

# EC744 Wireless Communication

## Fall 2008

Mohamed Essam Khedr

Department of Electronics and Communications

# Wireless Communication

## Fading Channel Overview

# Syllabus

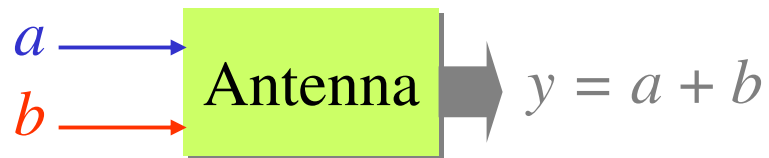
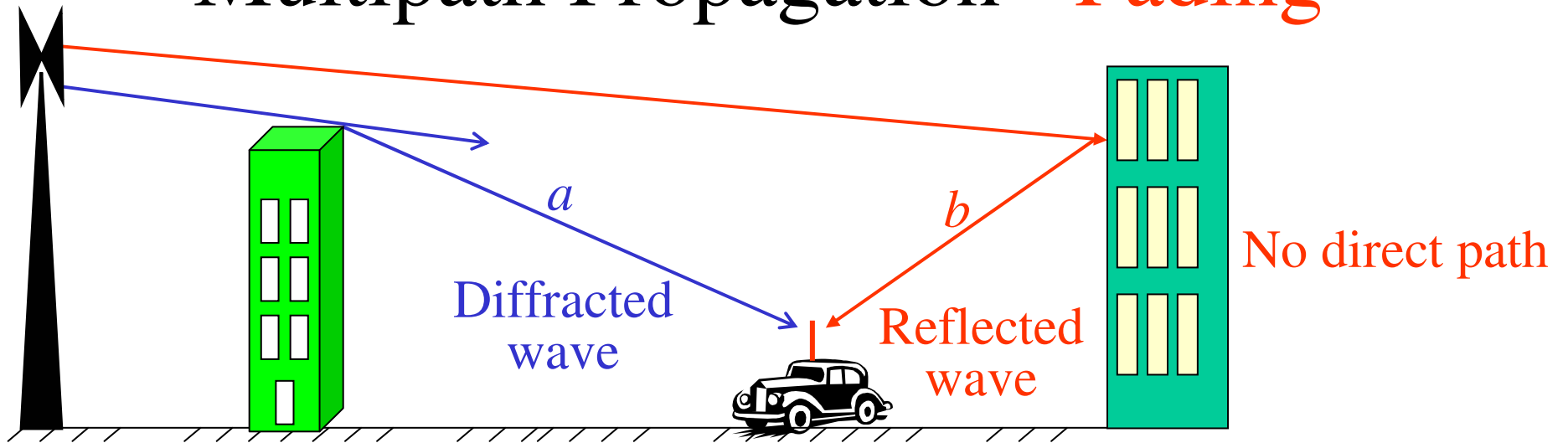
- Tentatively

Week 1	Overview wireless communications, Probabilities
Week 2	Digital Communication fundamentals
Week 3	Channel characteristics (AWGN, fading)
Week 4	Modulation techniques Demodulation techniques (coherent and non-coherent)
Week 5	Source coding techniques
Week 6	Channel coding techniques
Week 7	Mid Term exam (take home), Diversity techniques
Week 8	Equalization techniques
Week 9	Spread spectrum, MIMO and OFDM
Week 10	Wireless networking: 802.11, 802.16, UWB
Week 11	Hot topics
Week 12	Presentations
Week 13	Presentations
Week 14	Presentations
Week 15	Final Exam

# Fading

- Is due to multipath propagation.  
With respect to a stationary base station, multipath propagation creates a stochastic standing wave pattern, through which the mobile station moves.
- Caused by shadowing:  
when the propagation environment is changing significantly, but this fading is typically much slower than the multipath fading.

# Multipath Propagation - Fading



$a$  &  $b$  are in phase



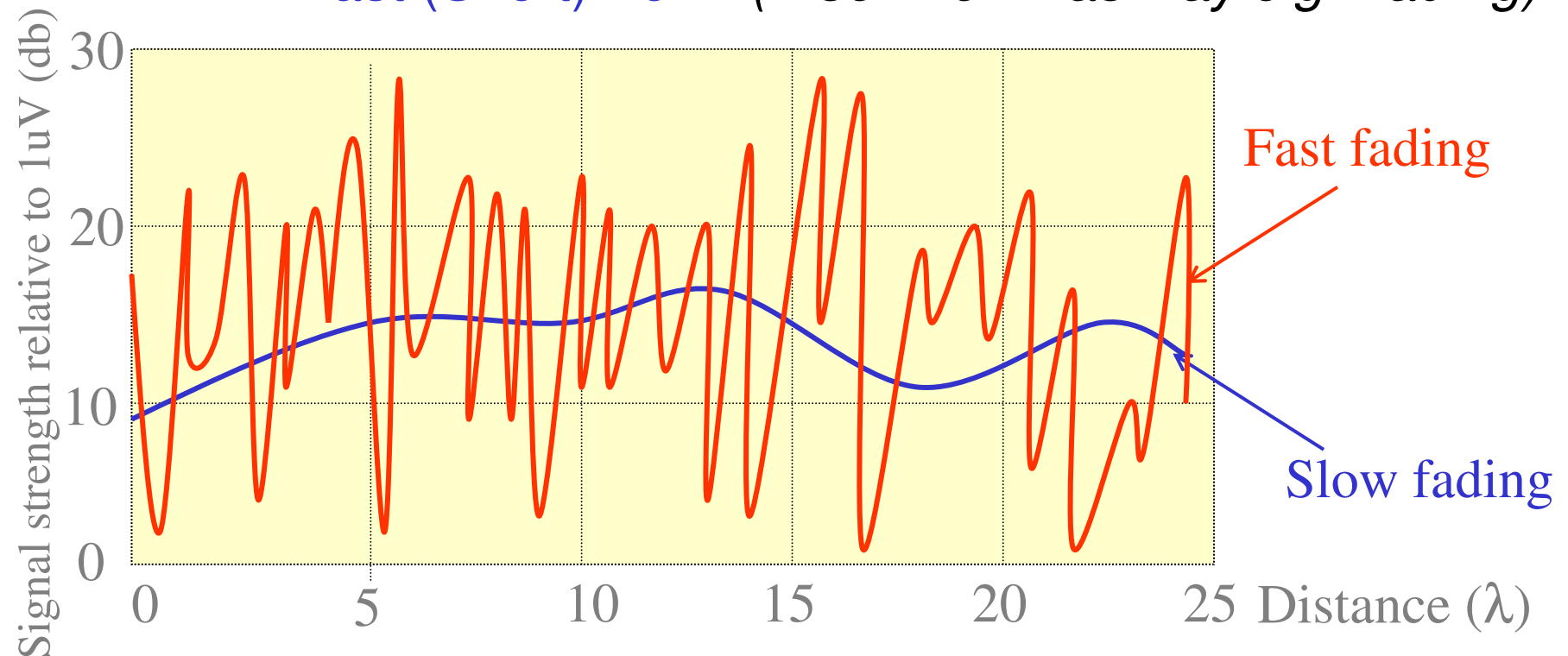
$a$  &  $b$  are out of phase by  $\pi$

**Complete fading when**  
 $2\pi d/\lambda = n\pi$ ,  $d$  is the path difference

# Fading - Types

- Slow (Long) Term

- Fast (Short) Term (*Also known as Rayleigh fading*)

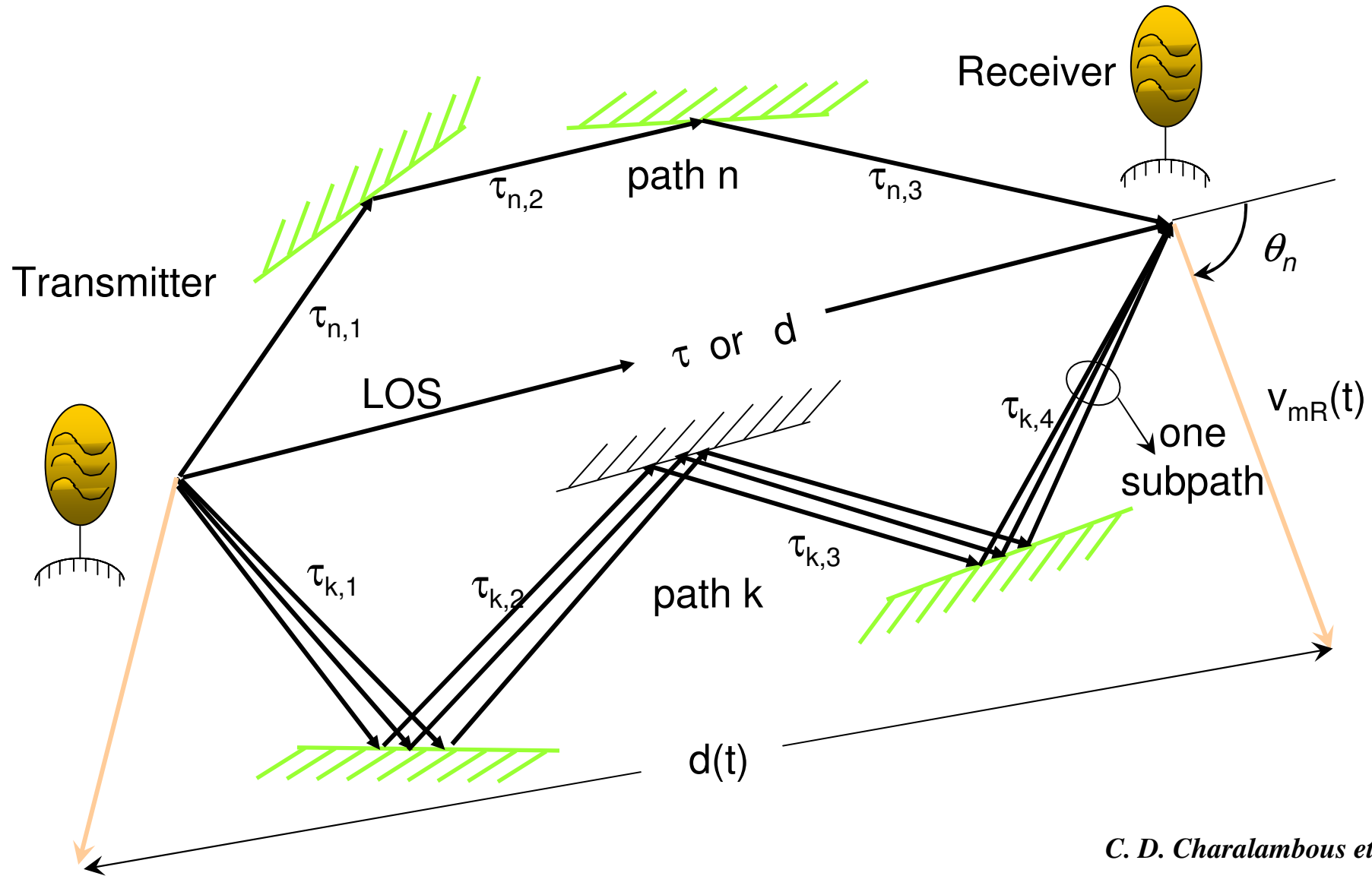


Exact representation of fading characteristics is not possible, because of infinite number of situation.

## Fading - **Slow (Long) Term**

- Slower variation in mean signal strength (distance 1-2 km)
- Produced by movement over much longer distances
- Caused by:
  - **Terrain configuration (hill, flat area etc.):**  
Results in local mean (long term fading) attenuation and fluctuation.
  - **The built environment (rural and urban areas etc.), between base station and the mobile unit:**  
Results in local mean attenuation

# Fading - Slow (Long) Term



# Fading- Fast (Short) Term

- **Describes the constant amplitude fluctuations in the received signal as the mobile moves.**
- Caused by multipath reflection of transmitted signal by local scatters (houses, building etc.)
- Observed over distances =  $\lambda/2$
- **Signal variation up to 30 dB.**
- Is a frequency selective phenomenon.
- Can be described using **Rayleigh statistics**, (no line of sight).
- Can be described using **Rician statistics**, (line of sight).
- Causes random fluctuations in the received power, and also distorts the pulse carrying the information.



## Fading- Fast (Short) Term - *contd.*

A received signal amplitude is given as the sum of delayed components. In terms of phasor notation it is given as:

$$S_r(t) = \sum_{i=1}^N a_i \cos(2\pi f_c t + \phi_i)$$

Or

$$S_r(t) = \cos(2\pi f_c t) \sum_{i=1}^N a_i \cos(\phi_i) - \sin(2\pi f_c t) \sum_{i=1}^N a_i \sin(\phi_i)$$

In-phase

Quadrature

## Fading- Fast (Short) Term - *contd.*

The phase  $\phi_i$  can be assumed to be uniformly distributed in the range  $(0, 2\pi)$ , provided the locations of buildings etc. are completely random.

This for large  $N$ , the amplitude of the received signal is:

$$S_r(t) = X \cos(2\pi f_c t) - Y \sin(2\pi f_c t)$$

where

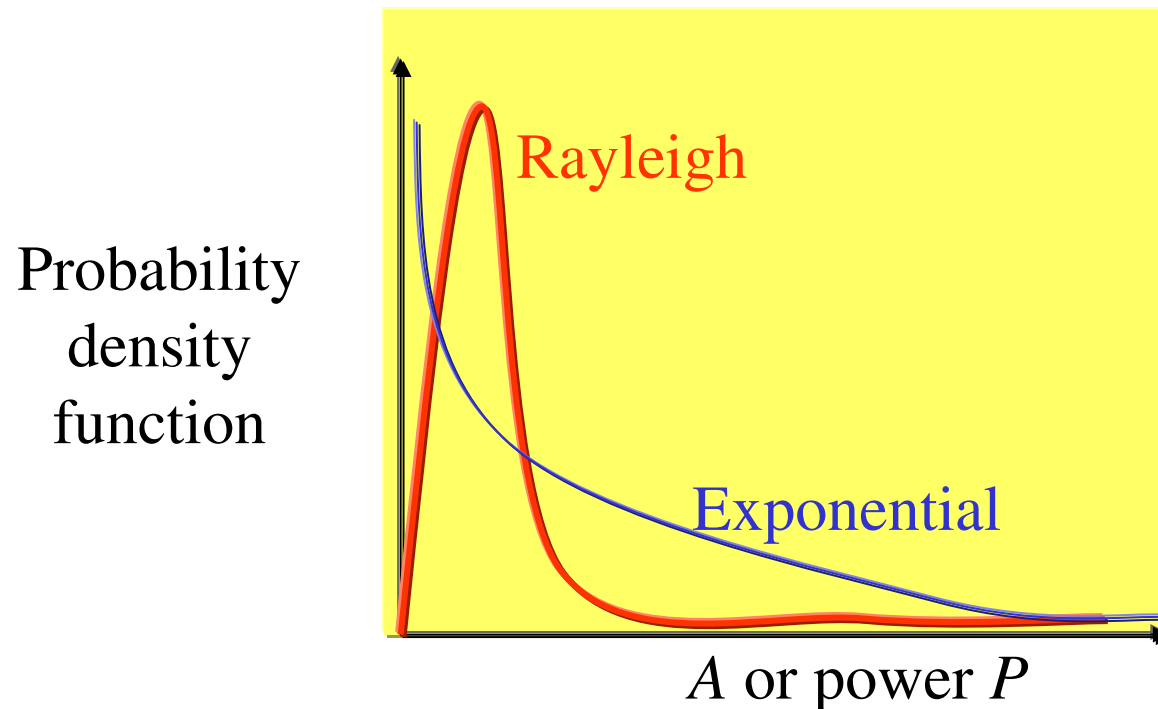
$$X = \sum_{i=1}^N a_i \cos(\phi_i), \quad Y = \sum_{i=1}^N a_i \sin(\phi_i)$$

$X$  and  $Y$  are independent, identically distributed Gaussian random variables.

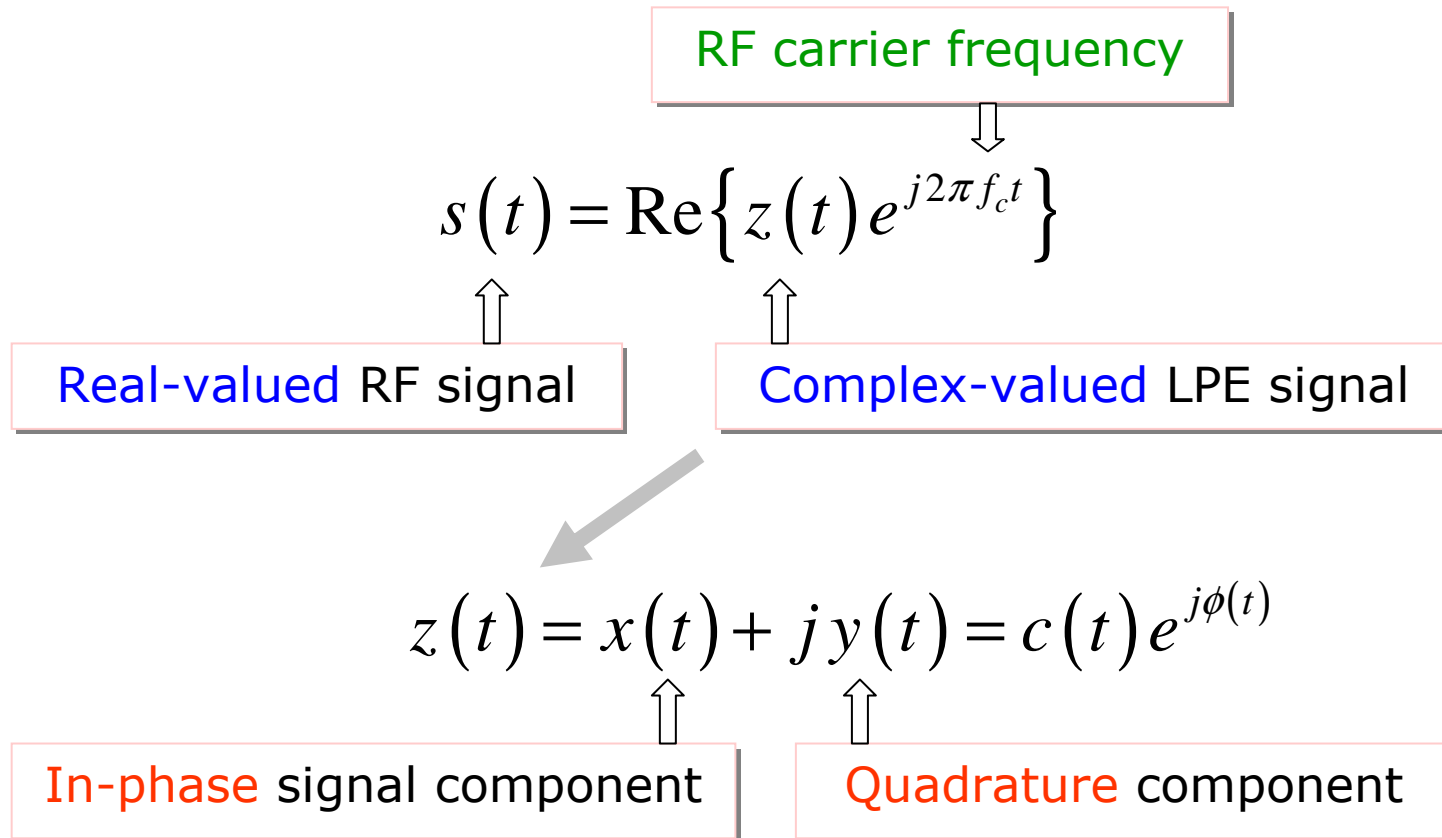
## Fading- Fast (Short) Term - *contd.*

The envelope of the received signal is:

$$A = (X^2 + Y^2)^{0.5} \quad \text{Which will be Rayleigh distributed.}$$



# Low-pass equivalent (LPE) signal



# Radio channel modelling

## Narrowband modelling

Calculation of path loss  
e.g. taking into account

- free space loss
- reflections
- diffraction
- scattering

Basic problem: **signal fading**

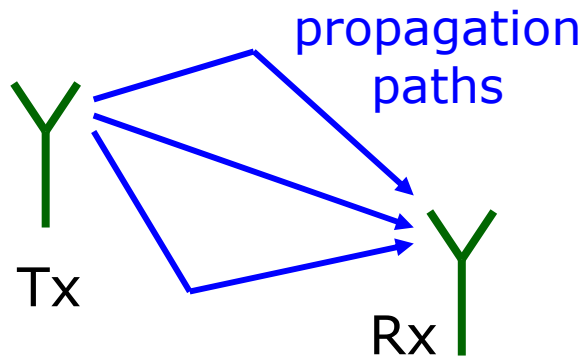
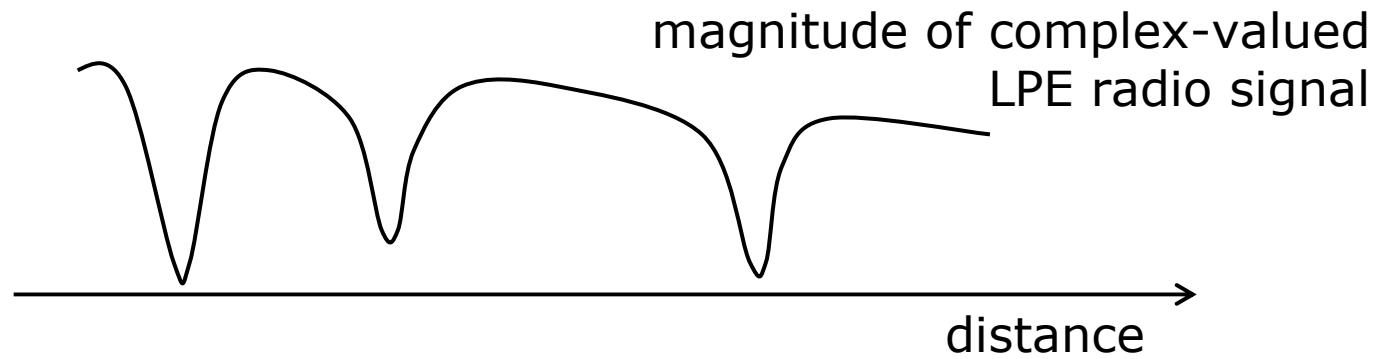
## Wideband modelling

Deterministic models  
(e.g. ray tracing)

Stochastic models  
(e.g. WSSUS)

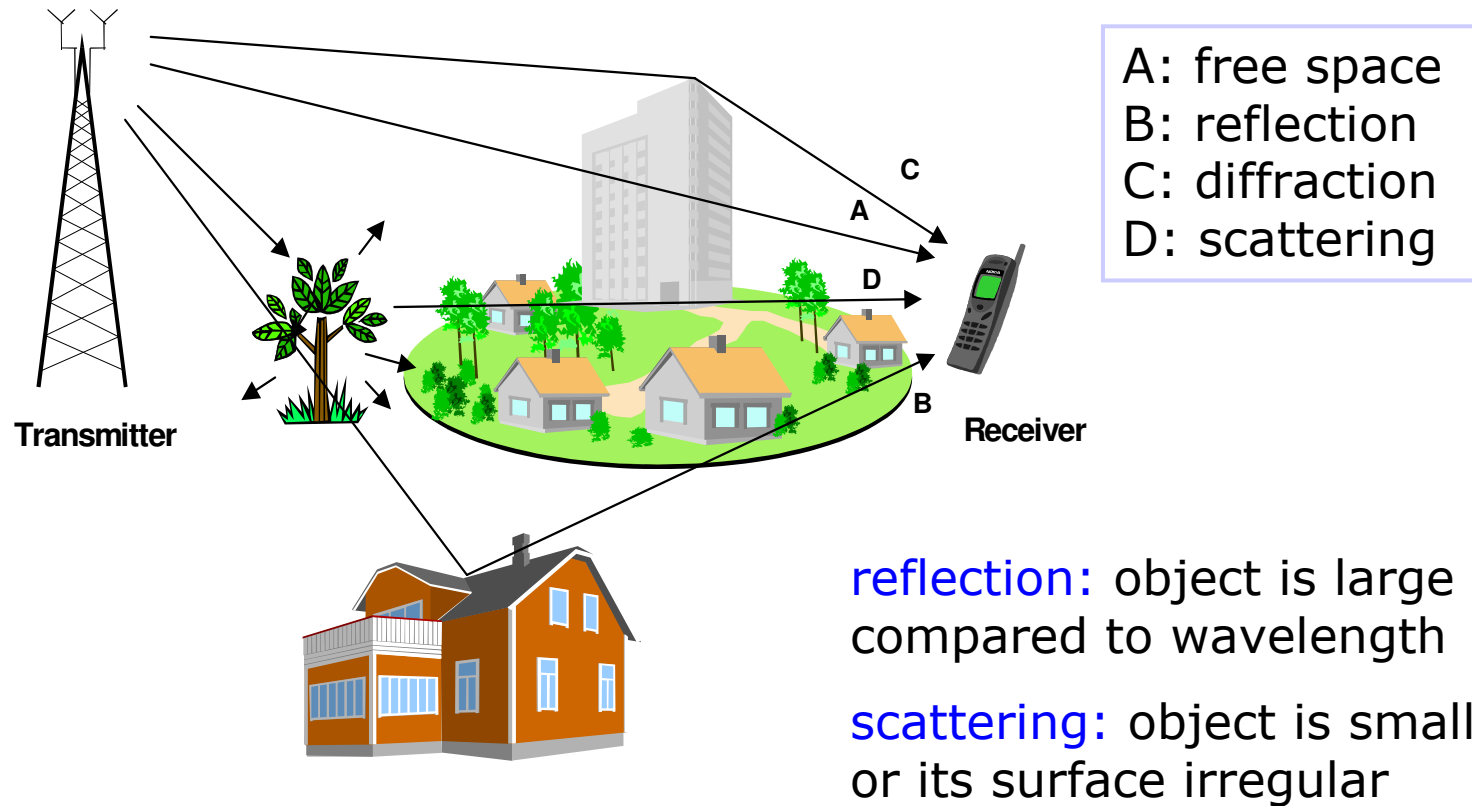
Basic problem: **signal dispersion**

# Signal fading in a narrowband channel



fade  $\Leftrightarrow$  signal replicas received via different propagation paths cause destructive interference

# Propagation mechanisms

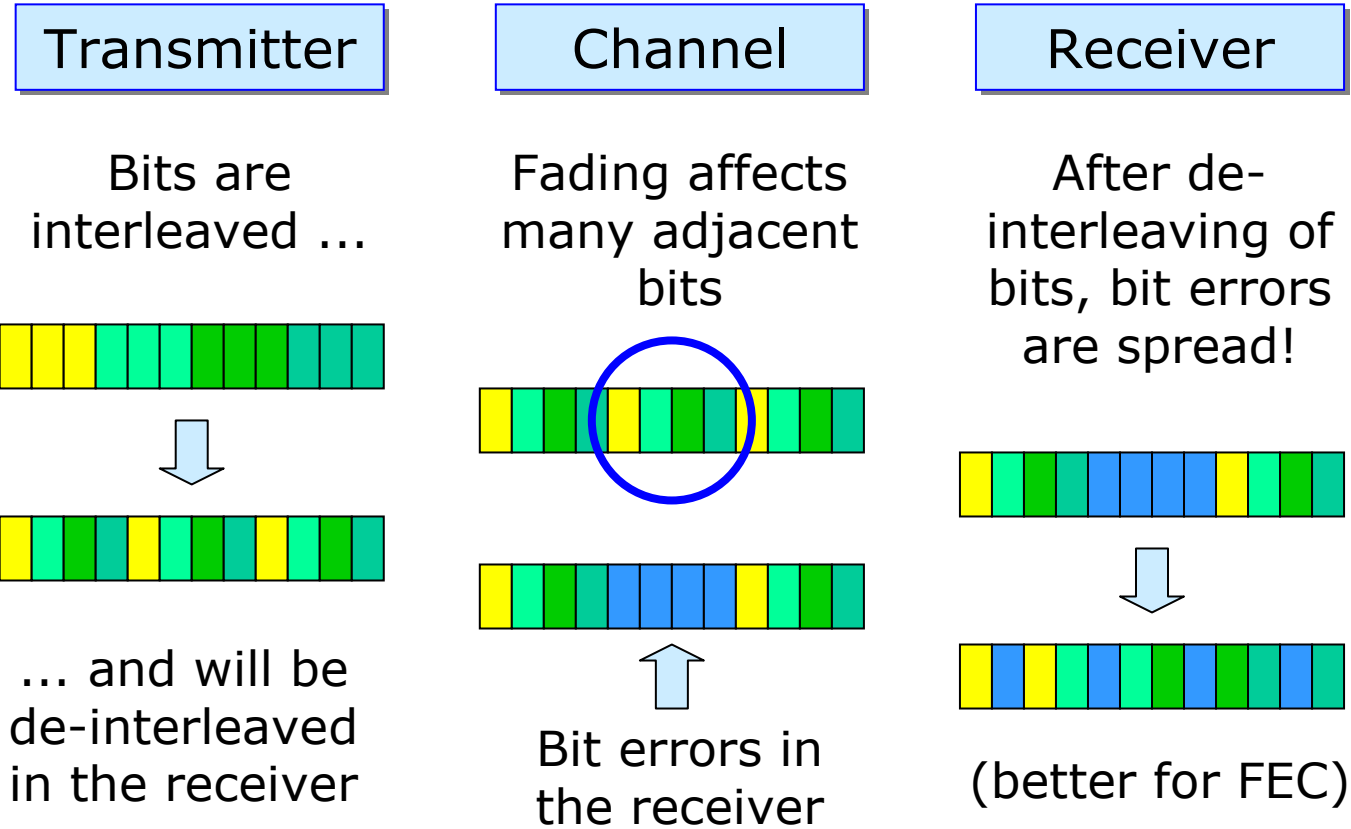


## Countermeasures: narrowband fading

- **Diversity** (transmitting the same signal at different frequencies, at different times, or to/from different antennas)
  - will be investigated in later lectures
  - wideband channels => **multipath diversity**
- **Interleaving** (efficient when a fade affects many bits or symbols at a time), **frequency hopping**
- **Forward Error Correction** (FEC, uses large overhead)
- **Automatic Repeat reQuest** schemes (ARQ, cannot be used for transmission of real-time information)

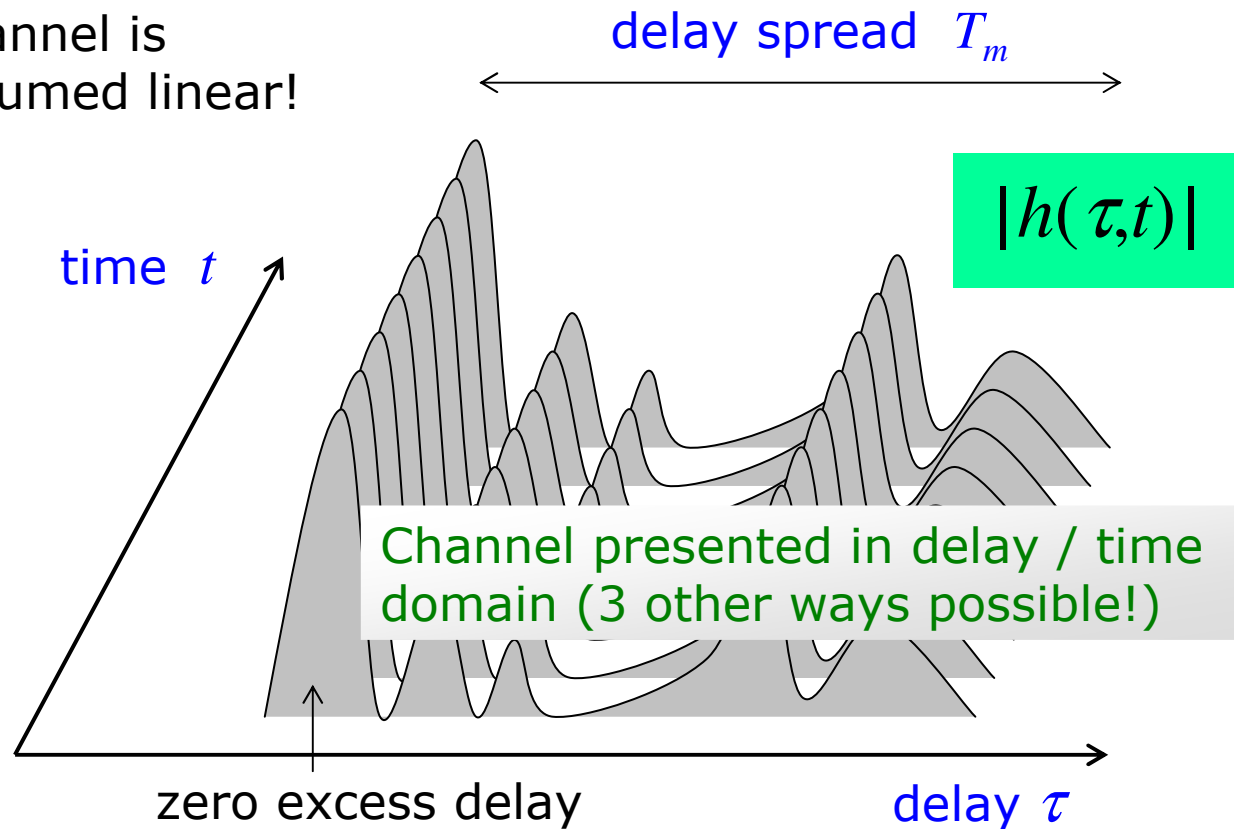


# Bit interleaving



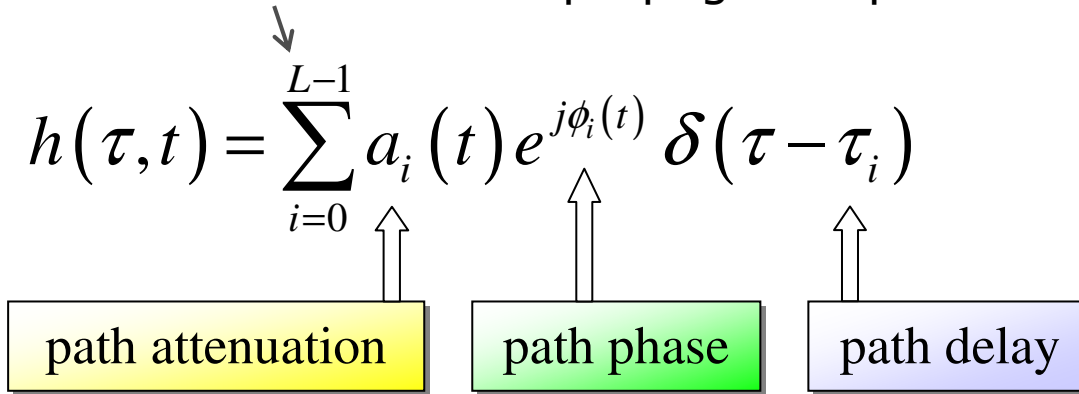
# Channel Impulse Response (CIR)

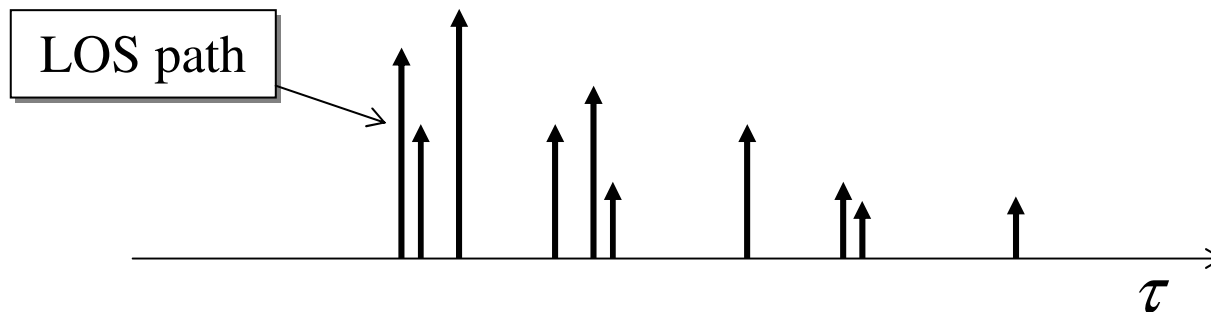
Channel is assumed linear!



# CIR of a wideband fading channel

The CIR consists of  $L$  resolvable propagation paths

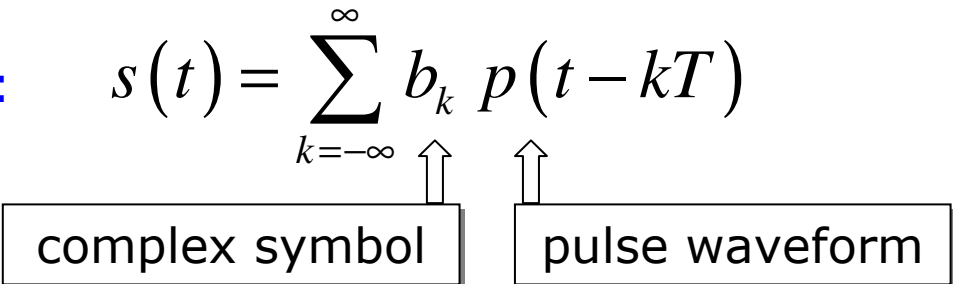
$$h(\tau, t) = \sum_{i=0}^{L-1} a_i(t) e^{j\phi_i(t)} \delta(\tau - \tau_i)$$




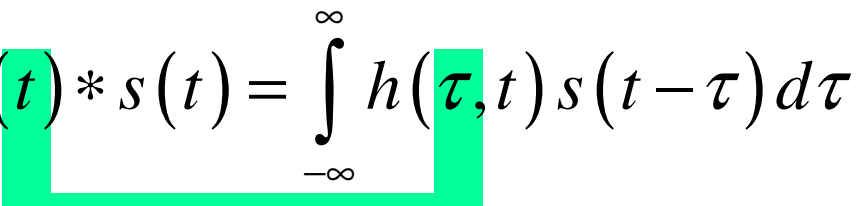
## Received multipath signal

Transmitted signal:  $s(t) = \sum_{k=-\infty}^{\infty} b_k p(t - kT)$

complex symbol      pulse waveform



Received signal:

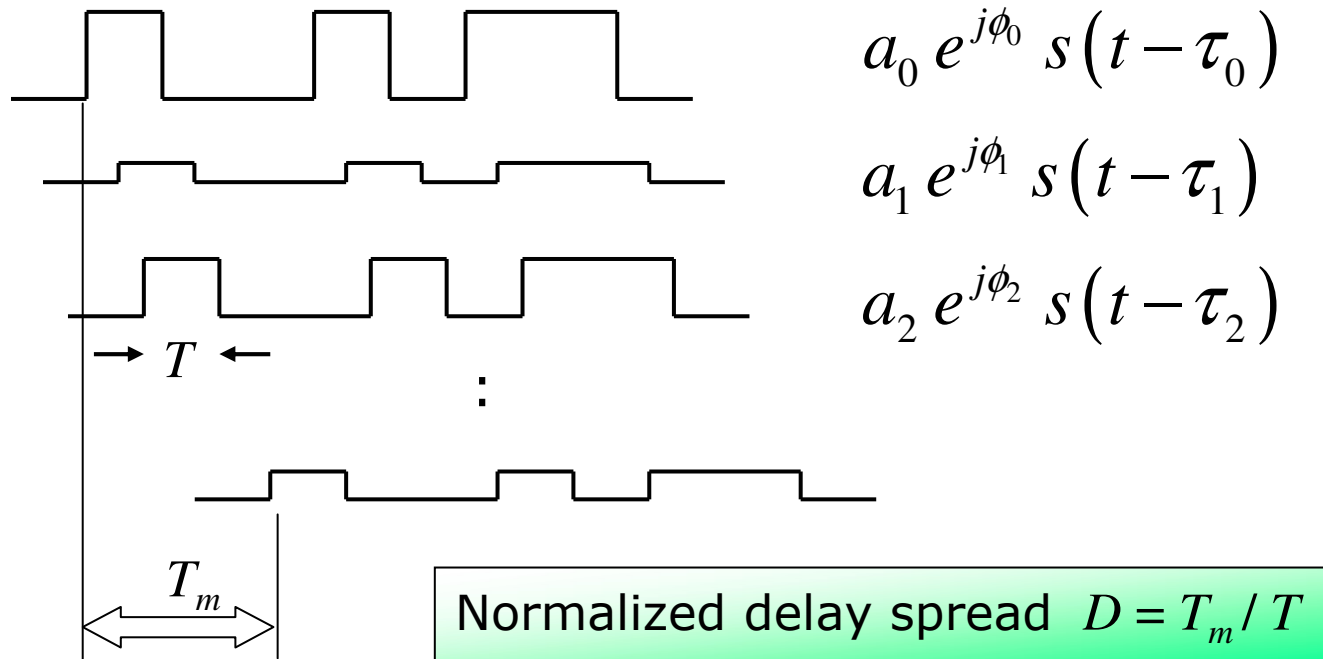
$$r(t) = h(t) * s(t) = \int_{-\infty}^{\infty} h(\tau, t) s(t - \tau) d\tau$$


$$= \sum_{i=0}^{L-1} a_i(t) e^{j\phi_i(t)} s(t - \tau_i)$$

$$\int f(t) \delta(t - t_0) dt = f(t_0)$$

## Received multipath signal

The received multipath signal is the sum of  $L$  attenuated, phase shifted and delayed replicas of the transmitted signal  $s(t)$



## Received multipath signal

The normalized delay spread is an important quantity.

When  $D \ll 1$ , the channel is

- narrowband
- frequency-nonselective
- flat

and there is no intersymbol interference (ISI).

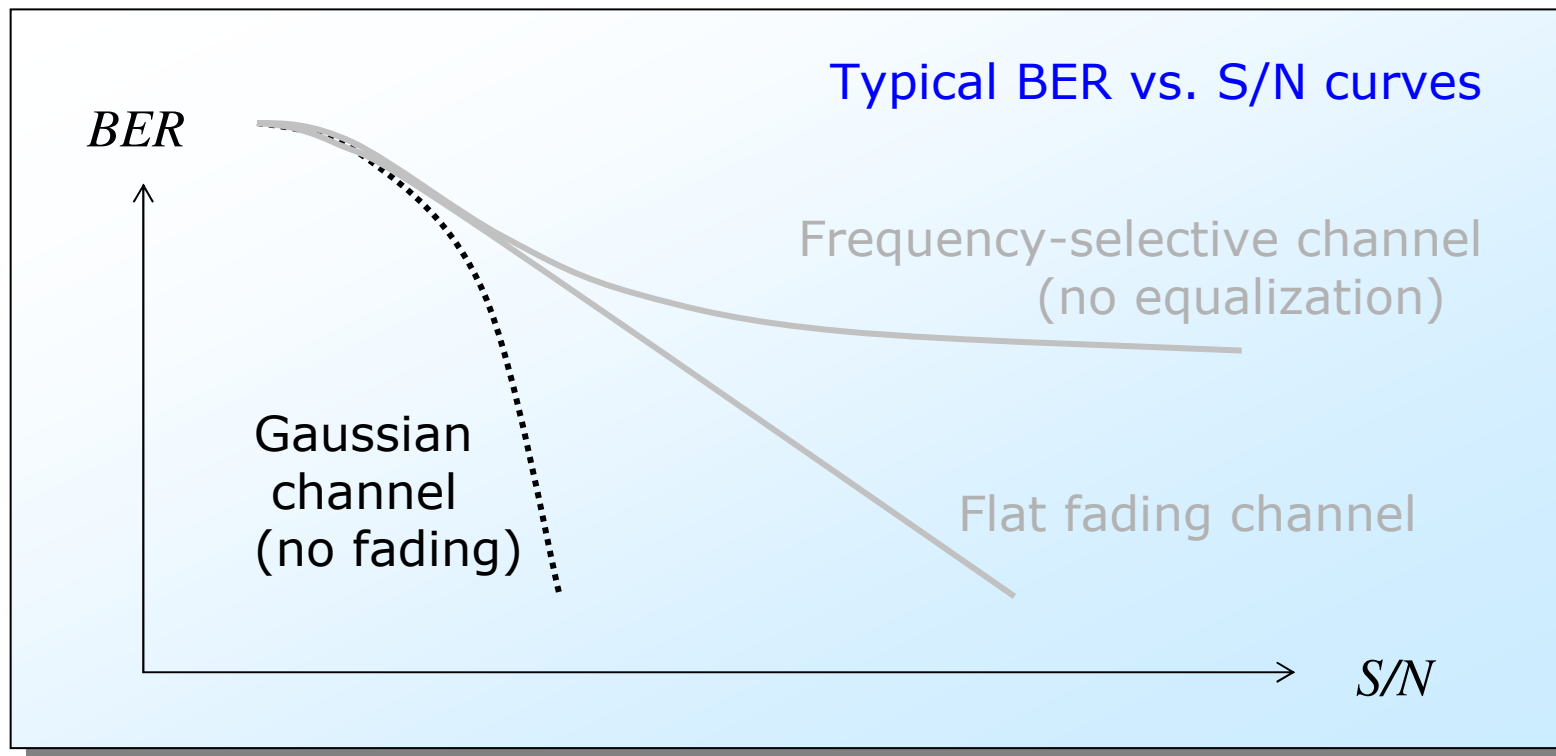
When  $D$  approaches or exceeds unity, the channel is

- wideband
- frequency selective
- time dispersive

Important feature  
has many names!

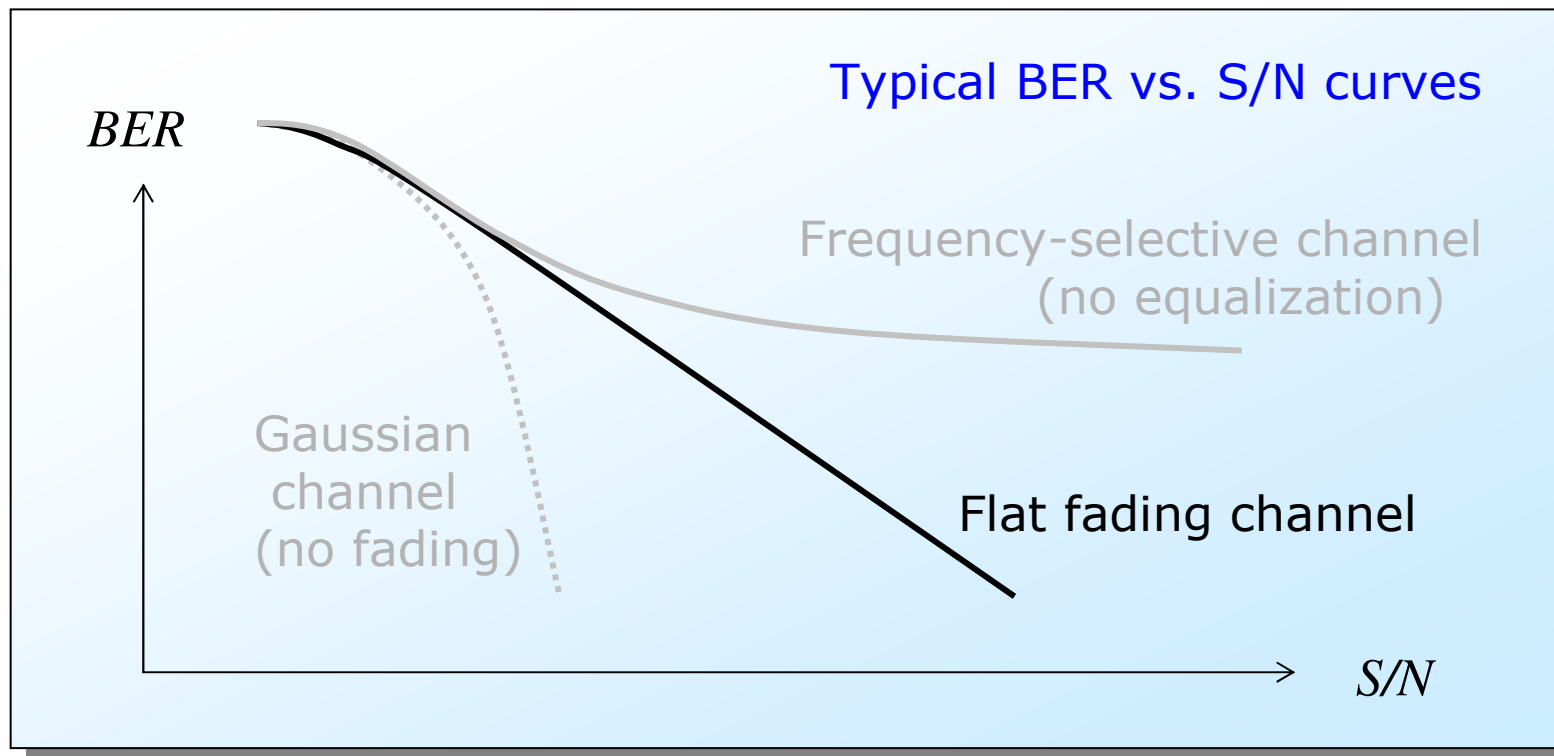
# BER vs. S/N performance

In a Gaussian channel (no fading)       $BER \Leftrightarrow Q(S/N)$   
erfc( $S/N$ )



## BER vs. S/N performance

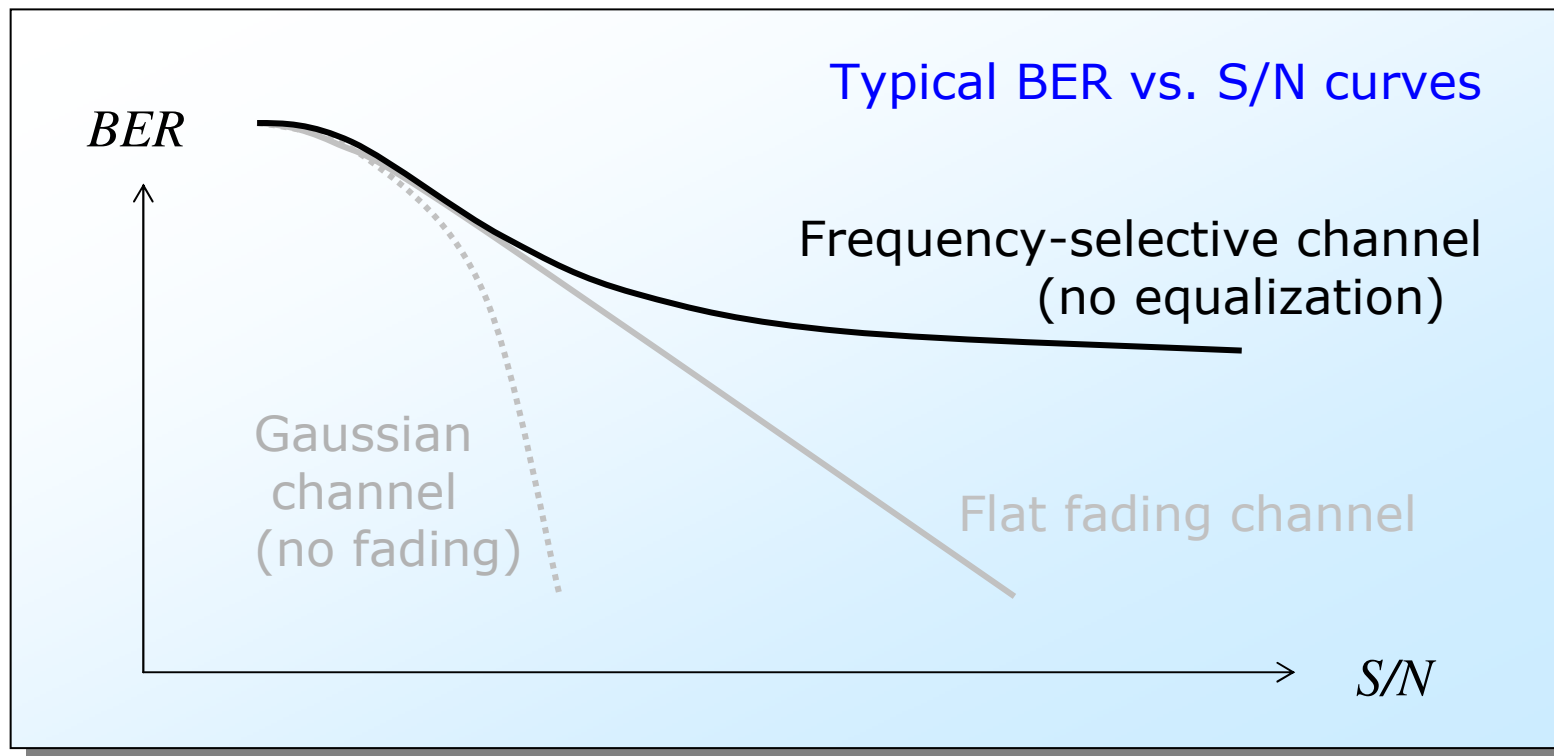
Flat fading (Proakis 7.3):  $BER = \int BER(S/N|z) p(z) dz$   
 $z = \text{signal power level}$





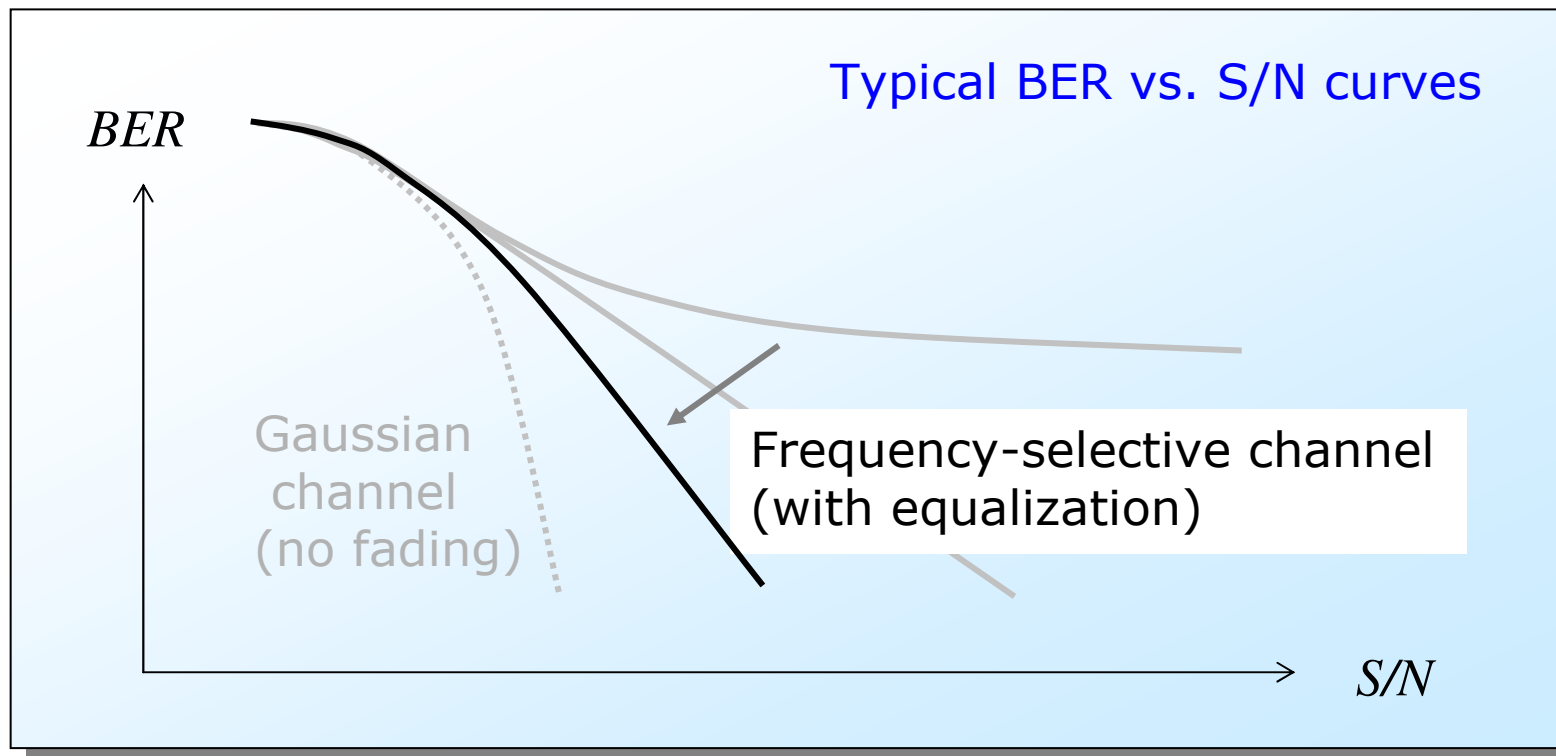
## BER vs. S/N performance

Frequency selective fading  $\Leftrightarrow$  irreducible BER floor



## BER vs. S/N performance

Diversity (e.g. multipath diversity)  $\Leftrightarrow$  improved performance



# Time-variant transfer function

Time-variant CIR: 
$$h(\tau, t) = \sum_{i=0}^{L-1} a_i(t) e^{j\phi_i(t)} \delta(\tau - \tau_i)$$

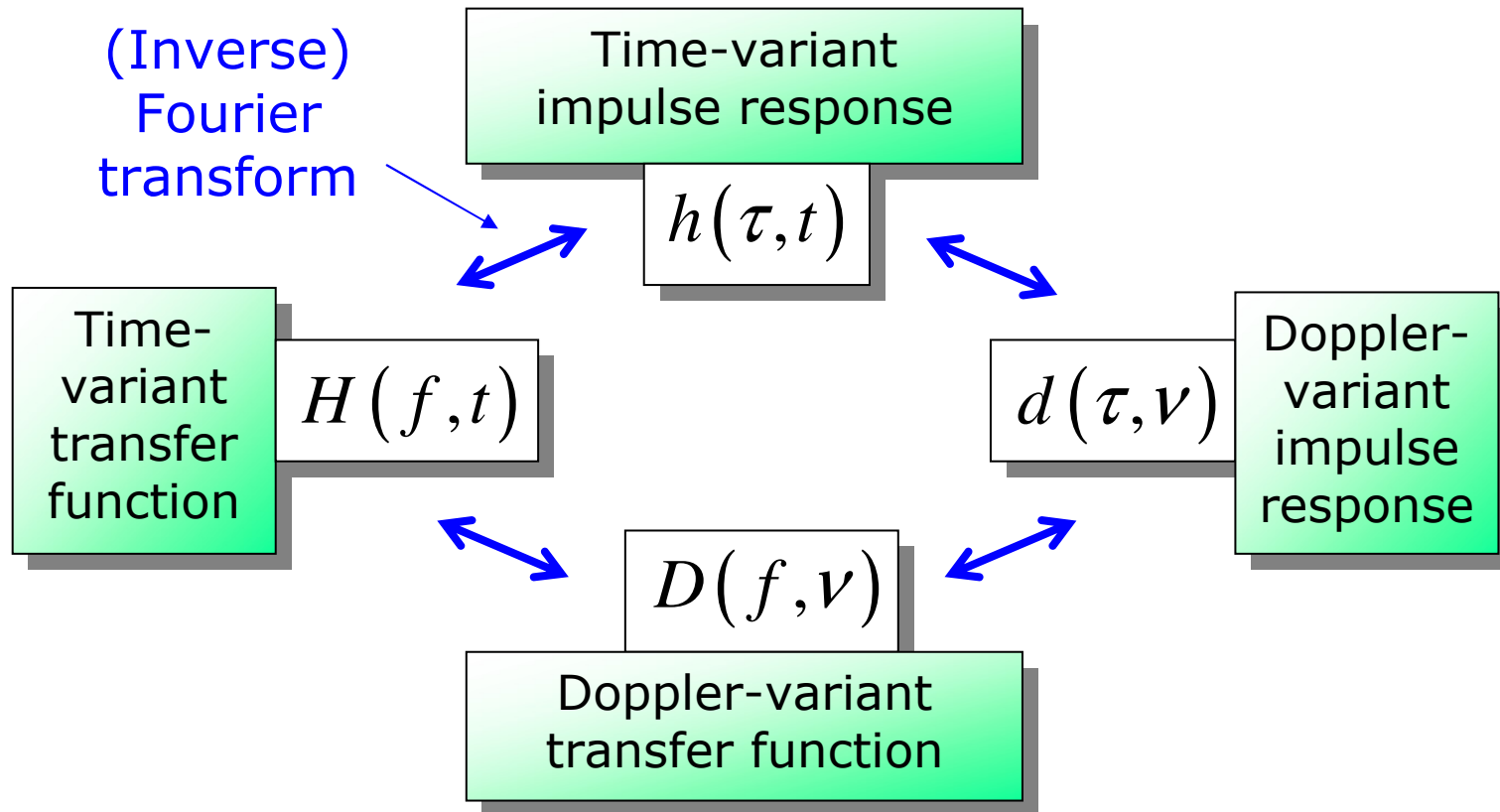
Time-variant transfer function (frequency response):

$$H(f, t) = \int_{-\infty}^{\infty} h(\tau, t) e^{-j2\pi f\tau} d\tau = \sum_{i=0}^{L-1} a_i(t) e^{j\phi_i(t)} e^{-j2\pi f\tau_i}$$

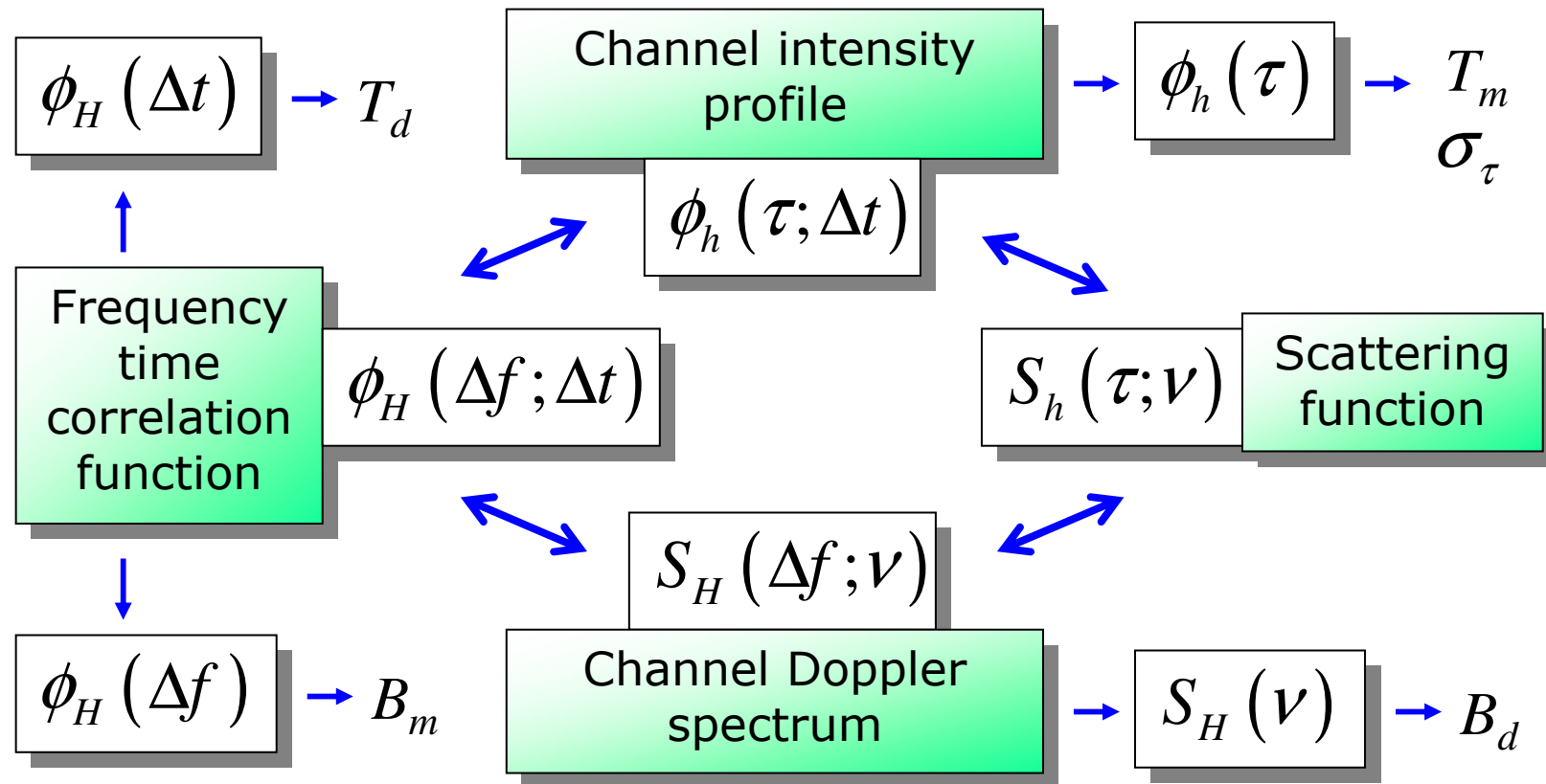
In a narrowband channel  
this reduces to:

$$H(f, t) = \sum_{i=0}^{L-1} a_i(t) e^{j\phi_i(t)}$$

# Deterministic channel functions



# Stochastical (WSSUS) channel functions

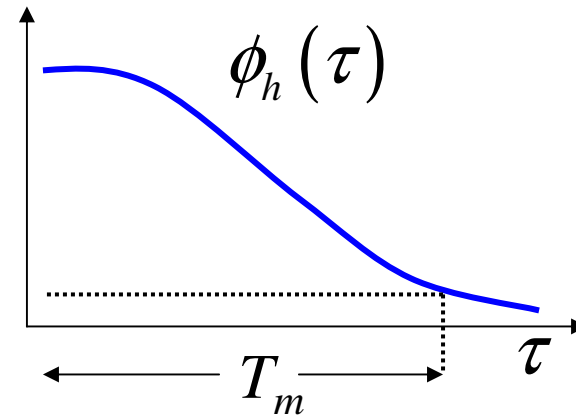


# Stochastic (WSSUS) channel variables

Maximum delay spread:  $T_m$

Maximum delay spread may be defined in several ways.

For this reason, the **RMS delay spread** is often used instead:



$$\sigma_\tau = \sqrt{\frac{\int \tau^2 \phi_h(\tau) d\tau}{\int \phi_h(\tau) d\tau} - \left[ \frac{\int \tau \phi_h(\tau) d\tau}{\int \phi_h(\tau) d\tau} \right]^2}$$

# Parameters of Mobile Multipath Channels

- Time Dispersion Parameters
  - Grossly quantifies the multipath channel
  - Determined from Power Delay Profile
  - Parameters include
    - Mean Access Delay
    - RMS Delay Spread
    - Excess Delay Spread (X dB)
- Coherence Bandwidth
- Doppler Spread and Coherence Time

# Measuring PDPs

- Power Delay Profiles
  - Are measured by channel sounding techniques
  - Plots of relative received power as a function of excess delay
  - They are found by averaging *instantaneous* power delay measurements over a local area
    - Local area: no greater than 6m outdoor
    - Local area: no greater than 2m indoor
      - » Samples taken at  $\lambda/4$  meters approximately
      - » For 450MHz – 6 GHz frequency range.



# Timer Dispersion Parameters

Determined from a power delay profile.

Mean excess delay ( $\bar{\tau}$ ):

$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)(\tau_k)}{\sum_k P(\tau_k)}$$

Rms delay spread ( $\sigma_\tau$ ):

$$\sigma_\tau = \sqrt{\bar{\tau}^2 - (\bar{\tau})^2}$$

$$\bar{\tau}^2 = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)(\tau_k^2)}{\sum_k P(\tau_k)}$$

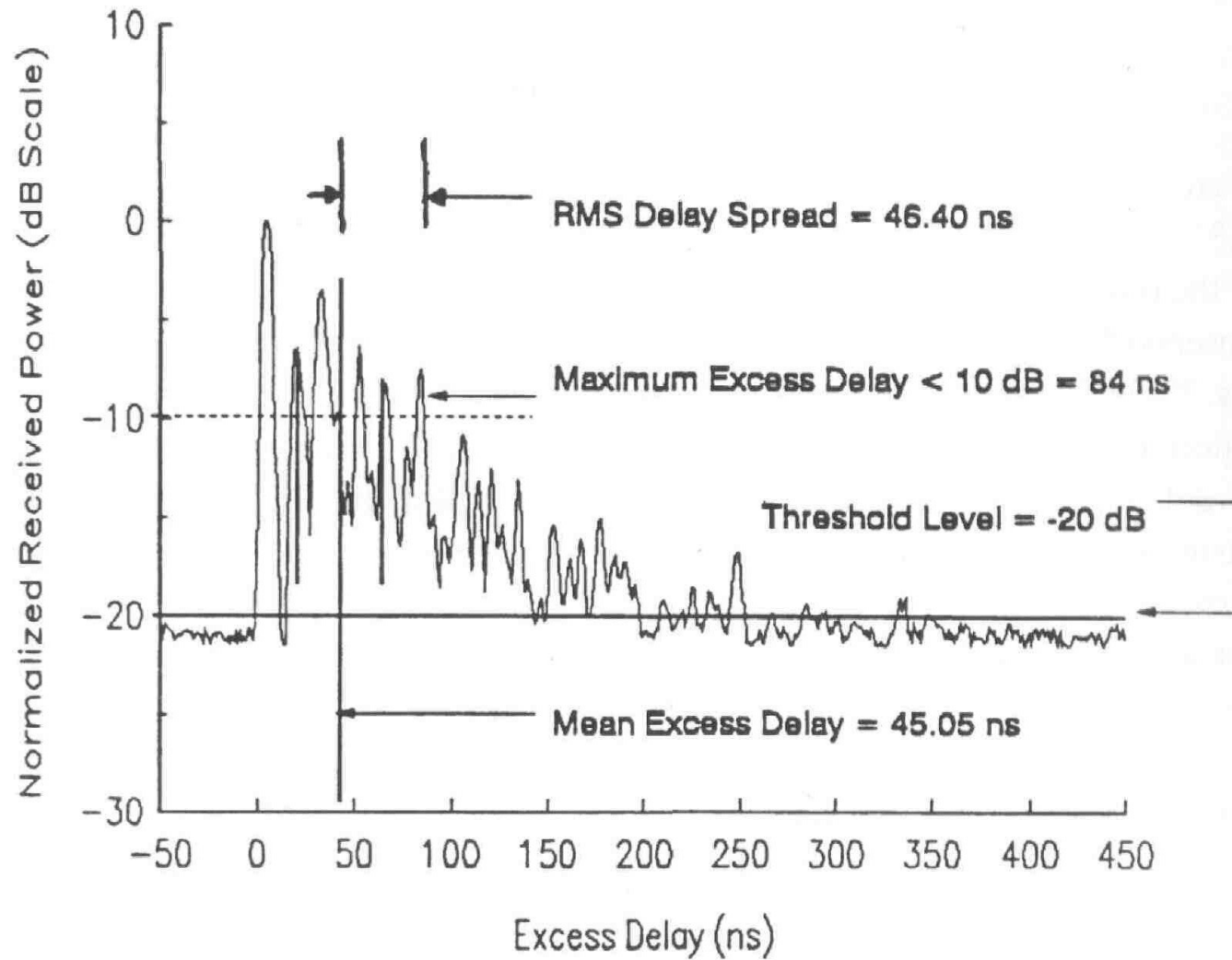
# Timer Dispersion Parameters

## **Maximum Excess Delay (X dB):**

Defined as the time delay value after which the multipath energy falls to X dB below the maximum multipath energy (not necessarily belonging to the first arriving component).

It is also called *excess delay spread*.

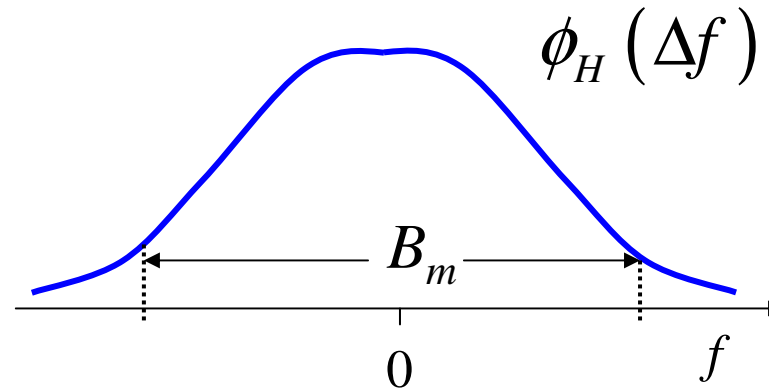
# RMS Delay Spread



# Stochastic (WSSUS) channel variables

Coherence bandwidth  
of channel:

$$B_m \approx 1/T_m$$



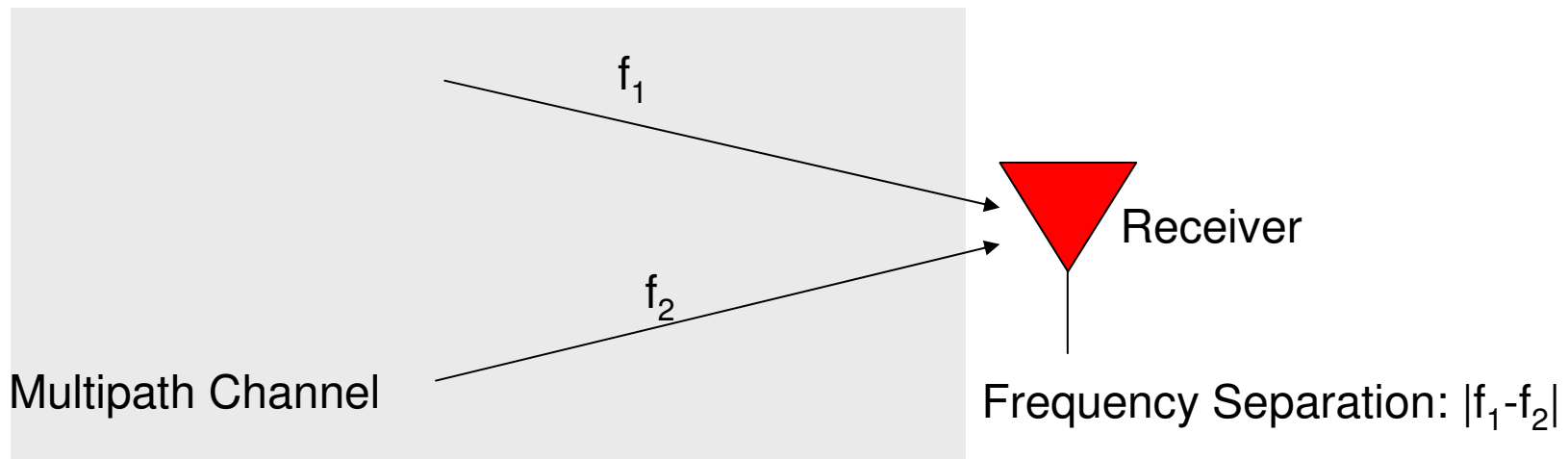
Implication of  
coherence bandwidth:

If two sinusoids (frequencies) are spaced much less apart than  $B_m$ , their fading performance is similar.

If the frequency separation is much larger than  $B_m$ , their fading performance is different.

# Coherence Bandwidth ( $B_c$ )

- Range of frequencies over which the channel can be considered flat (i.e. channel passes all spectral components with equal gain and linear phase).
  - It is a definition that depends on RMS Delay Spread.
- Two sinusoids with frequency separation greater than  $B_c$  are affected quite differently by the channel.



# Coherence Bandwidth

Frequency correlation between two sinusoids:  $0 \leq C_{r_1, r_2} \leq 1$ .

If we define Coherence Bandwidth ( $B_c$ ) as the range of frequencies over which the frequency correlation is above 0.9, then

$$B_c = \frac{1}{50\sigma}$$

$\sigma$  is rms delay spread.

If we define Coherence Bandwidth as the range of frequencies over which the frequency correlation is above 0.5, then

$$B_c = \frac{1}{5\sigma}$$

This is called 50% coherence bandwidth.

# Coherence Bandwidth

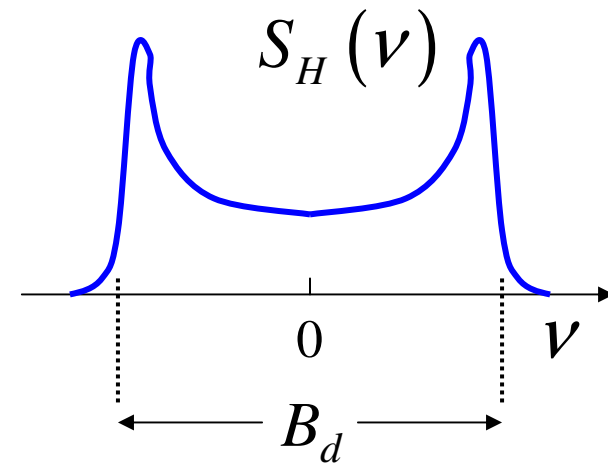
- Example:
  - For a multipath channel,  $\sigma$  is given as  $1.37\mu\text{s}$ .
  - The 50% coherence bandwidth is given as:  $1/5\sigma = \underline{146\text{kHz}}$ .
    - This means that, for a good transmission from a transmitter to a receiver, the range of transmission frequency (channel bandwidth) should not exceed 146kHz, so that all frequencies in this band experience the same channel characteristics.
    - Equalizers are needed in order to use transmission frequencies that are separated larger than this value.
    - This coherence bandwidth is enough for an AMPS channel (30kHz band needed for a channel), but is not enough for a GSM channel (200kHz needed per channel).

# Stochastical (WSSUS) channel variables

Maximum Doppler spread:  $B_d$

The Doppler spectrum is often U-shaped (like in the figure on the right). The reason for this behaviour is the relationship

$$\nu = \frac{V}{\lambda} \cos \alpha = f_d \cos \alpha$$





# Coherence Time

- **Delay spread** and **Coherence bandwidth** describe the time dispersive nature of the channel in a local area.
  - They don't offer information about the time varying nature of the channel caused by relative motion of transmitter and receiver.
- **Doppler Spread** and **Coherence time** are parameters which describe the time varying nature of the channel in a small-scale region.

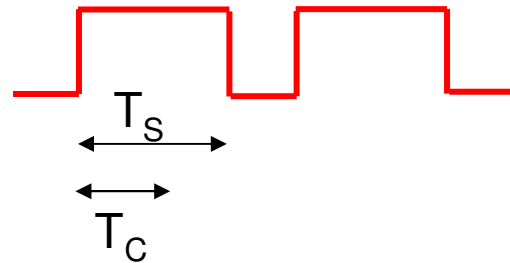
# Doppler Spread

- Measure of spectral broadening caused by motion
- We know how to compute Doppler shift:  $f_d$
- Doppler spread,  $B_D$ , is defined as the maximum Doppler shift:  $f_m = v/\lambda$
- If the baseband signal bandwidth is much greater than  $B_D$  then effect of Doppler spread is negligible at the receiver.

# Coherence Time

Coherence time is the time duration over which the channel impulse response is essentially invariant.

If the symbol period of the baseband signal (reciprocal of the baseband signal bandwidth) is greater than the coherence time, then the signal will distort, since the channel will change during the transmission of the signal.



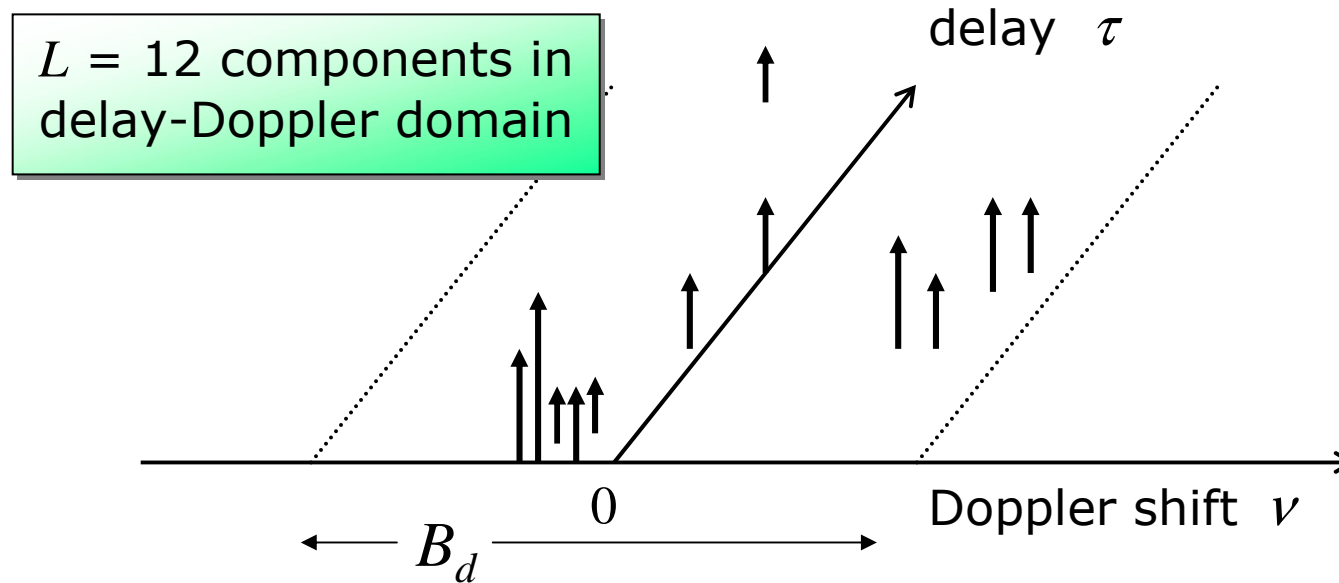
Coherence time ( $T_c$ ) is defined as:

$$T_c \approx \frac{1}{f_m}$$

Coherence time is also defined as:  $T_c \approx \sqrt{\frac{9}{16\pi f_m^2}} = \frac{0.423}{f_m}$

Coherence time definition implies that two signals arriving with a time separation greater than  $T_c$  are affected differently by the channel.

## Delay - Doppler spread of channel



$$h(\tau, t) = \sum_{i=0}^{L-1} a_i(t) e^{j(2\pi\nu_i t + \phi_i)} \delta(\tau - \tau_i)$$

## Fading distributions (Rayleigh)

In a flat fading channel, the (time-variant) CIR reduces to a (time-variant) complex channel coefficient:

$$c(t) = a(t) e^{j\phi(t)} = x(t) + j y(t) = \sum_i a_i(t) e^{j\phi_i(t)}$$

When the quadrature components of the channel coefficient are **independently and Gaussian distributed**, we get:

$$p(a) = \frac{a}{\sigma^2} e^{-a^2/2\sigma^2}$$

Rayleigh distribution

$$p(\phi) = \frac{1}{2\pi}$$

Uniform distribution

## Fading distributions (Rice)

In case there is a strong (e.g., LOS) multipath component in addition to the complex Gaussian component, we obtain:

$$c(t) = a_0 + a(t)e^{j\phi(t)} = a_0 + \sum_i a_i(t)e^{j\phi_i(t)}$$

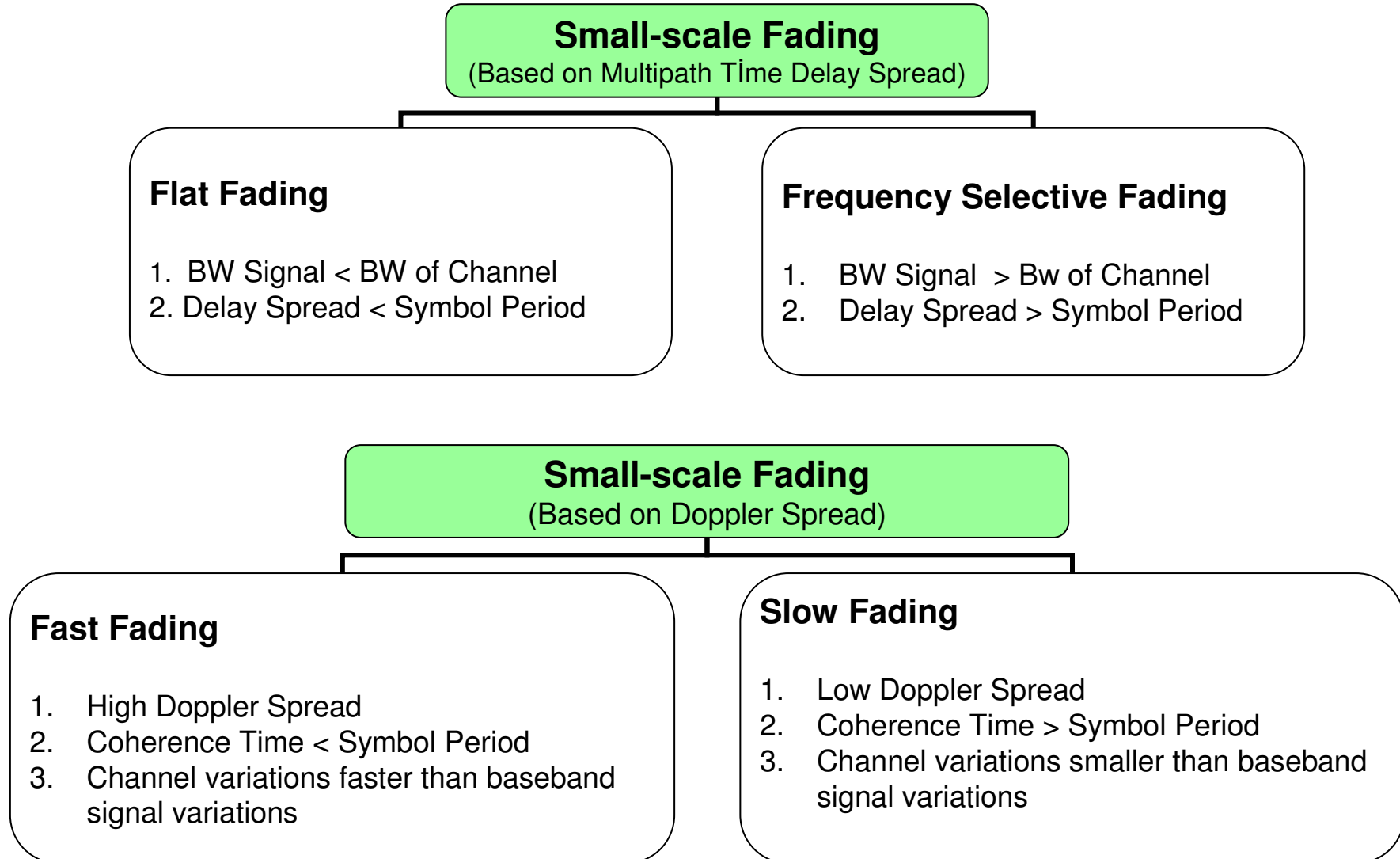
From the joint (magnitude and phase) pdf we can derive:

$$p(a) = \frac{a}{\sigma^2} e^{-(a^2 + a_0^2)/2\sigma^2} I_0\left(\frac{aa_0}{\sigma^2}\right)$$

Rice distribution

Modified Bessel function of first kind and order zero

# Types of Small-scale Fading

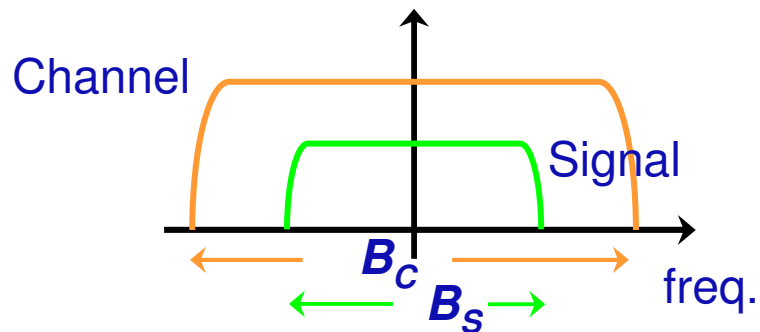


# Channel Classification

## Based on Time-Spreading

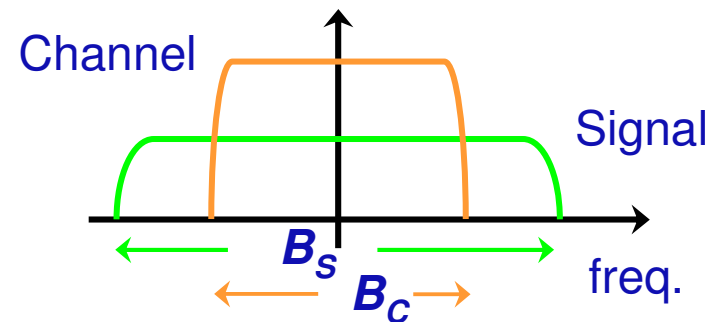
### Flat Fading

1.  $B_S < B_C \Leftrightarrow T_m < T_s$
2. Rayleigh, Ricean distrib.
3. Spectral chara. of transmitted signal preserved



### Frequency Selective

1.  $B_S > B_C \Leftrightarrow T_m > T_s$
2. Intersymbol Interference
3. Spectral chara. of transmitted signal not preserved
4. Multipath components resolved





# Fading in Digital Mobile Communications

- If  $B_s \gg B_c$ , then a notch appears in the spectrum. Thus resulting in **inter-symbol interference (ISI)**.
  - To overcome this, an **adaptive equaliser (AE)** with inverse response may be used at the receiver. Training sequences are transmitted to update AE.
- If  $B_s \ll B_c$ , then flat fading occurs, resulting in a **burst of error**.
  - **Error correction coding** is used to overcome this problem.

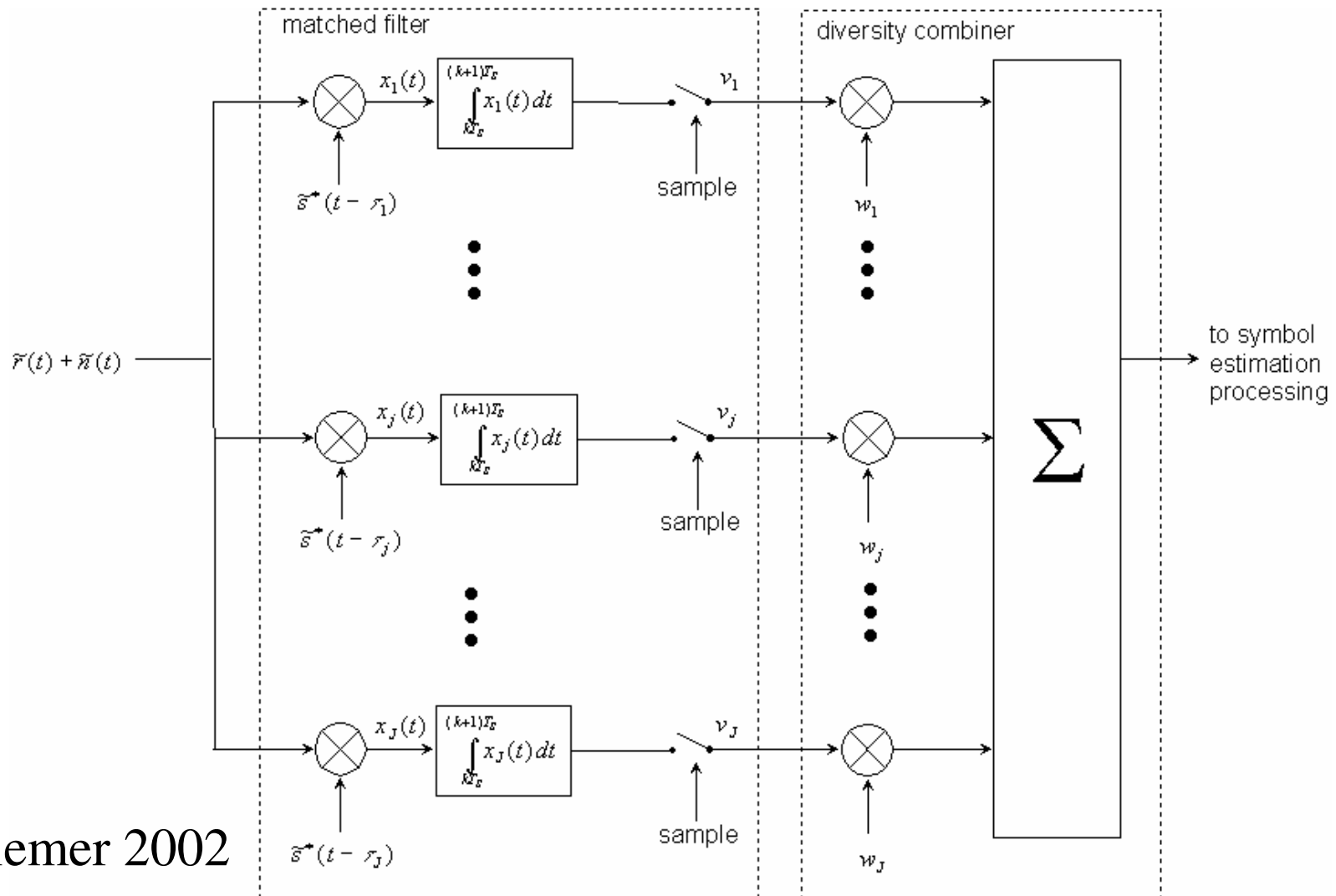
# Mitigation Techniques for the Multipath Fading Channel

- Space diversity –
  - Signals at the same frequency using two or three antennas located several wavelengths apart.
  - Antennas are connected to two or three radio receivers.
  - The receiver with the strongest signal is elected
  - **Disadvantage: Uses two or more antennas, therefore the need for a large site.**
- Frequency diversity –
  - Signals at different frequencies received by the same antenna very rarely fade simultaneously. Thus the use of several carrier frequencies or the use of a wideband signal to combat fading.
  - A single aerial connected to a number receiver, each tuned to a different frequency, whose outputs are connected in parallel. The receiver with the strongest instantaneous signal will provide the output.
  - **Disadvantage: Uses two or more frequencies to transmit the same signal.**

# Mitigation Techniques for the Multipath Fading Channel

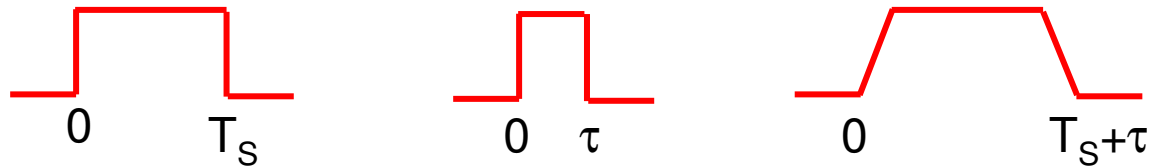
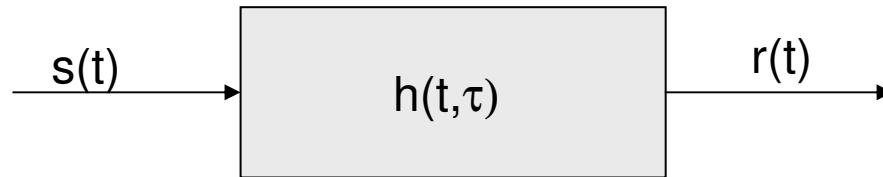
- Time diversity – Spread out the effects of errors through interleaving and coding
- Multipath diversity
  - Consider the tapped delay line model of a channel shown previously
  - If multipaths can be put together coherently at the receiver, diversity improvement results
  - This is what the RAKE receiver does (see next viewgraph)

# RAKE Multipath Signal Processing



R.E. Ziemer 2002

# Flat Fading



$$\tau \ll T_S$$

Occurs when:

$$B_S \ll B_C$$

and

$$T_S \gg \sigma_\tau$$

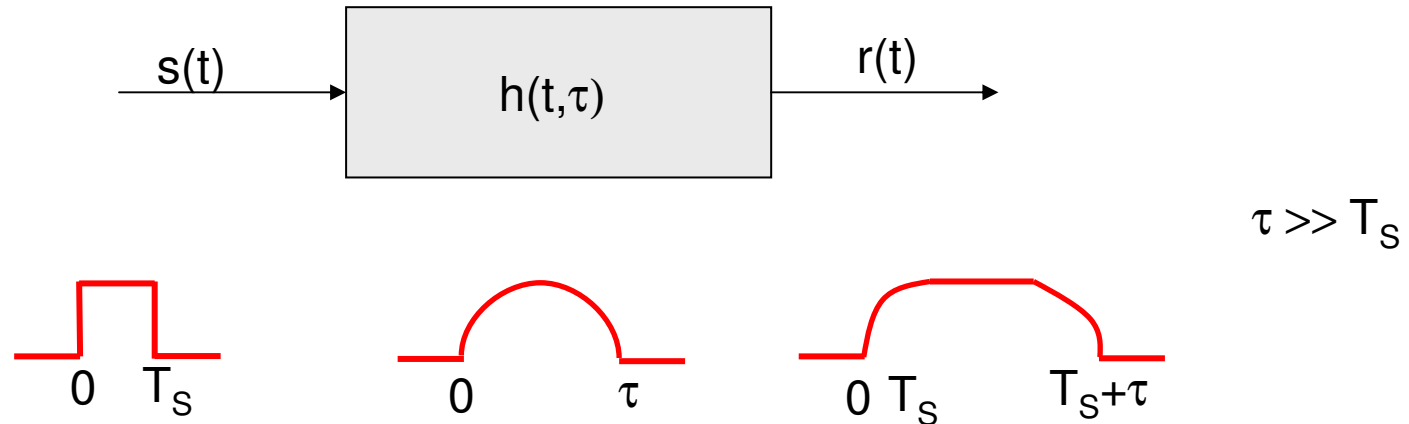
$B_C$ : Coherence bandwidth

$B_S$ : Signal bandwidth

$T_S$ : Symbol period

$\sigma_\tau$ : Delay Spread

# Frequency Selective Fading



Causes distortion of the received baseband signal

Causes Inter-Symbol Interference (ISI)

Occurs when:

$$B_S > B_C$$

and

$$T_S < \sigma_\tau$$

As a rule of thumb:  $T_S < \sigma_\tau$

# Fast Fading

- Due to Doppler Spread
  - Rate of change of the channel characteristics is **larger** than the Rate of change of the transmitted signal
  - The channel changes during a symbol period.
  - The channel changes because of receiver motion.
  - Coherence time of the channel is smaller than the symbol period of the transmitter signal

Occurs when:

$$B_S < B_D$$

and

$$T_S > T_C$$

$B_S$ : Bandwidth of the signal

$B_D$ : Doppler Spread

$T_S$ : Symbol Period

$T_C$ : Coherence Bandwidth

# Slow Fading

- Due to Doppler Spread
  - Rate of change of the channel characteristics is **much smaller** than the Rate of change of the transmitted signal

Occurs when:

$$B_S \gg B_D$$

and

$$T_S \ll T_C$$

$B_S$ : Bandwidth of the signal

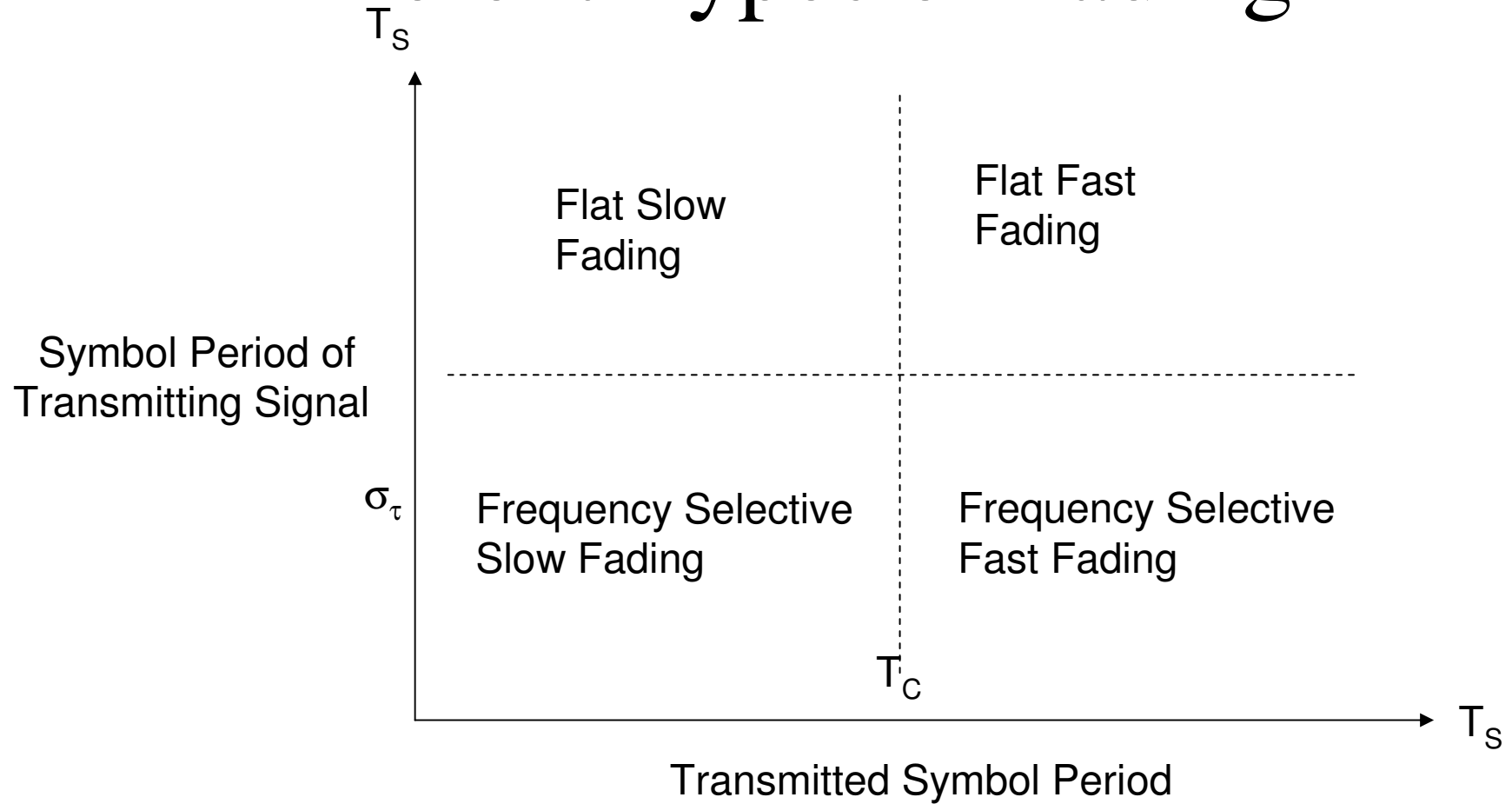
$B_D$ : Doppler Spread

$T_S$ : Symbol Period

$T_C$ : Coherence Bandwidth

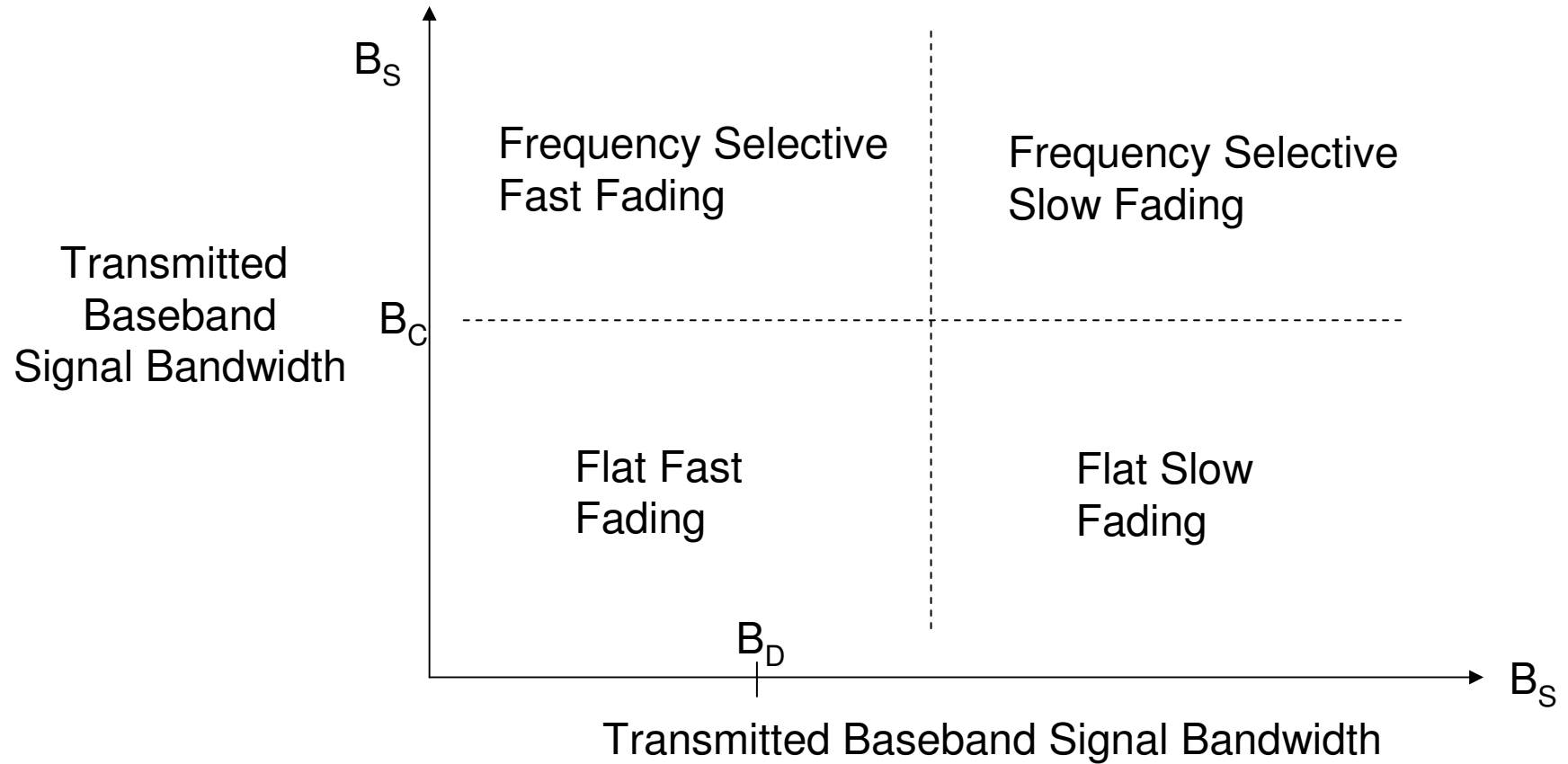


# Different Types of Fading



With Respect To SYMBOL PERIOD

# Different Types of Fading



With Respect To BASEBAND SIGNAL BANDWIDTH

# Average Fade Duration

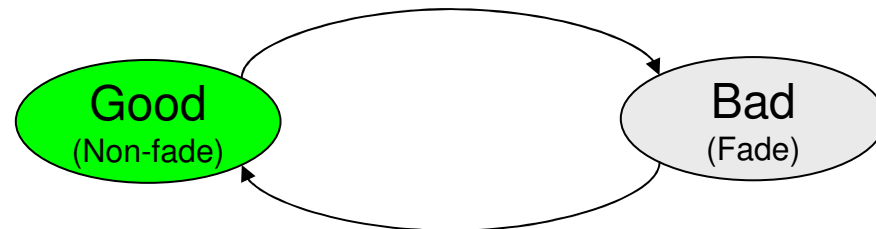
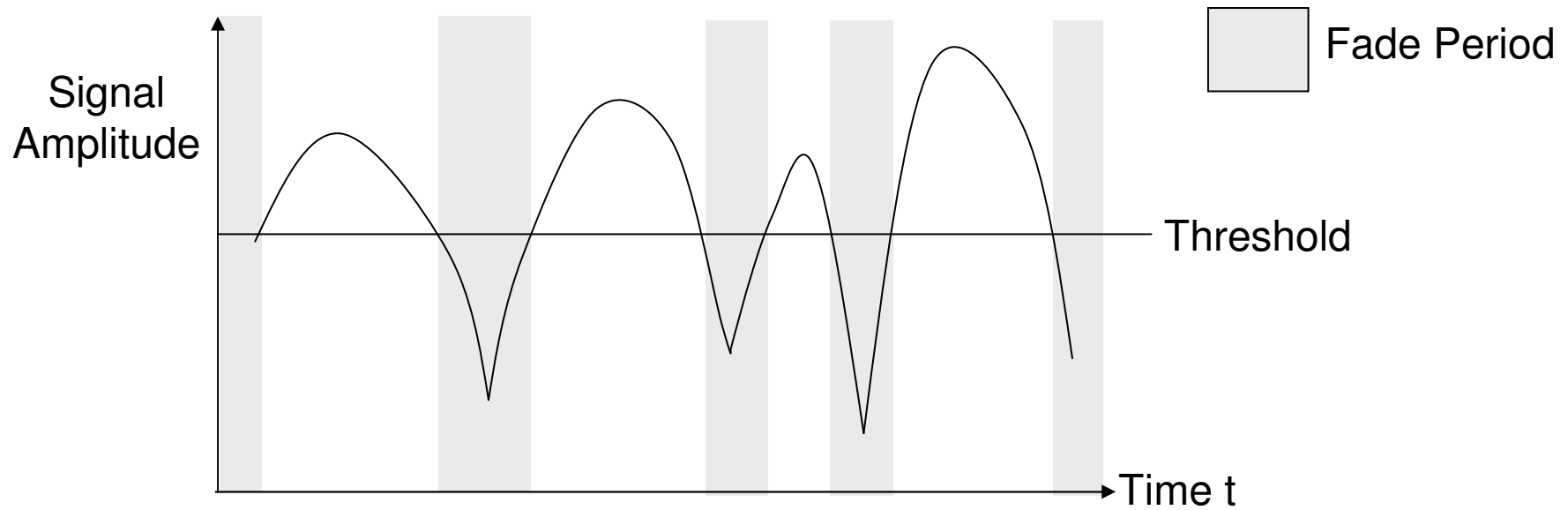
Defined as the average period of time for which the received signal is below a specified level  $R$ .

For Rayleigh distributed fading signal, it is given by:

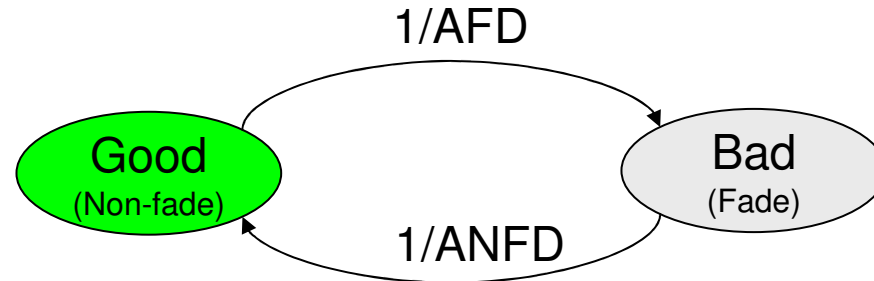
$$\bar{\tau} = \frac{1}{N_R} \Pr[r \leq R] = \frac{1}{N_R} (1 - e^{-\rho^2})$$

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}, \quad \rho = \frac{R}{r_{rms}}$$

# Fading Model – Gilbert-Elliot Model



# Gilbert-Elliot Model



The channel is modeled as a Two-State Markov Chain.  
Each state duration is memory-less and exponentially distributed.

The rate going from Good to Bad state is:  $1/AFD$  (AFD: Avg Fade Duration)  
The rate going from Bad to Good state is:  $1/ANFD$  (ANFD: Avg Non-Fade Duration)