EC 721 Advanced Digital Communications
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Shannon theorem

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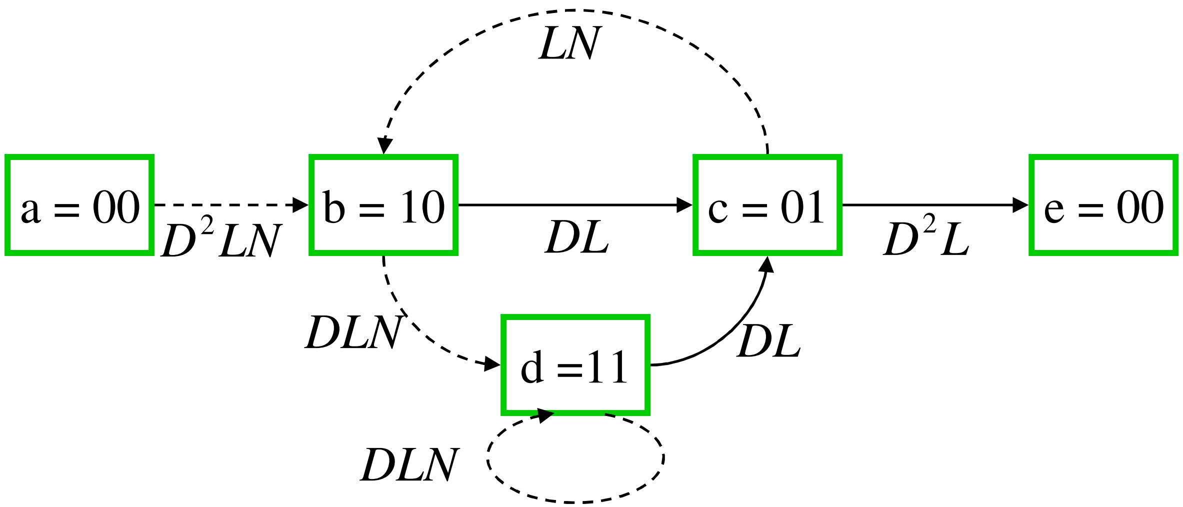
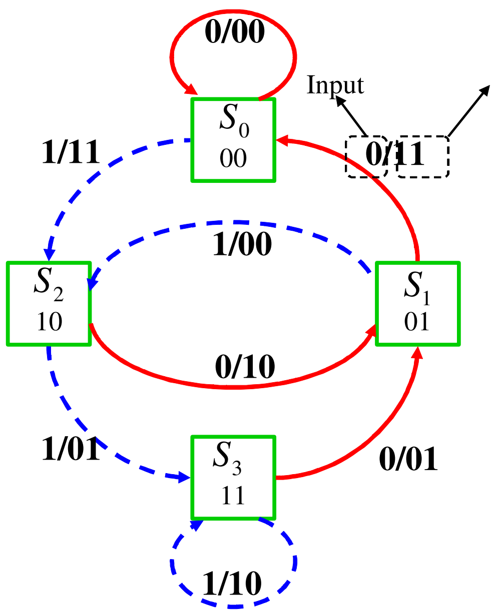
Transfer function …

- Example of transfer function for the rate ½ Convolutonal code.
  1. Redraw the state diagram such that the zero state is split into two nodes, the starting and ending nodes.
  2. Label each branch by the corresponding $D^i L^j N^l$





Transfer function …

3. Write the state equations ($X_a, \ldots, X_e$ dummy variables)

$$\begin{cases} X_b = D^2 LNX_a + LNX_c \\ X_c = DLX_b + DLX_d \\ X_d = DLNX_b + DLNX_d \\ X_e = D^2 LX_c \end{cases}$$

4. Solve $T(D, L, N) = X_e / X_a$

$$T(D,L,N) = \frac{D^5 L^3 N}{1 - DL(1+L)N} = D^5 L^3 N + D^6 L^4 N^2 + D^6 L^5 N^2 + \ldots.$$

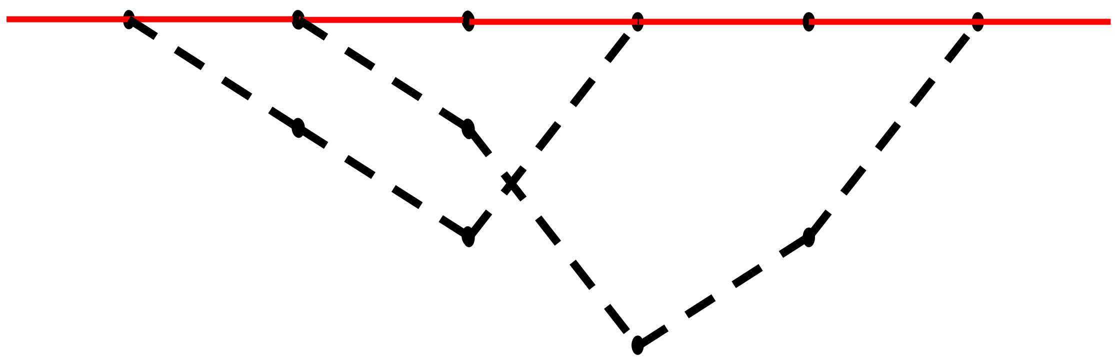
One path with weight 5, length 3 and data weight of 1

One path with weight 6, length 4 and data weight of 2

One path with weight 5, length 5 and data weight of 2

Performance bounds …

- Analysis is based on:
  - Assuming the all-zero codeword is transmitted
  - Evaluating the probability of an "**error event**" (usually using bounds such as union bound).
    - An "error event" occurs at a time instant in the trellis if a non-zero path leaves the all-zero path and remerges to it at a later time.



Performance bounds …

- Bounds on bit error probability for memoryless channels:
  - Hard-decision decoding:

$$P_B \leq \left. \frac{dT(D,L,N)}{dN} \right|_{N=1,L=1,D=2\sqrt{p(1-p)}}$$

Performance bounds …

- Error correction capability of Convolutional codes, given by $t = \lfloor (d_f - 1)/2 \rfloor$, depends on
  - If the decoding is performed long enough (within 3 to 5 times of the constraint length)
  - How the errors are distributed (bursty or random)
- For a given code rate, increasing the constraint length, usually increases the free distance.
- For a given constraint length, decreasing the coding rate, usually increases the free distance.
- The coding gain is upper bounded

$$\text{coding gain} \leq 10 \log_{10}(R_c d_f)$$

Performance bounds …

- Basic coding gain (dB) for soft-decision Viterbi decoding

Uncoded E_b/N_0 (dB)	Code rate		1/3		1/2	
	P_B	K	7	8	6	7
6.8	10^{-3}		4.2	4.4	3.5	3.8
9.6	10^{-5}		5.7	5.9	4.6	5.1
11.3	10^{-7}		6.2	6.5	5.3	5.8
Upper bound			7.0	7.3	6.0	7.0

Interleaving

- Convolutional codes are suitable for memoryless channels with random error events.

- Some errors have bursty nature:
  - Statistical dependence among successive error events (time-correlation) due to the channel memory.
    - Like errors in multipath fading channels in wireless communications, errors due to the switching noise, …

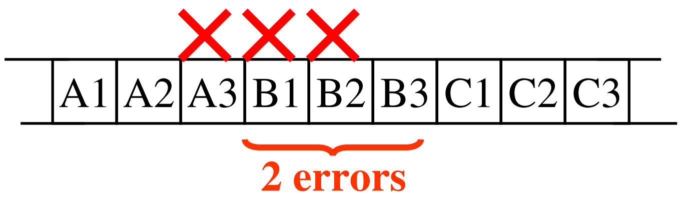
- “Interleaving” makes the channel looks like as a memoryless channel at the decoder.

Interleaving …

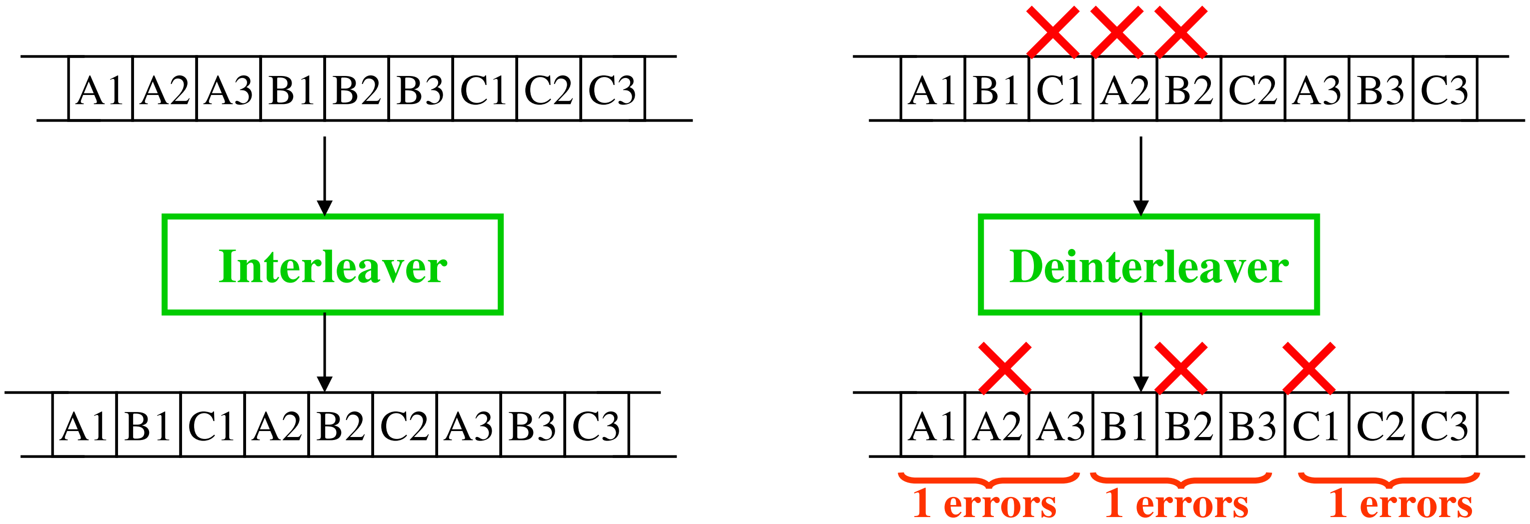
- Interleaving is done by spreading the coded symbols in time (interleaving) before transmission.
- The reverse in done at the receiver by deinterleaving the received sequence.
- "Interleaving" makes bursty errors look like random. Hence, Conv. codes can be used.
- Types of interleaving:
  - Block interleaving
  - Convolutional or cross interleaving

Interleaving …

- Consider a code with t=1 and 3 coded bits.
- A burst error of length 3 can not be corrected.

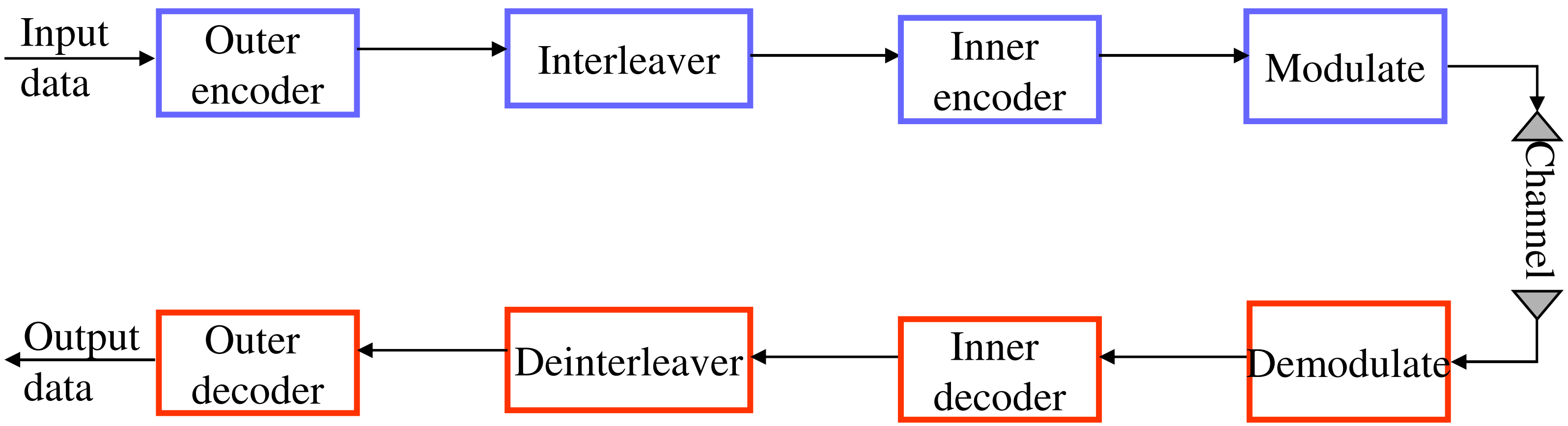


- Let us use a block interleaver 3X3



Concatenated codes

- A concatenated code uses two levels on coding, an inner code and an outer code (higher rate).
  - Popular concatenated codes: Convolutional codes with Viterbi decoding as the inner code and Reed-Solomon codes as the outer code
- The purpose is to reduce the overall complexity, yet achieving the required error performance.



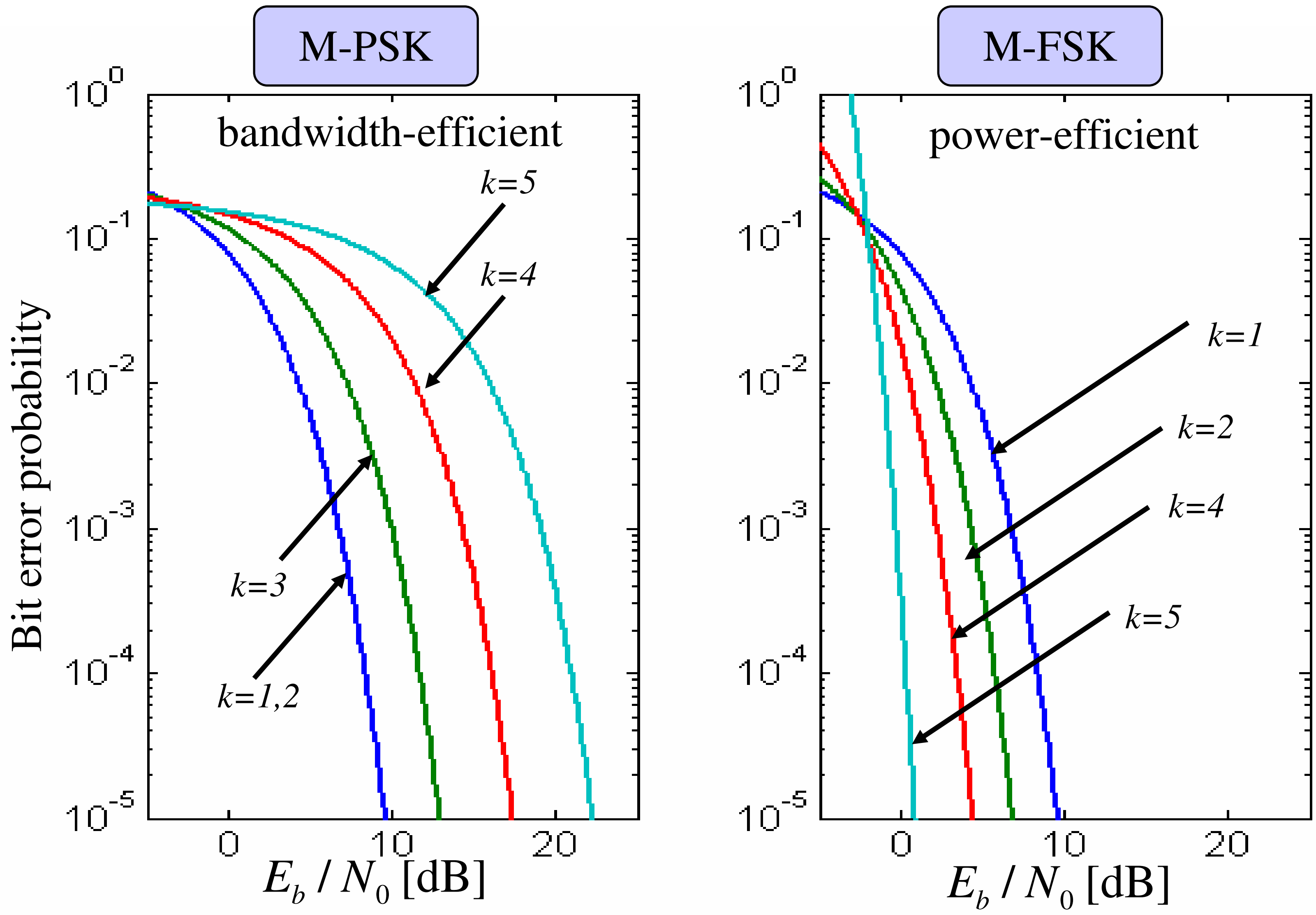
Today, we are going to talk about:

- Shannon limit
- Comparison of different modulation schemes
- Trade-off between modulation and coding

Goals in designing a DCS

- Goals:
  - Maximizing the transmission bit rate
  - Minimizing probability of bit error
  - Minimizing the required power
  - Minimizing required system bandwidth
  - Maximizing system utilization
  - Minimize system complexity

Error probability plane (example for coherent MPSK and MFSK)

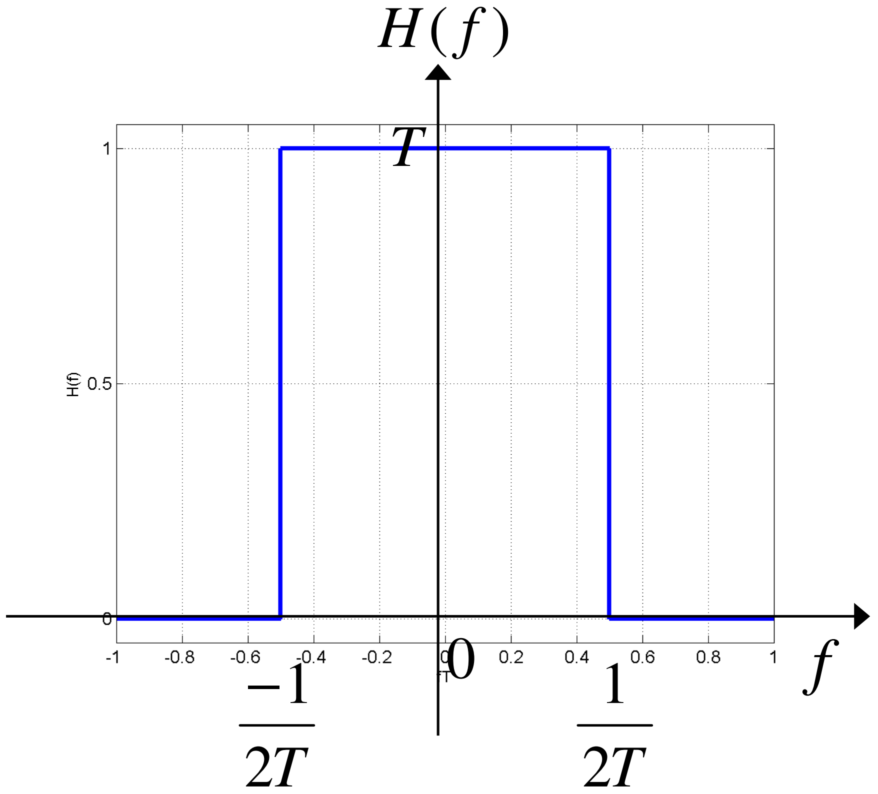


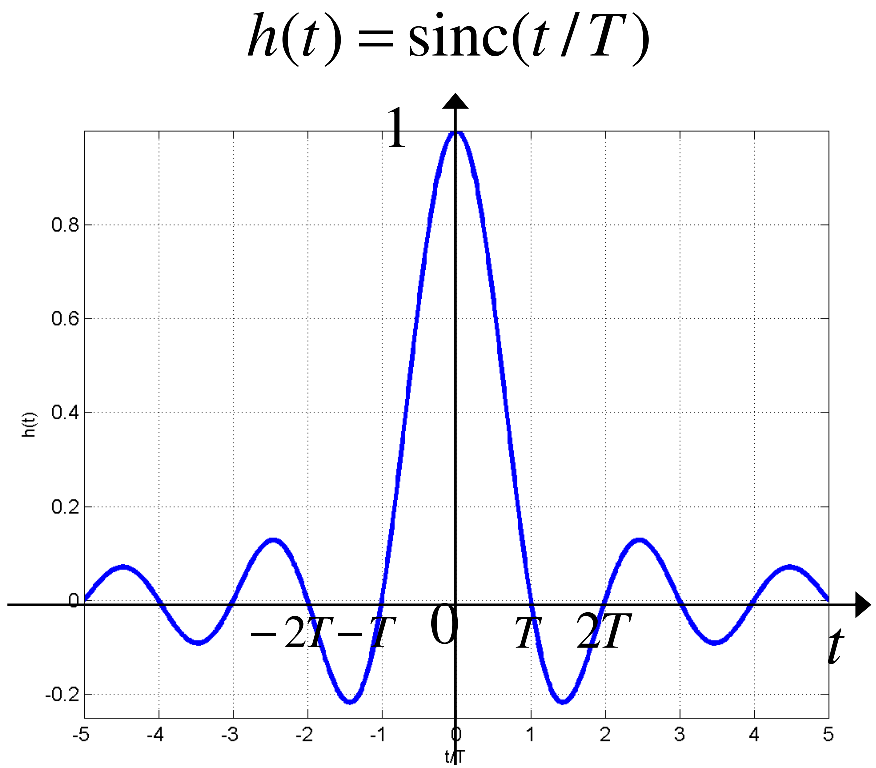
Limitations in designing a DCS

- Limitations:
  - The Nyquist theoretical minimum bandwidth requirement
  - The Shannon-Hartley capacity theorem (and the Shannon limit)
  - Government regulations
  - Technological limitations
  - Other system requirements (e.g satellite orbits)

Nyquist minimum bandwidth requirement

- The theoretical minimum bandwidth needed for baseband transmission of Rs symbols per second is $Rs/2$ hertz.





Shannon limit

- Channel capacity: The maximum data rate at which the error-free communication over the channel is performed.
- Channel capacity on AWGV channel (Shannon-Hartley capacity theorem):

$$C = W \log_2\left(1 + \frac{S}{N}\right) \quad \text{[bits/s]}$$

W [Hz]: Bandwidth

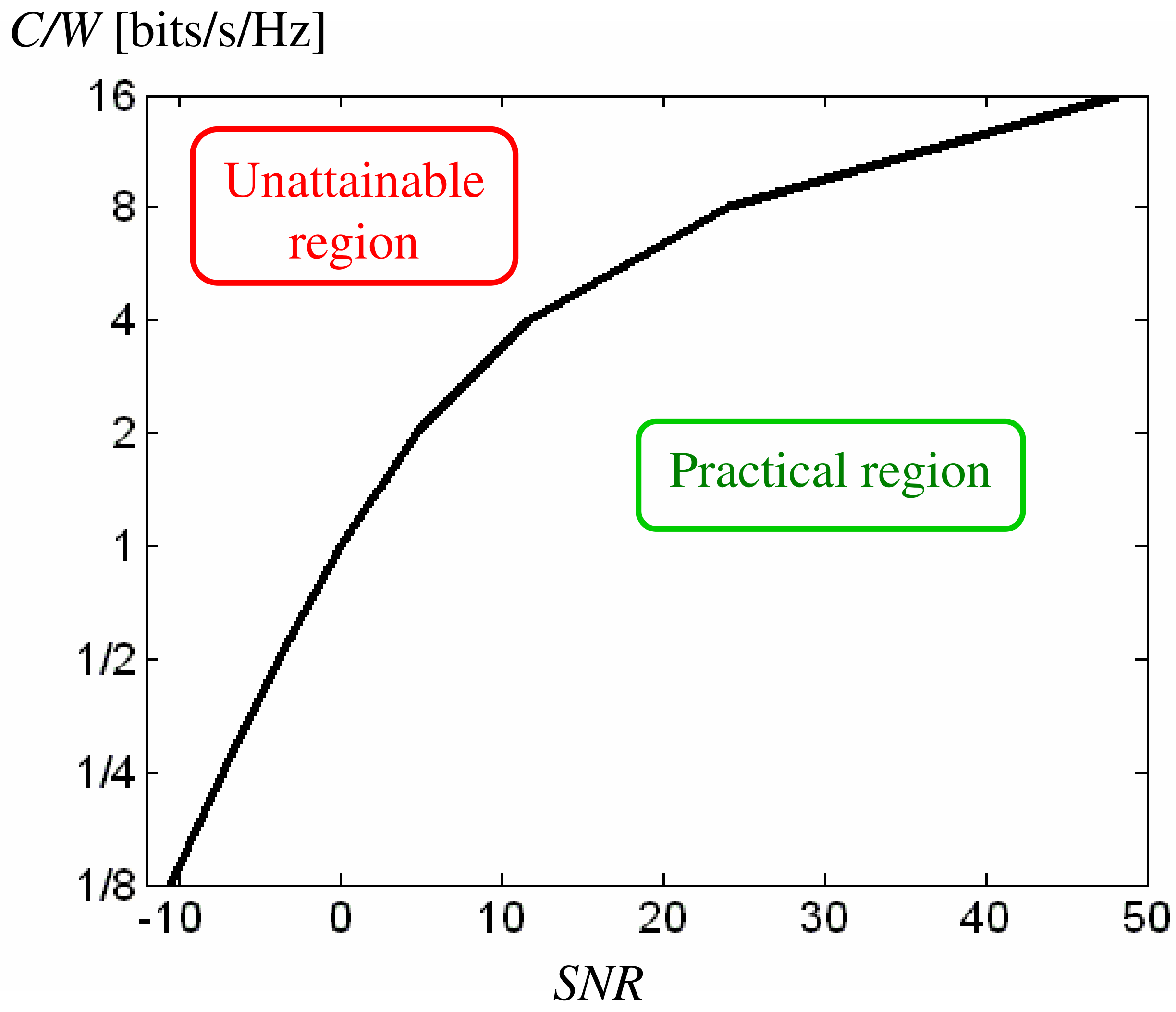
$S = E_b C$ [Watt]: Average received signal power

$N = N_0 W$ [Watt]: Average noise power

Shannon limit …

- Shannon theorem puts a limit on transmission data rate, not on error probability:
  - Theoretically possible to transmit R_b information at any rate $R_b \leq C$, where with an arbitrary small error probability by using a sufficiently complicated coding scheme
  - For an information rate $R_b > C$, it is not possible to find a code that can achieve an arbitrary small error probability.

Shannon limit …



Shannon limit …

$$C = W \log_2\left(1+\frac{S}{N}\right)$$

$$\begin{cases} S = E_b C \\ N = N_0 W \end{cases}$$

$$\Rightarrow \quad \boxed{\frac{C}{W} = \log_2\left(1+\frac{E_b}{N_0}\frac{C}{W}\right)}$$

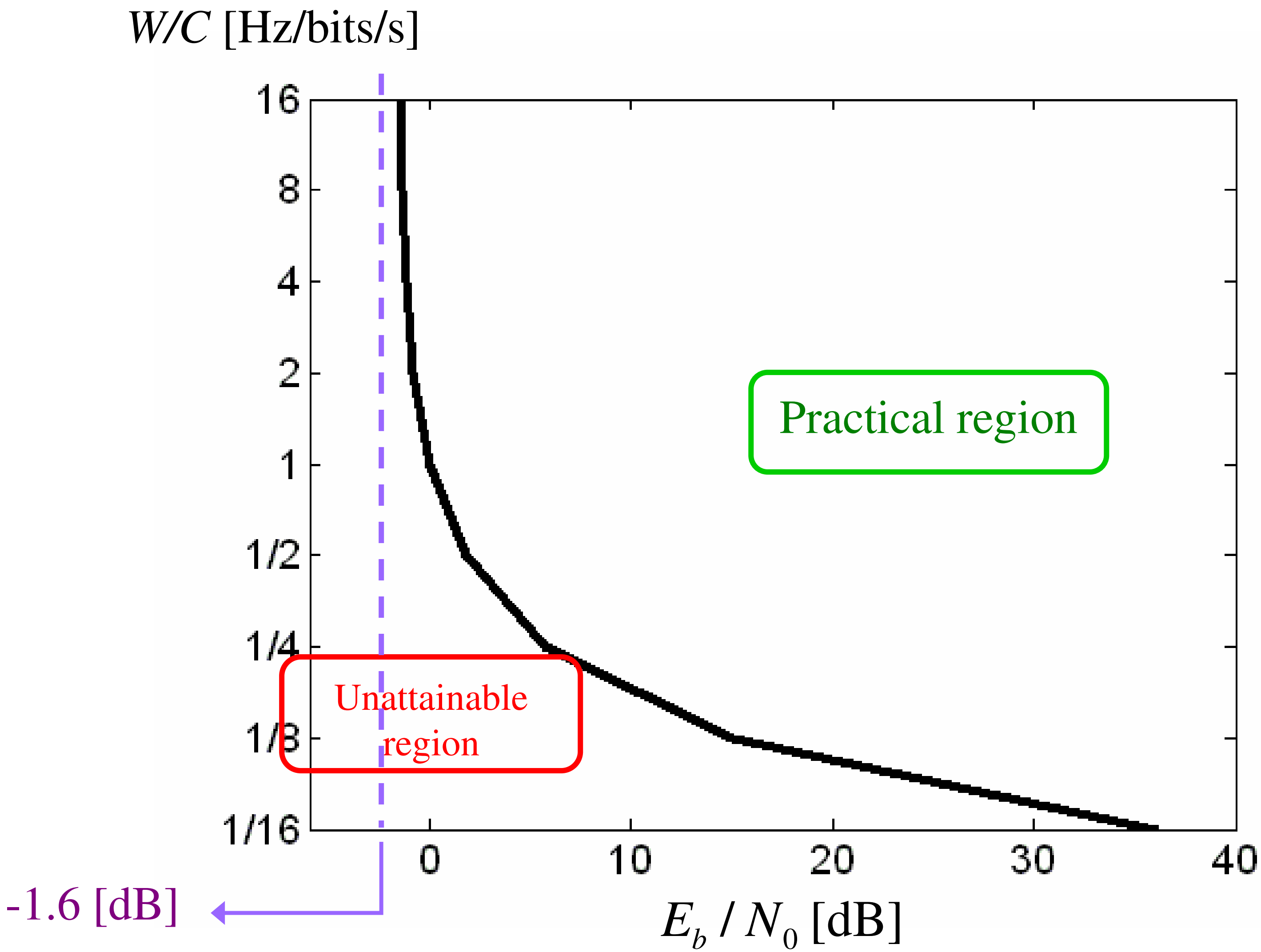
$$\text{As } W \to \infty \text{ or } \frac{C}{W} \to 0, \text{ we get :}$$

$$\frac{E_b}{N_0} \to \frac{1}{\log_2 e} = 0.693 \approx -1.6 \ [\text{dB}]$$

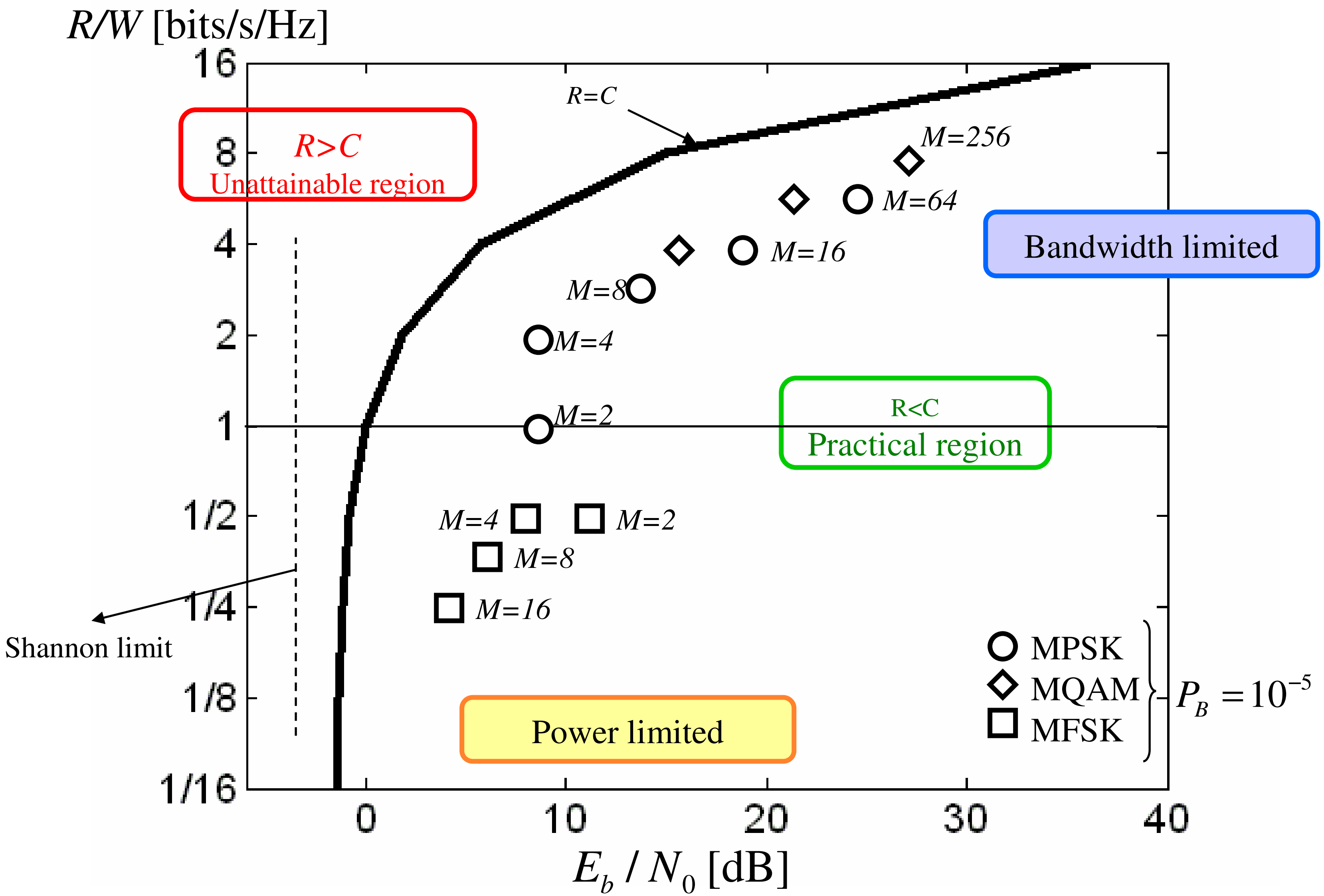
Shannon limit

- There exists a limiting value of E_b / N_0 below which there can be no error-free communication at any information rate.
- By increasing the bandwidth alone, the capacity can not be increased to any desired value.

Shannon limit …



Bandwidth efficiency plane



Power and bandwidth limited systems

- Two major communication resources:
  - Transmit power and channel bandwidth
- In many communication systems, one of these resources is more precious than the other. Hence, systems can be classified as:
  - **Power-limited systems:**
    - save power at the expense of bandwidth (for example by using coding schemes)
  - **Bandwidth-limited systems:**
    - save bandwidth at the expense of power (for example by using spectrally efficient modulation schemes)

M-ary signaling

- Bandwidth efficiency:

$$\frac{R_b}{W} = \frac{\log_2 M}{WT_s} = \frac{1}{WT_b} \quad \text{[bits/s/Hz]}$$

  - Assuming Nyquist (ideal rectangular) filtering at baseband, the required passband bandwidth is:

$$W = 1/T_s = R_s \quad \text{[Hz]}$$

- M-PSK and M-QAM (bandwidth-limited systems)

$$R_b / W = \log_2 M \quad \text{[bits/s/Hz]}$$

  - Bandwidth efficiency increases as M increases.

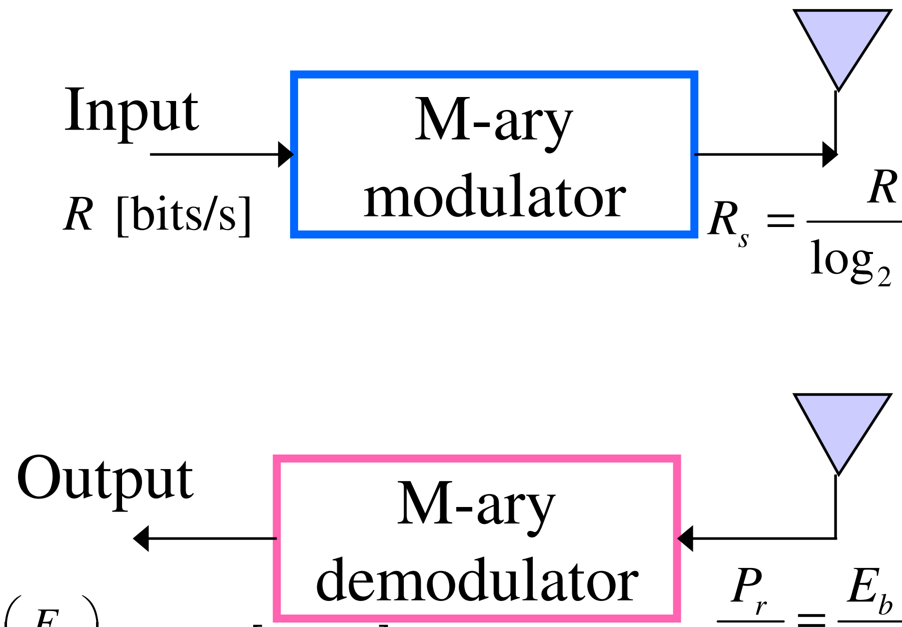
- MFSK (power-limited systems)

$$R_b / W = \log_2 M / M \quad \text{[bits/s/Hz]}$$

  - Bandwidth efficiency decreases as M increases.

Design example of uncoded systems

- Design goals:
  1. The bit error probability at the modulator output must meet the system error requirement.
  2. The transmission bandwidth must not exceed the available channel bandwidth.



$$R_s = \frac{R}{\log_2 M} \quad \text{[symbols/s]}$$

$$P_E(M) = f\left(\frac{E_s}{N_0}\right), \quad P_B = g[P_E(M)]$$

$$\frac{P_r}{N_0} = \frac{E_b}{N_0} R = \frac{E_s}{N_0} R_s$$

Design example of uncoded systems …

- Choose a modulation scheme that meets the following system requirements:

An AWGN channel with $W_C = 4000$ [Hz]

$$\frac{P_r}{N_0} = 53 \ \text{[dB.Hz]} \qquad R_b = 9600 \ \text{[bits/s]} \qquad P_B \le 10^{-5}$$

$$\Rightarrow R_b > W_C \Rightarrow \text{Band-limited channel} \Rightarrow \text{MPSK modulation}$$

$$\Rightarrow M = 8 \Rightarrow R_s = R_b / \log_2 M = 9600/3 = 3200 \,\text{[sym/s]} < W_C = 4000 \ \text{[Hz]}$$

$$\Rightarrow \frac{E_s}{N_0} = (\log_2 M)\frac{E_b}{N_0} = (\log_2 M)\frac{P_r}{N_0}\frac{1}{R_b} = 62.67$$

$$\Rightarrow P_E(M=8) \approx 2Q\left[\sqrt{2E_s/N_0}\sin(\pi/M)\right] = 2.2\times10^{-5}$$

$$\Rightarrow P_B \approx \frac{P_E(M)}{\log_2 M} = 7.3\times10^{-6} < 10^{-5}$$

Design example of uncoded systems …

- Choose a modulation scheme that meets the following system requirements:

An AWGN channel with $W_C = 45$ [kHz]

$$\frac{P_r}{N_0} = 48 \ \text{[dB.Hz]} \qquad R_b = 9600 \ \text{[bits/s]} \qquad P_B \le 10^{-5}$$

$$\Rightarrow \frac{E_b}{N_0} = \frac{P_r}{N_0}\frac{1}{R_b} = 6.61 = 8.2 \ \text{[dB]}$$

$$\Rightarrow R_b < W_C \ \text{ and relatively small } E_b/N_0 \Rightarrow \text{power-limited channel} \Rightarrow \text{MFSK}$$

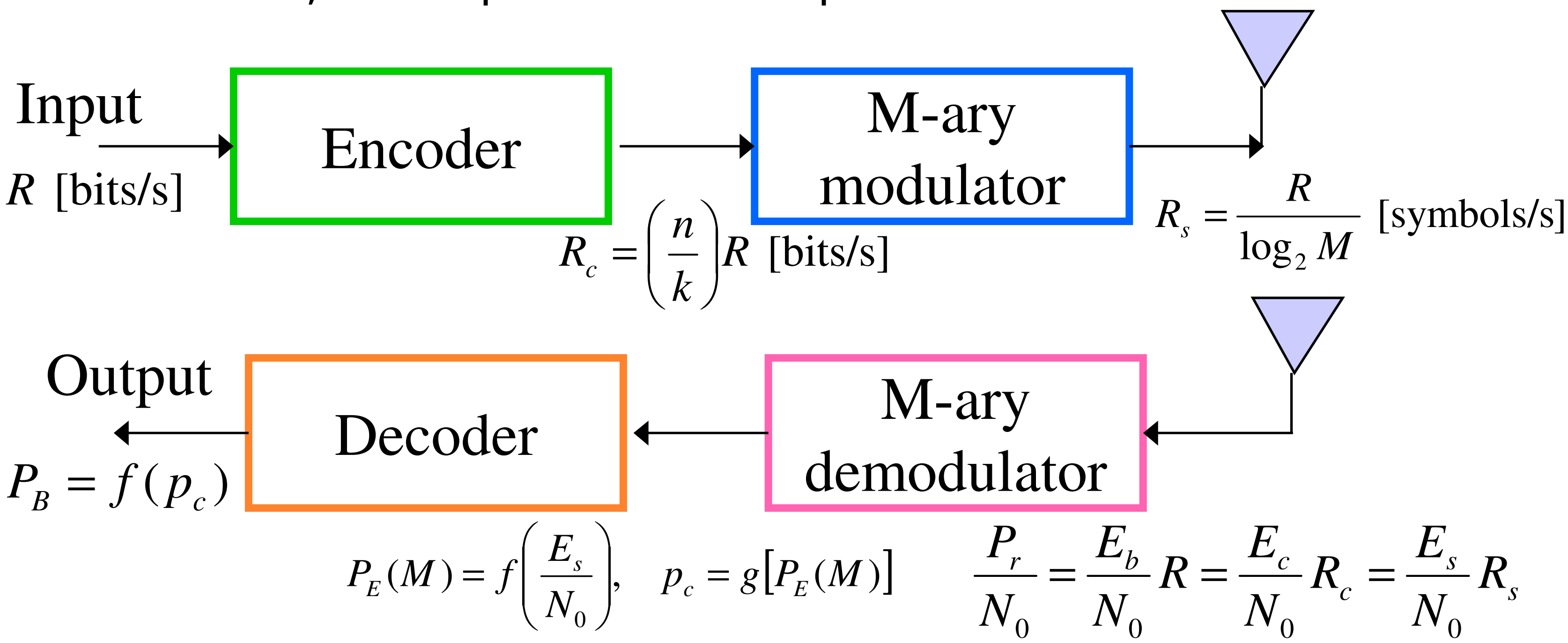
$$\Rightarrow M = 16 \Rightarrow W = MR_s = MR_b/(\log_2 M) = 16 \times 9600/4 = \underline{38.4 \ \text{[ksym/s]} < W_C = 45 \ \text{[kHz]}}$$

$$\Rightarrow \frac{E_s}{N_0} = (\log_2 M)\frac{E_b}{N_0} = (\log_2 M)\frac{P_r}{N_0}\frac{1}{R_b} = 26.44$$

$$\Rightarrow P_E(M=16) \le \frac{M-1}{2}\exp\left(-\frac{E_s}{2N_0}\right) = 1.4 \times 10^{-5} \Rightarrow P_B \approx \frac{2^{k-1}}{2^k - 1}P_E(M) = \underline{7.3 \times 10^{-6} < 10^{-5}}$$

Design example of coded systems

- Design goals:
  1. The bit error probability at the decoder output must meet the system error requirement.
  2. The rate of the code must not expand the required transmission bandwidth beyond the available channel bandwidth.
  3. The code should be as simple as possible. Generally, the shorter the code, the simpler will be its implementation.



Design example of coded systems …

- Choose a modulation/coding scheme that meets the following system requirements:

$$\text{An AWGN channel with } W_C = 4000 \text{ [Hz]}$$

$$\frac{P_r}{N_0} = 53 \text{ [dB.Hz]} \qquad R_b = 9600 \text{ [bits/s]} \qquad P_B \leq 10^{-9}$$

$$\Rightarrow R_b > W_C \Rightarrow \text{Band-limited channel} \Rightarrow \text{MPSK modulation}$$

$$\Rightarrow M = 8 \Rightarrow R_s = R_b / \log_2 M = 9600/3 = 3200 < 4000$$

$$\Rightarrow P_B \approx \frac{P_E(M)}{\log_2 M} = 7.3 \times 10^{-6} > 10^{-9} \Rightarrow \text{Not low enough : power-limited system}$$

- The requirements are similar to the bandwidth-limited uncoded system, except the target bit error probability is much lower.

Design example of coded systems

- Using 8-PSK, satisfies the bandwidth constraint, but not the bit error probability constraint. Much higher power is required for uncoded 8-PSK.

$$P_B \leq 10^{-9} \Rightarrow \left(\frac{E_b}{N_0}\right)_{uncoded} \geq 16 \quad \text{dB}$$

- The solution is to use channel coding (block codes or convolutional codes) to save the power at the expense of bandwidth while meeting the target bit error probability.

Design example of coded systems

- For simplicity, we use BCH codes.
- The required coding gain is:

$$G(\text{dB}) = \left(\frac{E_b}{N_0}\right)_{uncoded}(\text{dB}) - \left(\frac{E_c}{N_0}\right)_{coded}(\text{dB}) = 16 - 13.2 = 2.8\ \text{dB}$$

- The maximum allowable bandwidth expansion due to coding is:

$$R_s = \frac{R}{\log_2 M} = \left(\frac{n}{k}\right)\frac{R_b}{\log_2 M} \le W_C \Rightarrow \left(\frac{n}{k}\right)\frac{9600}{3} \le 4000 \Rightarrow \frac{n}{k} \le 1.25$$

- The current bandwidth of uncoded 8-PSK can be expanded still by 25% to remain below the channel bandwidth.

- Among the BCH codes, we choose the one which provides the required coding gain and bandwidth expansion with minimum amount of redundancy.

Design example of coded systems …

- Bandwidth compatible BCH codes

Coding gain in dB with MPSK

n	k	t	$P_B = 10^{-5}$	$P_B = 10^{-9}$
31	26	1	1.8	2.0
63	57	1	1.8	2.2
63	51	2	2.6	3.2
127	120	1	1.7	2.2
127	113	2	2.6	3.4
127	106	3	3.1	4.0

Design example of coded systems …

- Examine that combination of 8-PSK and (63,51) BCH codes meets the requirements:

$$\Rightarrow \quad R_s = \left(\frac{n}{k}\right)\frac{R_b}{\log_2 M} = \left(\frac{63}{51}\right)\frac{9600}{3} = \underline{3953\ \text{[sym/s]} < W_C = 4000\ \text{[Hz]}}$$

$$\Rightarrow \quad \frac{E_s}{N_0} = \frac{P_r}{N_0}\frac{1}{R_s} = 50.47 \quad \Rightarrow \quad P_E(M) \approx 2Q\left[\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{M}\right] = 1.2\times 10^{-4}$$

$$\Rightarrow \quad p_c \approx \frac{P_E(M)}{\log_2 M} = \frac{1.2\times 10^{-4}}{3} = 4\times 10^{-5}$$

$$\Rightarrow \quad P_B \approx \frac{1}{n}\sum_{j=t+1}^{n} j\binom{n}{j} p_c^j (1-p_c)^{n-j} \approx \underline{1.2\times 10^{-10} < 10^{-9}}$$

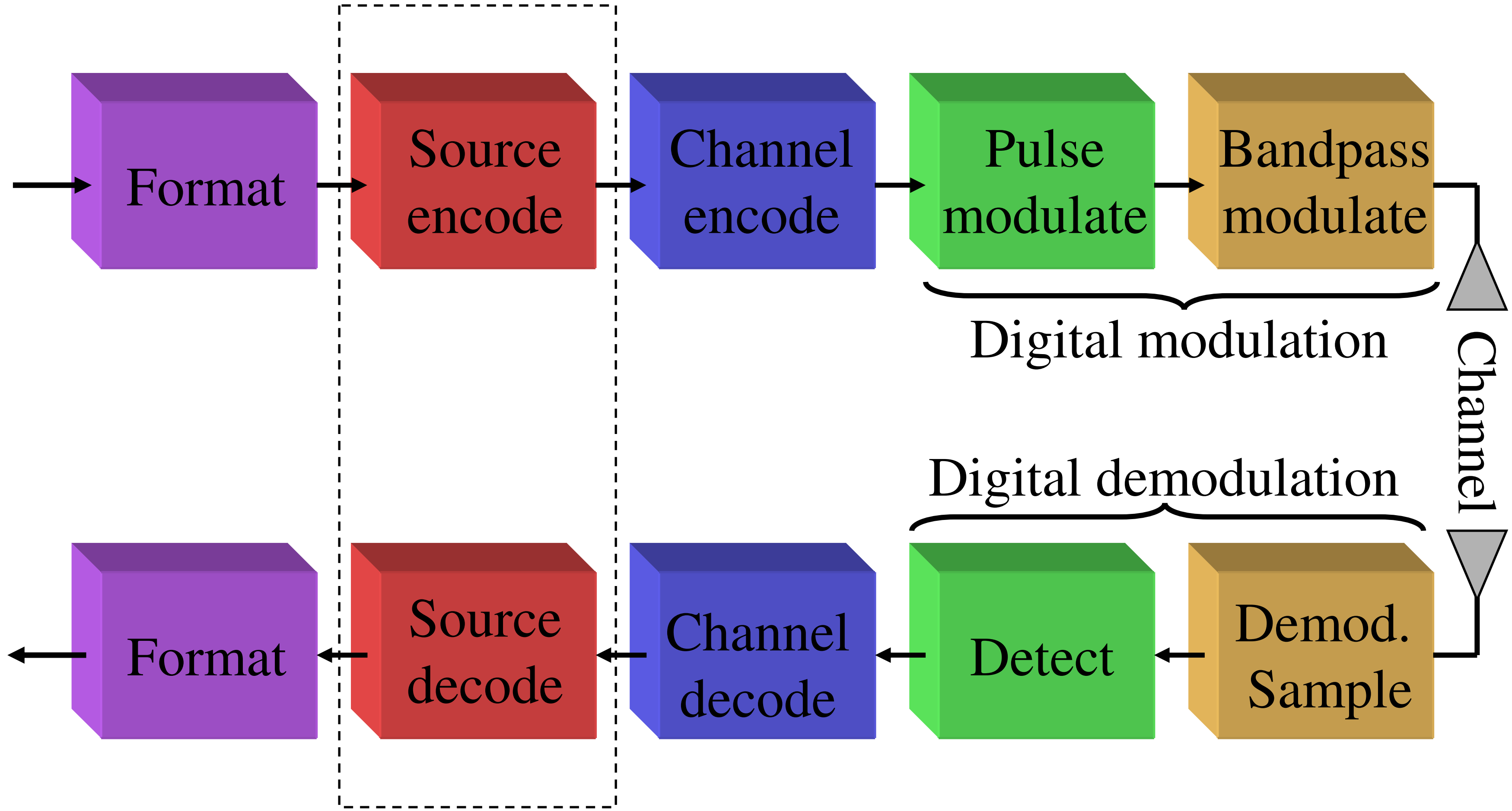
Effects of error-correcting codes on error performance

- Error-correcting codes at fixed SNR influence the error performance in two ways:
  1. Improving effect:
     - The larger the redundancy, the greater the error-correction capability
  2. Degrading effect:
     - Energy reduction per channel symbol or coded bits for real-time applications due to faster signaling.
  - The degrading effect vanishes for non-real time applications when delay is tolerable, since the channel symbol energy is not reduced.

Bandwidth efficient modulation schemes

- Offset QPSK (OQPSK) and Minimum shift keying
  - Bandwidth efficient and constant envelope modulations, suitable for non-linear amplifier
- M-QAM
  - Bandwidth efficient modulation
- Trellis coded modulation (TCM)
  - Bandwidth efficient modulation which improves the performance without bandwidth expansion

Block diagram of a DCS



Course summary – cont'd

- In details, we studies:
  1. Basic definitions and concepts
     - Signals classification and linear systems
     - Random processes and their statistics
       - WSS, and ergodic processes
       - Autocorrelation and power spectral density
     - Power and energy spectral density
     - Noise in communication systems (AWGN)
     - Bandwidth of signal

Course summary – cont'd

3. Channel coding
   - Linear block codes (cyclic codes and Hamming codes)
     - Encoding and decoding structure
       - Generator and parity-check matrices (or polynomials), syndrome, standard array
     - Codes properties:
       - Linear property of the code, Hamming distance, minimum distance, error-correction capability, coding gain, bandwidth expansion due to redundant bits, systematic codes

Course summary – cont'd

- Convolutional codes
  - Encoder and decoder structure
    - Encoder as a finite state machine, state diagram, trellis, transfer function
  - Minimum free distance, catastrophic codes, systematic codes
  - Maximum likelihood decoding:
    - Viterbi decoding algorithm with soft and hard decisions
  - Coding gain, Hamming distance, Euclidean distance, affects of free distance, code rate and encoder memory on the performance (probability of error and bandwidth)

Course summary – cont'd

4. Modulation
    - Baseband modulation
        - Signal space, Euclidean distance
        - Orthogonal basic function
        - Matched filter to improve ISI
        - Equalization to reduce ISI due to the channel
        - Pulse shaping to reduce ISI due to filtering at the transmitter and receiver
            - Minimum Nyquist bandwidth, ideal Nyquist pulse shapes, raise cosine pulse shape

Course summary – cont'd

- Baseband detection
  - Structure of optimum receiver
  - Optimum receiver structure
    - Optimum detection (MAP)
    - Maximum likelihood detection for equally likely symbols
  - Average bit error probability
  - Union bound on error probability
  - Upper bound on error probability based on minimum distance

Course summary – cont'd

- Passband modulation
  - Modulation schemes
    - One dimensional waveforms (ASK, M-PAM)
    - Two dimensional waveforms (M-PSK, M-QAM)
    - Multidimensional waveforms (M-FSK)
  - Coherent and non-coherent detection
  - Average symbol and bit error probabilities
  - Average symbol energy, symbol rate, bandwidth
  - Comparison of modulation schemes in terms of error performance and bandwidth occupation (power and bandwidth)

Course summary – cont'd

5. Trade-off between modulation and coding
   - Channel models
     - Discrete inputs, discrete outputs
       - Memoryless channels : BSC
       - Channels with memory
     - Discrete input, continuous output
       - AWGN channels
   - Shannon limits for information transmission rate
   - Comparison between different modulation and coding schemes
     - Probability of error, required bandwidth, delay
   - Trade-offs between power and bandwidth
     - Uncoded and coded systems