



COLLEGE OF ENGINEERING & TECHNOLOGY

Department : Graduate Studies

Electronics & Communications Engineering

Lecturer : Prof. Mohamed Essam Khedr

Course : Wireless Communications

Course Code : EC 744

Midterm Exam (Take home). Only lecture slides and textbooks, No Internet is allowed
Time : 3-5 hours, Please be punctual

Question one: (probability, random variables and random process)

I- Consider the following experiment involving four urns {A, B, C, D}. A ball is chosen from urn A, which contains six balls labelled B, three balls labelled C and three balls labelled D. The chosen ball specifies the urn from which the second drawing is made. Urn B contains five red balls and five white balls. Urn C contains four red and six white balls and urn D contains two red and eight white balls.

a- Construct the sample space and probability assignment that describes the experiment.

b- Given that the second ball drawn is red, what is the conditional probability that the first drawing yielded B?

II- Let X, Y be two random variables with finite second moments and define $Z=X+Y$

$$\text{Prove } \sigma_x^2 + \sigma_y^2 - 2\sigma_x\sigma_y \leq \sigma_z^2 \leq \sigma_x^2 + \sigma_y^2 + 2\sigma_x\sigma_y$$

III- Let X(t) and Y(t) be independent Gaussian random process with zero means and same covariance function $C(t_1, t_2)$, define $Z(t) = X(t)\cos(\omega t) + Y(t)\sin(\omega t)$

Find the mean and autocovariance of Z(t)

Question Two: (Cellular systems)

A cellular system with a frequency reuse of 28 is considered.

a- What are the values of i, j, D, and D/R

b- Sketch the system layout on the attached figure

c- Calculate the C/I ratio for path loss equal to 3

d- If the total coverage area is 300 km², 2800 duplex channels, Cell radius = 1 km, what is the capacity of the system.

Question three: (Wireless channels)

I- The scattering function $S(\tau, \lambda)$ for a fading multipath channel is non zero for the range of values $0 \leq \tau \leq 1\text{ms}$ and $-0.1\text{Hz} \leq \lambda \leq 0.1\text{Hz}$. Assume that the scattering function is approximately uniform in the two variables

a- find the value of the multipath spread of the channel

b- the Doppler spread of the channel

c- the coherence time of the channel

d- the coherence bandwidth of the channel

e- when the channel will be frequency selective

f- when the channel will be flat fading

g- when the channel will be slowly fading

II- Suppose that a binary signal $\pm s_i(t)$ is transmitted over a fading channel and that the received signal is

$$r(t) = \pm a s_i(t) + n(t) \quad 0 \leq t \leq T, \text{ where } n(t) \text{ is zero mean white Gaussian noise with PSD} = N_0.$$

The channel gain "a" is specified by the probability density function

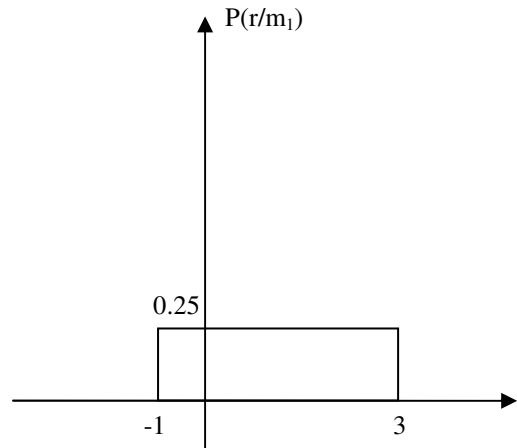
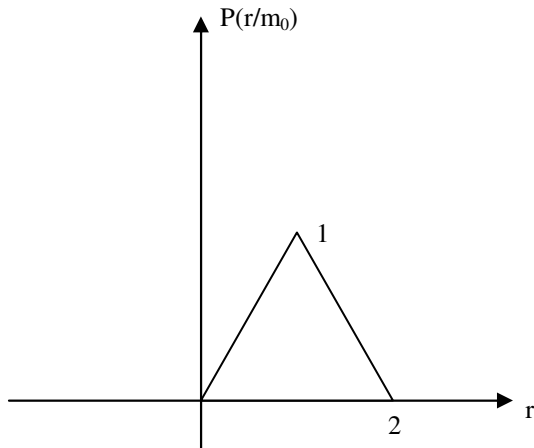
$$p(a) = 0.1\delta(a) + 0.9\delta(a - 2)$$

a- Determine the average probability of error for a demodulated matched to $s_i(t)$

b- What value does P[E] approaches as E_b/N_0 approaches infinity

Question Four: (Modulation/Demodulation)

- I- A communication system is used to transmit one of the two equally likely messages, m_0, m_1 . the channel output is a continuous random variable "r". The conditional density function of which is shown in the following figure. Determine the optimum receiver decision rule and compute the resulting probability of error.



- II- Consider the octal signal point constellations in the following figure
- The nearest neighbour signal points in the signal constellations are separated in distance by A unites, Determine the radii r of the first circle and a and b of the inner and outer circles.
 - Determine the average transmitted powers of the two signal constellations and compare the two powers.
 - Sketch the decision boundaries for both constellations
 - Find the average probability of error of both constellations.

