EC744 Wireless Communication
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Wireless Communication
OFDM

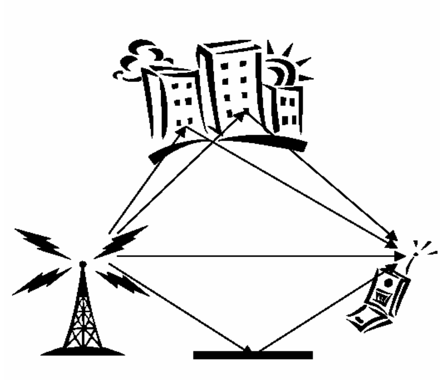
Syllabus

- Tentatively

Week 1	Overview wireless communications, Probabilities
Week 2	Digital Communication fundamentals
Week 3	Channel characteristics (AWGN, fading)
Week 4	Modulation techniques Demodulation techniques (coherent and non-coherent)
Week 5	OFDM
Week 6	Channel coding techniques
Week 7	Mid Term exam (take home), Diversity techniques
Week 8	Equalization techniques
Week 9	Spread spectrum, MIMO
Week 10	Wireless networking: 802.11, 802.16, UWB
Week 11	Hot topics
Week 12	Presentations
Week 13	Presentations
Week 14	Presentations
Week 15	Final Exam

What is OFDM?

- OFDM stands for Orthogonal Frequency Division Multiplexing. and is a modulation technique for transmitting large amounts of digital data over a radio wave.



OFDM history

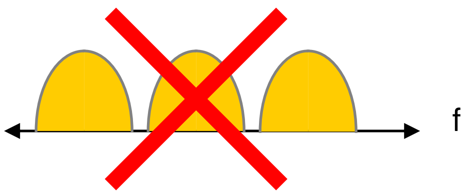
- The concept of using parallel data transmission by means of frequency division multiplexing (FDM) was published in mid 60s
- Some early development can be traced back in the 50s. A U.S. patent was filled and issued in January, 1970
- In the 1980s, OFDM has been studied for high-speed modems .

How OFDM differs from other multiplexing techniques?

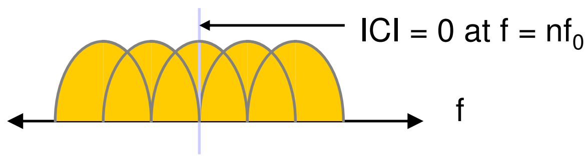
- ***Orthogonality**:
- The “orthogonal” part of the OFDM name indicates that there is a precise mathematical relationship between the frequencies of the carriers in the system
- **Wireless** The OFDM modulation scheme offers many advantages for broadband wireless transport. -It supports high data rates

Subcarrier Orthogonality

- Orthogonality simplifies recovery of the N data streams
  - Orthogonal subcarriers = No inter-carrier-interference (ICI)
- Time Domain Orthogonality:
  - Every subcarrier has an integer number of cycles within T_{OFDM}
    - Satisfies precise mathematical definition of orthogonality for complex exponential (and sinusoidal) functions over the interval $[0, T_{OFDM}]$
- Frequency Domain Orthogonality:

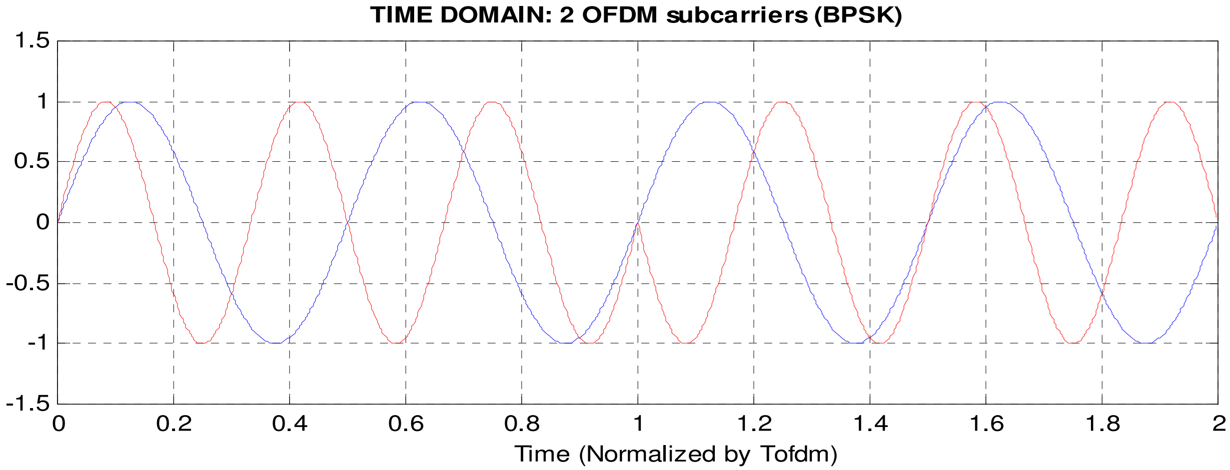


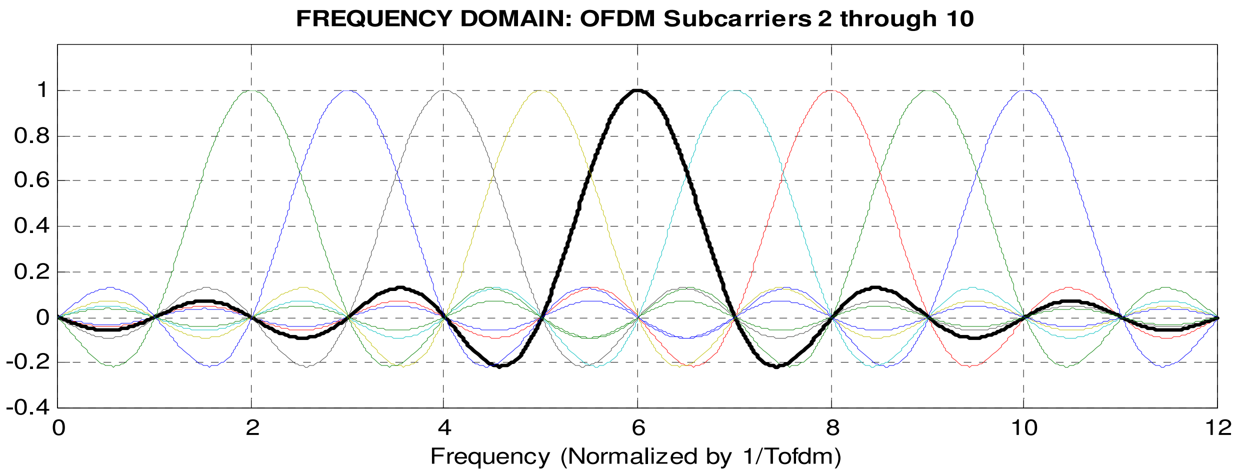
Some FDM systems achieve orthogonality through zero spectral overlap
⇒ BW inefficient!



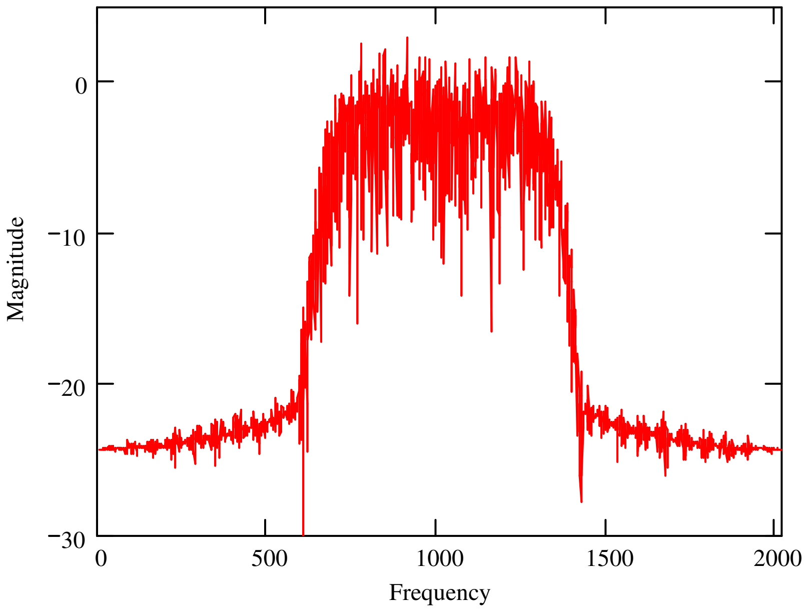
OFDM systems have overlapped spectra with each subcarrier spectrum having a Nyquist "zero ISI pulse shape" (really zero ICI in this case).
⇒ BW efficient!

OFDM Signal (Time & Frequency)





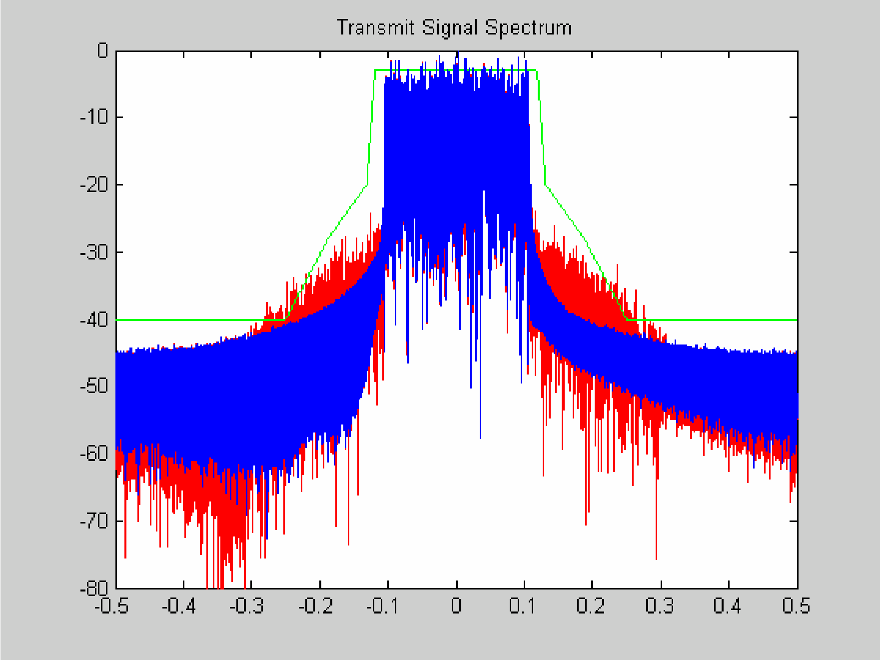
Practical Signal Spectra



Single carrier signals require filtering for spectral containment. This signal has narrow rolloff regions which requires long filters.

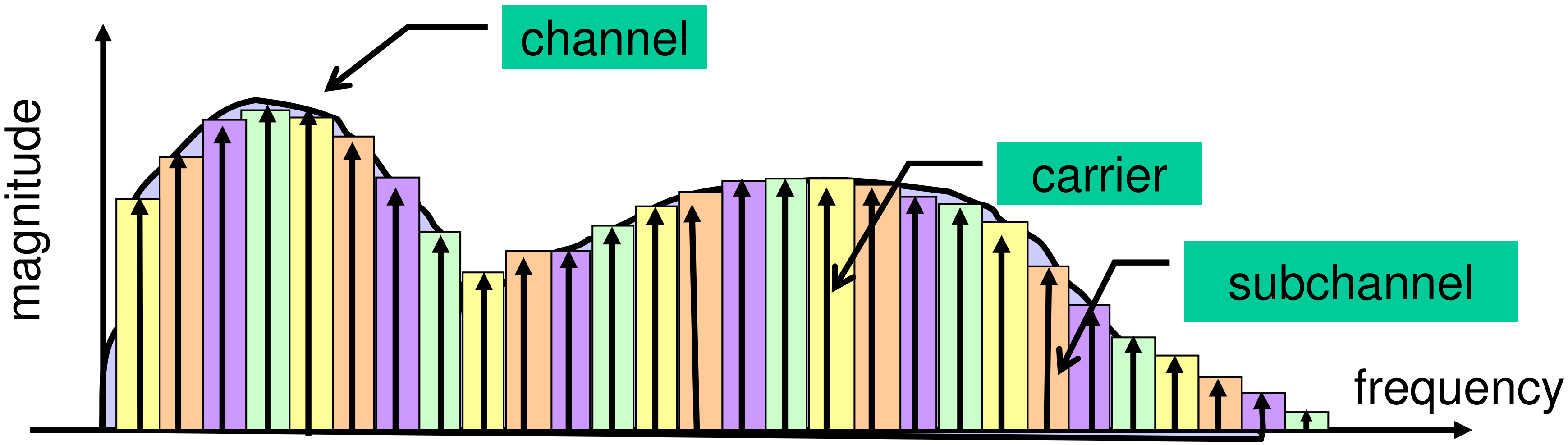
OFDM spectra have naturally steep sides, especially with large N. The PAPR is often higher, which may result in more spectral regrowth.

The blue trace is an unfiltered OFDM signal with 216 subcarriers. The red trace includes the effects of a non-linear Power Amplifier.



Multicarrier Modulation

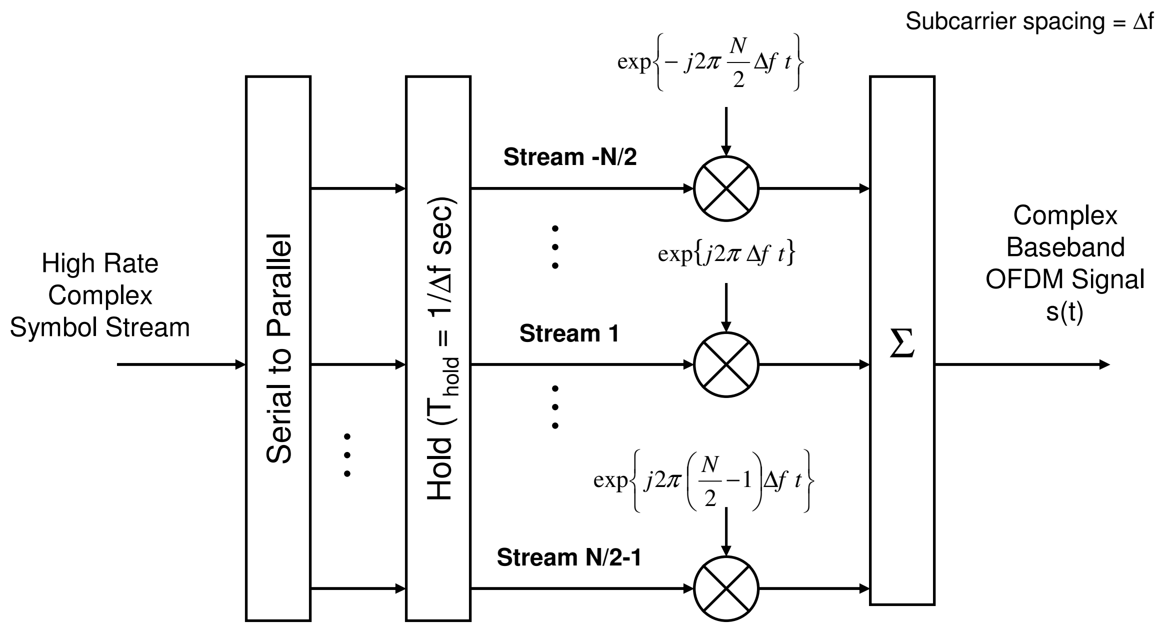
- **Divide broadband channel into narrowband subchannels**
  - No ISI in *subchannels* if constant gain in every subchannel and if ideal sampling
- **Orthogonal Frequency Division Multiplexing**
  - Based on the fast Fourier transform
  - Standardized for DAB, DVB-T, IEEE 802.11a, 802.16a, HyperLAN II
  - Considered for fourth-generation mobile communication systems



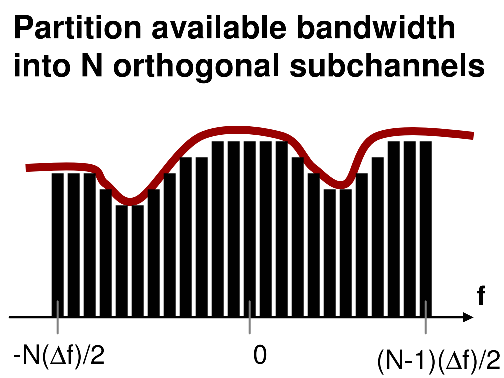
Subchannels are 312 kHz wide in 802.11a and HyperLAN II

OFDM System

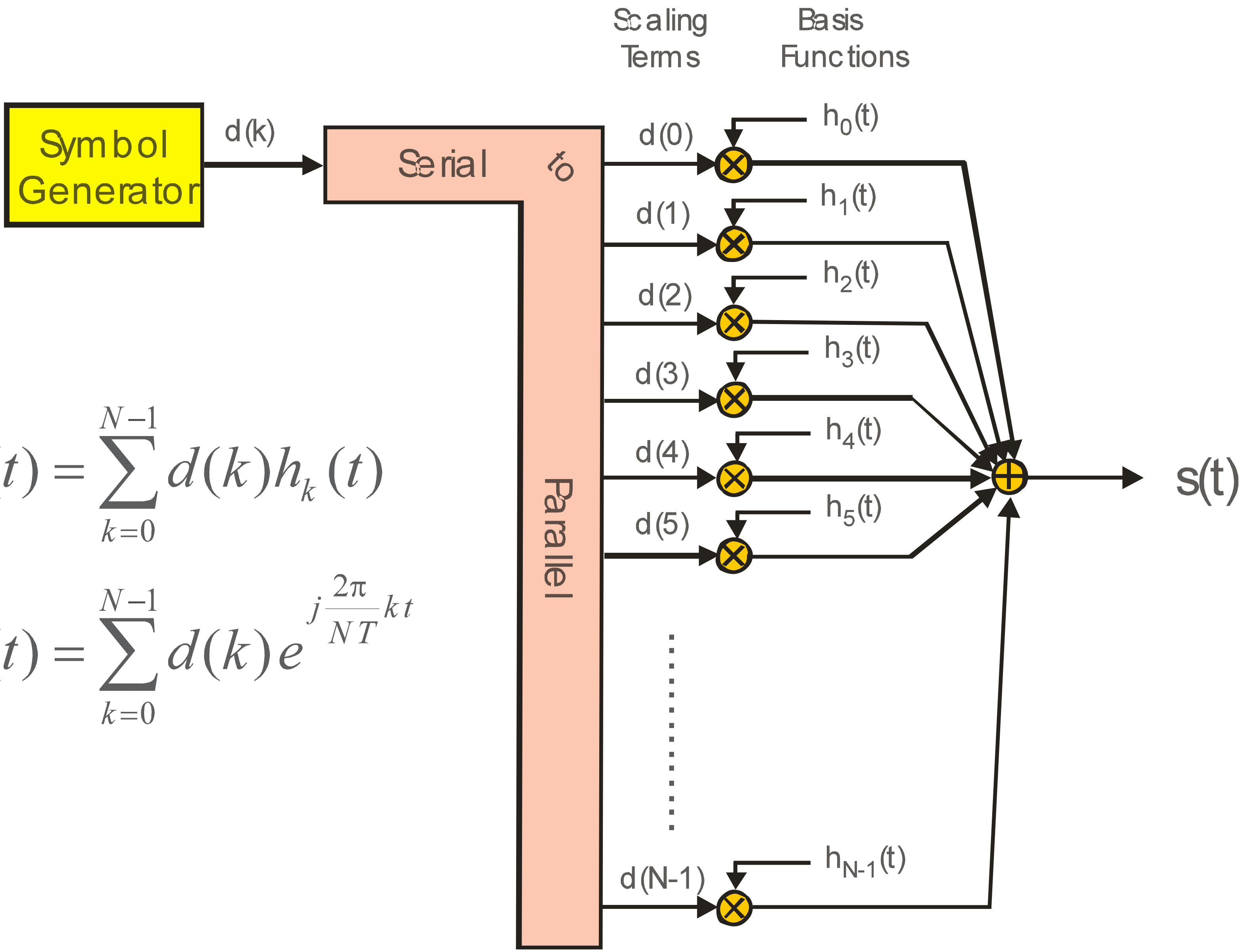
- Orthogonal Frequency Division Multiplexing
  - Split a high symbol rate data stream into N lower rate streams
  - Transmit the N low rate data streams using N subcarriers
    - Frequency Division Multiplexing (FDM) & Multi-Carrier Modulation (MCM)
  - N subcarriers must be mutually orthogonal



OFDM Conceptual Block Diagram



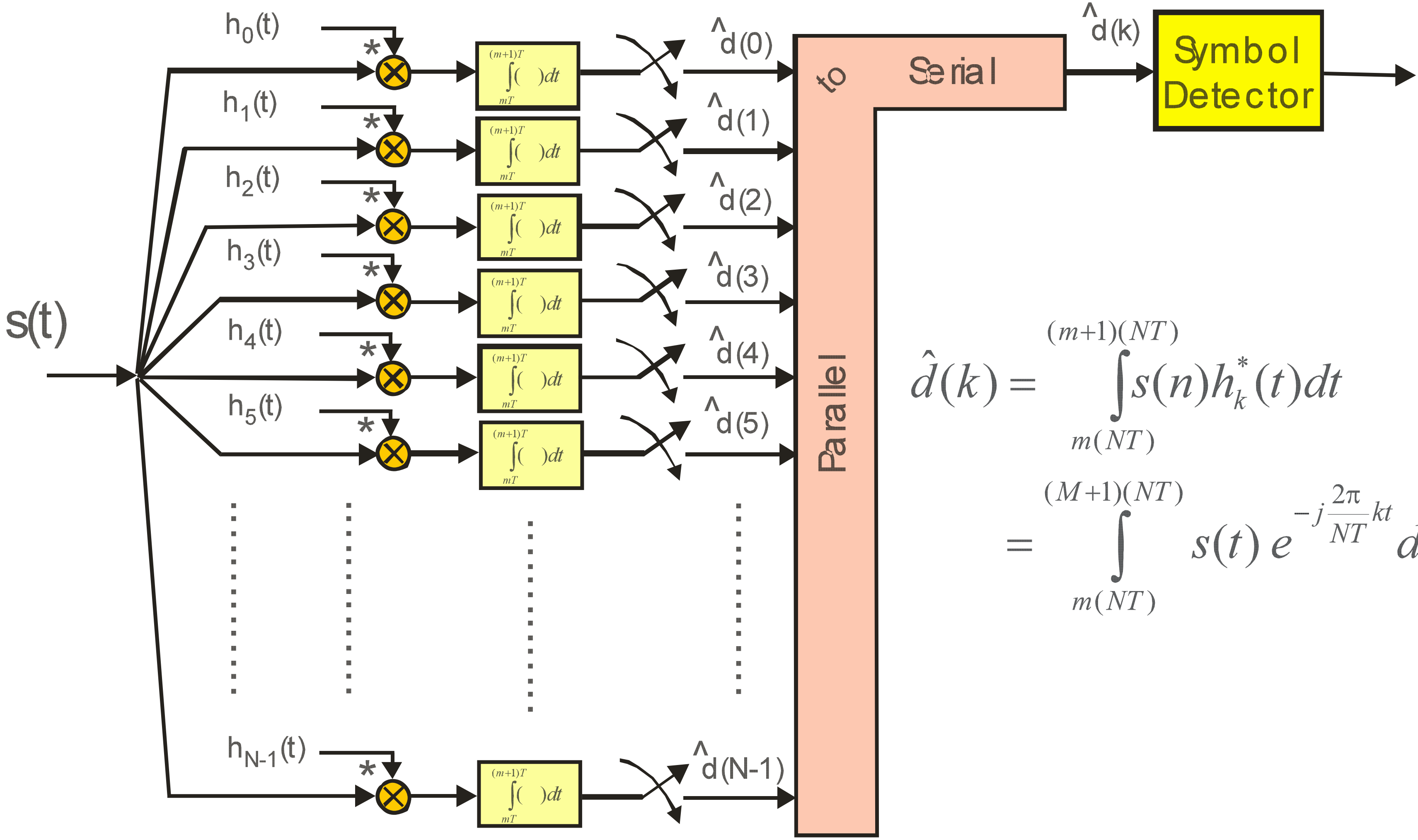
OFDM Modulator



$$s(t) = \sum_{k=0}^{N-1} d(k) h_k(t)$$

$$s(t) = \sum_{k=0}^{N-1} d(k) e^{j\frac{2\pi}{NT}kt}$$

OFDM Demodulator

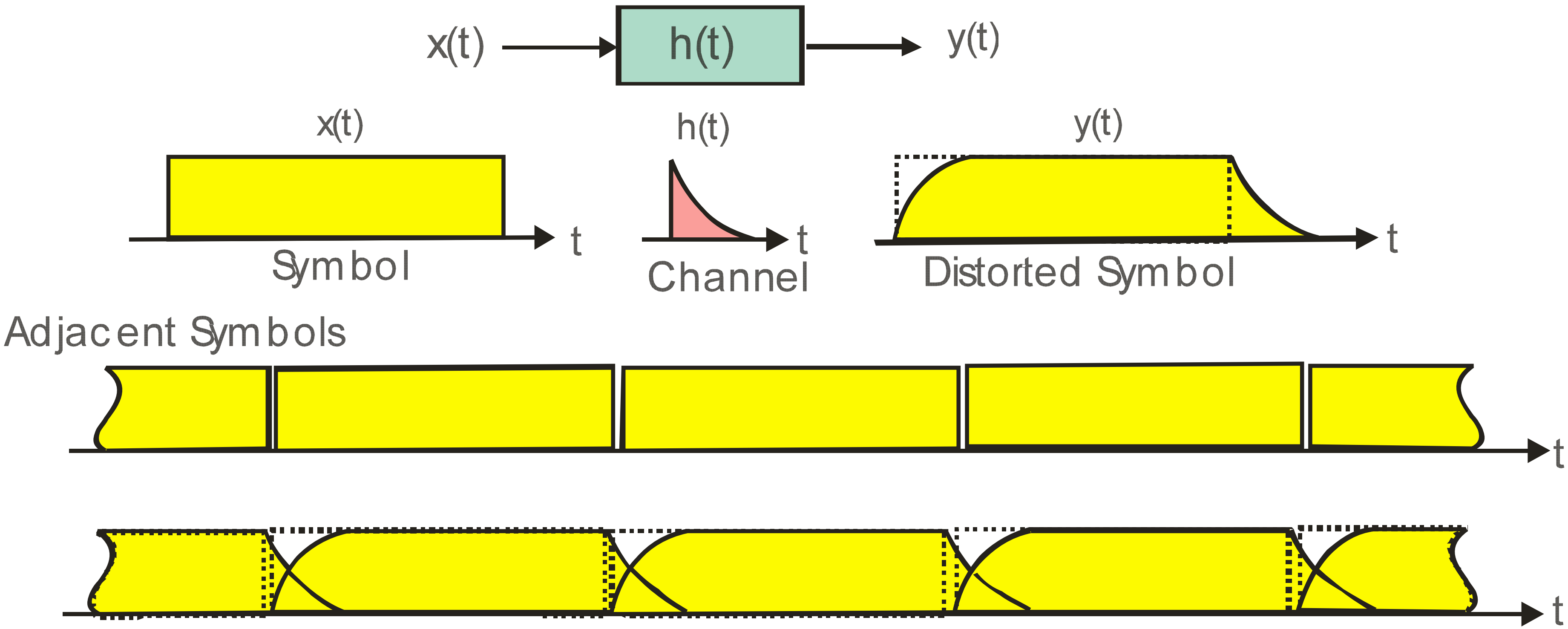


$$\hat{d}(k) = \int_{m(NT)}^{(m+1)(NT)} s(n) h_k^*(t) dt$$

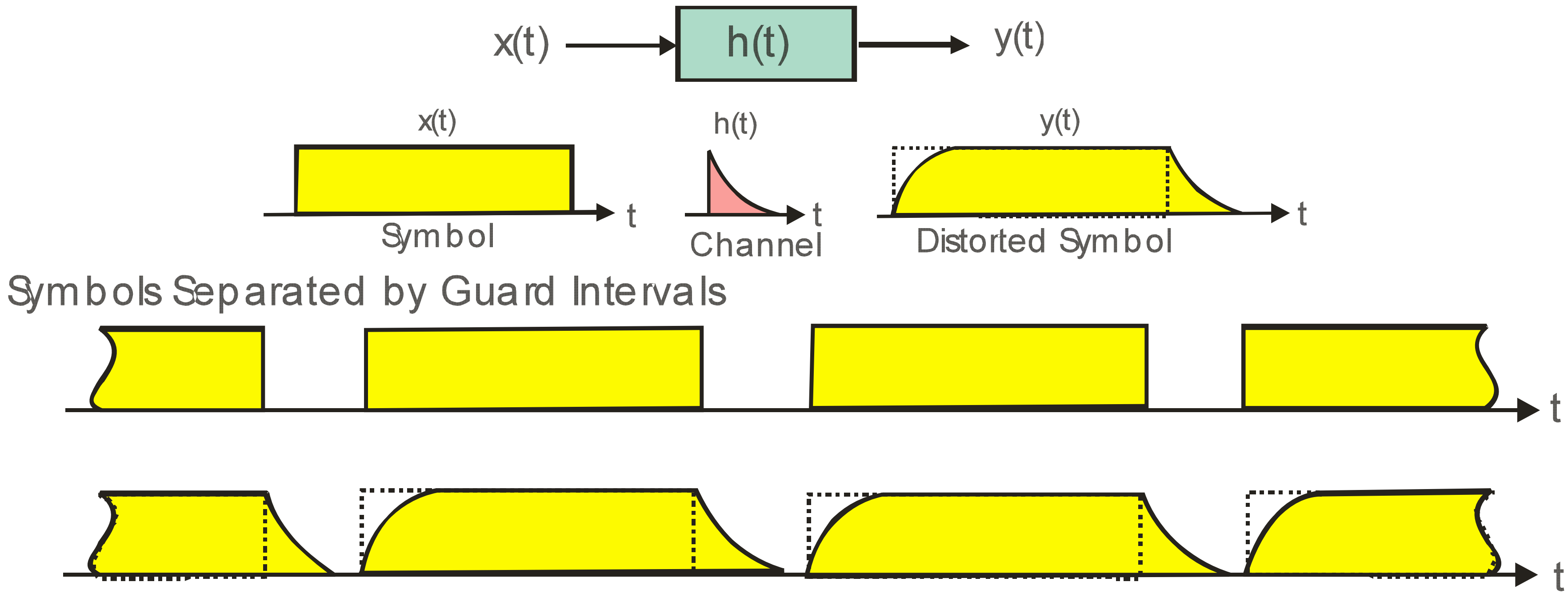
$$= \int_{m(NT)}^{(M+1)(NT)} s(t)\, e^{-j\frac{2\pi}{NT}kt} dt$$

Adjacent Symbol Interference (ASI)

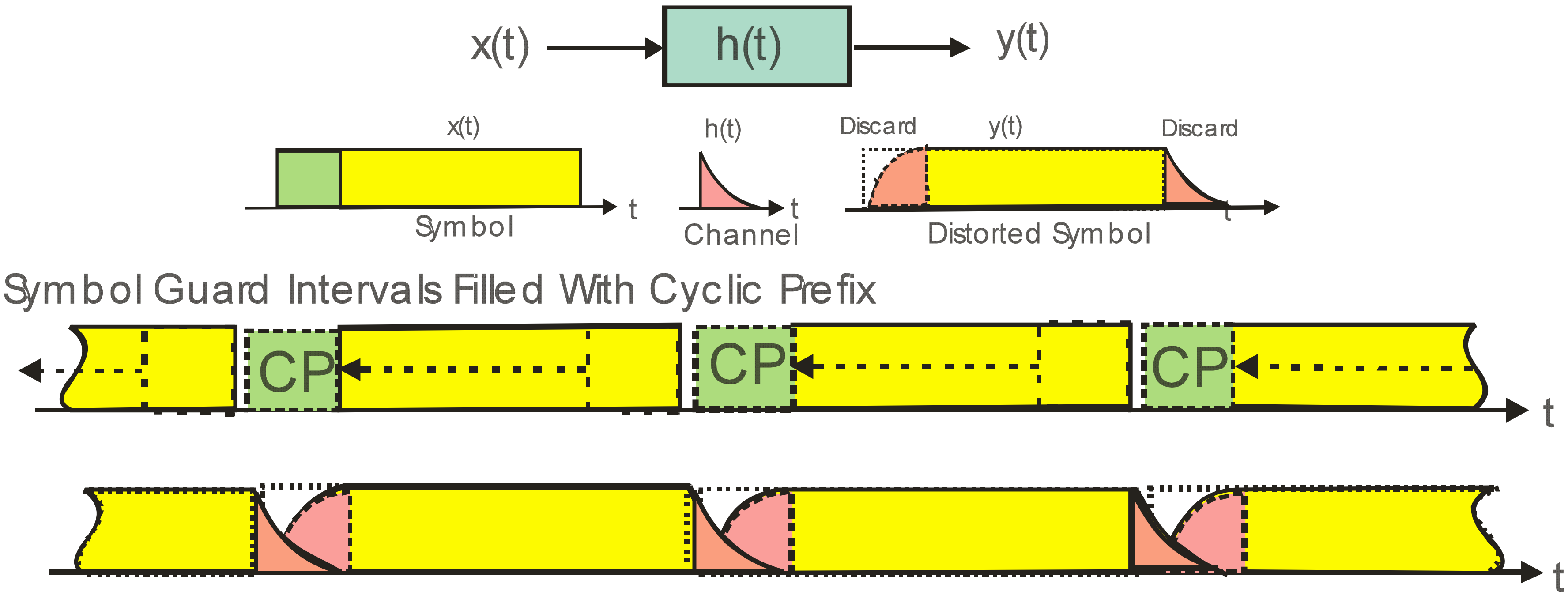
Symbol Smearing Due to Channel



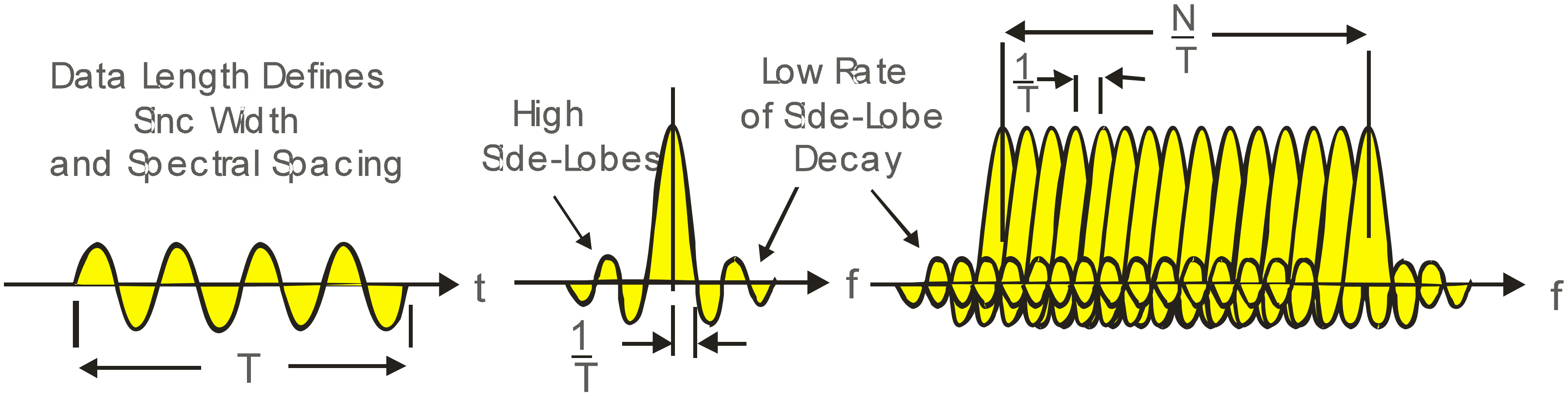
Guard Interval Inserted Between Adjacent Symbols to Suppress ASI



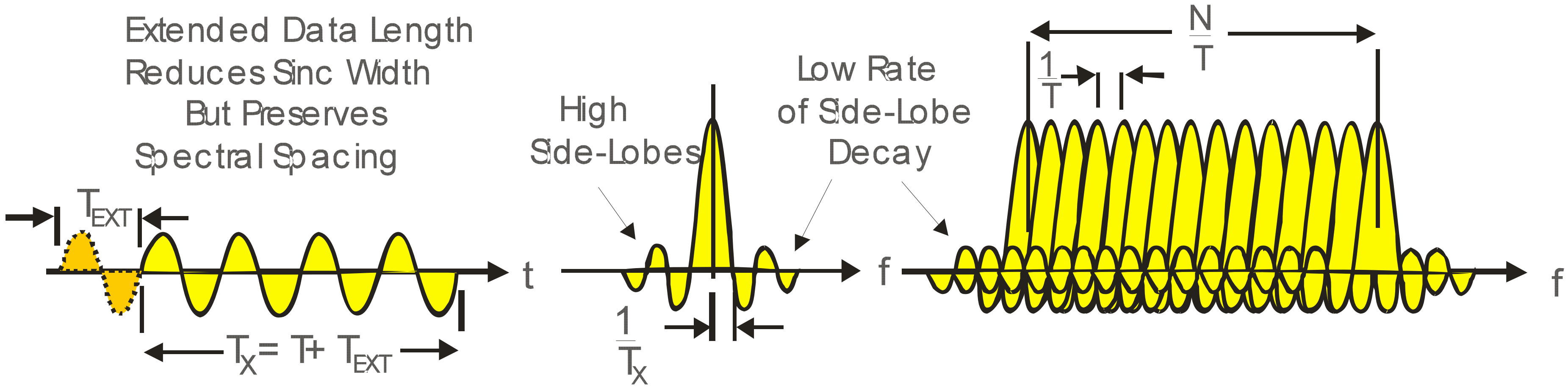
Cyclic Prefix Inserted in Guard Interval to Suppress Adjacent Channel Interference (ACI)



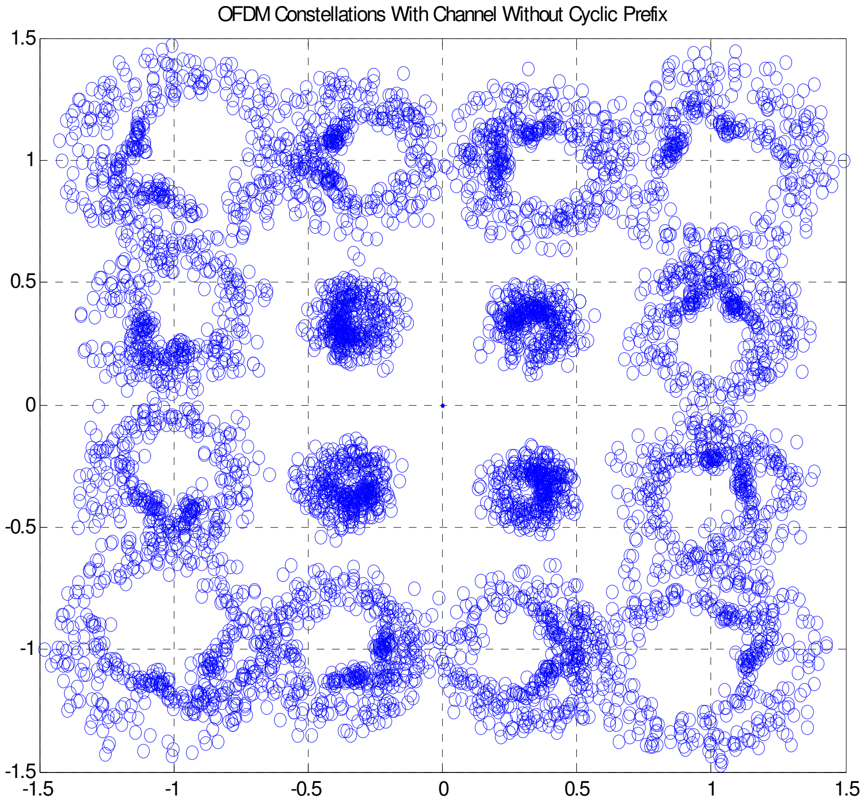
Data Length Defines Sinc Width: Spectral Spacing Matches Width

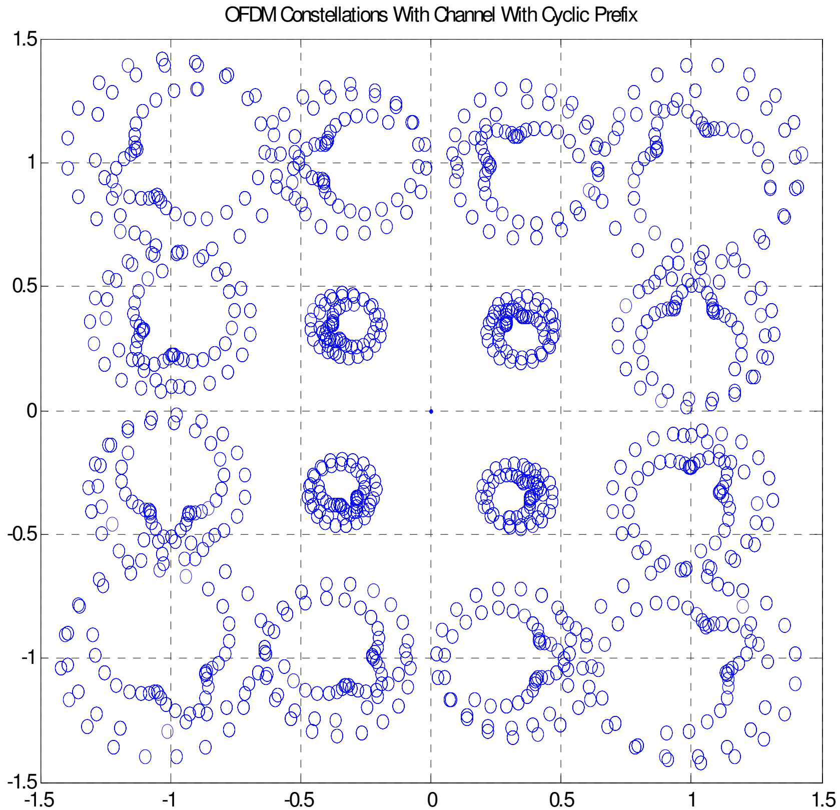


Extended Data Length Reduces Sinc Width: Spectral Spacing Preserved

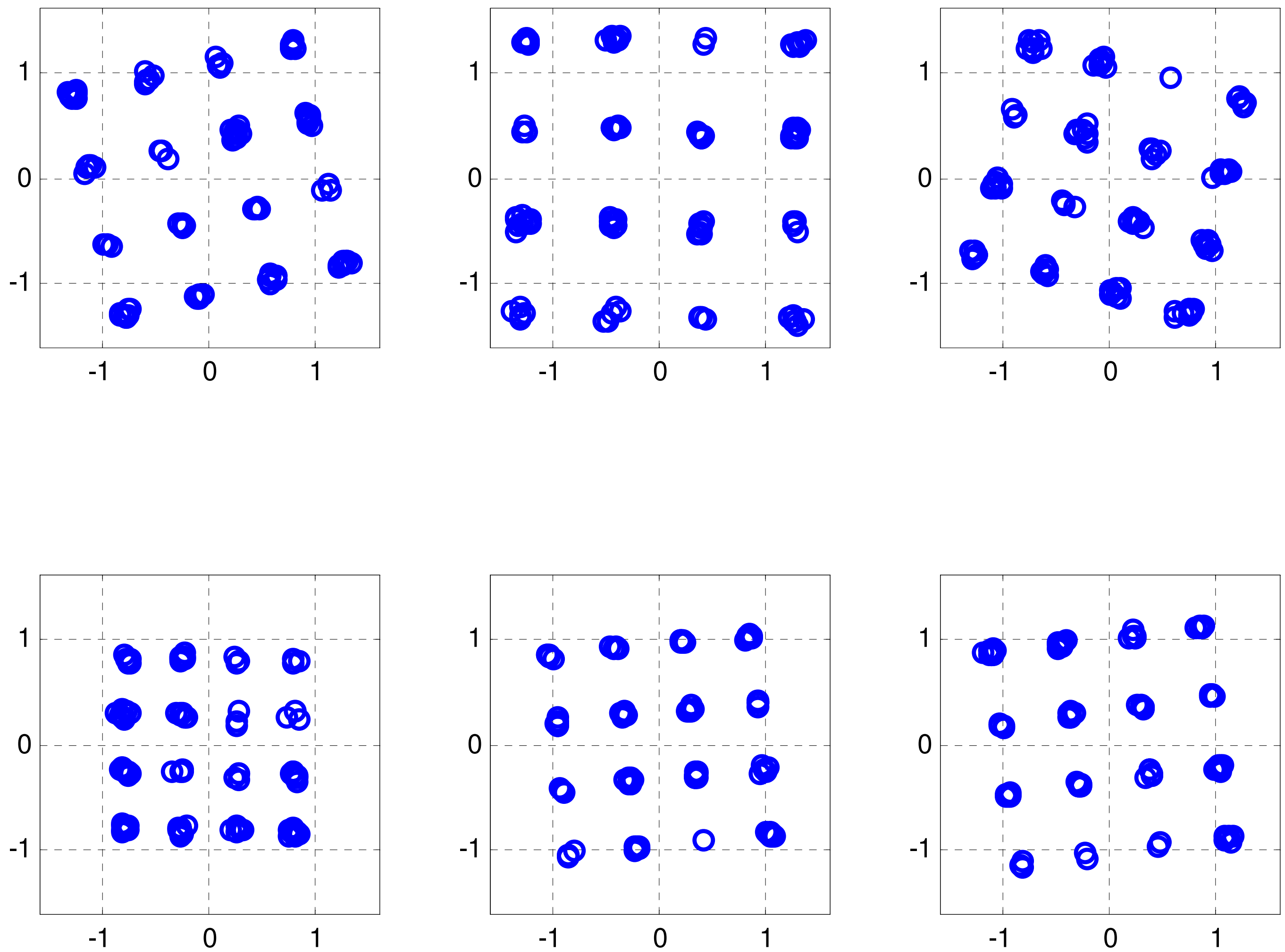


Overlaid Constellations , All Frequencies, Without and With Cyclic Prefix



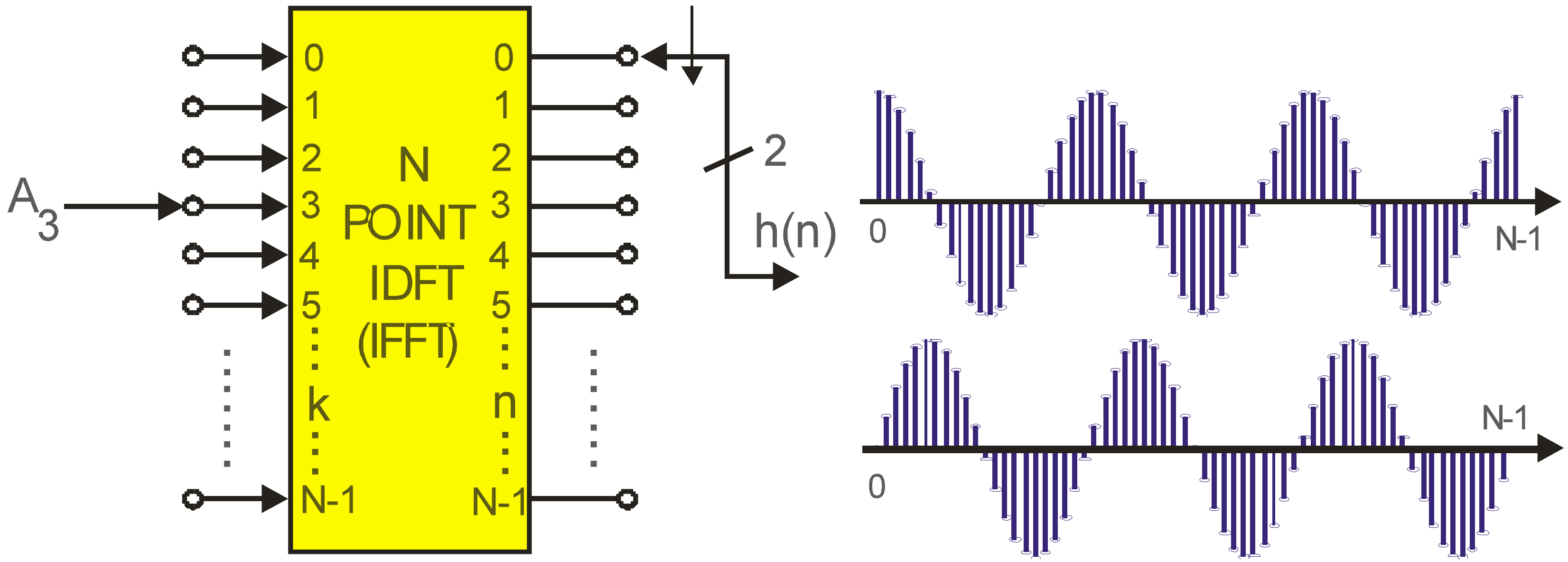


Constellations: Different OFDM Bins Without Cyclic Prefix



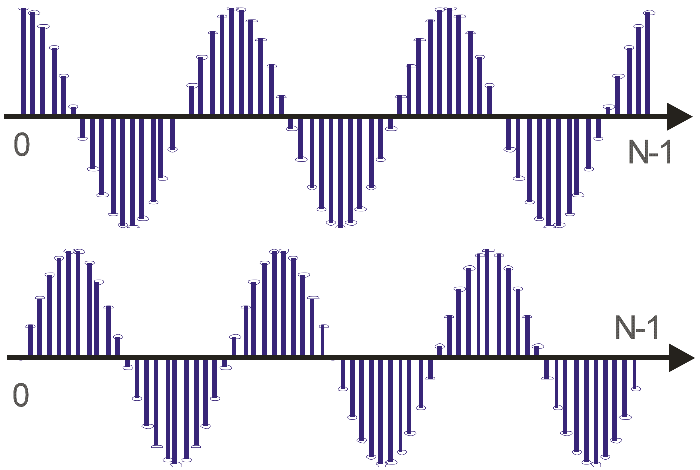
Constellations: Different OFDM Bins
With Cyclic Prefix

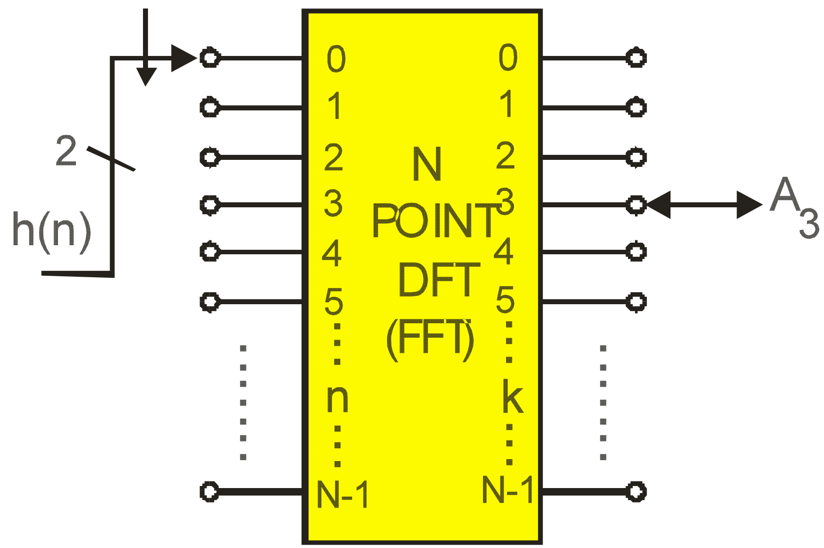
DFT (FFT) as Signal Generator for Complex Sinusoids



$$h(n) = \frac{1}{N}\sum_{n=0}^{N-1} H(k)\, e^{j\frac{2\pi}{N}nk} \quad : \quad n = 0,1,2,\ldots, N-1$$

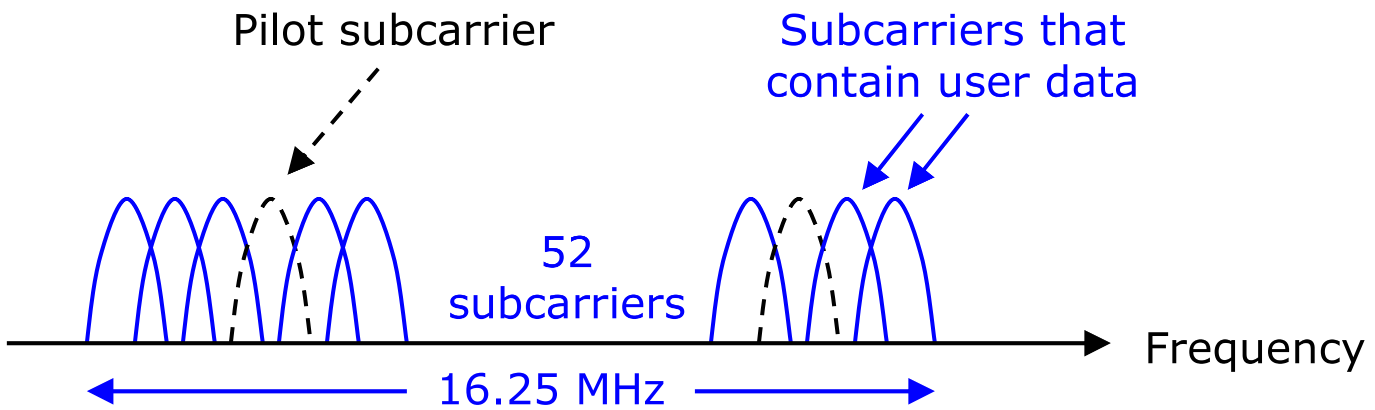
DFT (FFT) As Signal Analyzer for Complex Sinusoids





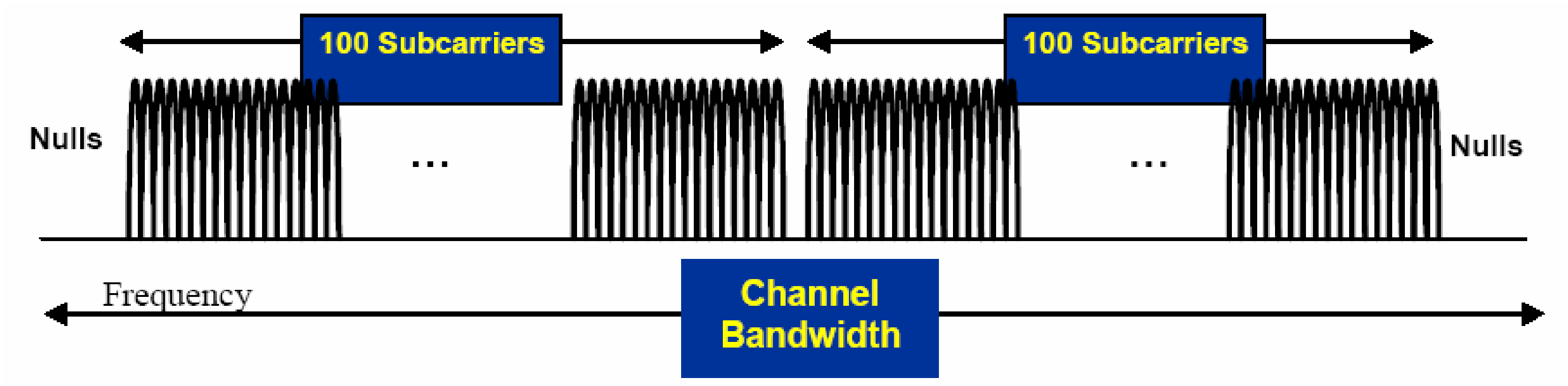
$$H(k) = \sum_{n=0}^{N-1} h(n)\, e^{-j\frac{2\pi}{N}nk} \quad : \quad k = 0, 1, 2, \dots, N-1$$

OFDM example 1: IEEE 802.11a&g (WLAN)



48 data subcarriers + 4 pilot subcarriers. There is a ”null” at the center carrier. Around each data subcarrier is centered a subchannel carrying a low bitrate data signal (low bitrate => no intersymbol interference).

OFDM example 2: IEEE 802.16a (WiMAX)



Only 200 of 256 subcarriers are used: 192 data subcarriers + 8 pilot subcarriers. There are 56 ”nulls” (center carrier, 28 lower frequency and 27 higher frequency guard carriers).

Usage of OFDM

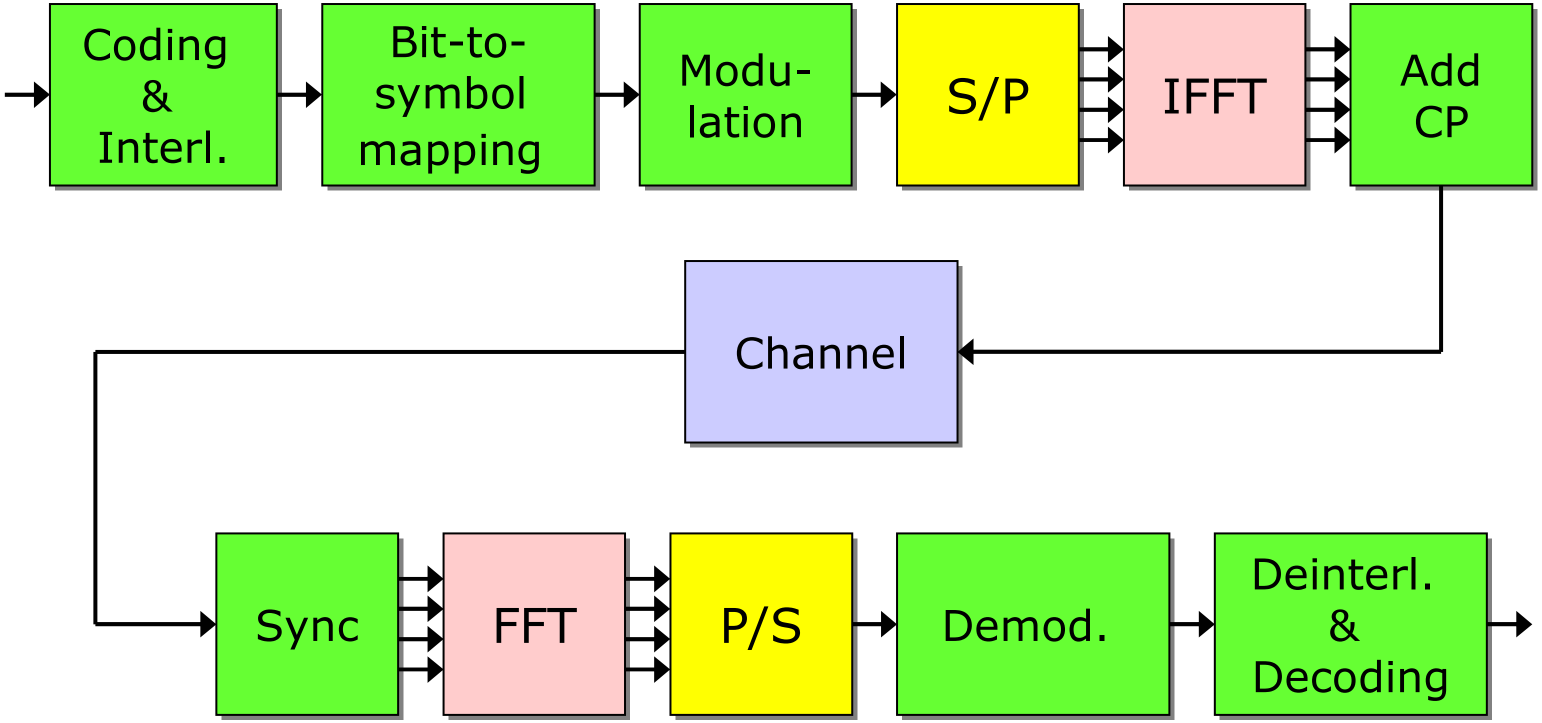
OFDM is used (among others) in the following systems:

- IEEE 802.11a&g (WLAN) systems
- IEEE 802.16a (WiMAX) systems
- ADSL (DMT = Discrete MultiTone) systems
- DAB (Digital Audio Broadcasting)
- DVB-T (Digital Video Broadcasting)

OFDM is spectral efficient, but not power efficient (due to linearity requirements of power amplifier).

OFDM is primarily a modulation method; OFDMA is the corresponding multiple access scheme.

OFDM system block diagram



Subcarrier modulation (IEEE 802.11a&g)

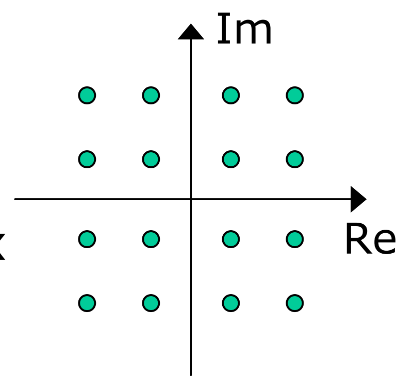
Modulation	Bit rate
BPSK	6 Mbit/s
BPSK	9 Mbit/s
QPSK	12 Mbit/s
QPSK	18 Mbit/s
16-QAM	24 Mbit/s
16-QAM	36 Mbit/s
64-QAM	48 Mbit/s
64-QAM	54 Mbit/s

BPSK = Binary Phase Shift Keying (PSK)

QPSK = Quaternary PSK

QAM = Quadrature Amplitude Modulation

16-QAM signal constellation in the complex plane



Why (for instance) 54 Mbit/s ?

Symbol duration = 4 μs

Data-carrying subcarriers = 48

Bits / subchannel = 6 (64-QAM)

Bits / OFDM symbol = 6 x 48 = 288

Channel coding: number reduced to 3/4 x 288
= 216 bits/symbol

=> Bit rate = 216 bits / 4 μs = 54 Mbit/s

Subcarrier modulation and coding

N data subcarriers or subchannels carry *N* data symbols in parallel (= transmitted at the same time). A symbol carries 1 bit (BPSK), 2 bits (4-PSK), 4 bits (16-QAM), or 6 bits of user data (64-QAM). *N* data symbols in parallel form one OFDM symbol.

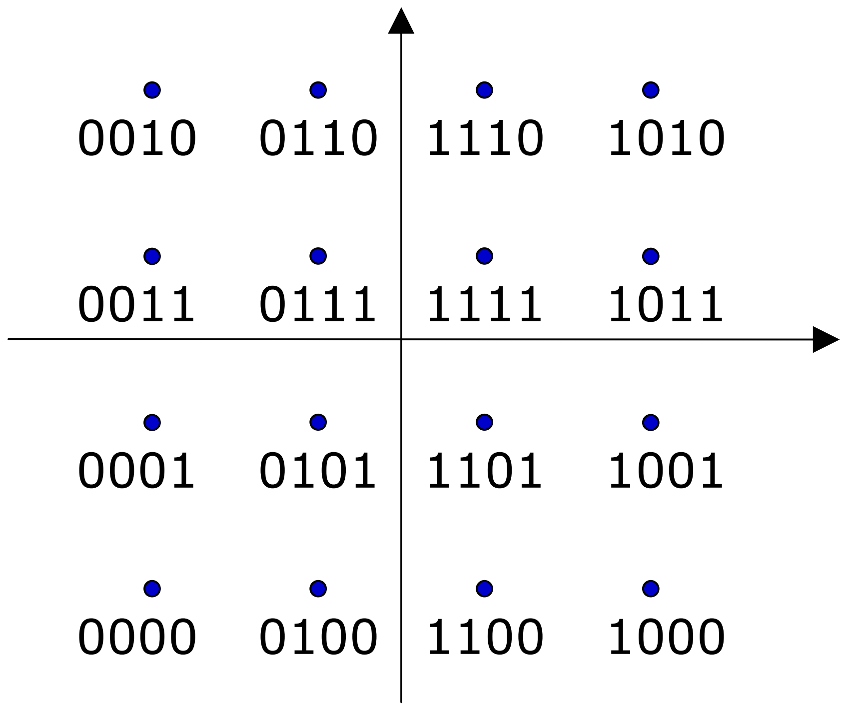
For each modulation method, there are several coding options for FEC (Forward Error Control). They must be taken into account when calculating user data rates, as shown on the previous slide. Typical coding options: 1/2 (convolutional encoding), 2/3 and 3/4 (puncturing) coding rates.

Gray bit-to-symbol mapping in QAM

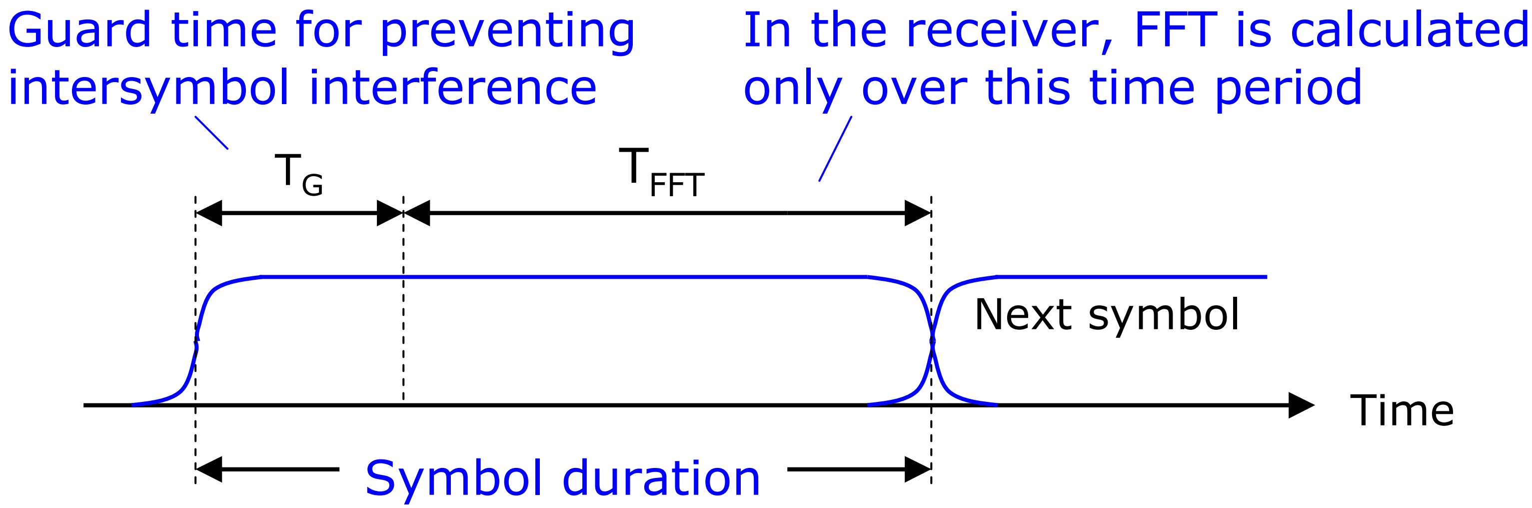
Gray bit-to-symbol mapping is usually used in QAM systems.

The reason: it is optimal in the sense that a symbol error (involving two adjacent symbols in the QAM signal constellation) results in a single bit error.

Example for 16-QAM



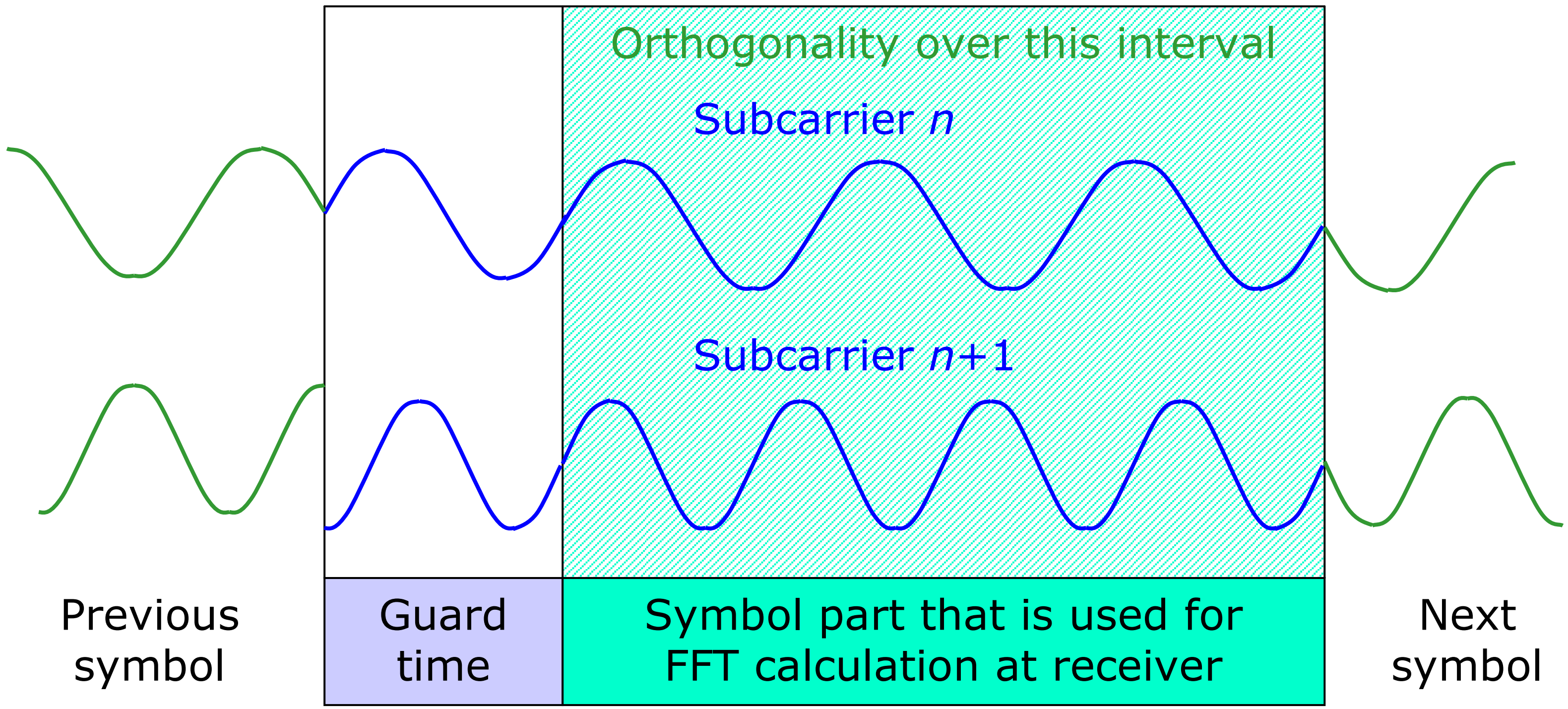
Subcarrier signal in time domain



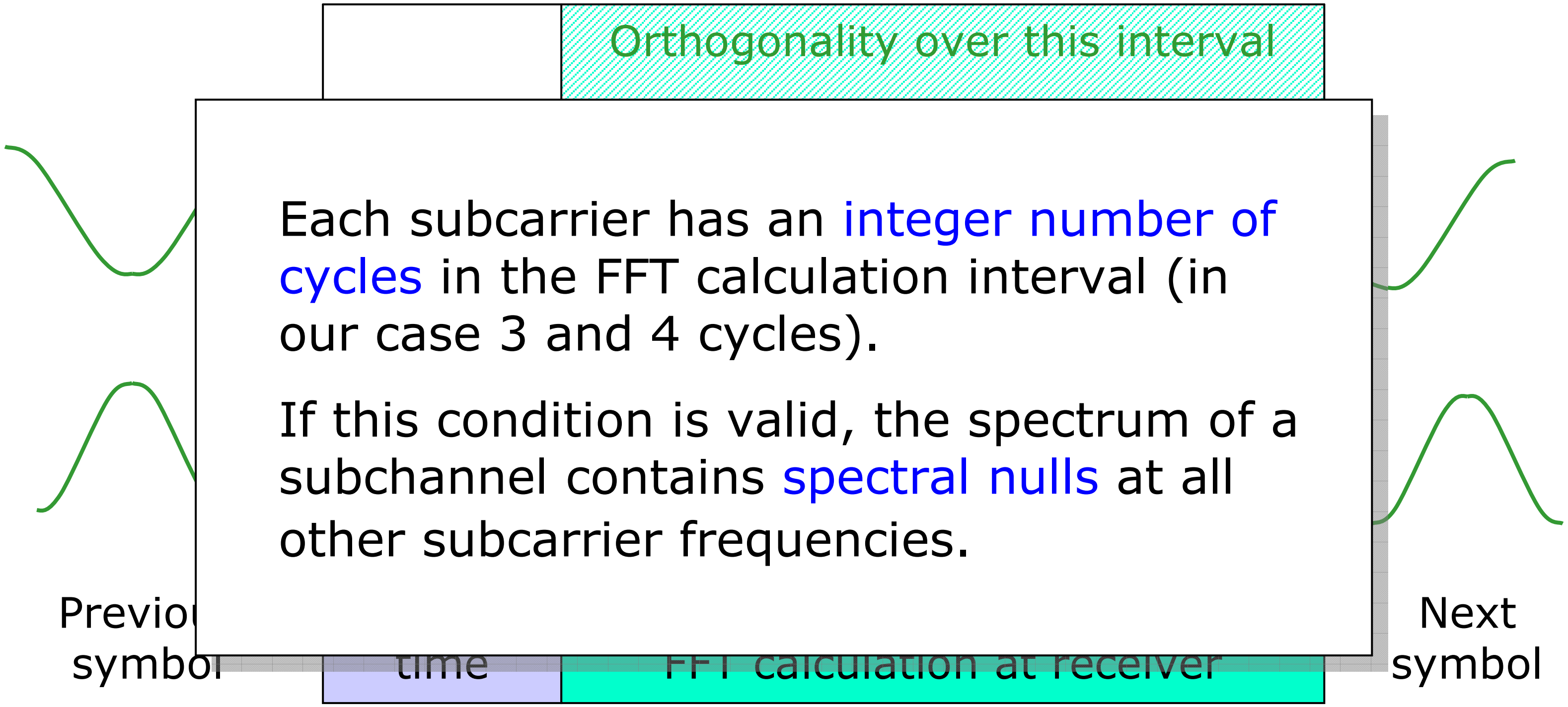
IEEE 802.11a&g: T_G = 0.8 μs, T_{FFT} = 3.2 μs

IEEE 802.16a offers flexible bandwidth allocation (i.e. different symbol lengths) and T_G choice: T_G/T_{FFT} = 1/4, 1/8, 1/16 or 1/32

Orthogonality between subcarriers (1)



Orthogonality between subcarriers (2)



Each subcarrier has an integer number of cycles in the FFT calculation interval (in our case 3 and 4 cycles).

If this condition is valid, the spectrum of a subchannel contains spectral nulls at all other subcarrier frequencies.

Orthogonality between subcarriers (2)

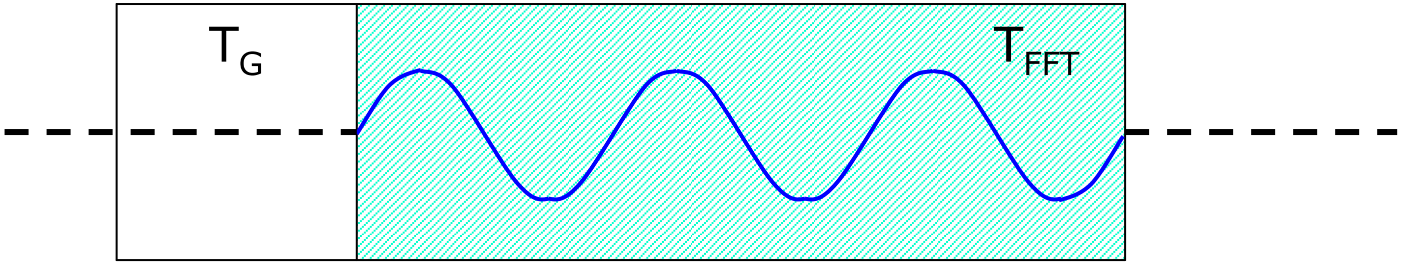
Orthogonality over the FFT interval:

$$\int_0^{T_{FFT}} \cos\left(2\pi mt/T_{FFT}\right)\cos\left(2\pi nt/T_{FFT}\right)dt = \begin{cases} T_{FFT}/2 & m = n \\ 0 & m \neq n \end{cases}$$

Phase shift in either subcarrier - orthogonality over the FFT interval is still retained:

$$\int_0^{T_{FFT}} \cos\left(2\pi mt/T_{FFT} + \phi\right)\cos\left(2\pi nt/T_{FFT}\right)dt = 0 \qquad m \neq n$$

Time vs. frequency domain



Square-windowed sinusoid in time domain

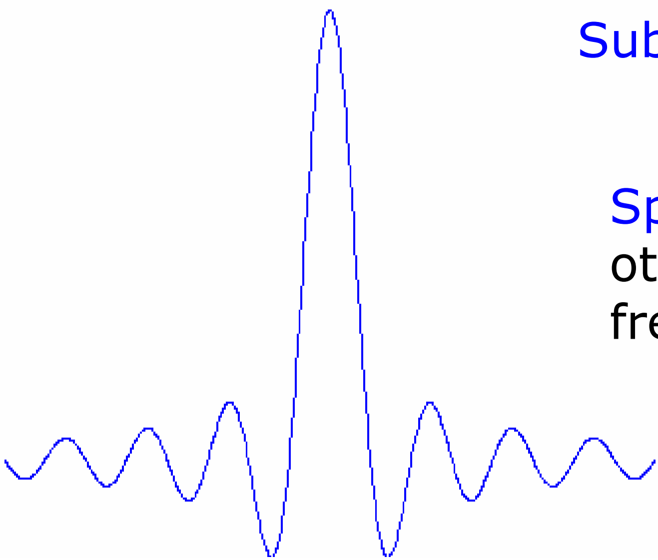
=>

"sinc" shaped subchannel spectrum in frequency domain

$$\mathrm{sinc}\left(fT_{FFT}\right)=\left[\sin\left(\pi fT_{FFT}\right)\right]/\left(\pi fT_{FFT}\right)$$

Subchannels in frequency domain

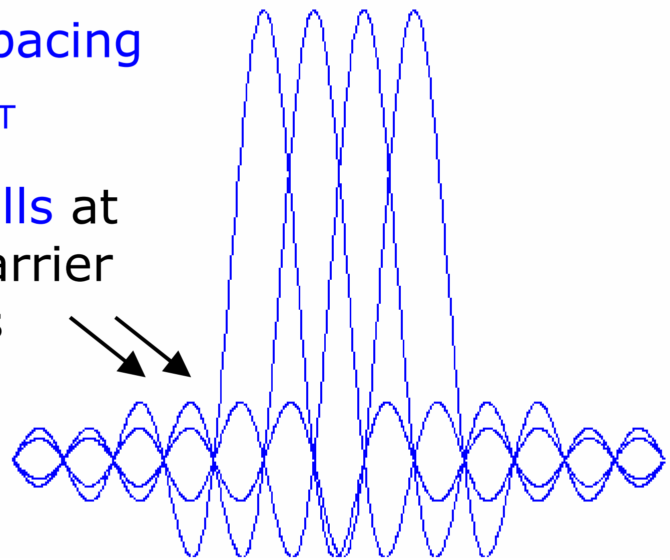
Single subchannel



OFDM spectrum

Subcarrier spacing
= $1/T_{FFT}$

Spectral nulls at
other subcarrier
frequencies



Presentation of OFDM signal

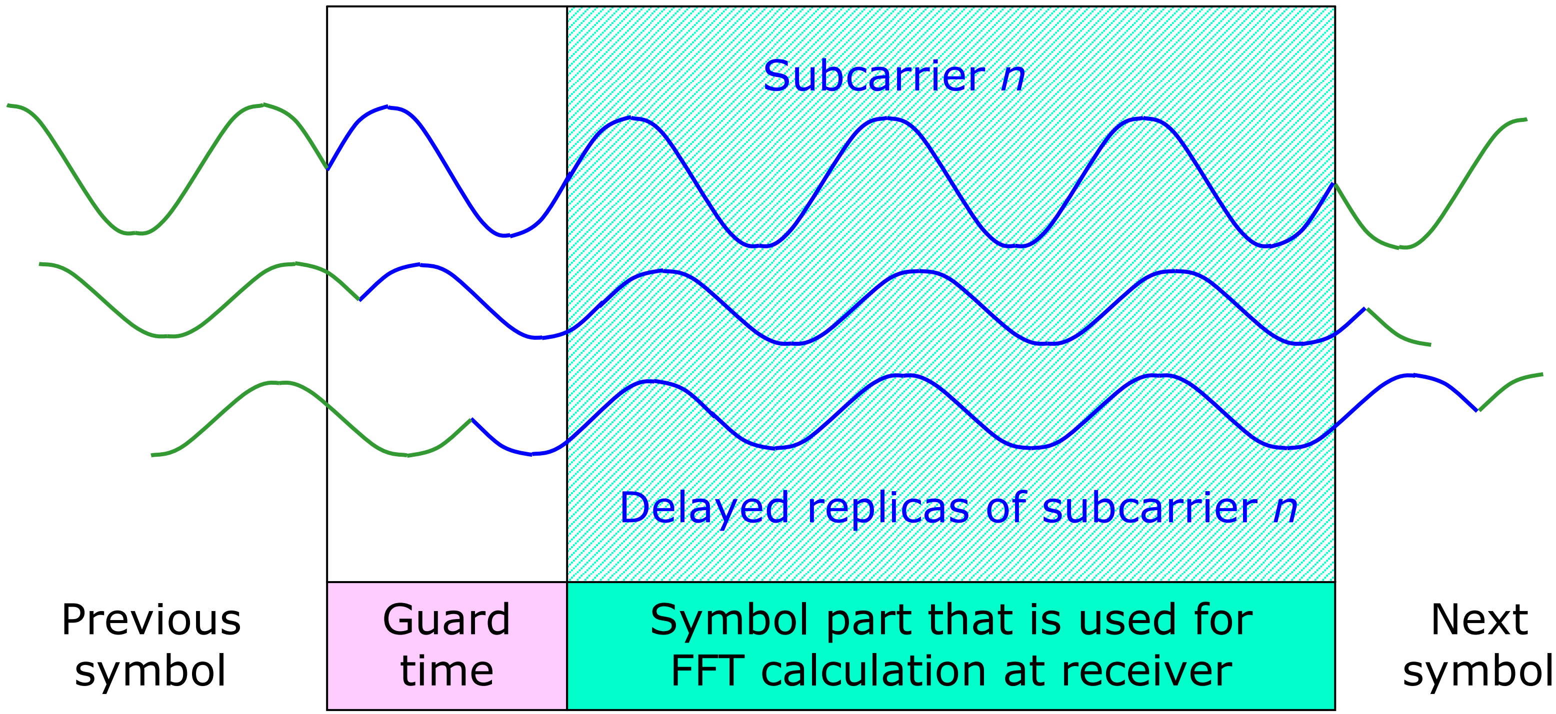
$$s(t) = \sum_{k=-\infty}^{\infty} g_k(t - kT)$$ Sequence of OFDM symbols

The *k*:th OFDM symbol (in complex LPE form) is

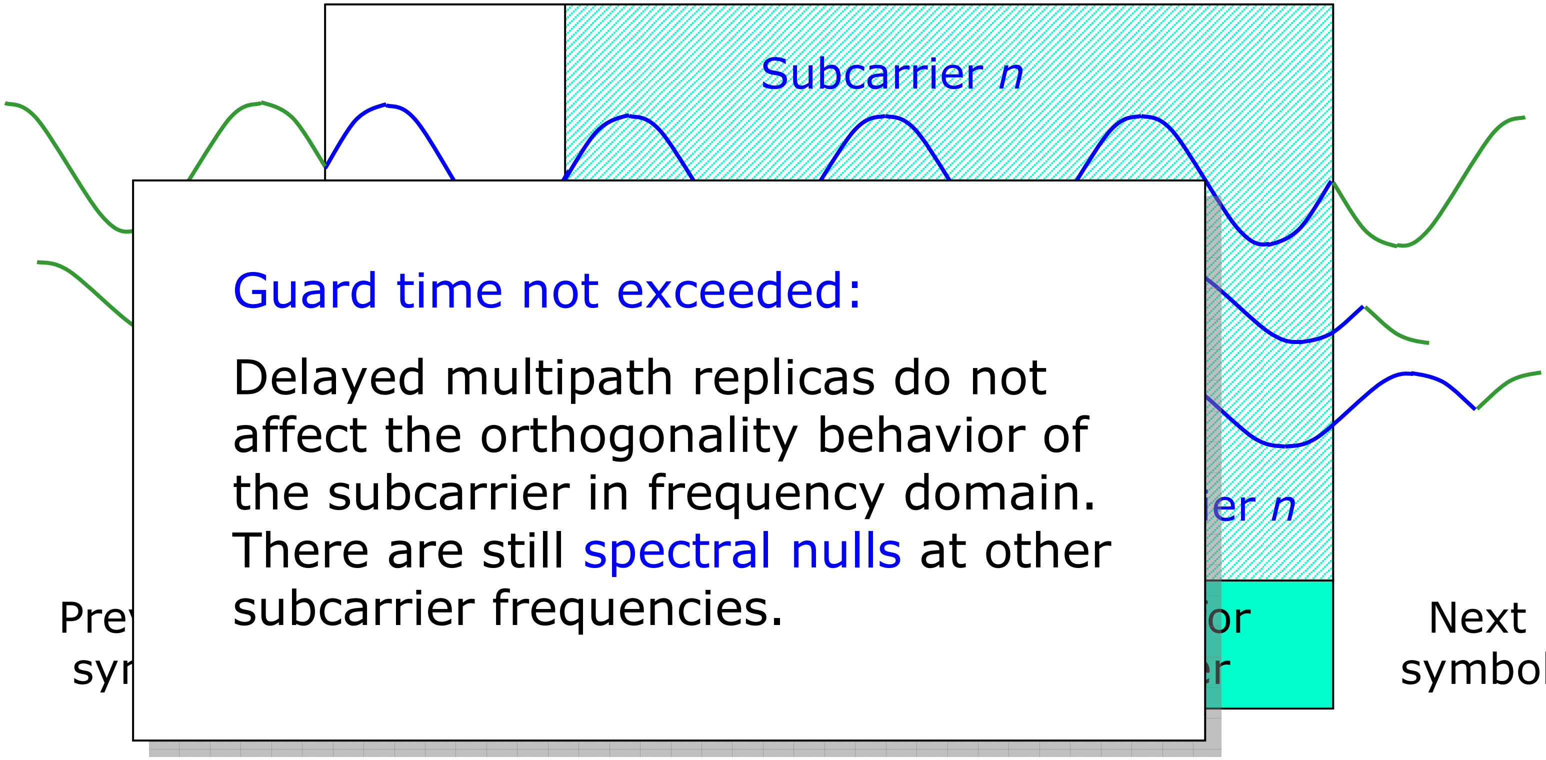
$$g_k(t) = \sum_{\substack{n=-N/2 \\ n \neq 0}}^{N/2} a_{n,k} \exp\left(j2\pi \frac{n}{T_{FFT}} t \right) \qquad (k-1)T < t < kT$$

where N = number of subcarriers, $T = T_G + T_{FFT}$ = symbol period, and $a_{n,k}$ is the complex data symbol modulating the *n*:th subcarrier during the *k*:th symbol period.

Multipath effect on subcarrier *n* (1)



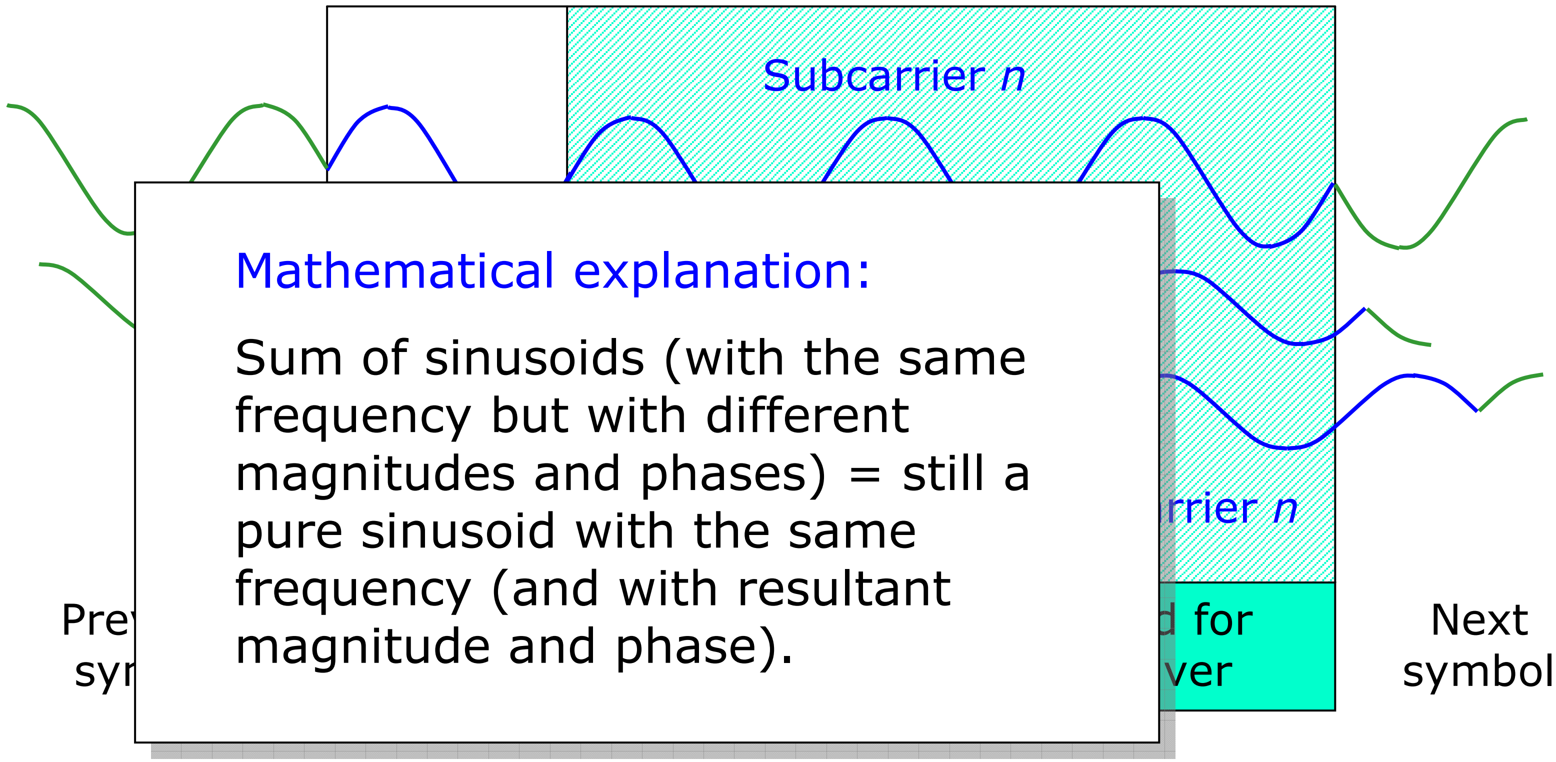
Multipath effect on subcarrier *n* (2)



Guard time not exceeded:

Delayed multipath replicas do not affect the orthogonality behavior of the subcarrier in frequency domain. There are still spectral nulls at other subcarrier frequencies.

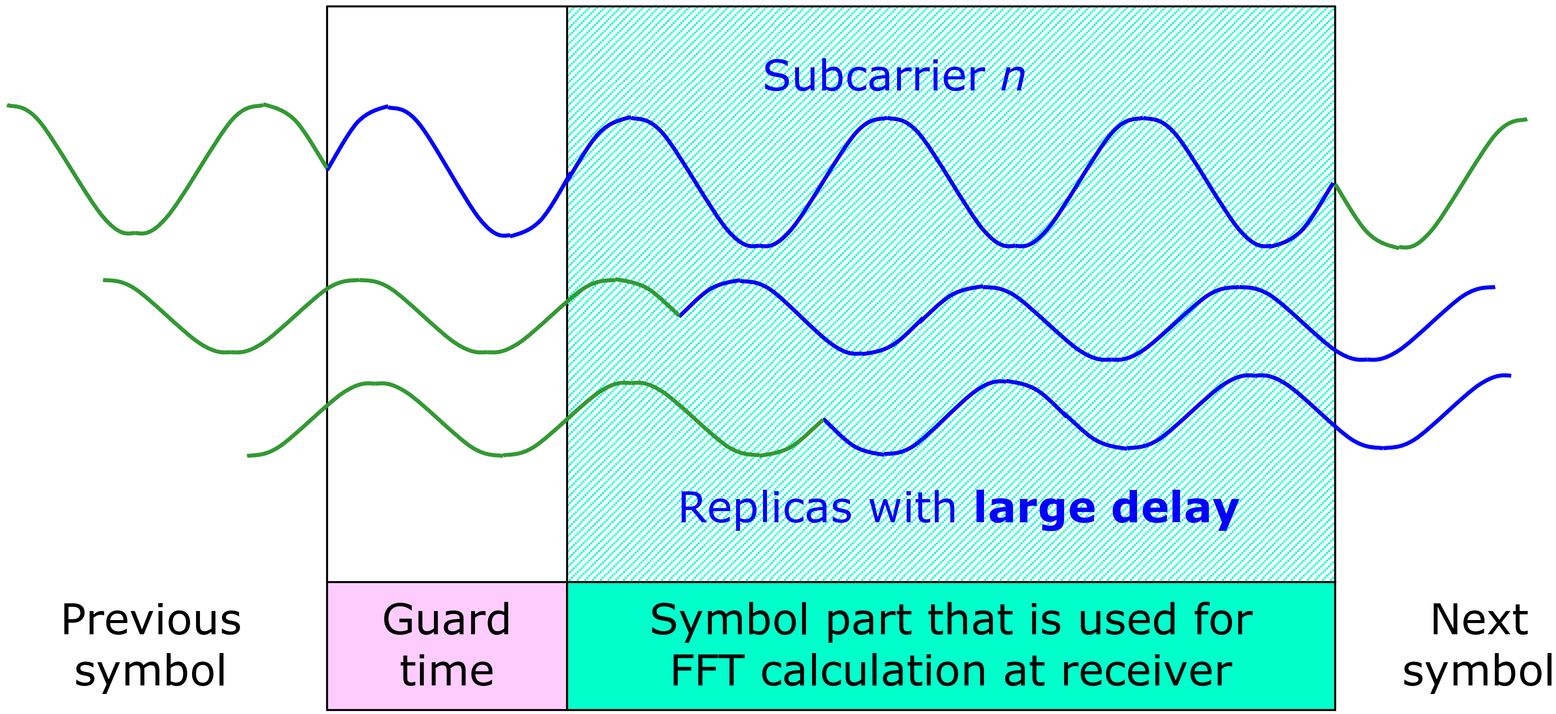
Multipath effect on subcarrier *n* (3)



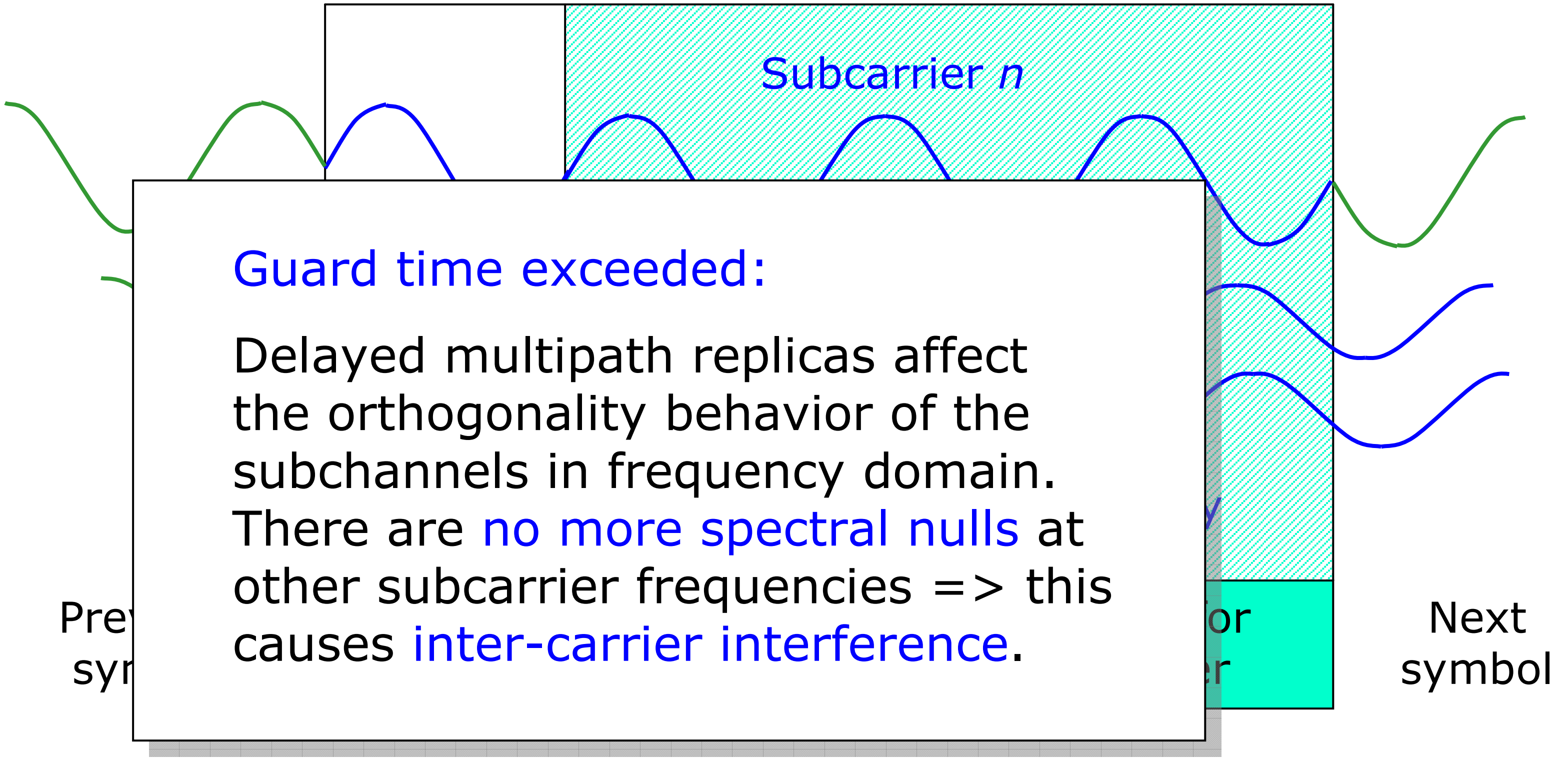
Mathematical explanation:

Sum of sinusoids (with the same frequency but with different magnitudes and phases) = still a pure sinusoid with the same frequency (and with resultant magnitude and phase).

Multipath effect on subcarrier *n* (4)



Multipath effect on subcarrier *n* (5)



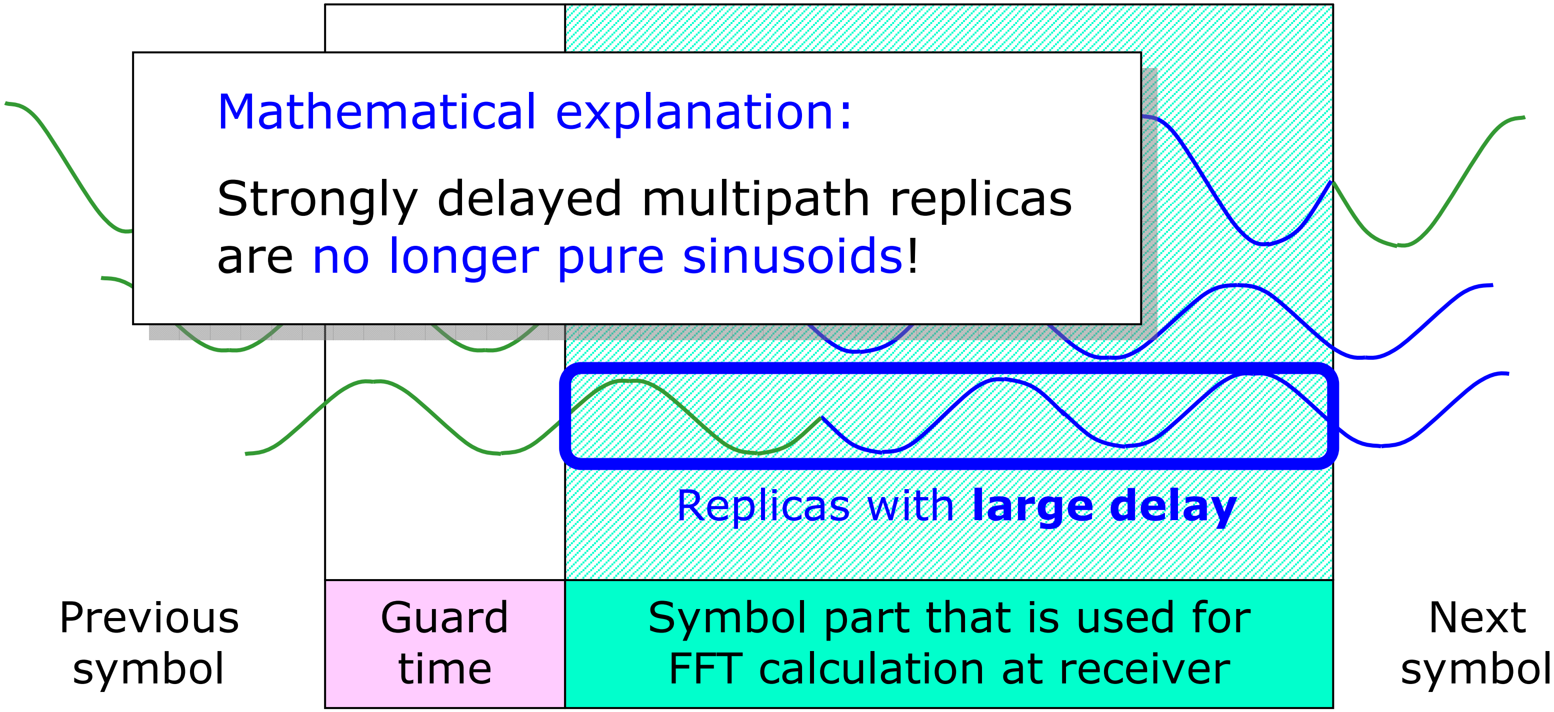
Guard time exceeded:

Delayed multipath replicas affect the orthogonality behavior of the subchannels in frequency domain. There are no more spectral nulls at other subcarrier frequencies => this causes inter-carrier interference.

Multipath effect on subcarrier *n* (6)

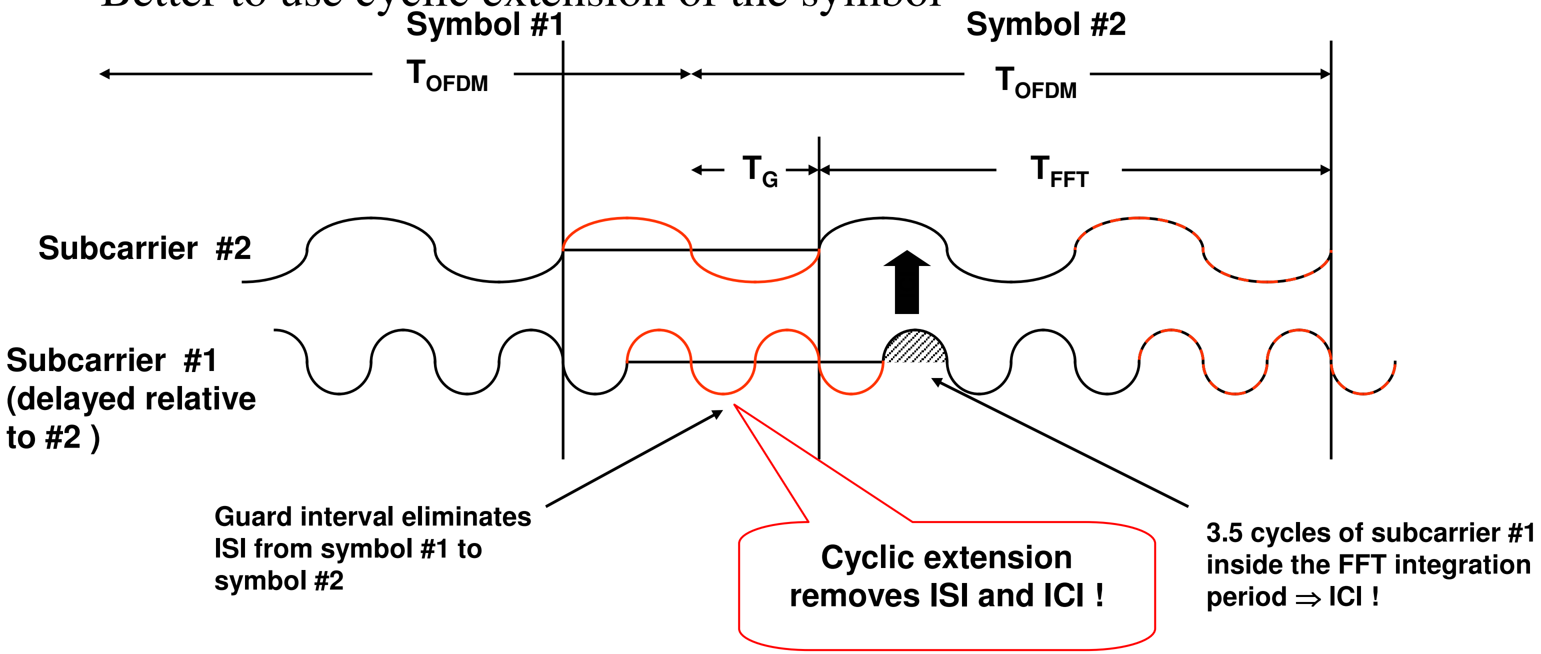
Mathematical explanation:

Strongly delayed multipath replicas are no longer pure sinusoids!



Cyclic Prefix (Guard Interval)

- Delay Spread Causes Inter-Symbol-Interference (ISI) and Inter-Carrier-Interference (ICI)
  - Non-linear phase implies different subcarriers experience different delay (virtually all real channels are non-linear phase)
  - Adding a guard interval between OFDM symbols mitigates this problem
    - Zero valued guard interval will eliminate ISI but causes ICI
    - Better to use cyclic extension of the symbol



Discrete multitone (DMT) modulation

DMT is a special case of OFDM where the different signal-to-noise ratio values of different subcarriers are utilised constructively in the following way:

- Subcarriers with high S/N carry more bits (for instance by using a modulation scheme with more bits/symbol or by using a less heavy FEC scheme)

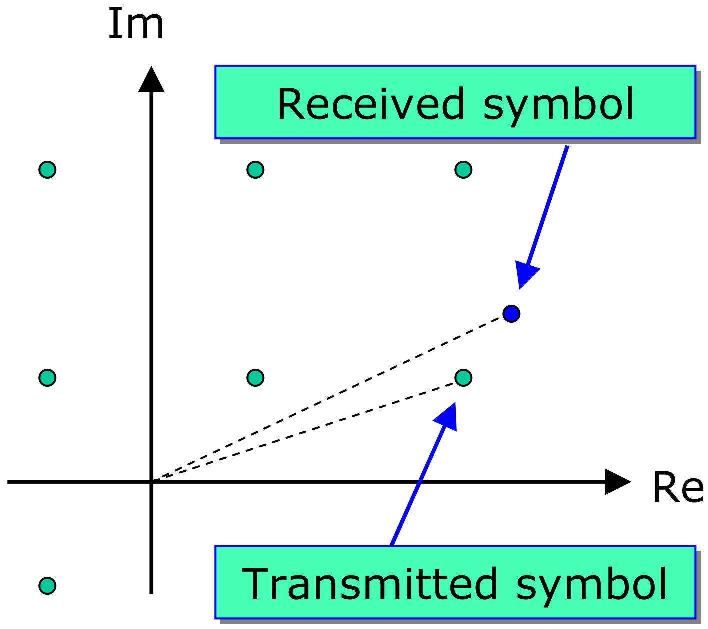
- Subcarriers with low S/N (due to frequency selective fading) carry less bits.

Note the requirement of a feedback channel.

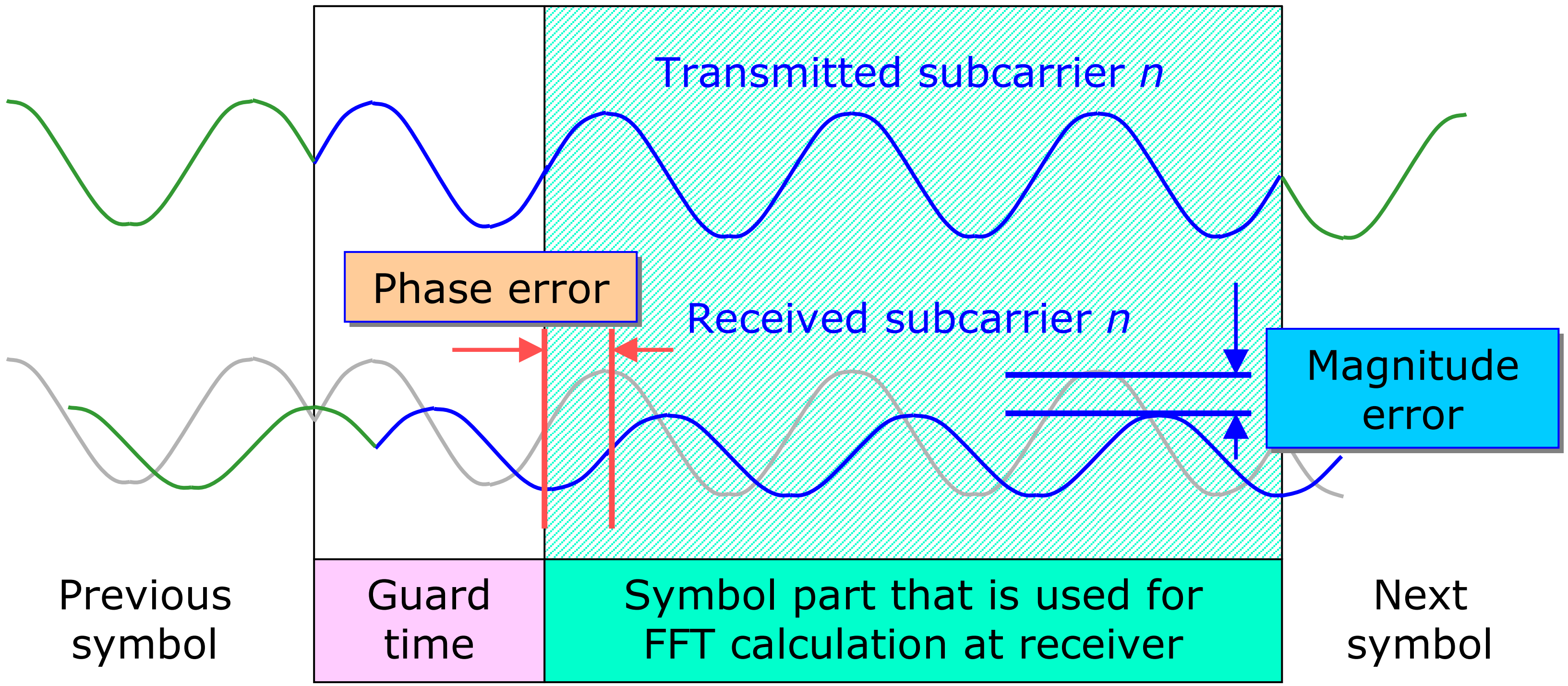
Task of pilot subcarriers

Pilot subcarriers contain signal values that are known in the receiver.

These pilot signals are used in the receiver for correcting the magnitude (important in QAM) and phase shift offsets of the received symbols (see signal constellation example on the right).

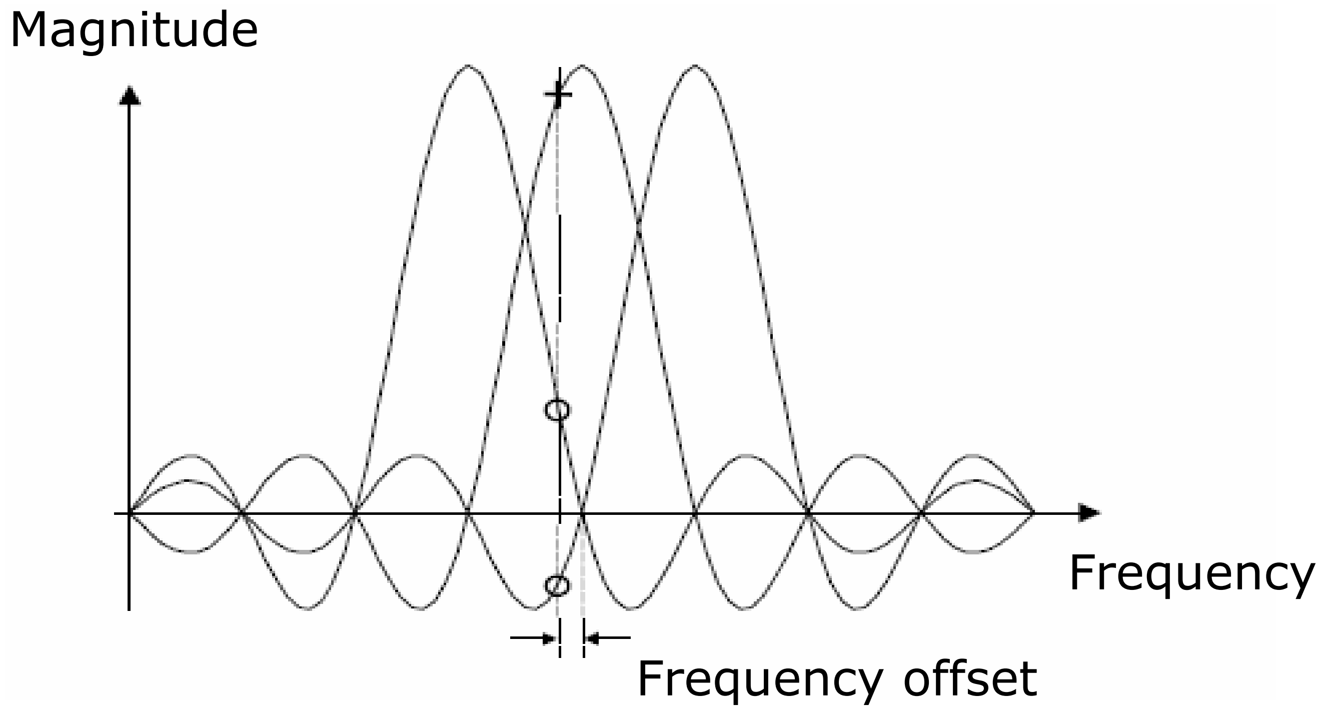


Transmitted and received subcarrier *n*



Frequency offset at receiver

Frequency offset causes inter-carrier interference (ICI)



Summary: Inter-carrier interference

Inter-carrier interference (ICI) means that the orthogonality between different subchannels in the OFDM signal is destroyed.

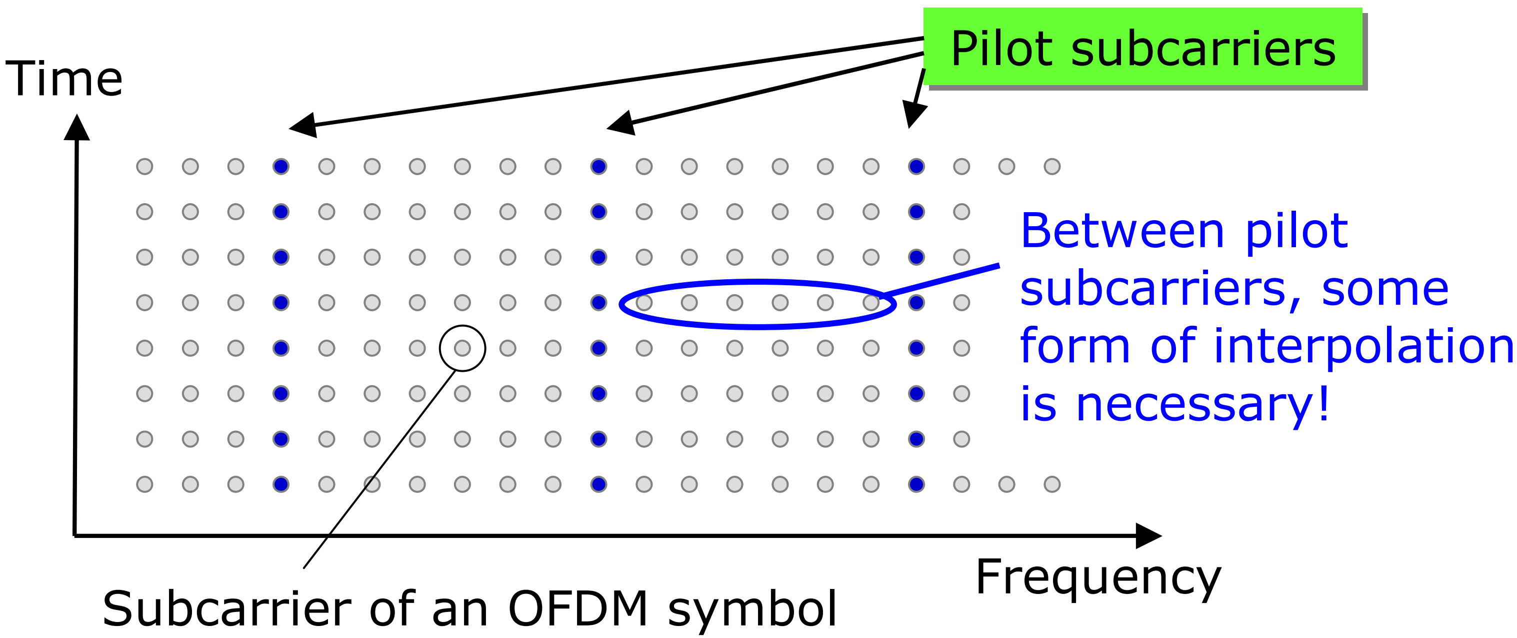
There are two causes of inter-carrier interference:

Delay spread of radio channel exceeds guard interval

Frequency offset at the receiver

Pilot allocation example 1 (1)

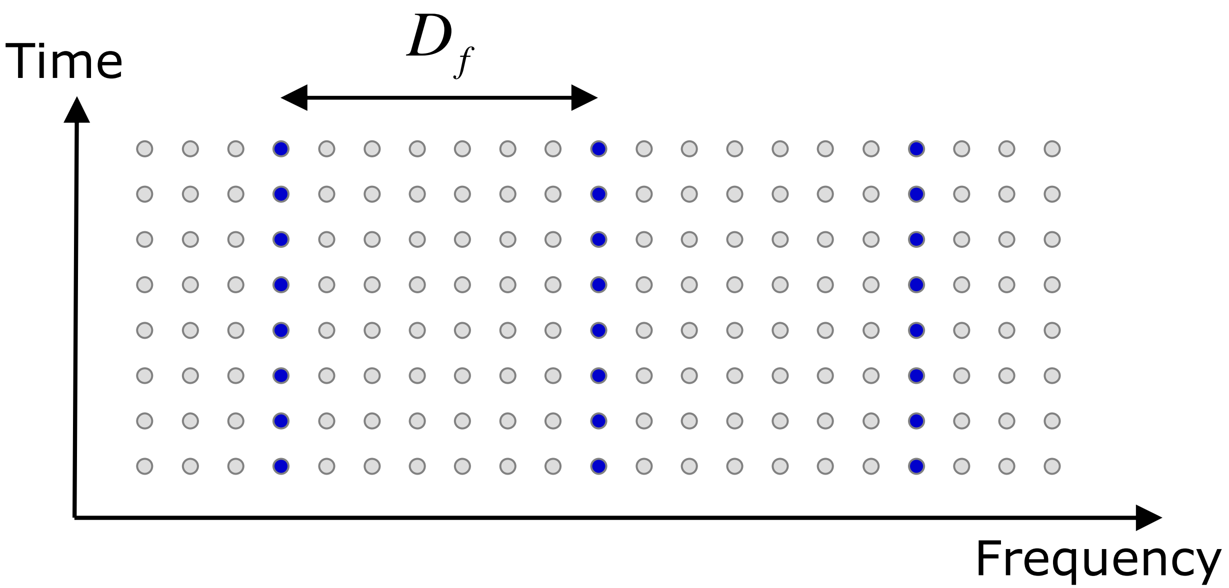
To be able to equalize the frequency response of a frequency selective channel, pilot subcarriers must be inserted at certain frequencies:



Pilot allocation example 1 (2)

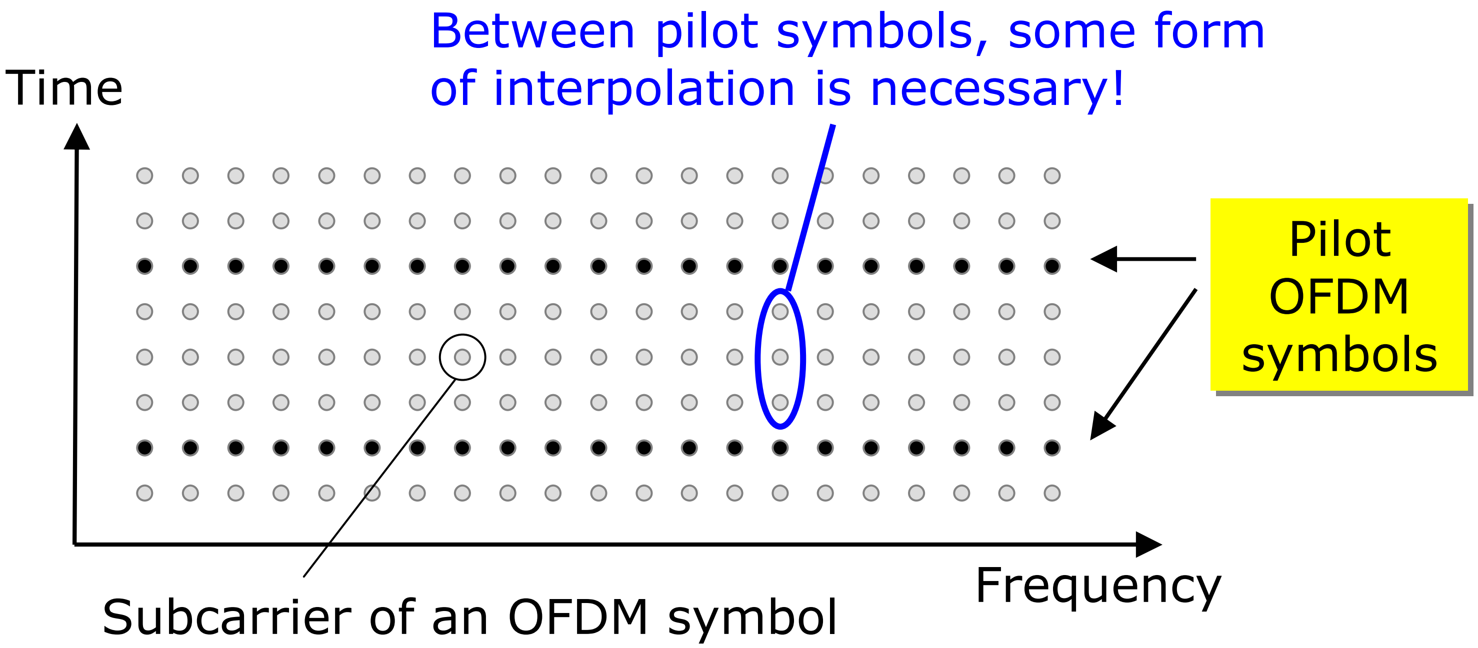
The Shannon sampling theorem must be satisfied, otherwise error-free interpolation is not possible:

$$D_f \leq 1/2T_m \qquad T_m = \text{maximum delay spread}$$



Pilot allocation example 2 (1)

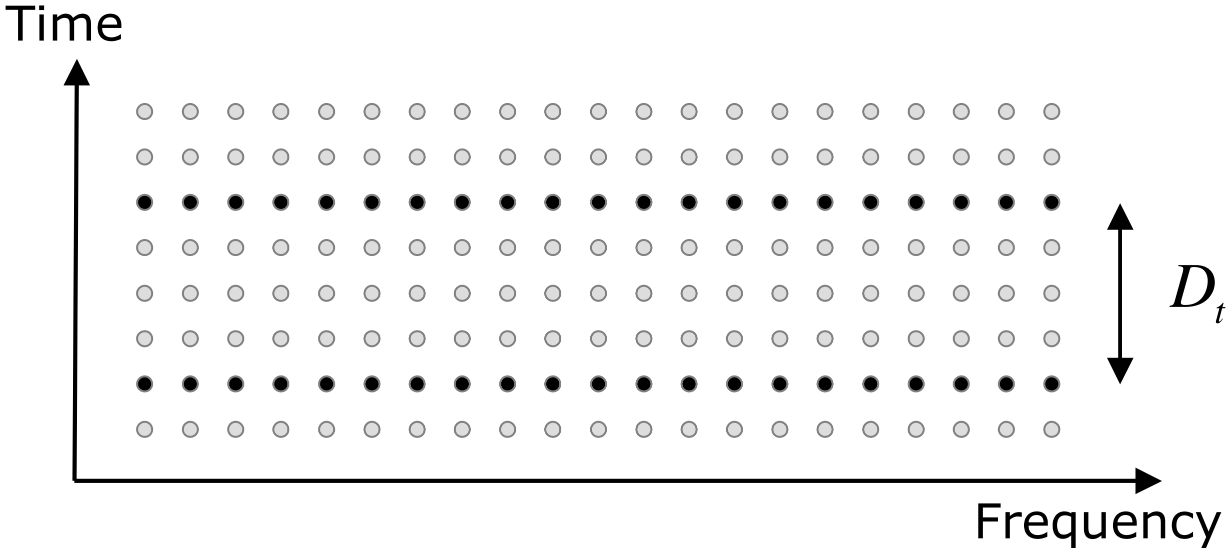
An alternative pilot scheme for equalizing the frequency response of a frequency selective channel:



Pilot allocation example 2 (2)

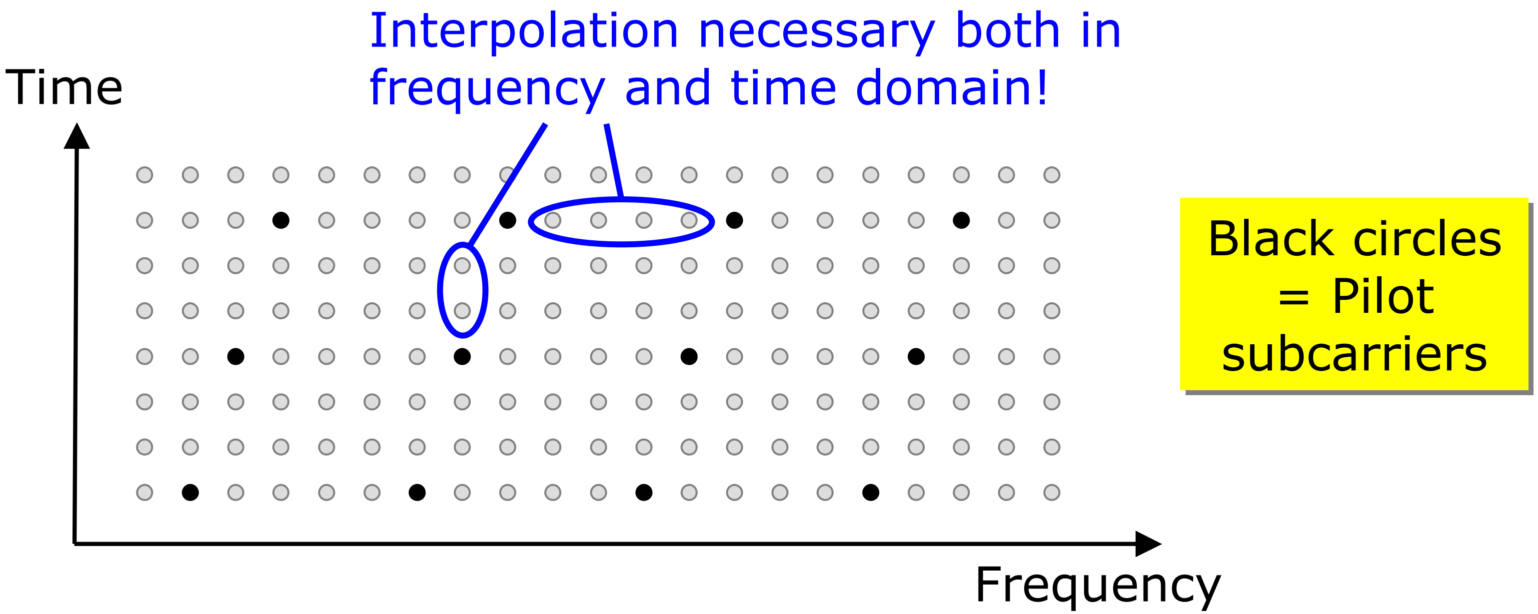
The Shannon sampling theorem must again be satisfied, otherwise error-free interpolation is not possible:

$$D_t \le 1/B_D \qquad B_D = \text{ maximum p-p Doppler spread}$$
$$\le 1/2\nu_{\max} \qquad \nu_{\max} = \text{ maximum Doppler frequency}$$



Pilot allocation example 3

An efficient pilot scheme (used in DVB-T) makes use of interpolation both in frequency and time domain:



Summary: OFDM features

In summary, OFDM offers the following features:

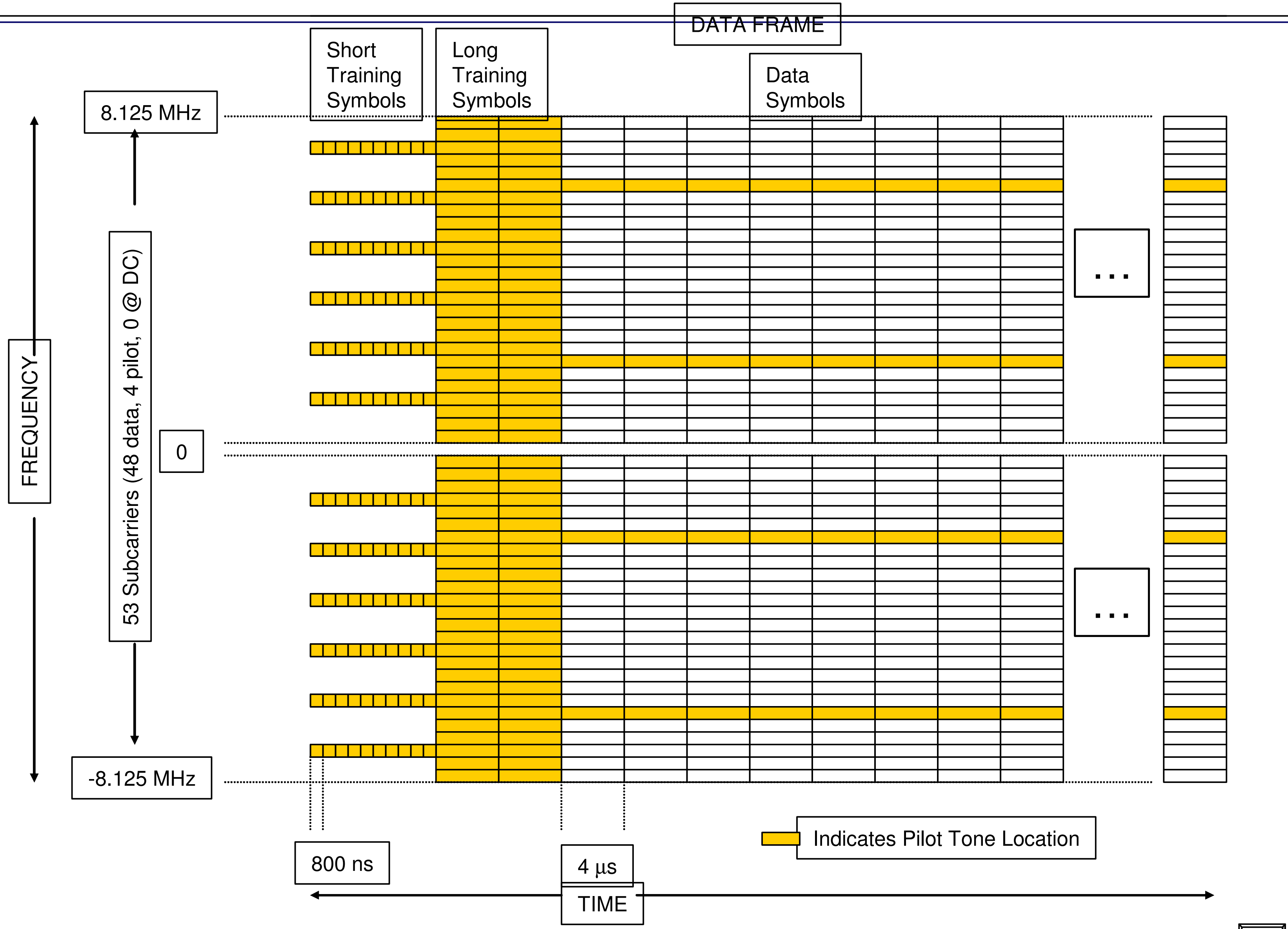
Multipath propagation (fading) does not cause intersymbol or intercarrier interference if the guard interval is sufficiently large and there is no frequency offset at the receiver.

Multipath fading, however, causes frequency selectivity in the transmission bandwidth. Pilot signals are employed for correcting (equalizing) the magnitude and phase of the received subcarriers at the pilot subcarrier frequencies.

Some form of interpolation is necessary for equalization at other than pilot subcarrier frequencies. Many pilot allocation schemes have been proposed in the literature, see e.g.

www.s3.kth.se/signal/grad/OFDM/URSIOFDM9808.htm

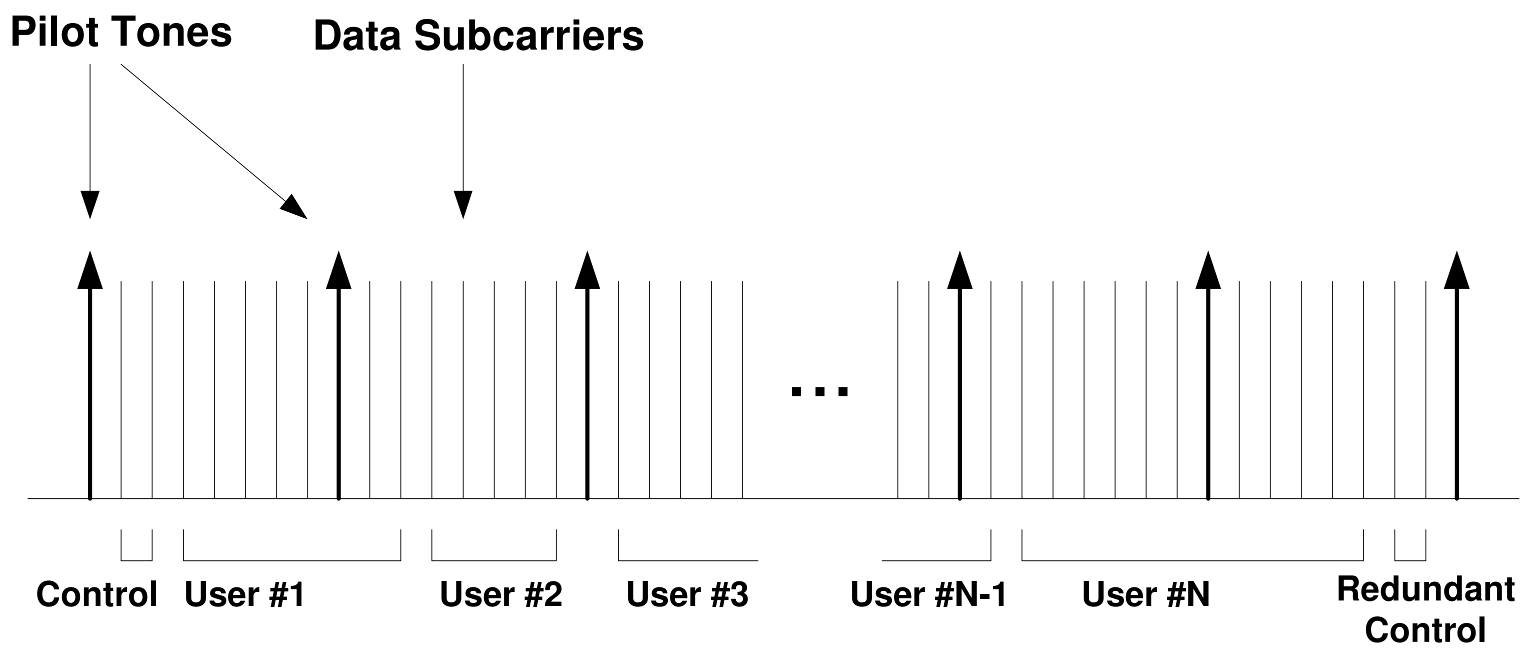
802.11a Time/Frequency Signal Structure



Cool and Interesting Tricks

- OFDMA
  - Different users on different subcarriers
- Adaptive Bit Loading
  - Seeking water filling capacity
  - Adaptation to Channel Fading
  - Adaptation to Interference

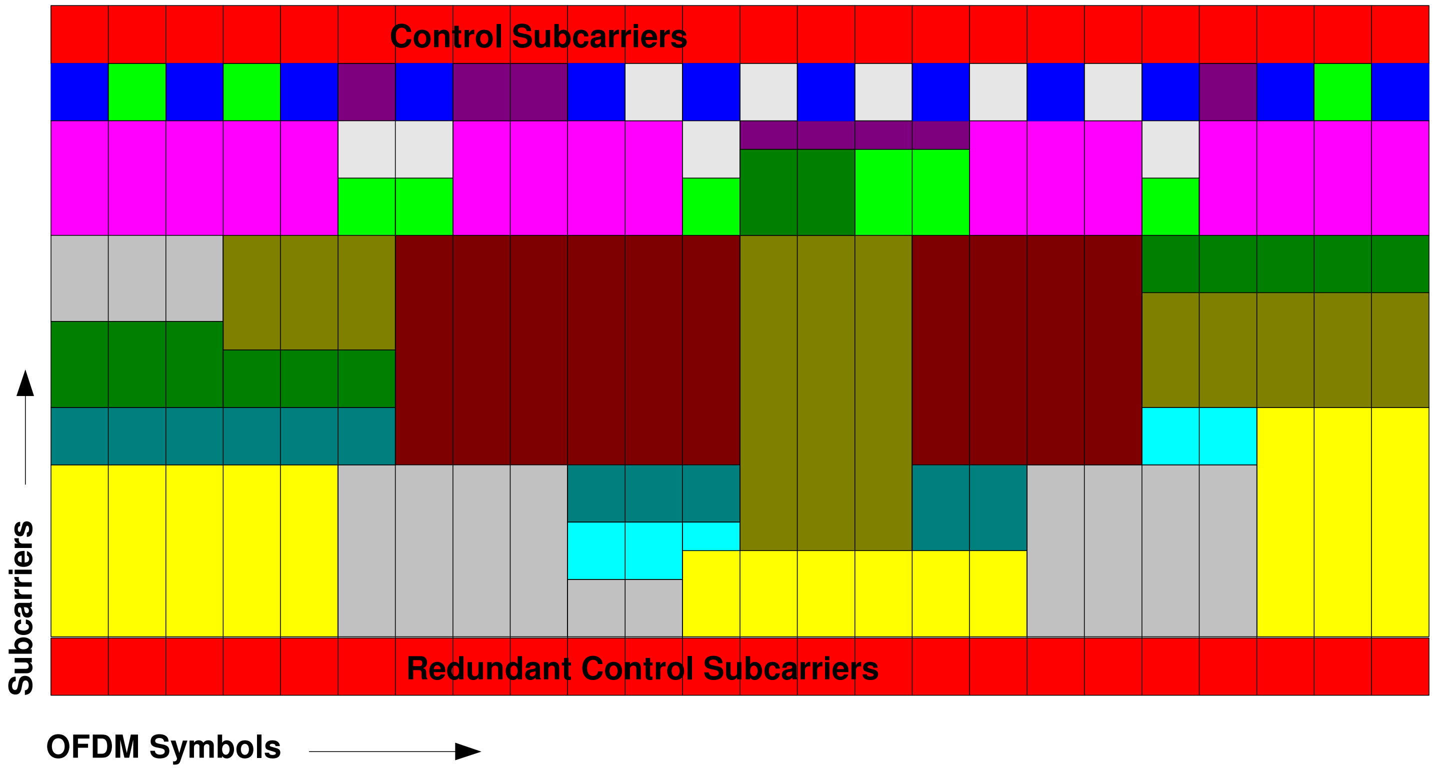
OFDMA Subcarrier Division



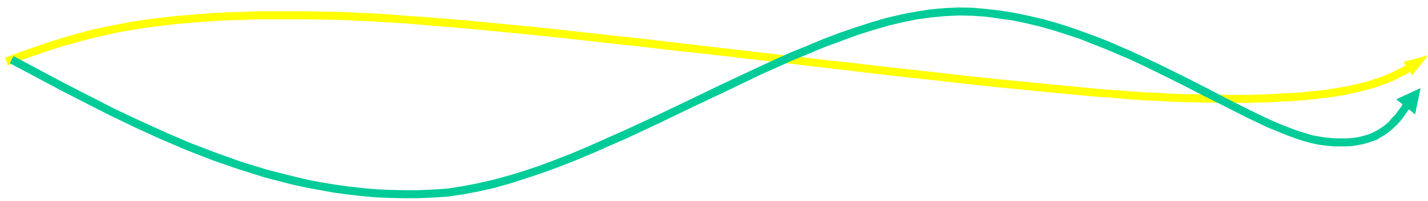
The 802.16 standard describes multiple means to implement OFDMA. In one mode each user's signal occupies contiguous subcarriers which can be independently modulated. Another mode permutes each user's subcarriers across the band in a spreading scheme so that all user's subcarriers are interlaced with other user's subcarriers. The first method allows for adaptive modulation and the second method increases frequency diversity.

Subcarrier Division with TDM

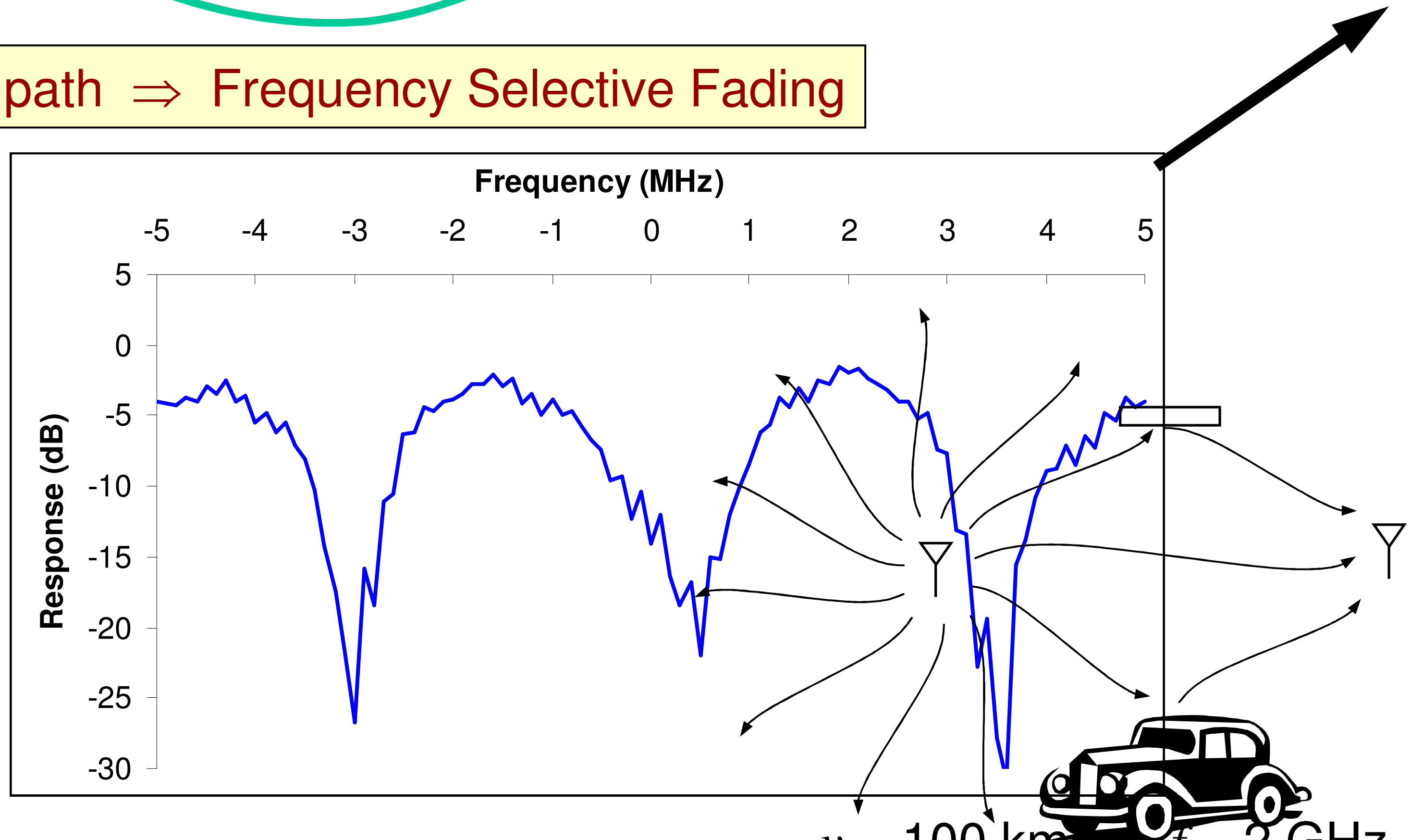
Each color is for a distinct terminal.



Channel Frequency Response



Multipath $\Rightarrow$ Frequency Selective Fading

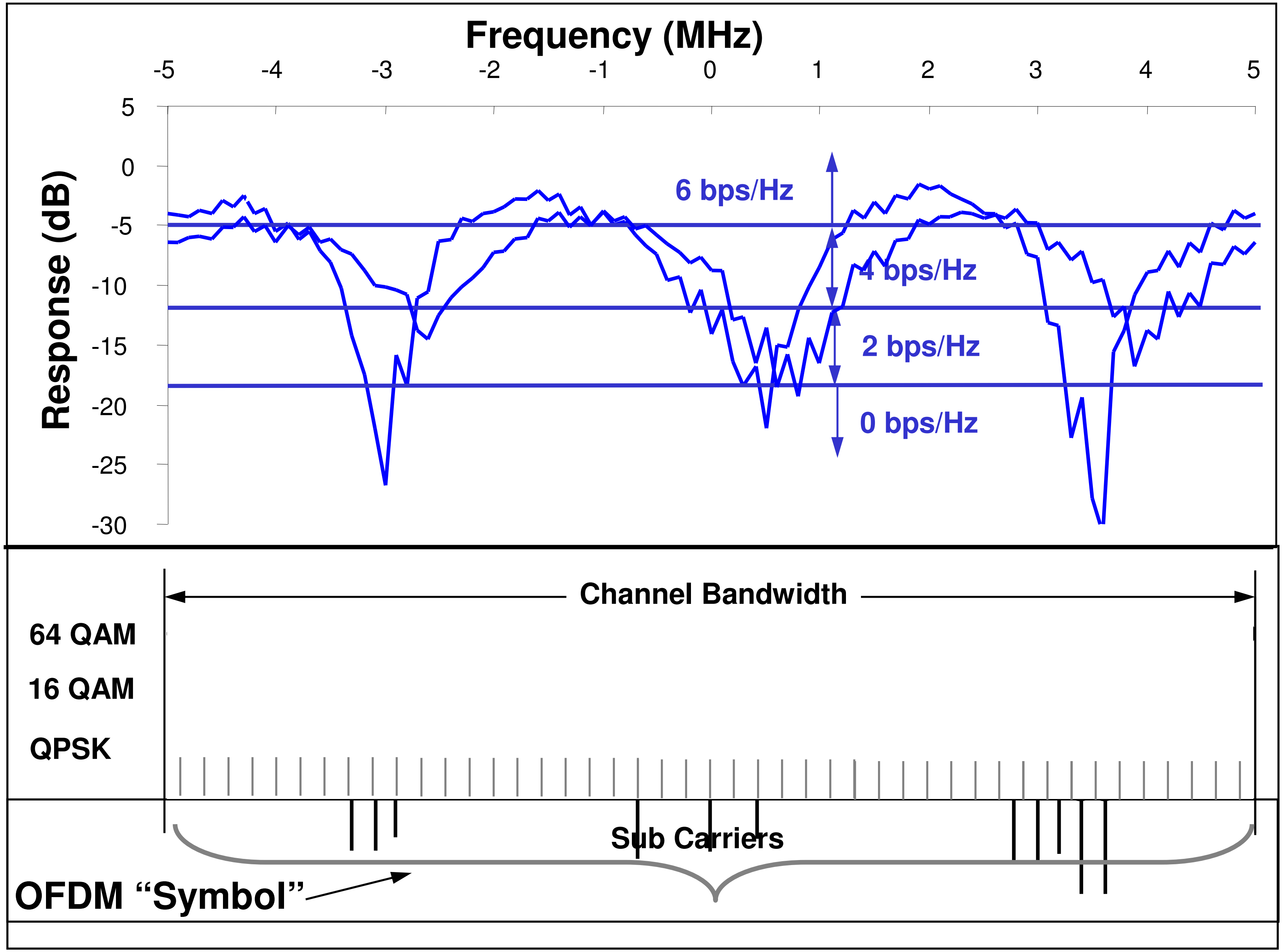


v = 100 km/hr f = 2 GHz

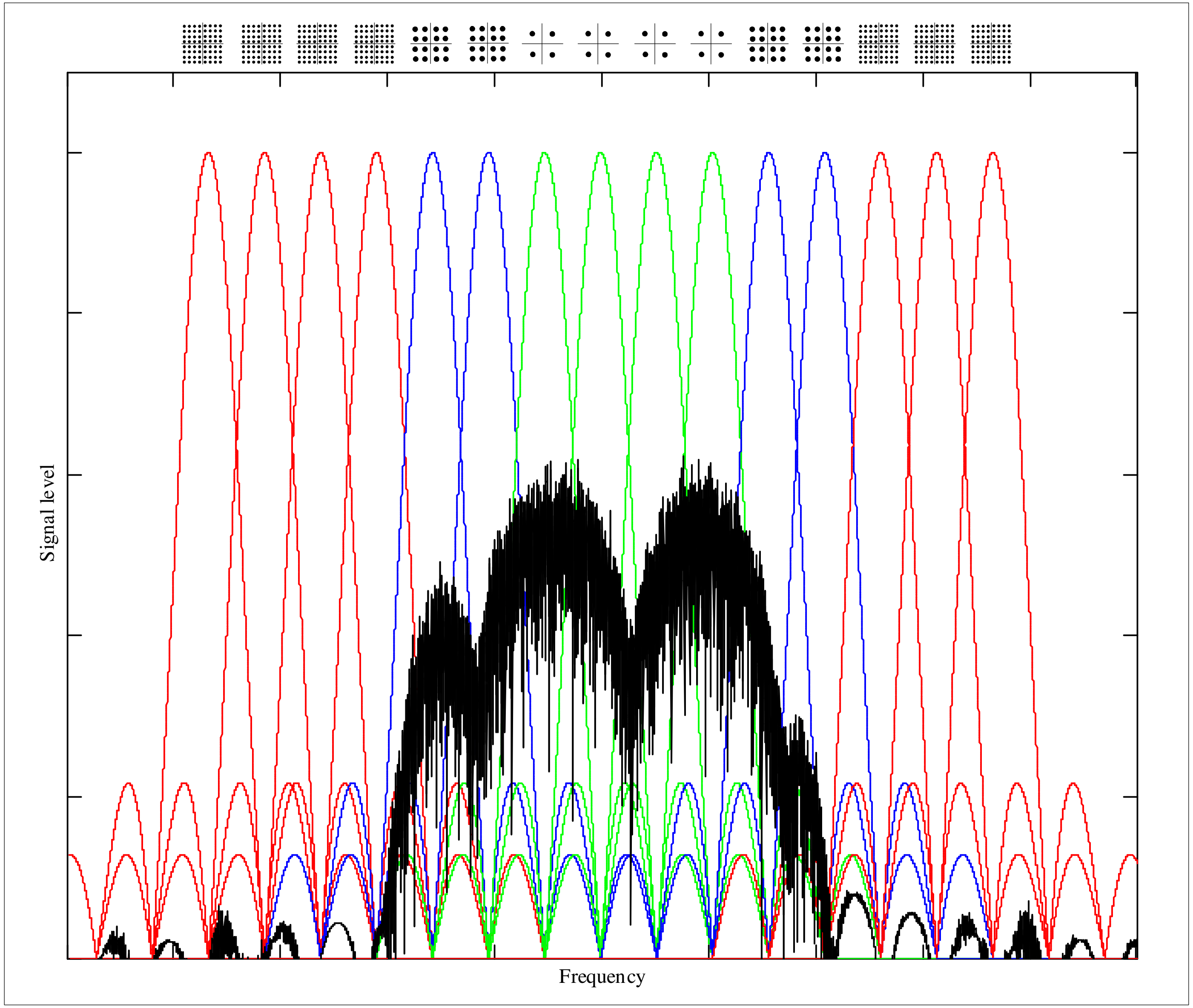
Δt = 0.5 m sec

Shannon's Law applies in each "flat" subinterval

Adaptive Bit Loading



Per-Subcarrier Adaptive Modulation



OFDM Advantages

- Efficiently Deals With Multi-path Fading
- Efficiently Deals With Channel Delay Spread
- Enhanced Channel Capacity
- Adaptively Modifies Modulation Density
- Robustness to Narrowband Interference

Downsides of OFDM

- Complexity
  - FFT for modulation, demodulation
    - Must be compared to complexity of equalizer
  - Synchronization
- Overhead
  - Cyclic Extension
    - Increases the length of the symbol for no increase in capacity
  - Pilot Tones
    - Simplify equalization and tracking for no increase in capacity
- PAPR
  - Depending on the configuration, the PAPR can be ~3dB-6dB worse than a single-carrier system
- Phase noise sensitivity
  - The subcarriers are N-times narrower than a comparable single-carrier system
- Doppler Spread sensitivity
  - Synchronization and EQ tracking can be problematic in high doppler environments

Example: IEEE 802.11a

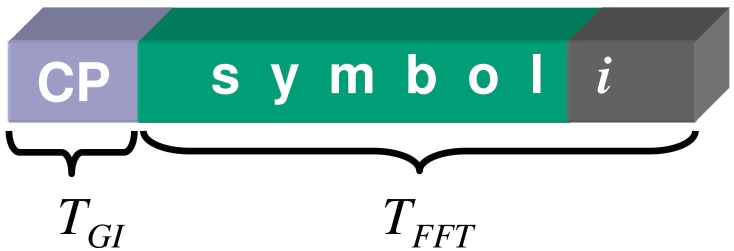
Data rate (Mbits/s)	Modulation	Coding rate (R)	Coded bits per subcarrier (N_{BPSC})	Coded bits per OFDM symbol (N_{CBPS})	Data bits per OFDM symbol (N_{DBPS})
6	BPSK	1/2	1	48	24
9	BPSK	3/4	1	48	36
12	QPSK	1/2	2	96	48
18	QPSK	3/4	2	96	72
24	16-QAM	1/2	4	192	96
36	16-QAM	3/4	4	192	144
48	64-QAM	2/3	6	288	192
54	64-QAM	3/4	6	288	216

- IEEE 802.11 employs adaptive modulation
  - Code rate & modulation depends on distance from base station
  - Overall data rate varies from 6 Mbps to 54 Mbps

Reference: IEEE Std 802.11a-1999

Case Study: IEEE 802.11a Wireless LAN

- System parameters
  - FFT size: 64
  - Number of tones used 52 (12 zero tones)
  - Number of pilots 4 (data tones = 52-4 = 48 tones)
  - Bandwidth: 20MHz
  - Subcarrier spacing : Δ_f = 20 MHz / 64 = 312.5 kHz
  - OFDM symbol duration: $T_{FFT} = 1/\Delta_f$ = 3.2us
  - Cyclic prefix duration: T_{GI} = 0.8us
  - Signal duration: $T_{signal} = T_{FFT} + T_{GI}$



OFDM Systems

System	Transform Size	Number Carriers	Channel Spacing kHz	Bandwidth MHz	Sample Rate MHz	Symbol Duration µsec	Data Rate Mbits/s
HyperLAN/2	64	52 4	312.5	16.25	20	3.2 0.8	6-54
802.11a	64	52 4	312.5	16.56	20	3.2 0.8	6-54
DVB-T	2048 1024	1712 842	4.464	7.643	9.174	224	0.68- 14.92
DAB	2048 8192	1536	1.00	1.536	2.048	24/48/96 msec	3.072
ADSL	256 (down) 64 (up)	36-127 7-28	4.3125	1.104	1.104	231.9	0.64- 8.192