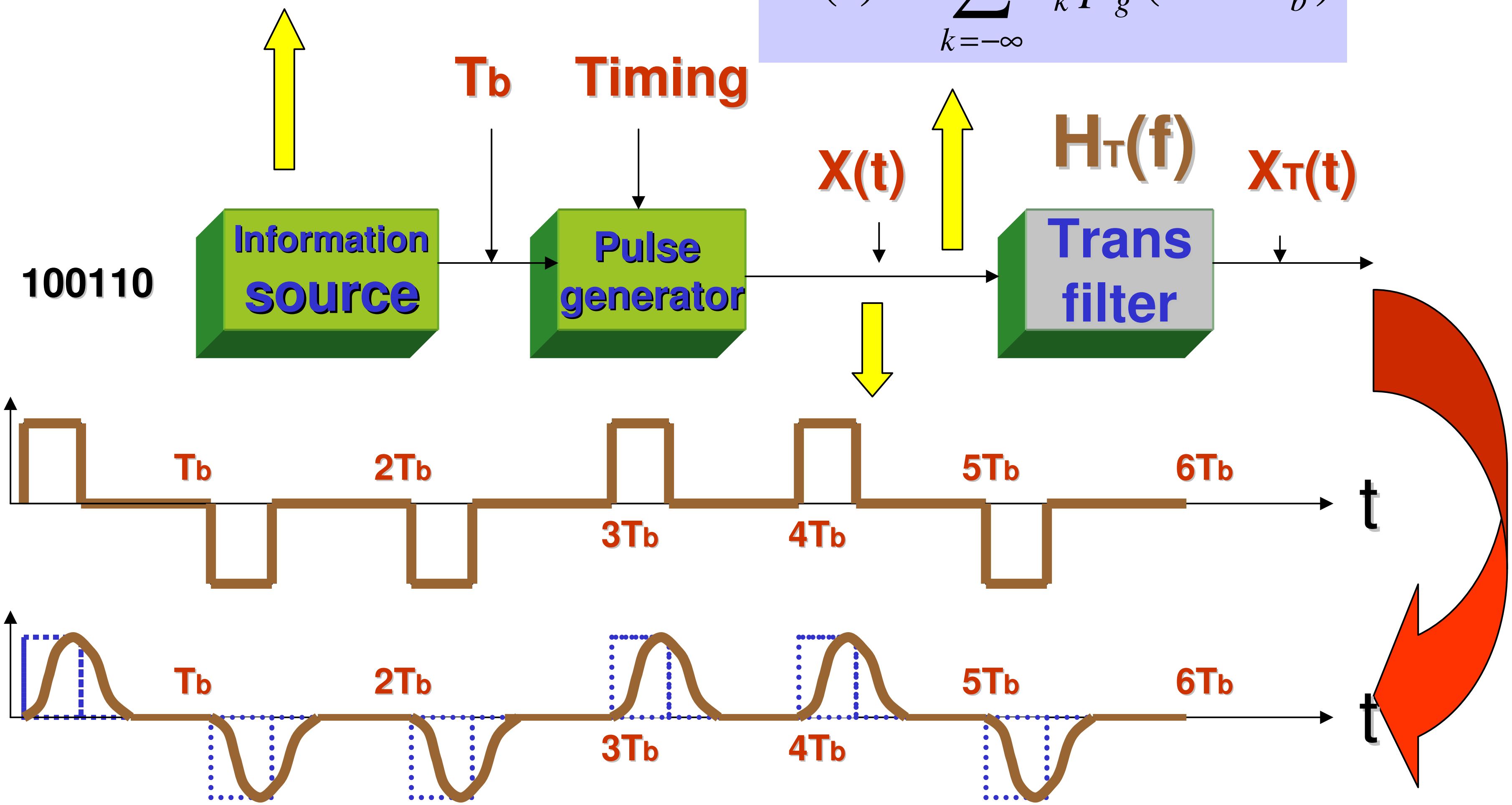
$$X(t) = \sum_{k=-\infty}^{\infty} a_k p_g(t - kT_b)$$



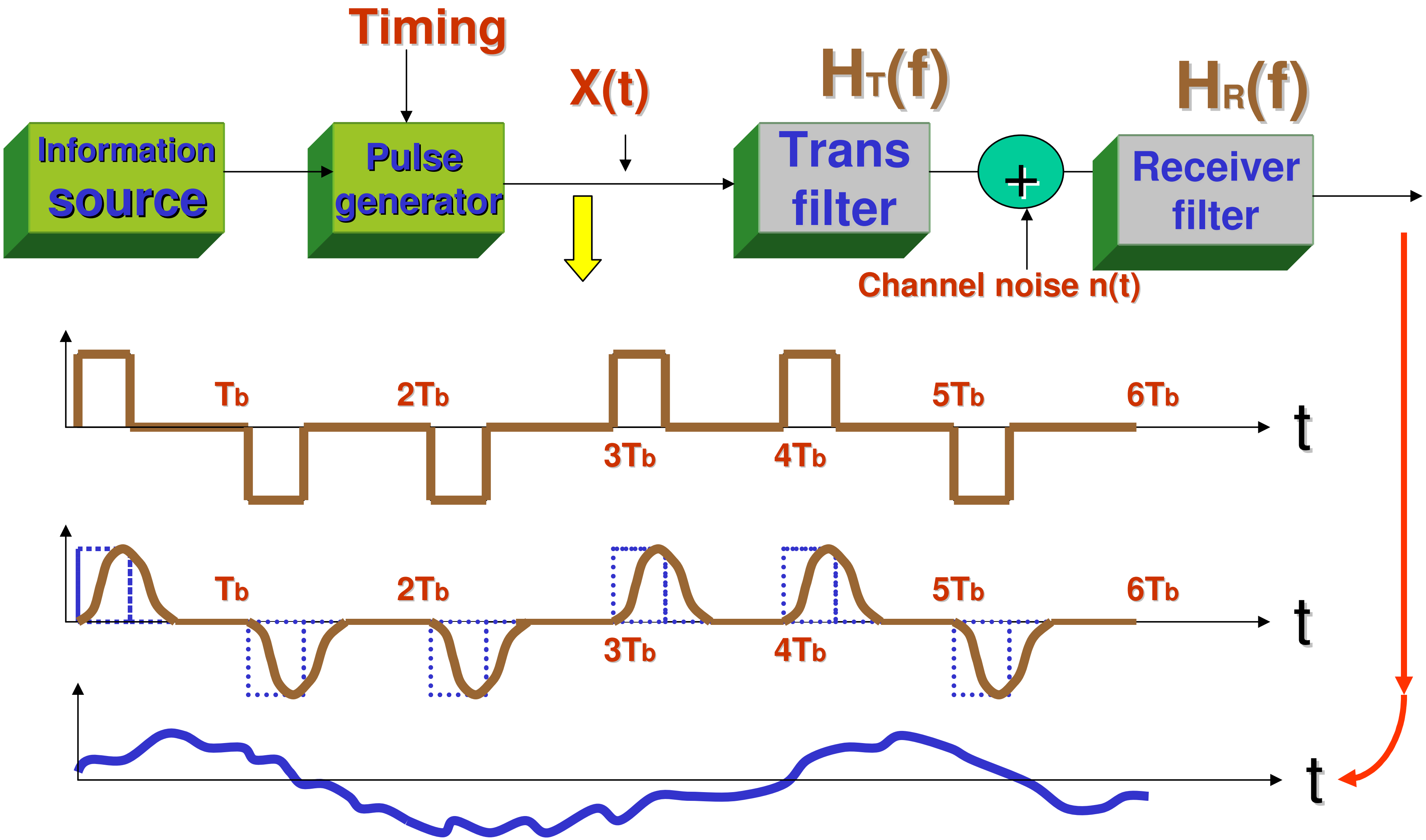
Block diagram Description (Continue - 3)

$\{d_k\}=\{1,1,1,1,0,0,1,1,0,0,0,1,1,1\}$

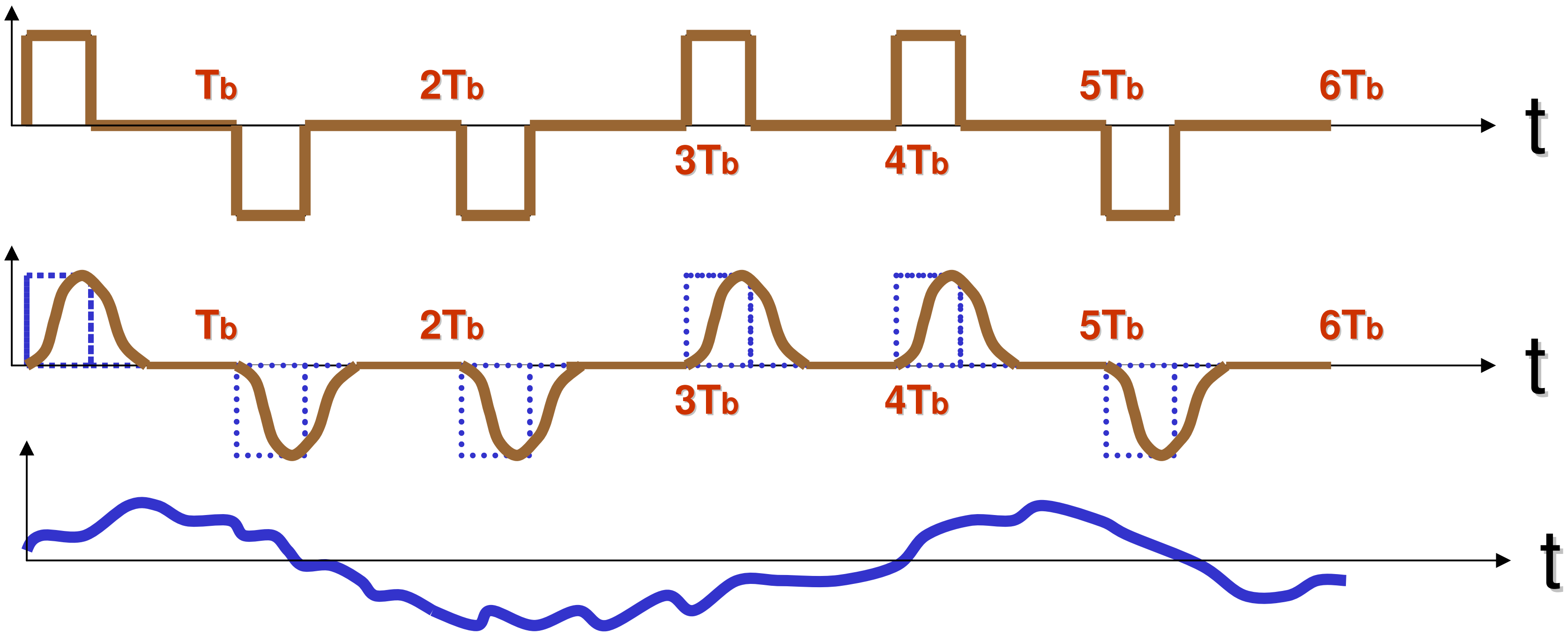
$$X(t) = \sum_{k=-\infty}^{\infty} a_k \, p_g(t - kT_b)$$



Block diagram Description (Continue - 4)

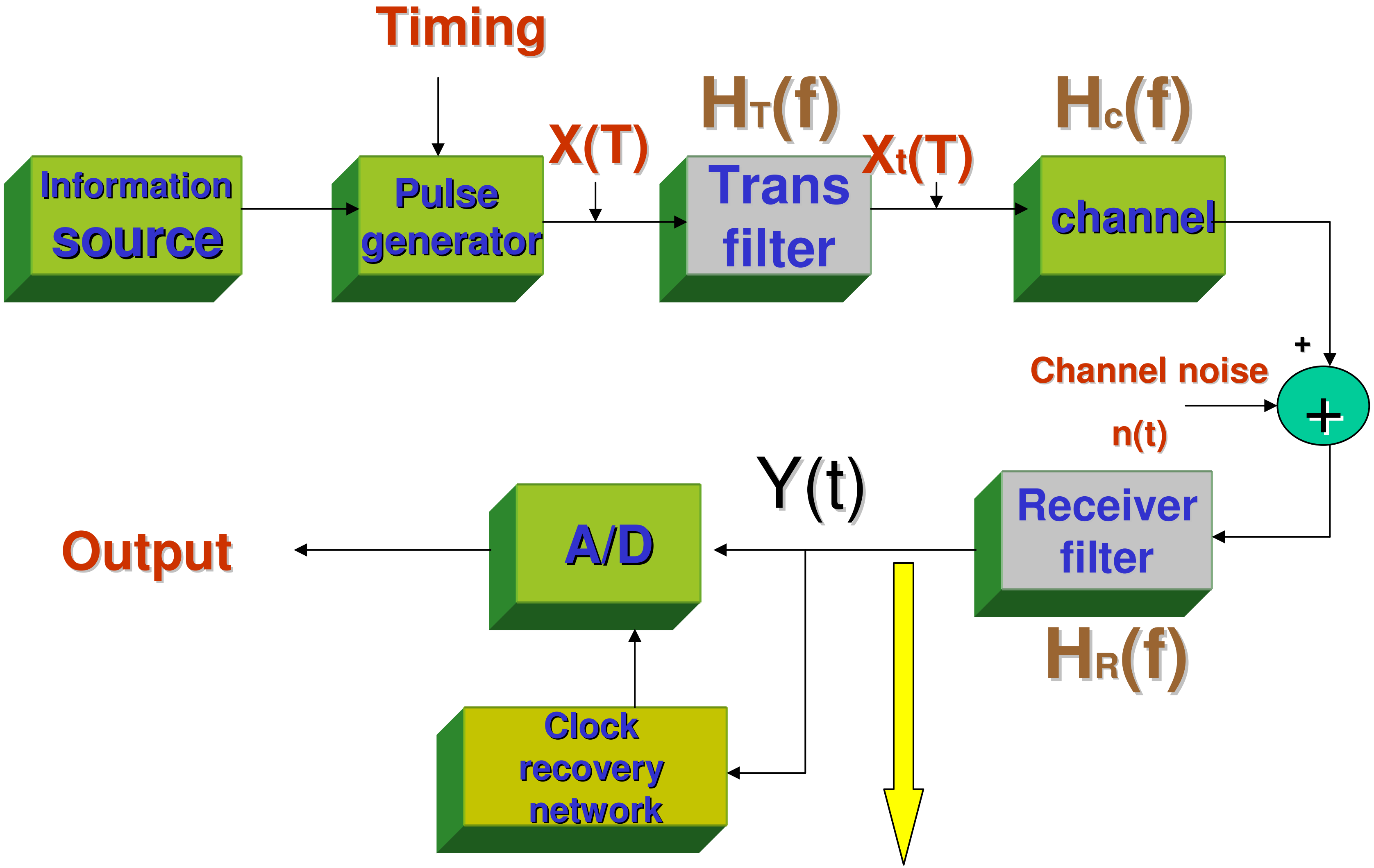


Block diagram Description (Continue - 5)



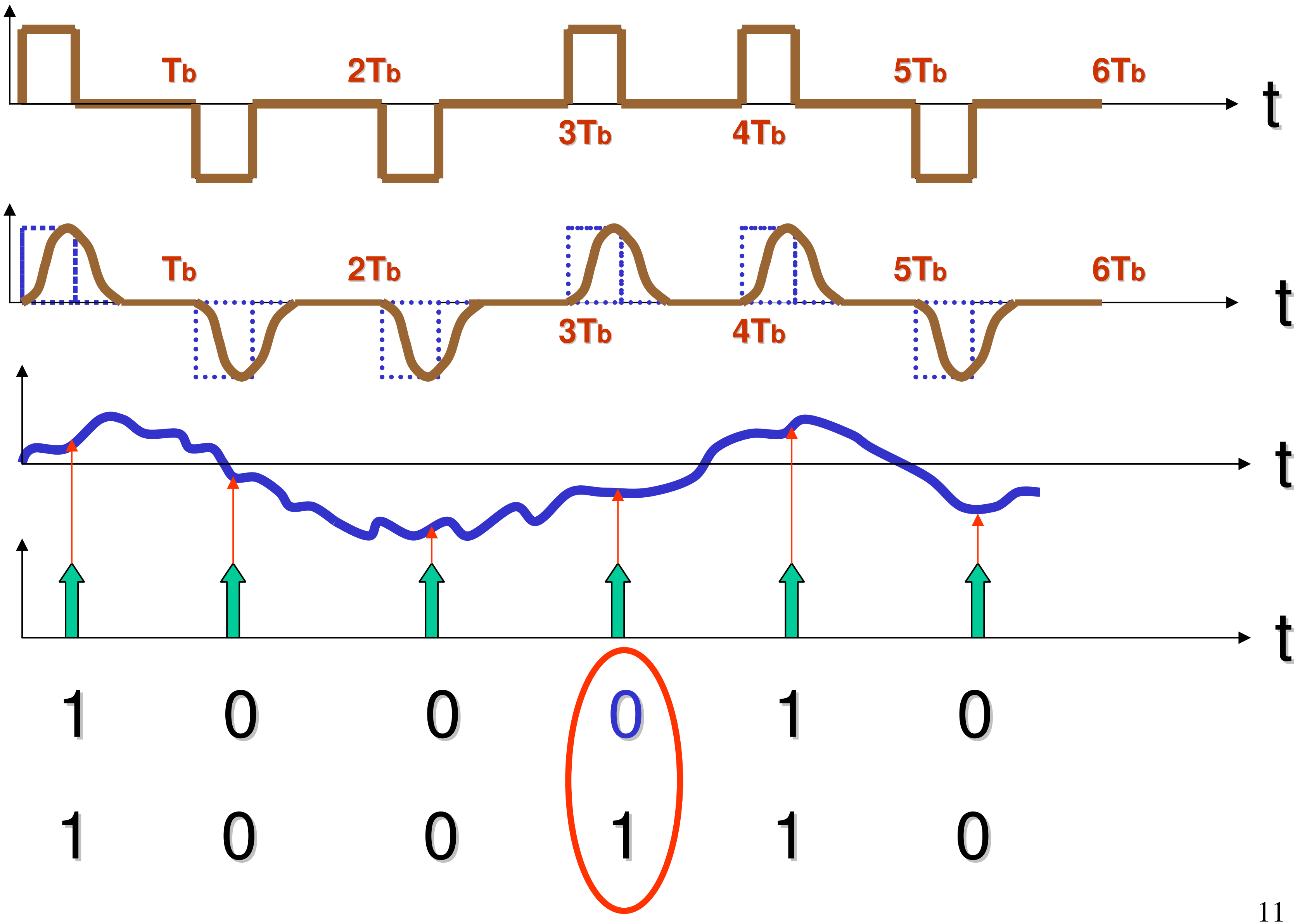
$$Y(t) = \sum_k A_k p_r(t - t_d - kT_b) + n_0(t)$$

Block diagram of an Binary/M-ary signaling scheme

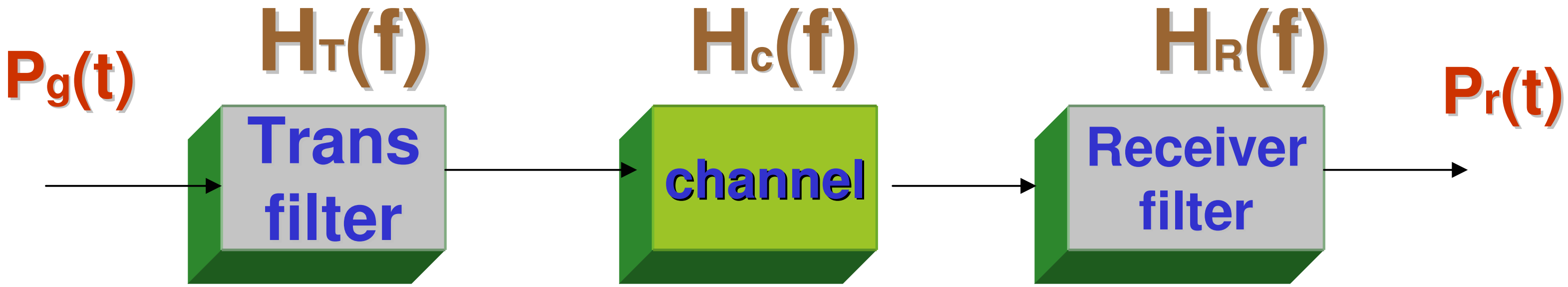


$$Y(t) = \sum_k A_k p_r(t - t_d - kT_b) + n_0(t)$$

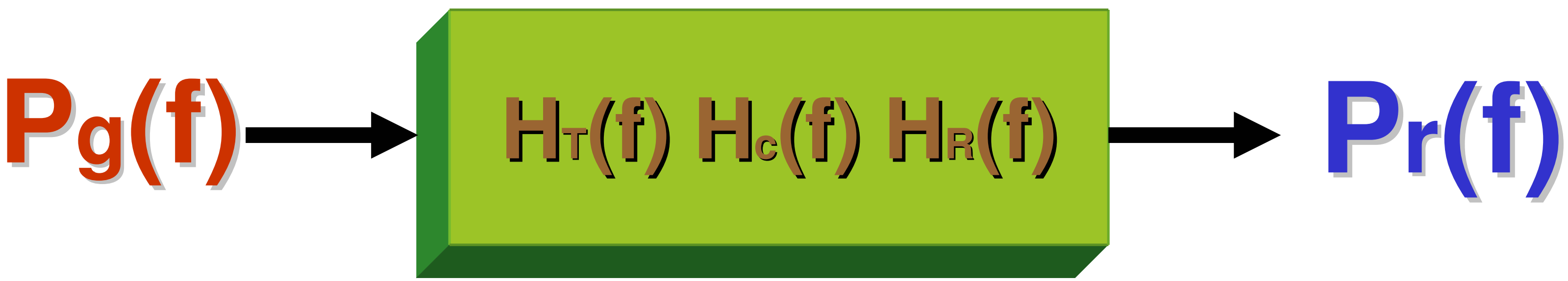
Block diagram Description



Explanation of $P_r(t)$



$$Y(t) = \sum_k A_k p_r(t - t_d - kT_b) + n_0(t)$$



$$p_r(0) = 1$$

Analysis and Design of Binary Signal

The output of the pulse generator X(t),is given by

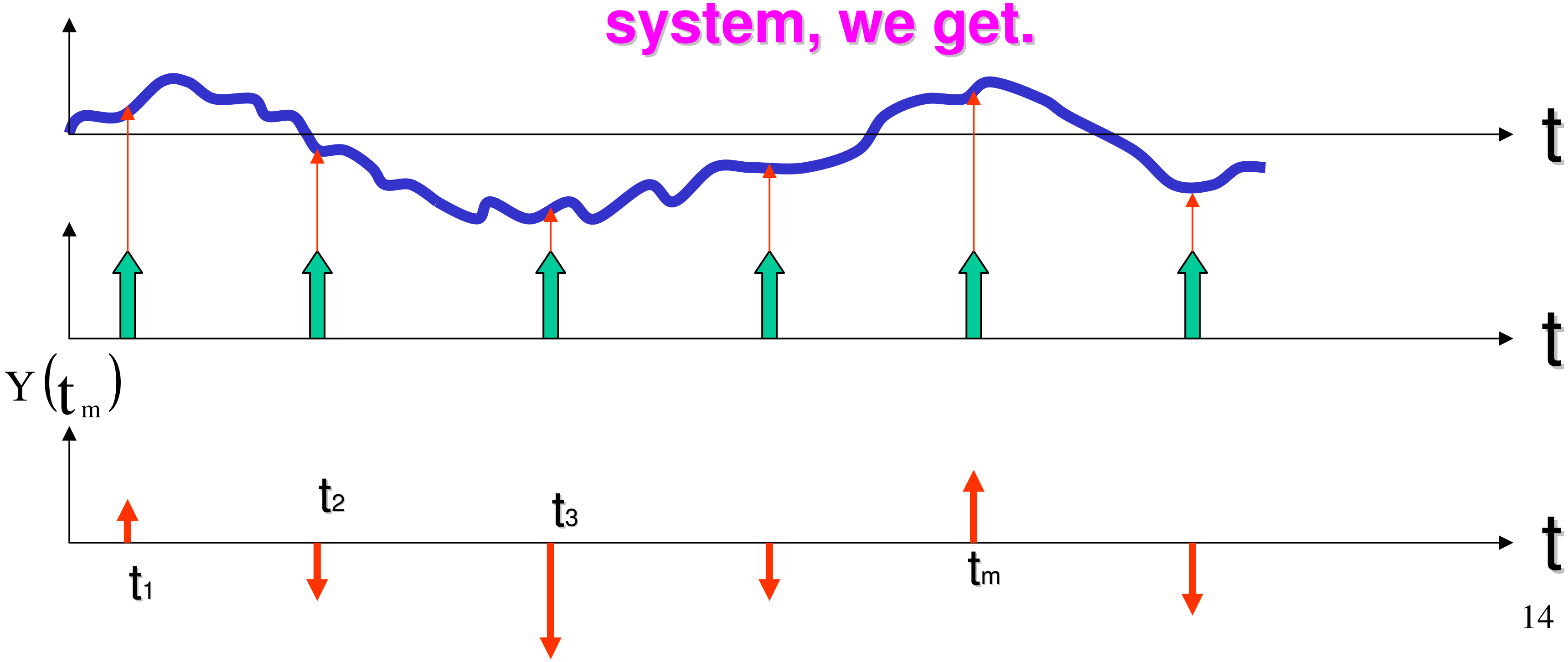
$$X(t) = \sum_{k=-\infty}^{\infty} a_k \, p_g(t - kT_b)$$

$P_g(t)$ is the basic pulse whose amplitude a_k depends on .the k_{th} input bit

The input to the A/D converter is

$$Y(t) = \sum_k A_k p_r(t - t_d - kT_b) + n_0(t)$$

For $t_m = mT_b + t_d$ and t_d is the total time delay in the system, we get.

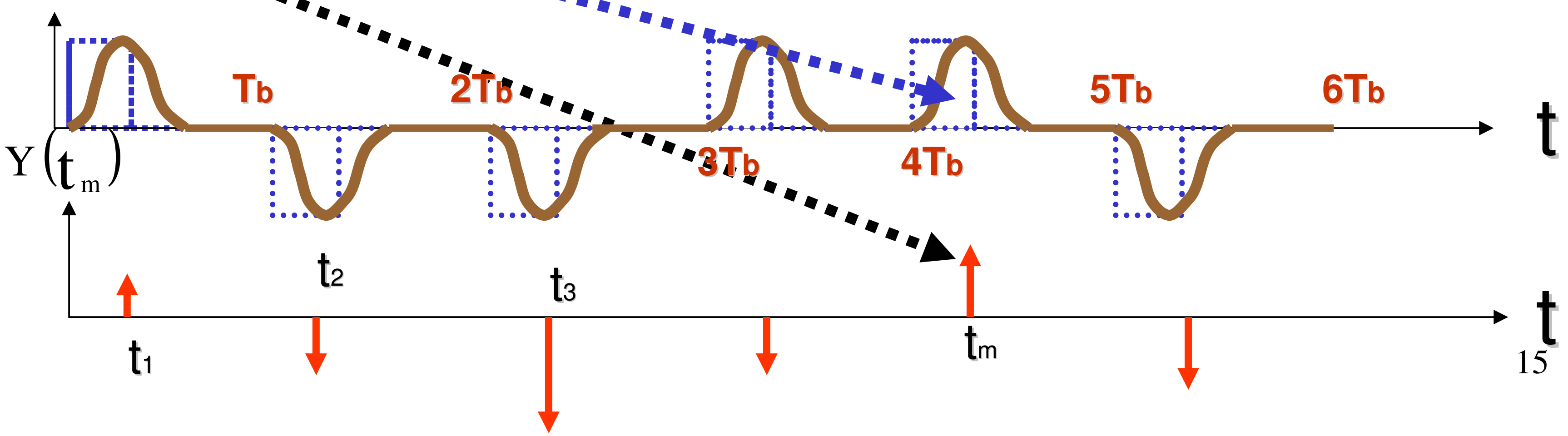


The output of the A/D converter at the sampling time

$$t_m = mT_b + t_d$$

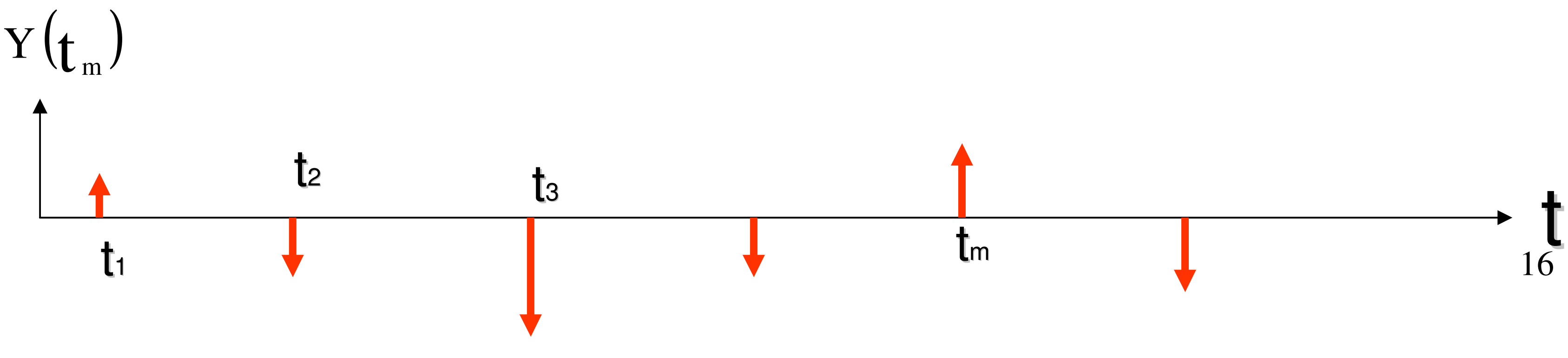
$$Y(t) = \sum_k A_k p_r(t - t_d - kT_b) + n_0(t)$$

$$Y(t_m) = A_m + \sum_{K \neq m} A_k p_r[(m-k)T_b] + n_0(t_m)$$

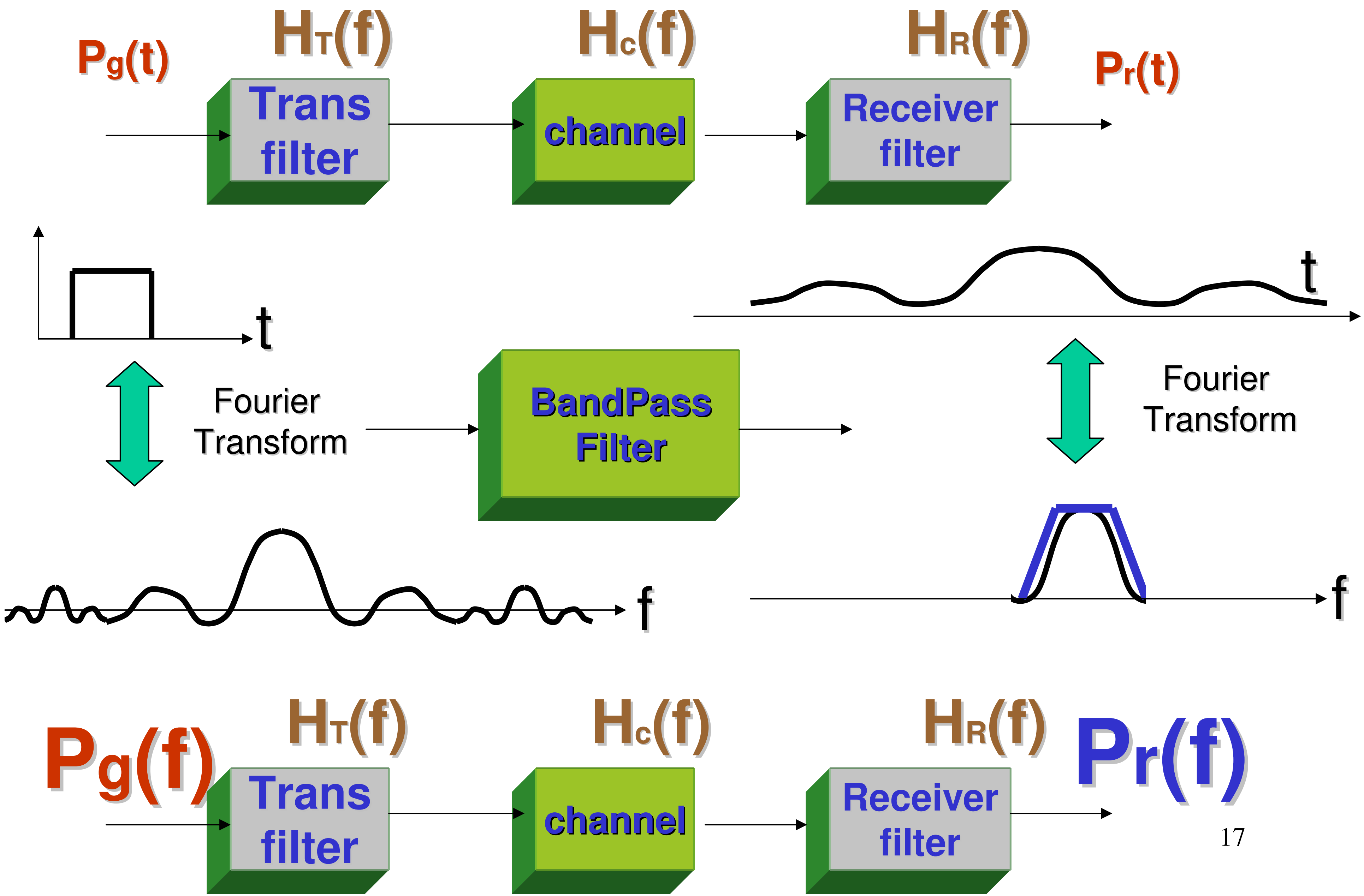


$$Y(t_m) = A_m + \sum_{K \neq m} A_k p_r[(m-k)T_b] + n_0(t_m)$$

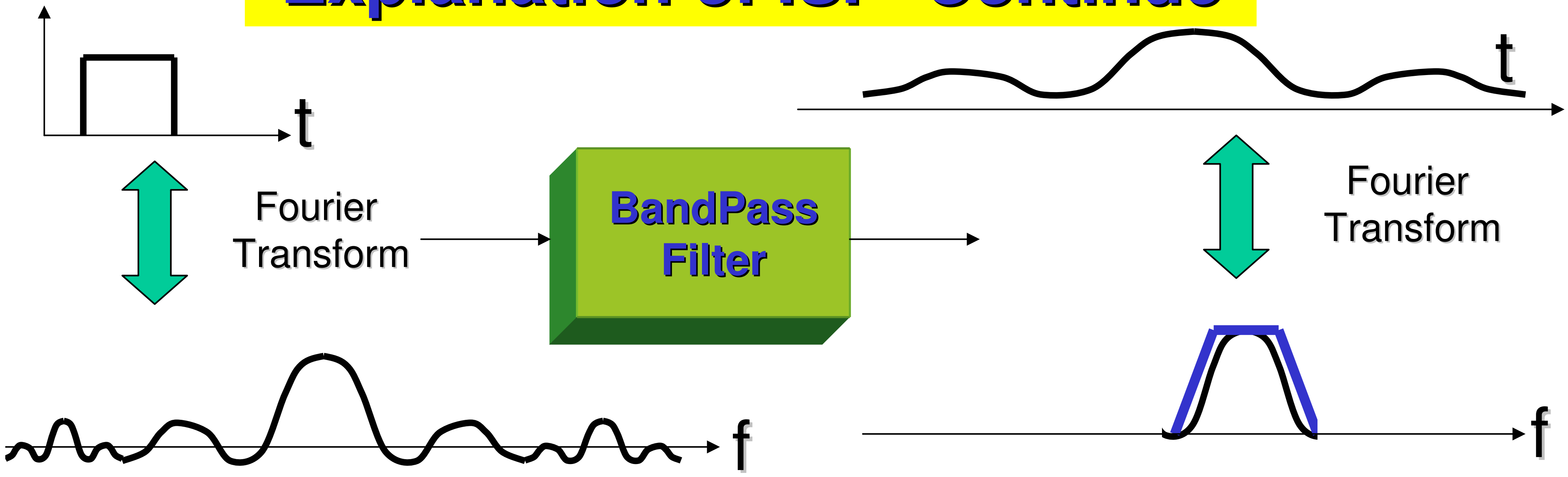
ISI - Inter Symbol Interference

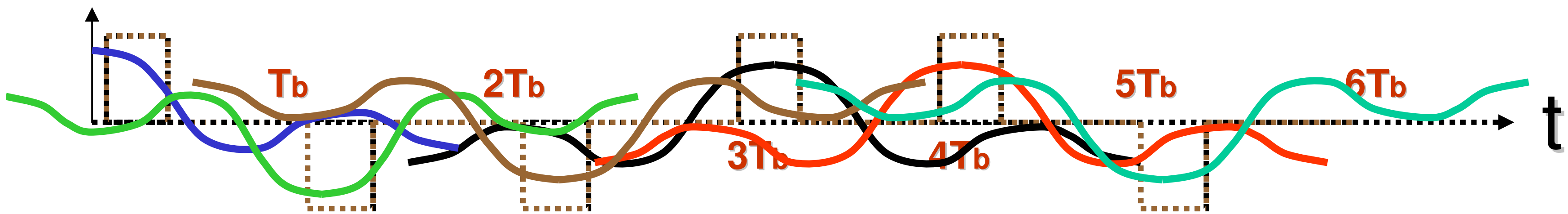


Explanation of ISI



Explanation of ISI - Continue





5.2.1 Baseband pulse shaping

The ISI can be eliminated by proper choice of received pulse shape $p_r(t)$.

$$p_r(nT_b) = \begin{cases} 1 & \text{for} \quad n = 0 \\ 0 & \text{for} \quad n \neq 0 \end{cases}$$

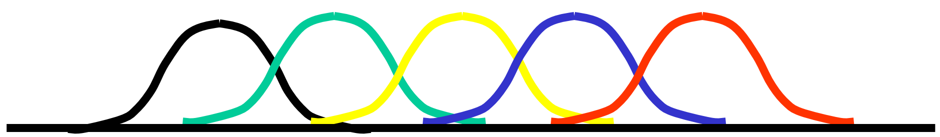
Doe's not Uniquely Specify Pr(t) for all values of t.

To meet the constraint, Fourier Transform Pr(f) of Pr(t), should satisfy a simple condition given by the following theorem

Theorem

if $$\sum_{k=-\infty}^{\infty} P_r\left(f + \frac{k}{T_b}\right) = T_b \quad \text{for} \quad |f| < 1/2T_b$$

Then $$p_r(nT_b) = \begin{cases} 1 & \text{for} \quad n = 0 \\ 0 & \text{for} \quad n \neq 0 \end{cases}$$



Proof

$$p_r(t) = \int_{-\infty}^{\infty} p_r(f)\exp(j2\pi ft)\,df$$

$$p_r(t) = \sum_{k=-\infty}^{\infty} \int_{(2k-1)/2T_b}^{(2k+1)/2T_b} p_r(f)\exp(j2\pi ft)\,df$$

$$p_r(nT_b) = \sum_k \int_{(2k-1)/2T_b}^{(2k+1)/2T_b} p_r(f)\exp(\ j2\pi fnT_b t)\,df$$

$$p_r(nT_b) = \sum_k \int_{-1/2T_b}^{1/2T_b} p_r\left(f' + \frac{k}{T_b}\right)\exp(\ j2\pi f' nT_b)\,df'$$

$$p_r(nT_b) = \int_{-1/2T_b}^{1/2T_b} \left(\sum_k p_r\left(f + \frac{k}{T_b}\right)\right)\exp(\ j2\pi fnT_b)\,df$$

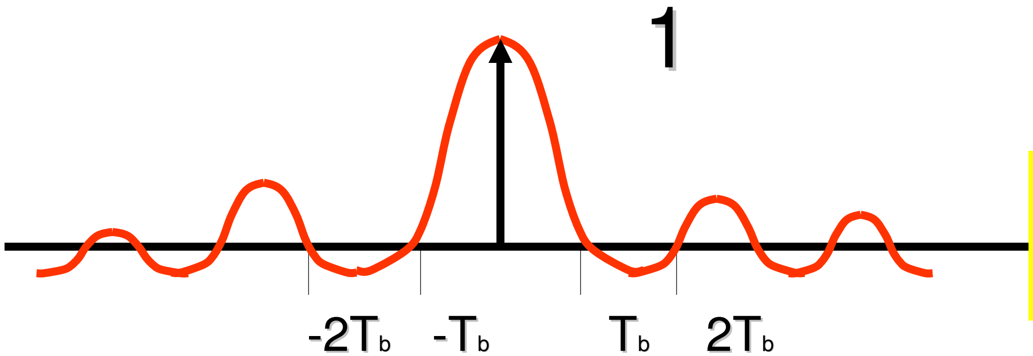
$$p_r(nT_b) = \int_{1/2T_b}^{1/2T_b} T_b \exp(\ j2\pi fnT_b)\,df = \frac{\sin(n\pi)}{n\pi}$$

Which verify that the Pr(t) with a transform Pr(f) Satisfy ZERO ISI

The condition for removal of ISI given in the theorem is called
Nyquist (Pulse Shaping) Criterion

$$Y(t_m) = A_m + \sum_{k \neq m} A_k P_r((m-k)T_b) + n_0(t_m)$$

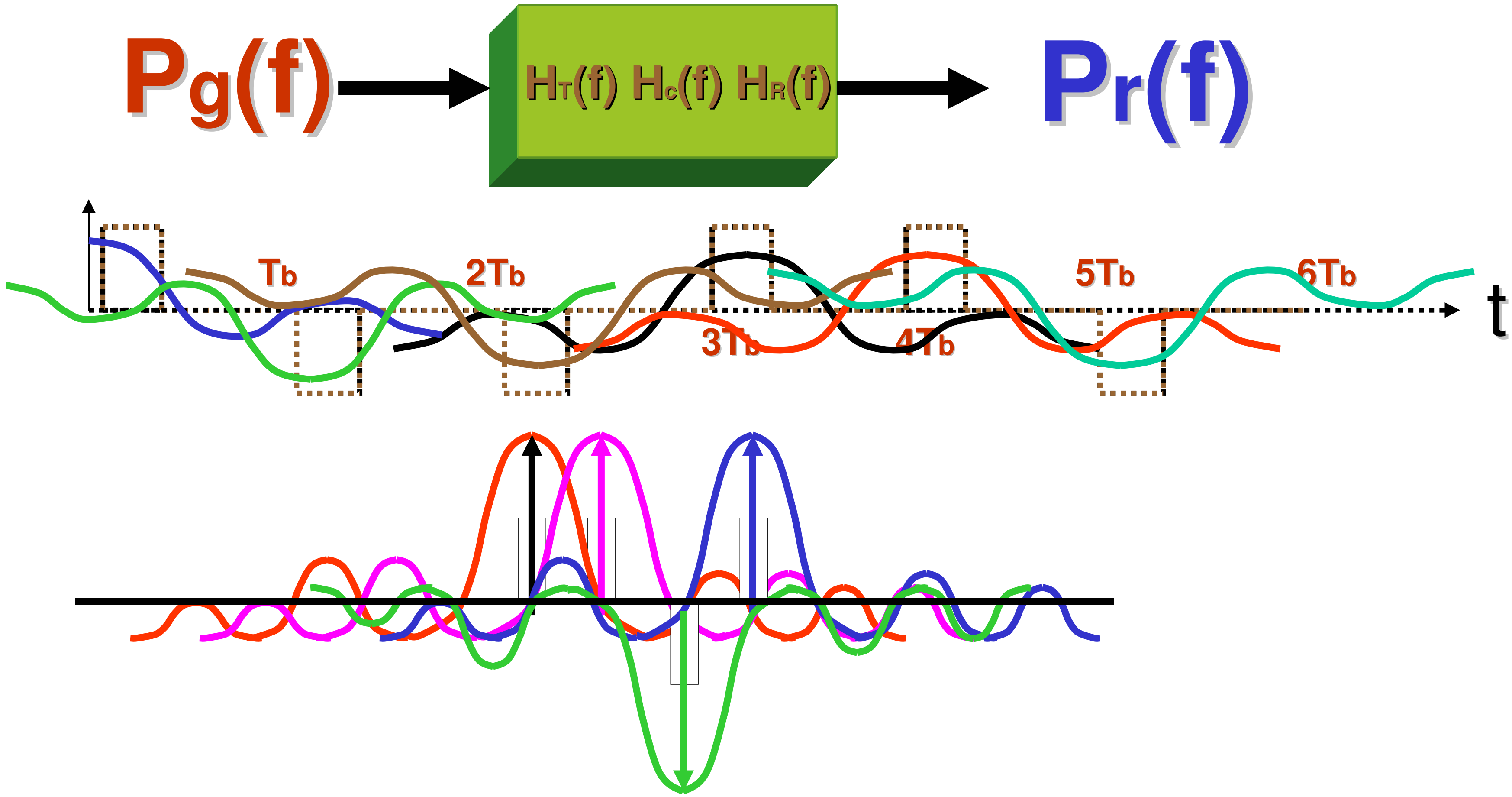
$$p_r(nT_b) = \begin{cases} 1 & \text{for} \quad n = 0 \\ 0 & \text{for} \quad n \neq 0 \end{cases}$$



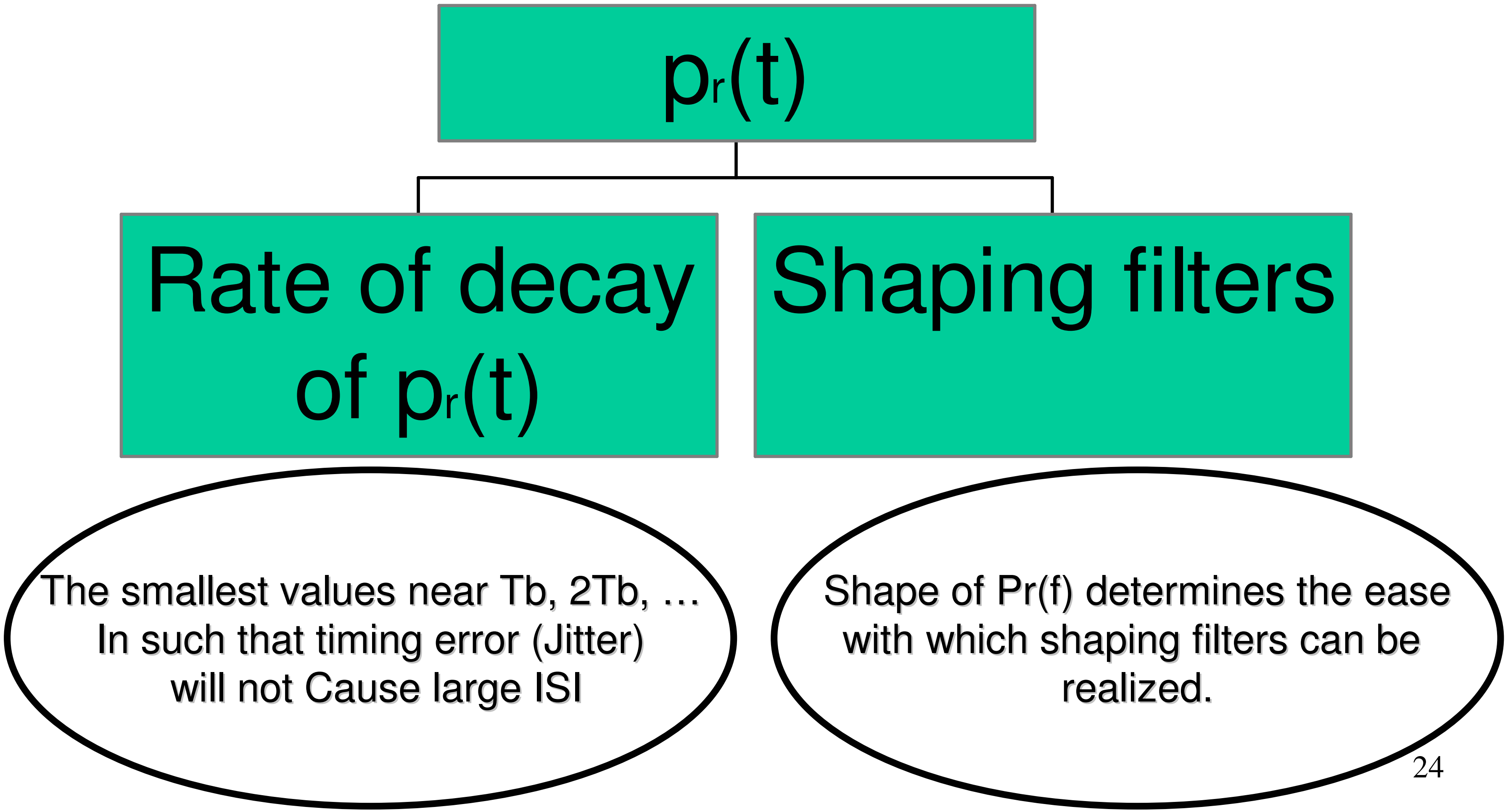
$$p_r(nT_b) = \frac{\sin(n\pi)}{n\pi}$$

The Theorem gives a condition for the removal of ISI using a Pr(f) with a bandwidth larger then rb/2/.

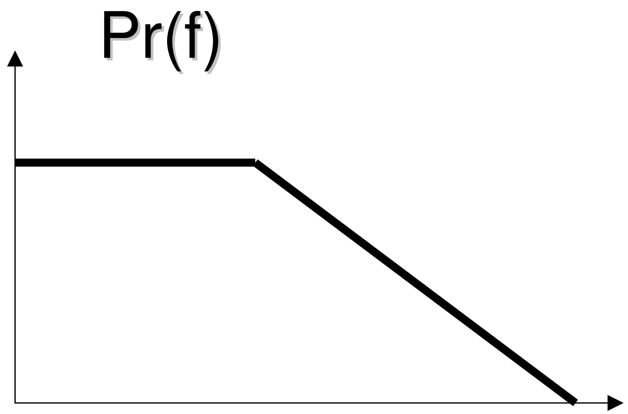
ISI can't be removed if the bandwidth of Pr(f) is less then rb/2.

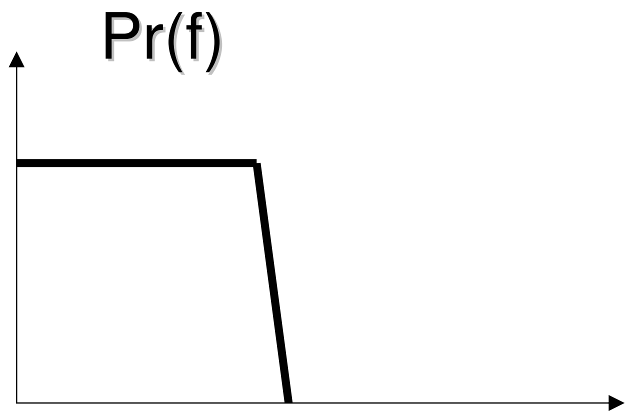


Particular choice of Pr(t) for a given application



A Pr(f) with a smooth roll - off characteristics is preferable over one with arbitrarily sharp cut off characteristics.





In practical systems where the bandwidth available for transmitting data at a rate of r_b bits\sec is between r_b\2 to r_b Hz, a class of $p_r(t)$ with a *raised cosine frequency characteristic* is most commonly used.

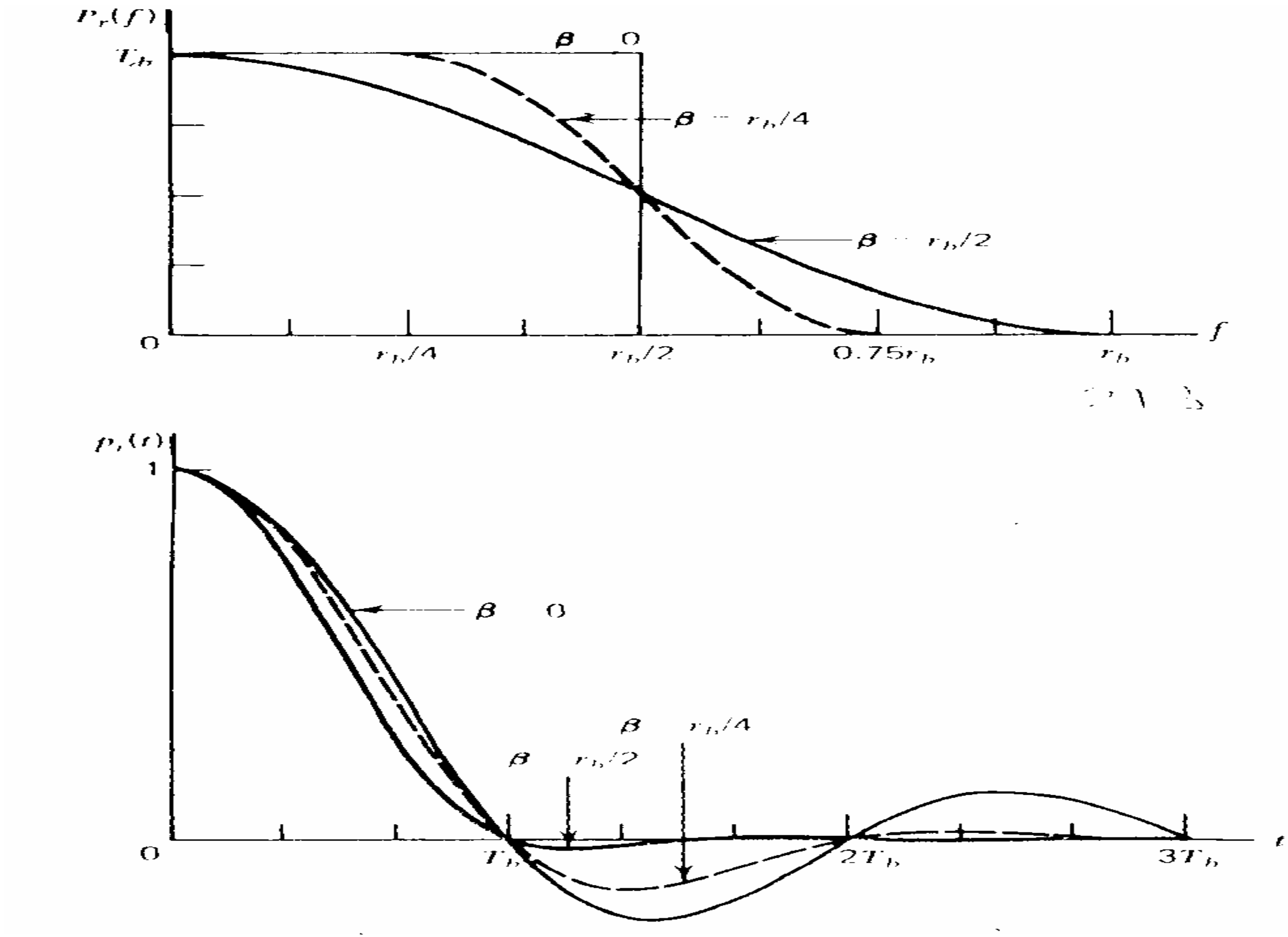
A raise Cosine Frequency spectrum consist of a flat amplitude portion and a roll off portion that has a sinusoidal form.

$$P_r(f) = \begin{cases} T_b, |f| \le r_b/2 - \beta \\ T_b \cos^2 \frac{\pi}{4\beta}\left(|f| - \frac{r_b}{2} + \beta\right), \frac{r_b}{2} - \beta < |f| \le \frac{r_b}{2} + \beta \\ 0, |f| < r_b/2 + \beta \end{cases}$$

$$FT^{-1}\{P_r(f)\} =$$

$$= P_r(t) = \frac{\cos 2\pi\beta t}{1 - (4\beta t)^2}\left(\frac{\sin \pi r_b t}{\pi r_b t}\right)$$

raised cosine frequency characteristic



Summary

The BW occupied by the pulse spectrum is **$B=r_b/2+\beta$**.
The minimum value of B is rb/2 and the maximum value is **r_b.**

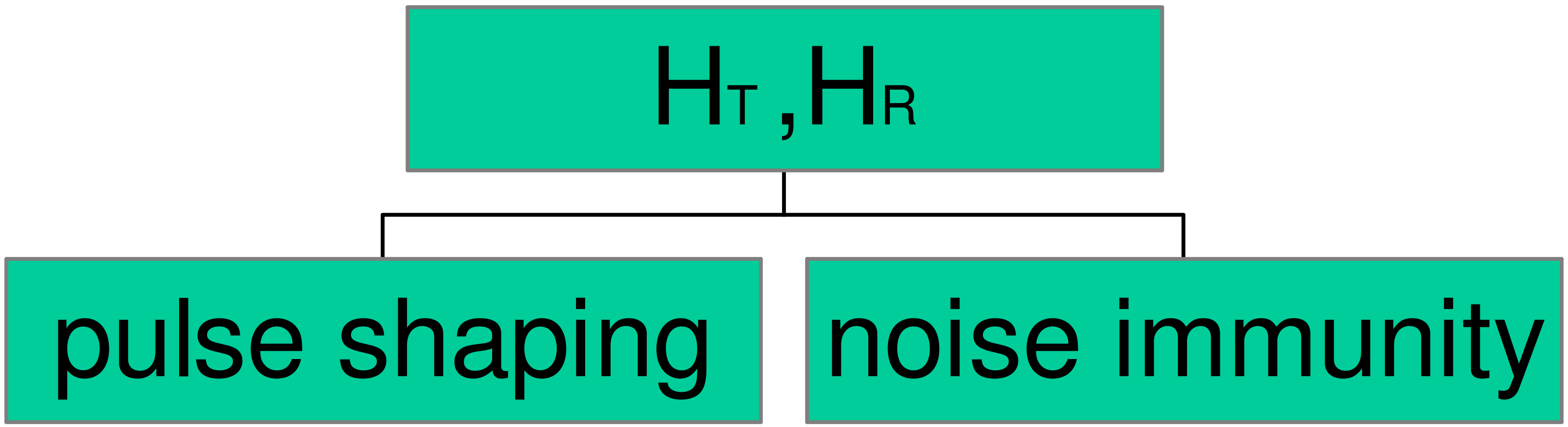
Larger values of β imply that more bandwidth is required for a given bit rate, however it lead for faster decaying pulses, which means that synchronization will be less critical and will not cause
large ISI.

β =rb/2 leads to a pulse shape with two convenient properties. The half amplitude pulse width is equal to Tb, and there are zero crossings at t=3/2Tb, 5/2Tb…. In addition to the zero crossing at Tb, 2Tb, 3Tb,…...

5.2.2

Optimum transmitting and receiving filters

The transmitting and receiving filters are chosen to provide a proper



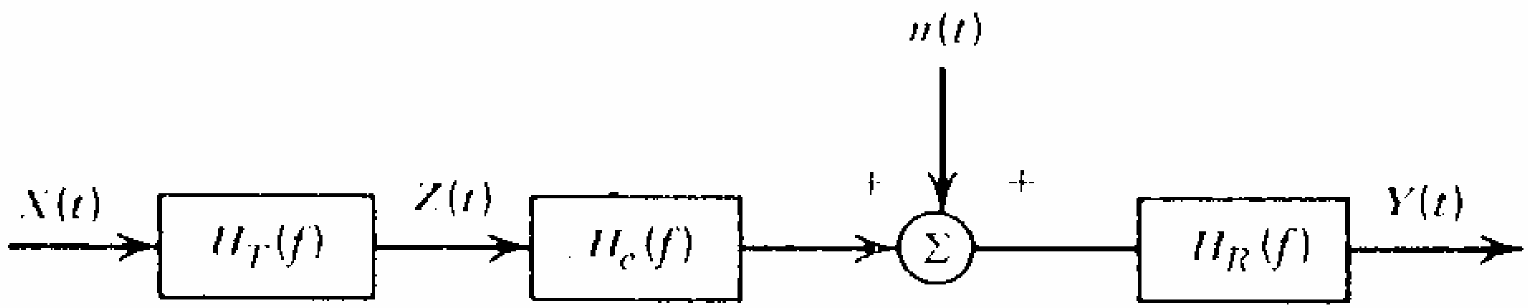
-One of design constraints that we have for selecting the filters is the relationship between the Fourier transform of $p_r(t)$ and $p_g(t)$.

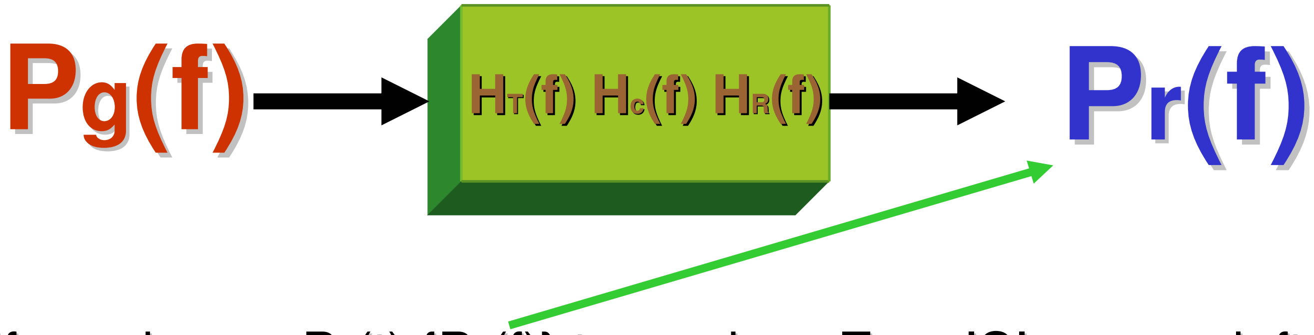
$$p_g(f)H_T(f)H_R(f) = K_c P_r(f)\exp(-2j2\pi f t_d)$$

Where t_d, is the time delay Kc normalizing constant.

In order to design optimum filter Ht(f) & Hr(f), we will assume that Pr(f), Hc(f) and Pg(f) are known.

Portion of a baseband PAM system





If we choose Pr(t) {Pr(f)} to produce Zero ISI we are left only to be concerned with noise immunity, that is will choose

$$\{H_T(f)\} \text{ and } \{H_R(f)\} \Rightarrow \textit{minimum of noise effects}$$

Noise Immunity

Problem definition:

For a given :

- Data Rate - r_b
- Transmission power - S_T
- Noise power Spectral Density - $G_n(f)$
- Channel transfer function - $H_c(f)$
- Raised cosine pulse - $P_r(f)$

Choose

$$\{H_T(f)\} \text{ and } \{H_R(f)\} \Rightarrow \textit{minimum of noise effects}$$

Error probability Calculations

At the m-th sampling time the input to the A/D is:

$$Y(t_m) = A_m + \sum_{k \neq m} A_k P_r((m-k)T_b) + n_0(t_m)$$

We decide:

$$\begin{aligned} &"1" \quad \textit{if} \quad Y(t_m) > 0 \\ &"0" \quad \textit{if} \quad Y(t_m) \le 0 \end{aligned}$$

$$P_{error} = \Pr ob\left[Y(t_m) > 0 \middle| \, "0" \; was \; sent\right] \bullet \Pr ob\left(To \; sent \, "0"\right) +$$
$$\Pr ob\left[Y(t_m) < 0 \middle| \, "1" \; was \; sent\right] \bullet \Pr ob\left(To \; sent \, "1"\right)$$

$$Y(t_m) = +A + n_0(t_m) \quad if \quad "1"$$
$$Y(t_m) = -A + n_0(t_m) \quad if \quad "0"$$

A=aKc

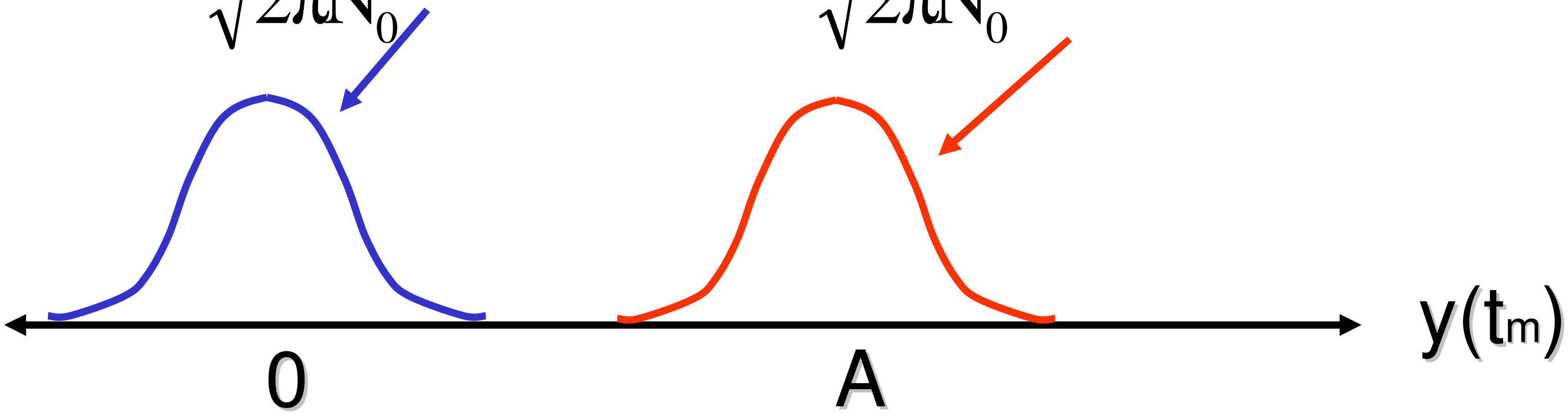
- $\Pr ob\left(To\ sent\ "0"\right) = \Pr ob\left(To\ sent\ "1"\right) = 0.5$

$$P_{error} = \frac{1}{2}\left\{\Pr ob\left[n_0(t_m) < -A\right] + \Pr ob\left[n_0(t_m) > A\right]\right\}$$

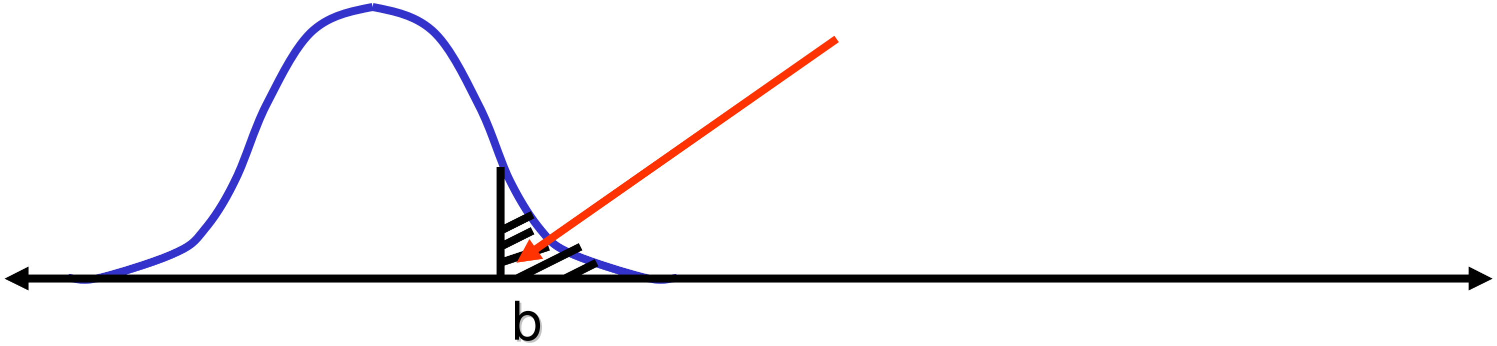
The noise is assumed to be zero mean Gaussian at the receiver input then the output should also be Zero mean Gaussian with variance No given by:

$$N_0 = \int_{-\infty}^{\infty} G_n(f)\left|H_R(f)\right|^2 df$$

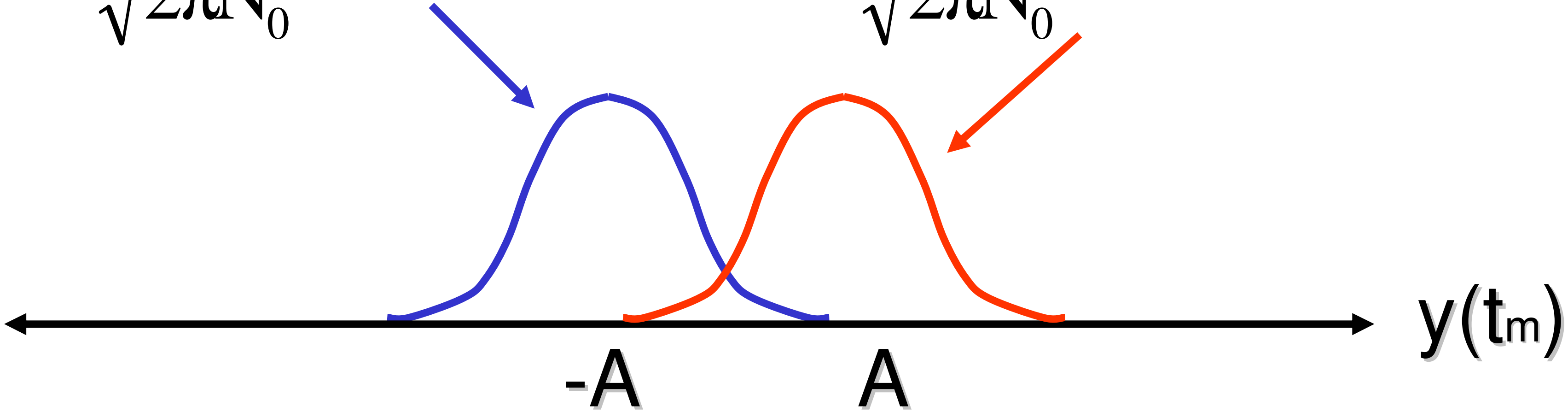
$$\frac{1}{\sqrt{2\pi N_0}} e^{\left(-[n]^2/2N_0\right)} \qquad \frac{1}{\sqrt{2\pi N_0}} e^{\left(-[n-A)]^2/2N_0\right)}$$



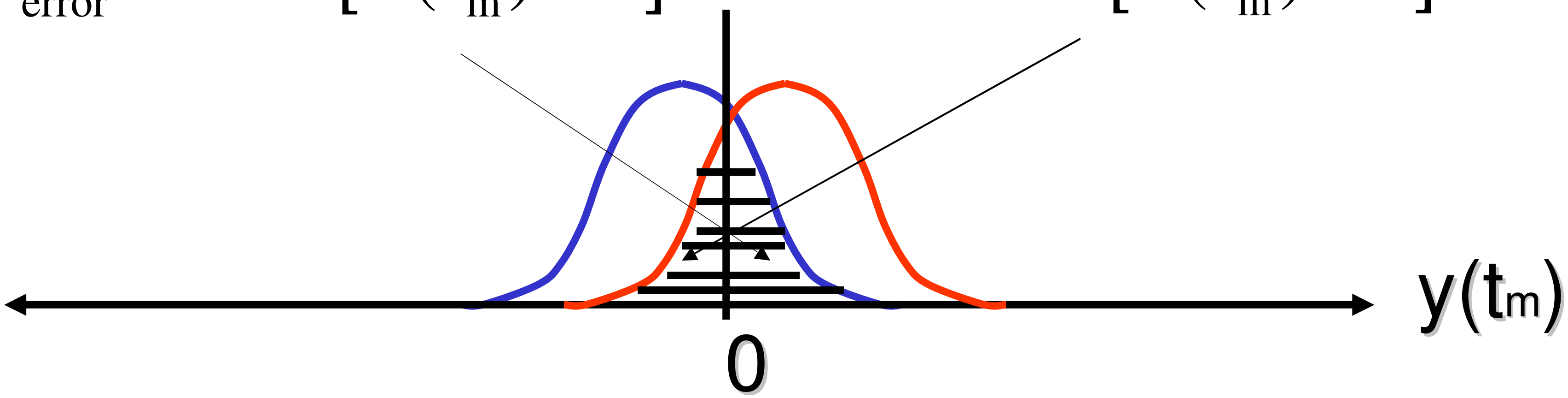
$$P_{error} = \Pr ob[n > b] = \int_b^{\infty} \frac{1}{\sqrt{2\pi N_0}} e^{\left(-[z)]^2/2N_0\right)} dz$$



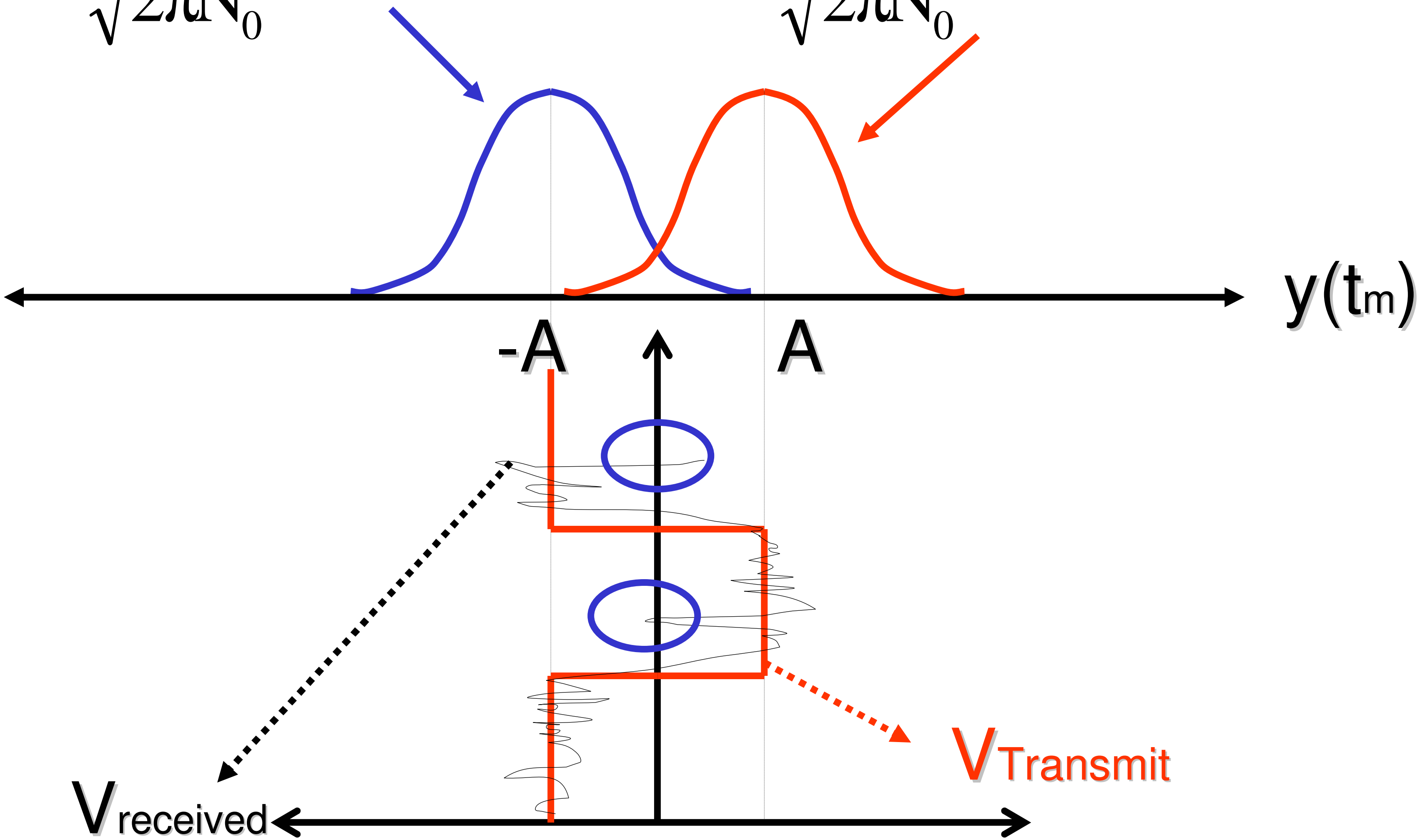
$$\frac{1}{\sqrt{2\pi N_0}} e^{\left(-[y(t_m)+A)]^2/2N_0\right)} \qquad \frac{1}{\sqrt{2\pi N_0}} e^{\left(-[y(t_m)-A)]^2/2N_0\right)}$$



$$P_{error} = \mathrm{Prob}\left[Y(t_m) > 0\right] \qquad \mathrm{Prob}\left[Y(t_m) < 0\right]$$



$$\frac{1}{\sqrt{2\pi N_0}} e^{\left(-[y(t_m)+A)]^2/2N_0\right)} \qquad \frac{1}{\sqrt{2\pi N_0}} e^{\left(-[y(t_m)-A)]^2/2N_0\right)}$$



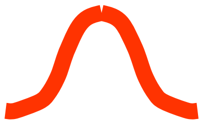
$$P_e = 1/2 \int_{|x|>A}^{\infty} \frac{1}{\sqrt{2\pi N_0}} \exp\left(-x^2/2N_0\right) dx =$$

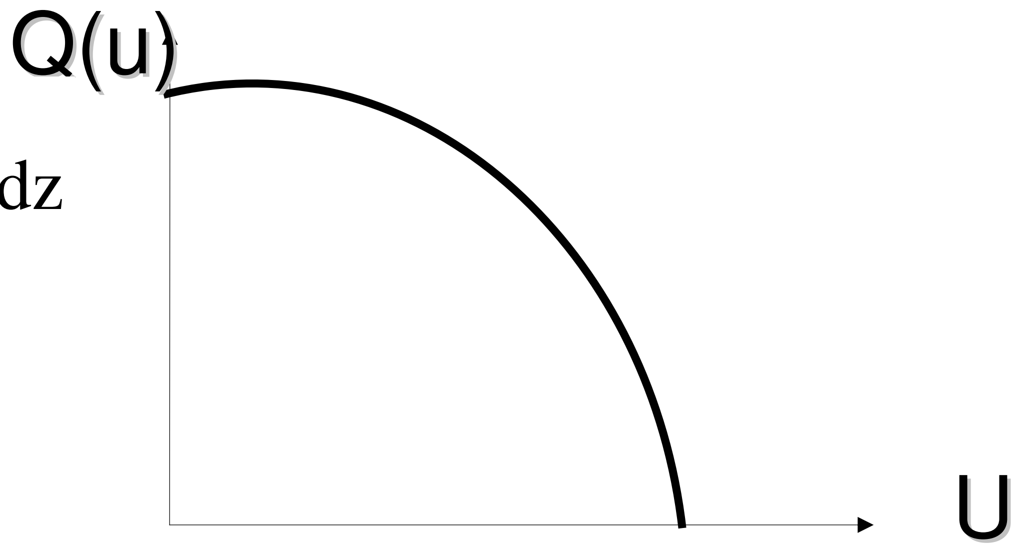
$$= \int_{A}^{\infty} \frac{1}{\sqrt{2\pi N_0}} \exp\left(-x^2/2N_0\right) dx \Big|_{z=x/\sqrt{N_0}} =$$

$$P_e = \int_{A/\sqrt{N_0}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-z^2/2\right) dz = Q\left(\frac{A}{\sqrt{N_0}}\right)$$

$$Q(u) = \int_{u}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-z^2/2\right) dz$$

$$Q(u)=\int_u^\infty \frac{1}{\sqrt{2\pi}}\exp\left(-z^2/2\right)dz$$

$= \int_u^\infty$  dz=



$$P_e=\int_{A/\sqrt{N_0}}^\infty \frac{1}{\sqrt{2\pi}}\exp\left(-z^2/2\right)dz = Q\left(\frac{A}{\sqrt{N_0}}\right)$$

$$Q(u)=\int_u^\infty \frac{1}{\sqrt{2\pi}}\exp\left(-z^2/2\right)dz$$

$$\frac{A}{\sqrt{N_0}} = \text{Signal to Noise Ratio}$$

P_{error} decreases as $A/\sqrt{N_0}$ increase

Hence we need to maximize the signal to noise Ratio

Thus for maximum noise immunity the filter transfer functions $H_T(f)$ and $H_R(f)$ must be chosen to maximize the SNR

Optimum filters design calculations

We will express the SNR in terms of $H_T(f)$ and $H_R(f)$

We will start with the signal:

$$X(t) = \sum_{k=-\infty}^{\infty} a_k p_g(t - kT_b)$$

$$G_X(f) = \frac{p_g(f)}{T_b} E\{a_k^2\} = \frac{a^2 |p_g(f)|^2}{T_b}$$

The psd of the transmitted signal is given by::

$$G_X(f) = |H_T(f)|^2 \cdot G_X(f)$$

And the average transmitted power ST is

$$S_T = \frac{a^2}{T_b} \int_{-\infty}^{\infty} |P_g(f)|^2 \cdot |H_T(f)|^2 df \Bigg|_{\substack{A_k = K_c a_k \\ A = K_c a}} =$$

$$= S_T = \frac{A^2}{K_c^2 T_b} \int_{-\infty}^{\infty} |P_g(f)|^2 \cdot |H_T(f)|^2 df$$

$$A^2 = \frac{S_T K_c^2 T_b}{\int_{-\infty}^{\infty} |P_g(f)|^2 \cdot |H_T(f)|^2 df}$$

The average output noise power of $n_0(t)$ is given by:

$$N_o = \int_{-\infty}^{\infty} G_n(f) |H_R(f)|^2 df$$

The SNR we need to maximize is

$$\frac{A^2}{N_o} = \frac{S_T T_b}{\left[\int_{-\infty}^{\infty} G_n(f)\left|H_R(f)\right|^2 df \cdot \int_{-\infty}^{\infty} \frac{\left|P_r(f)\right|^2}{\left|H_c(f)H_R(f)\right|^2} df\right]}$$

where $\left|P_r(f)\right| = \left|H_c(f)H_R(f)H_T(f)\right|$

Or we need to minimize

$$\min\left\{\left[\int_{-\infty}^{\infty} G_n(f)\left|H_R(f)\right|^2 df \cdot \int_{-\infty}^{\infty} \frac{\left|P_r(f)\right|^2}{\left|H_c(f)H_R(f)\right|^2} df\right]\right\} = \min\left\{\gamma^2\right\}$$

Using Schwartz's inequality

$$\int_{-\infty}^{\infty} |V(f)|^2 df \cdot \int_{-\infty}^{\infty} |W(f)|^2 df \geq \left| \int_{-\infty}^{\infty} V(f) W(f) df \right|^2$$

The minimum of the left side equaity is reached when

V(f)=const*W(f)

If we choose :

$$|V(f)| = |H_R(f)| G_n^{1/2}(f)$$

$$|W(f)| = \frac{|P_r(f)|}{|H_R(f)||H_c(f)|}$$

$$\gamma^2 \textit{ is minimized when}$$

$$\left|H_R(f)\right|^2 = \frac{K\left|P_r(f)\right|}{\left|H_c(f)\right| G_n^{1/2}(f)}$$

$$\left|H_T(f)\right|^2 = \frac{K_c^{\,2}\left|P_r(f)\right| G_n^{1/2}(f)}{K\left|H_c(f)\right|\left|P_g(f)\right|^2}$$

$$\mathrm{K} - \mathrm{an} \ \textit{arbitrary positive constant}$$

The filter should have alinear phase response in a total time delay of td

Finally we obtain the maximum value of the SNR to be:

$$\left(\frac{A^2}{N_o}\right)_{max} = \frac{S_T T_b}{\left[\int_{-\infty}^{\infty} \frac{|P_r(f)| G_n^{1/2}(f)}{|H_c(f)|} df\right]^2}$$

$$P_{error} = Q\left(\sqrt{\left(\frac{A^2}{N_o}\right)_{max}}\right)$$

For AWGN with $G_n(f) = \eta / 2$

and

$p_g(f)$ is chosen such that it does not change much over the bandwidth of interest we get.

$$\left|H_R(f)\right|^2 = K_1 \frac{\left|p_r(f)\right|}{\left|H_c(f)\right|}$$

$$\left|H_T(f)\right|^2 = K_2 \frac{\left|p_r(f)\right|}{\left|H_c(f)\right|}$$

Rectangular pulse can be used at the input of $H_T(f)$.

$$p_g(t) = \begin{cases} 1 & \text{for} \quad |t| < \tau / 2 ; \tau \ll T_b \\ 0 & \text{elsewhere} \end{cases}$$