EC 721 Advanced Digital Communications
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Modulation with memory

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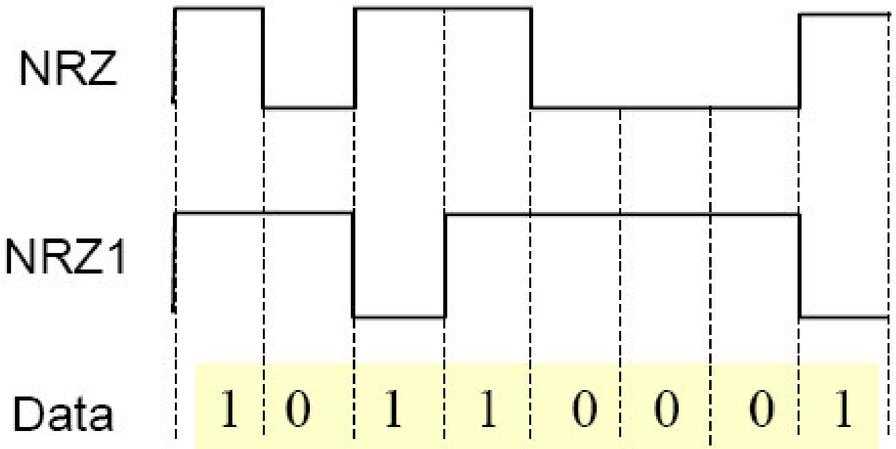
Representation of Digital Modulation with Memory

Linear Digital Modulation with Memory



NRZ modulation is equivalent to binary PAM or PSK. It is memoryless system

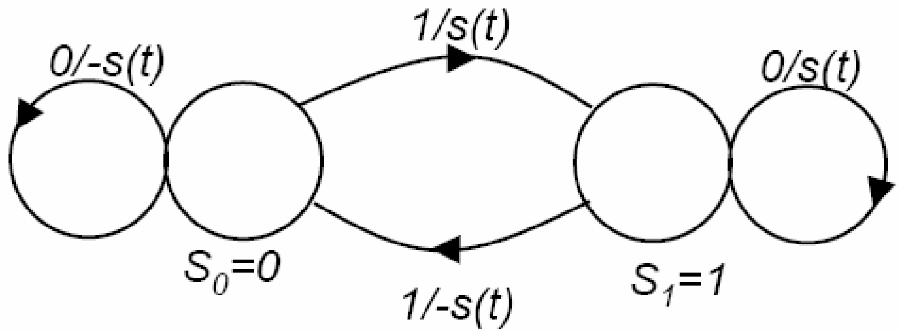
NRZ1 is called differential encoding.
If *A* is pulse amplitude,
A change when data=1 and
A doesn't change when data=0

$$b_k = a_k \oplus b_{k-1}$$

where $\{a_k\}$ s binary information sequence, $\{b_k\}$ is the output sequence, $\oplus$ addition modulo 2

Linear Digital Modulation with Memory

State diagram for the NRZ1 represents the encoder and modulator operations. s(t) represents the waveform to transmit the binary information Data s(t) for Data “1” and –s(t) for Data “0”



Input bit/channel symbol

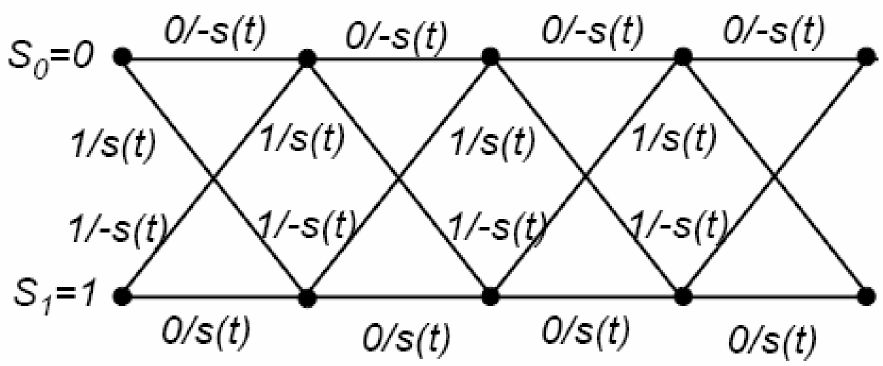
State diagram, It is also called Markow Chain

State transition matrix for input 0

$$T_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$T_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

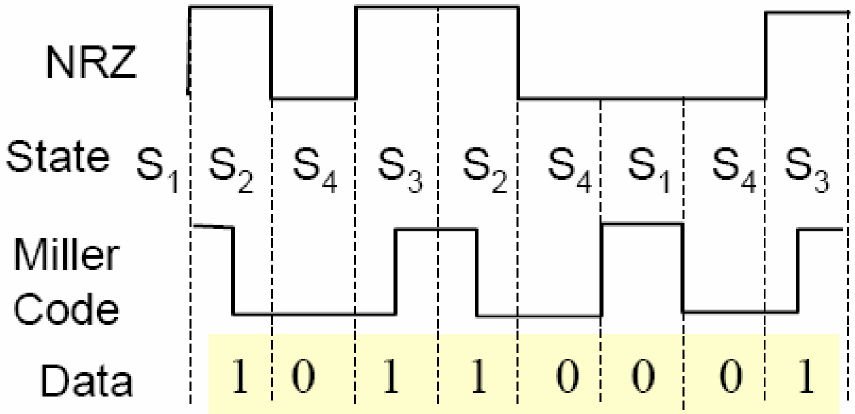
State transition matrix for input 1

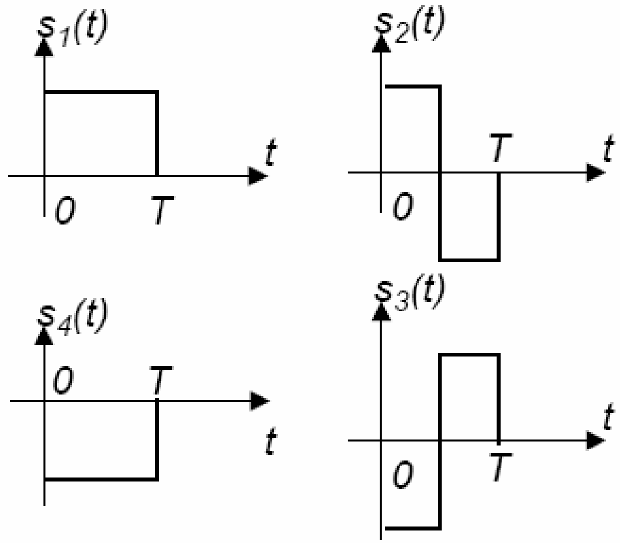


The trellis diagram for NRZ1

Linear Digital Modulation with Memory

Delay Modulation Miller Code

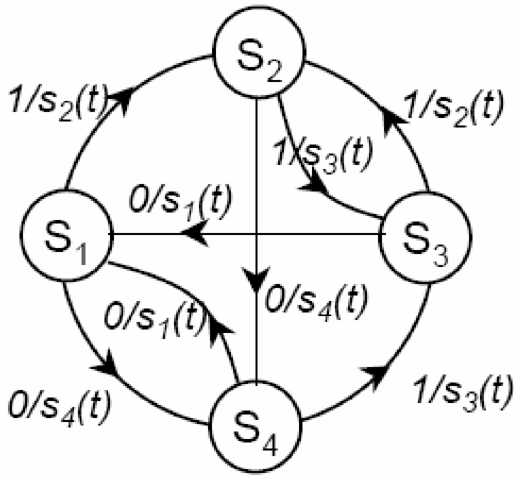




$s_4(t) = -s_1(t) \quad \text{for } 0<t<T$

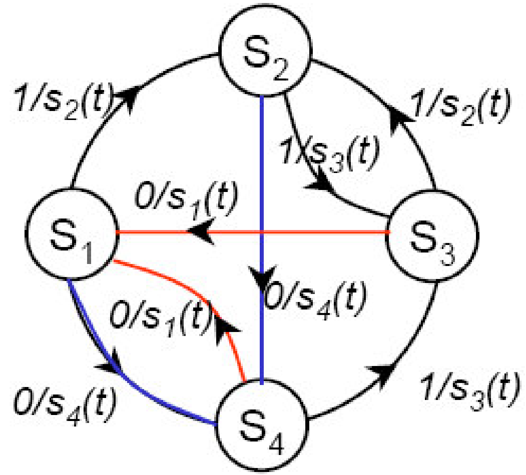
$s_3(t) = -s_2(t) \quad \text{for } 0<t<T$

Basic Waveform for Miller Code



State Diagram for Miller Code

State transition matrix



$$\begin{bmatrix} S_1(t) \\ S_2(t) \\ S_3(t) \\ S_4(t) \end{bmatrix}^T = \begin{bmatrix} S_1(t-1) \\ S_2(t-1) \\ S_3(t-1) \\ S_4(t-1) \end{bmatrix}^T \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{T}_0 = \begin{bmatrix} p(S_1 \mid S_1, I=0) & p(S_2 \mid S_1, I=0) & p(S_3 \mid S_1, I=0) & p(S_4 \mid S_1, I=0) \\ p(S_1 \mid S_2, I=0) & p(S_2 \mid S_2, I=0) & p(S_3 \mid S_2, I=0) & p(S_4 \mid S_2, I=0) \\ p(S_1 \mid S_3, I=0) & p(S_2 \mid S_3, I=0) & p(S_3 \mid S_3, I=0) & p(S_4 \mid S_3, I=0) \\ p(S_1 \mid S_4, I=0) & p(S_2 \mid S_4, I=0) & p(S_3 \mid S_4, I=0) & p(S_4 \mid S_4, I=0) \end{bmatrix}$$

$$\mathrm{T}_0 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Similarly

$$\mathrm{T}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Linear Digital Modulation with Memory

- State or channel Symbol) transition matrix

Let $p_{ij}=P(S_i|S_j)$. Then the matrix $\mathbf{P}=[p_{ij}]$ is called the transition probability matrix which is give for delay modulation as

$$\mathbf{P}=\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{bmatrix}$$

$$\mathbf{P}=P(I=0)T_0+P(I=1)T_1$$

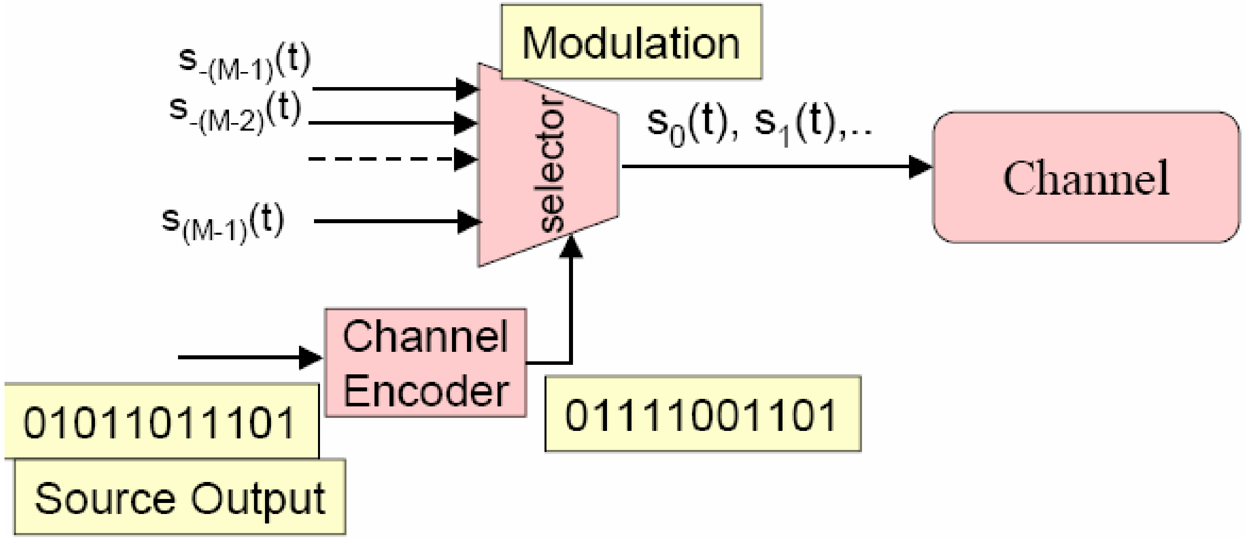
Example: The transition matrix with equal likely symbols P(0)=P(1)=1/2

$$\mathbf{P}=\frac{1}{2}\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}+\frac{1}{2}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}=\begin{bmatrix} 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \\ 1/2 & 1/2 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \end{bmatrix}$$

Non-Linear Modulation with Memory

Continuous Phase FSK (CPFSK)

First, let's look at the FSK



FSK

$$s_m(t) = \sqrt{\frac{2\varepsilon}{T}} \cos\left[2\pi f_c t + 2\pi(\frac{1}{2}\Delta f\, I_n)t\right]$$

where $I_n = \pm 1, \pm 3, \ldots \pm (M-1)$

Non-Linear Modulation with Memory

- FSK signal is generated by shifting carrier frequency to represent the digital information.

$$f_n = \frac{1}{2}\Delta f \; I_n, \quad I_n = \pm 1, \pm 3, \ldots, \pm(M-1)$$

- Needs to have $M=2^k$ separate oscillator to to represent each signal. The sudden switching from one signal frequency to another needs large bandwidth for transmission

- To avoid the problem, continuous-phase FSK is used

- Singe carrier whose frequency is changed continuously
- This types of FSK signal has memory because the phase of the carriers is constrained to be continuous

Continuous Phase FSK (CPFSK)

To represent CPFSK, let's use PAM signal for each k-bits block

$$d(t) = \sum_n I_n g(t - nT)$$

is called delta function

Where I_n represent the amplitude values ±1, ±3,…, ±(M-1) and each of them maps k-bit blocks of information sequence

g(t) is rectangular pulse, amplitude 1/2T and duration T second.

d(t) signal is used to frequency modulate the carrier and the carrier -modulated signal is

$$s(t) = \sqrt{\frac{2\varepsilon}{T}} \cos\left[2\pi f_c t + 4\pi T f_d \int_{-\infty}^{t} d(\tau)\, d\tau \right]$$

f_d: peak frequency deviation

The function $\int_{-\infty}^{t} d(\tau)\, d\tau$ Not consists of jump, this makes the result phase shift with memory

Continuous Phase FSK (CPFSK)

For simplicity

Let g(t) be a rectangular pulse of amplitude 1/2T at [0.T]

$$s(t) = \sqrt{\frac{2\varepsilon}{T}} \cos\left[2\pi f_c t + \phi(t;\mathbf{I})\right]$$

where

$$\begin{aligned}
\phi(t;\mathbf{I}) &= 4\pi T f_d \int_{-\infty}^{t} d(\tau)\, d\tau \\
&= 4\pi T f_d \int_{-\infty}^{t} \left[\sum_n I_n g(\tau - nT)\right] d\tau \\
&= 4\pi T f_d \left(\sum_{k=-\infty}^{n-1} I_k \frac{T}{2T} + I_n \frac{(t-nT)}{2T}\right), \quad t \in [nT, (n+1)T] \\
&= 2\pi f_d T \sum_{k=-\infty}^{n-1} I_k + 2\pi f_d (t - nT) I_n
\end{aligned}$$

Continuous Phase FSK (CPFSK)

The phase of carrier in $nT \le t \le (n+1)T$ is

$$\phi(t;\mathbf{I}) = 2\pi T f_d \sum_{k=-\infty}^{n-1} I_k + 2\pi f_d (t - nT) I_n$$
$$= \theta_n + 2\pi h I_n q(t - nT)$$

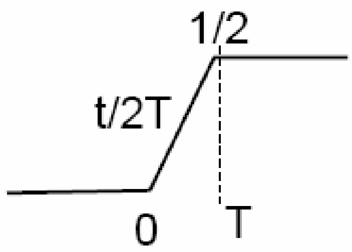
Special case of CPM explained in next slide

where

$h = 2Tf_d$ Modulation index

$\theta_n = \pi h \sum_{k=-\infty}^{n-1} I_k$ Represents the accumulator (memory) up to time $(n-1)T$

$$q(t) = \begin{cases} 0 & (t < 0) \\ t/2T & (0 \le t \le T) \\ 1/2 & (t > 0) \end{cases}$$



Continues-phase Modulation (CPM)

The carrier phase of continuous-phase modulated signal is

$$\phi(t;\mathbf{I}) = 2\pi \sum_{k=-\infty}^{n} I_k h_k q(t-kT) \qquad nT \le t \le (n+1)T$$

where

$\{I_k\}$ $\pm1, \pm3, \ldots, \pm(M-1)$ sequence of M-ary information symbols
$\{h_k\}$ is a sequence of modulation index
q(t) is normalized waveform shape.

When h_k is not fixed, the CPM signal is called multi-h

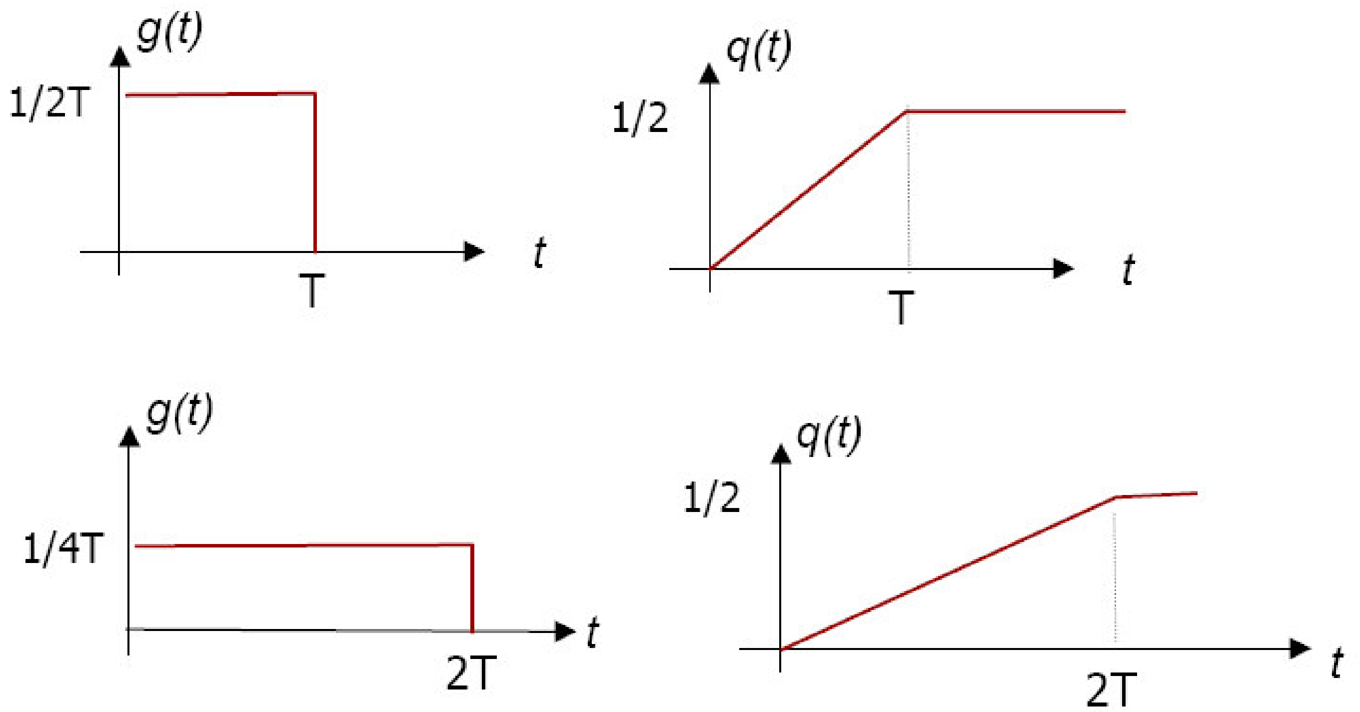
The normalized waveform q(t) can be represented as

$$q(t) = \int_0^t g(\tau)\, d\tau$$

- CPM signal is called full response CPM, if $g(t)=0$ for $t>T$
- CPM signal is called partial response CPM, if $g(t)\neq 0$ for some $t>T$

Continues-phase Modulation (CPM)

Example of full response CPM; Some shapes of g(t) and q(t)

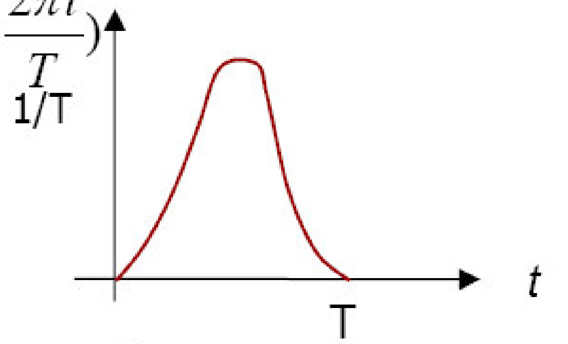


$$g(t) = \begin{cases} \dfrac{1}{2LT} & 0 \le 1 \le LT \\ 0 & (o.w) \end{cases}$$

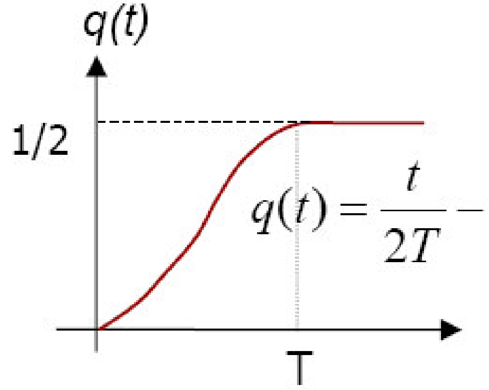
Continues-phase Modulation (CPM)

Example of partial response CPM;Some shapes of g(t) and q(t)

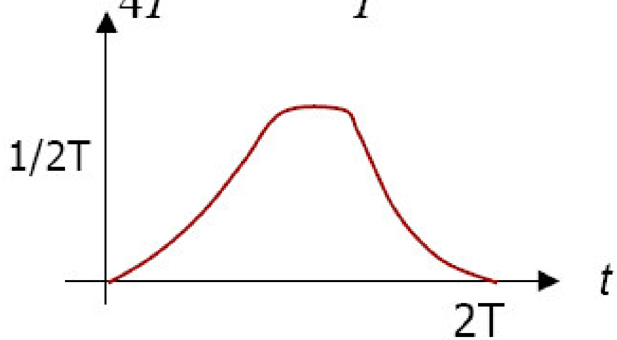
$$g(t) = \frac{1}{2T}\left(1 - \cos\frac{2\pi t}{T}\right)$$



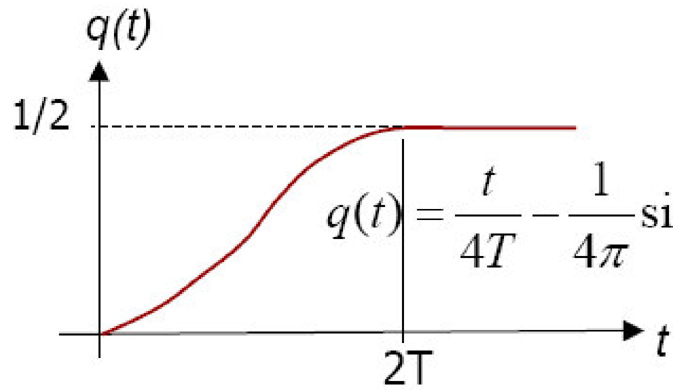
$$q(t) = \frac{t}{2T} - \frac{1}{4\pi}\sin\frac{2\pi t}{T}$$



$$g(t) = \frac{1}{4T}\left(1 - \cos\frac{\pi t}{T}\right)$$



$$q(t) = \frac{t}{4T} - \frac{1}{4\pi}\sin\frac{\pi t}{T}$$



$$g(t) = \begin{cases} \frac{1}{2LT}\left(1 - \cos\frac{2\pi t}{LT}\right) & 0 \le 1 \le LT \\ 0 & (o.w) \end{cases}$$

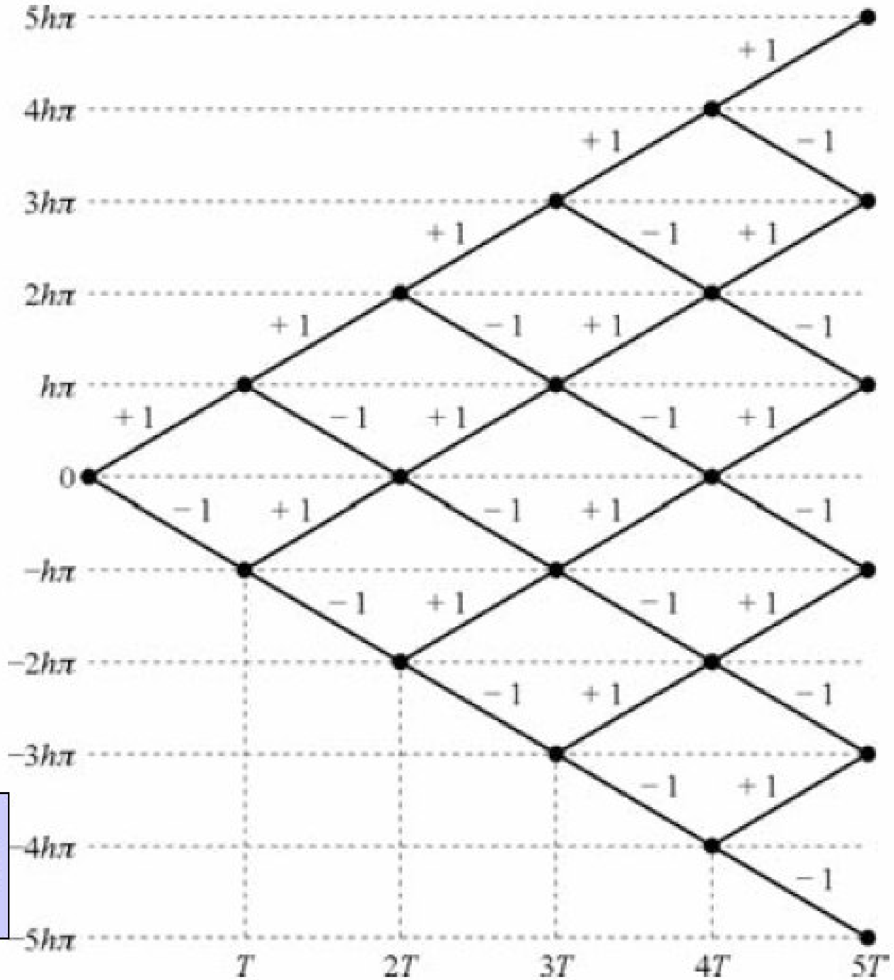
Continues-phase Modulation (CPM)

Example 1.

The case of Binary CPFSK with $I_n = \pm 1$ and g(t) is a full response rectangular function.

The set of phase trajectories starting t=0

$$\phi(t;\mathbf{I}) = \pi h \sum_{k=-\infty}^{n-1} I_k + 2\pi h I_n q(t-nT)$$

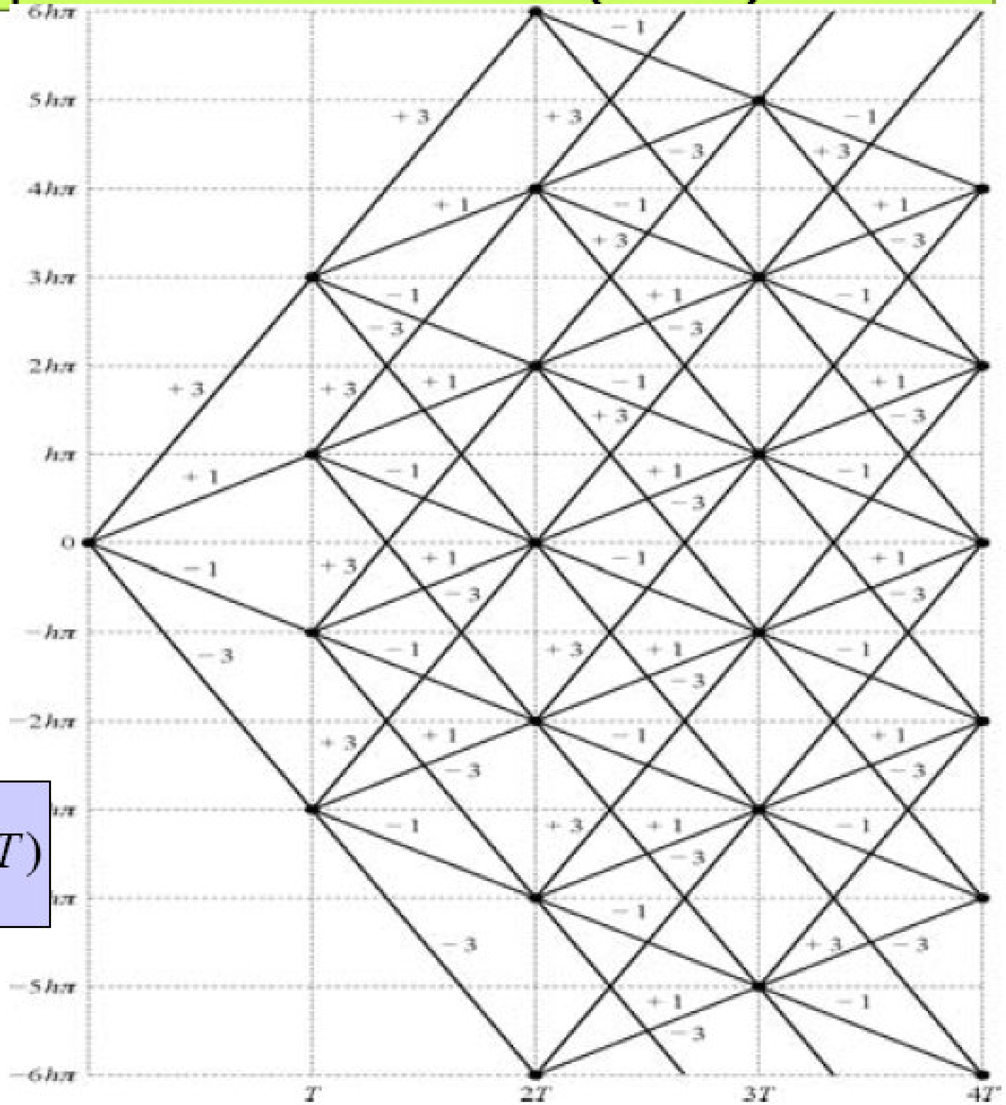


Continues-phase Modulation (CPM)

Example 2:

The case of Quaternary CPFSK with $I_n = \pm 1$ ± 3 and full response rectangular function, the set of phase trajectories starting t=0

$$\phi(t;\mathbf{I}) = \pi h \sum_{k=-\infty}^{n-1} I_k + 2\pi h I_n q(t - nT)$$



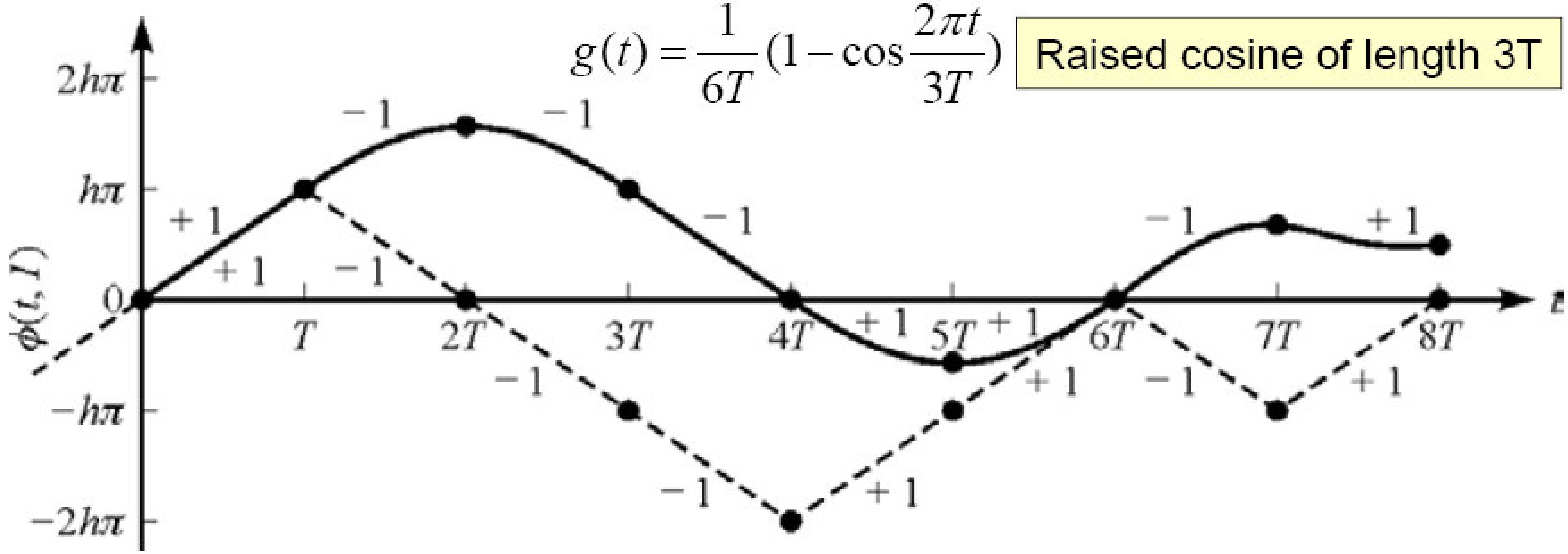
Continues-phase Modulation (CPM)

Example:

A phase trajectories generated by the sequence I_n=(1,-1,-1,-1,1,1,01,1) for the partial response

$$g(t) = \frac{1}{6T}(1 - \cos\frac{2\pi t}{3T})$$

Raised cosine of length 3T



Phase trajectories for binary CPFSK (dashed) and binary, partial response CPM based on raised cosine pulse of length 3*T* (solid). [*From Sundberg (1986), © 1986 IEEE.*]

Continues-Phase Modulation (CPM)

Phase State trellis

Simple way to represent the phase trajectories is concern only those phase values at $t=nT$. Range from $\phi=0$ to $\phi=\pi$.

$$\phi(nT,\mathbf{I}) \in \Theta_s = \{0, h\pi, 2h\pi, 3h\pi, \ldots\}$$

Example: For a full response CPM and h=m/p

$$\Theta_s = \left\{0, \frac{\pi m}{p}, \frac{2\pi m}{p}, \ldots, \frac{(p-1)\pi m}{p}\right\}$$

For m is even

There are p terminal phase state

$$\Theta_s = \left\{0, \frac{\pi m}{p}, \frac{2\pi m}{p}, \ldots, \frac{(2p-1)\pi m}{p}\right\}$$

For m is odd

There are $2p$ terminal phase state

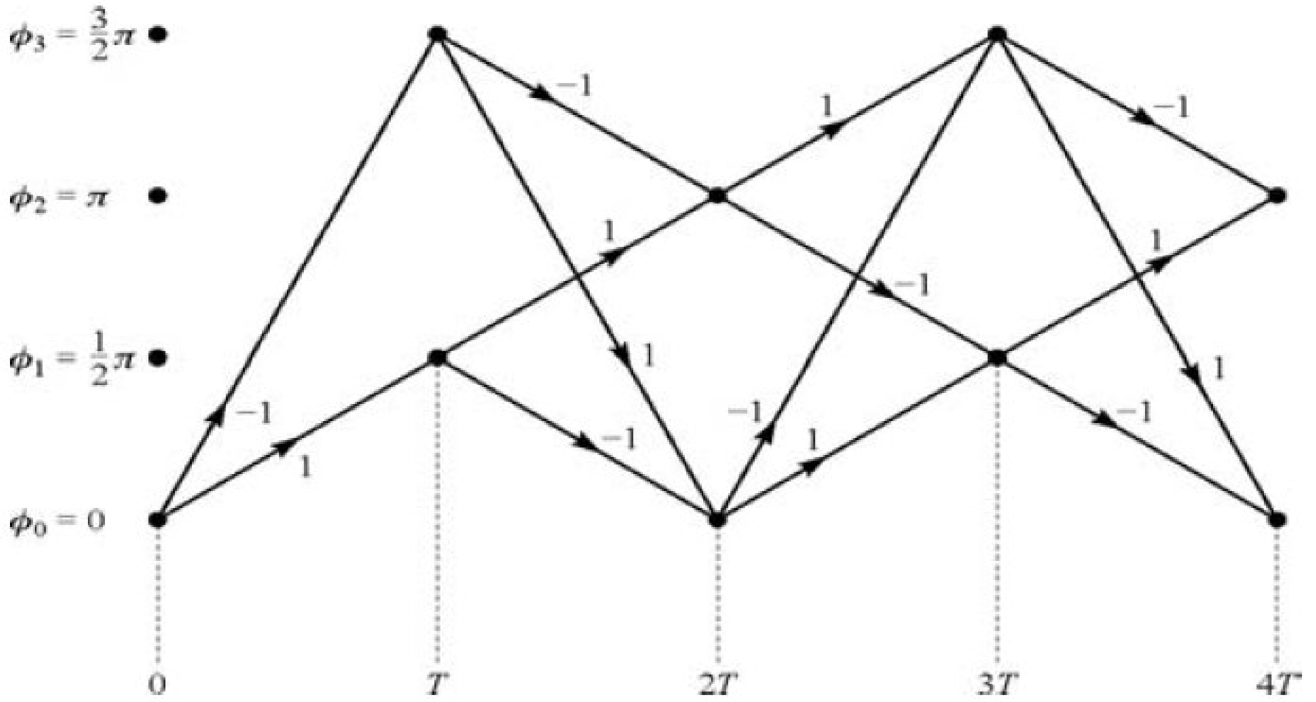
The maximum number of phase state is

$$S_t = \begin{cases} pM^{L-1} & m \text{ even} \\ 2pM^{L-1} & m \text{ odd} \end{cases}$$

M is alphabet size
L is an integer # extends the pulse shape

Continues-phase Modulation (CPM)

- Example: The phase state of the binary CPFSK (full response) with h=1/2 and S_t=4



Difference between phase state trellis and phase trellis is:

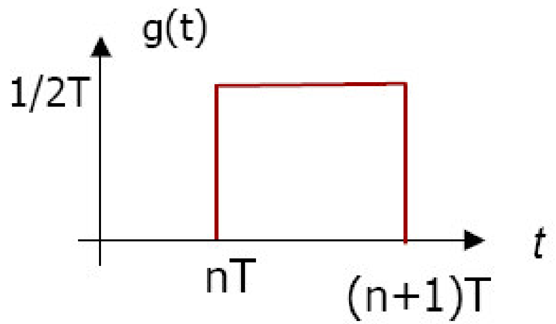
- The connection between state are made by drawing straight lines for phase state trellis. This is not true for phase trajectories from one state to another

Minimum-shift Keying (MSK)

MSK is special case of CPFSK and modulation index h=1/2.

The phase of the carrier in the interval $nT \le t \le (n+1)T$ is

$$\begin{aligned}\phi(t;\mathrm{I}) &= \frac{1}{2}\pi \sum_{k=-\infty}^{n-1} I_k + \pi I_n q(t-nT) \\ &= \theta_n + \frac{1}{2}\pi I_n\left(\frac{t-nT}{T}\right), \quad nT \le t \le (n+1)T\end{aligned}$$



The modulated carrier signal is

$$\begin{aligned}s(t) &= A\cos\left[2\pi f_c t + \theta_n + \frac{1}{2}\pi I_n\left(\frac{t-nT}{T}\right)\right] \\ &= A\cos\left[2\pi\left(f_c + \frac{1}{4T}I_n\right)t - \frac{1}{2}n\pi I_n + \theta_n\right]\end{aligned}$$

Minimum-shift Keying (MSK)

- The binary CPFSK signal will have two frequencies in the interval $nT \leq t \leq (n+1)T$

$$f_1 = f_c - \frac{1}{4T} \qquad f_2 = f_c + \frac{1}{4T}$$

- The binary CPFSK signal can be also written as

$$s_i(t) = A\cos\left[2\pi f_i t + \theta_n + \frac{1}{2}n\pi(-1)^{i-1}\right], \quad i = 1, 2$$

The frequency separation $\Delta f = f_2 - f_1 = 1/2T$

This the minimum frequency separation that is necessary to ensure the orthogonality of the signal s1(t) and s2(t) over a signaling interval of the length T. That is why binary CPFSK with h=1/2 is MSK

Minimum-shift Keying (MSK)

Also, MSK map be represents as a form of four-phase PSK.

The equivalent low-pass digitally modulated signal is

$$v(t) = A \sum_{n=-\infty}^{\infty} \left[I_{2n} g(t - 2nT) + jI_{2n+1} g(t - (2n+1)T) \right]$$

where

$$g(t) = \begin{cases} \sin \dfrac{\pi t}{2T}, & t \in [0, 2T) \\ 0, & o.w. \end{cases}$$

Viewed as a four-phase PSK signal with pulse shape is one-half cycle of sinusoidal. The even numbered binary valued symbols I_{2n} of the information sequence are transmitted via cos of the carrier. The odd-numbered symbols $\{I_{2n+1}\}$ are transmitted via the sin carrier

Minimum-shift Keying (MSK)

Transmission rate for each is 1/2T bits/ss, combine transmission rate will be 1/T bit/s.

MSK

$$s(t) = A \sum_{n=-\infty}^{\infty} \left[I_{2n} g(t-2nT) \cos 2\pi f_c t + I_{2n+1} g(t-(2n+1)T) \sin 2\pi f_c t \right]$$

s(t), a constant amplitude and frequency modulated signal

Minimum-shift Keying (MSK)

$$A\sum_{n=-\infty}^{\infty}[I_{2n}g(t-2nT)\cos 2\pi f_c t]$$

In phase signal component

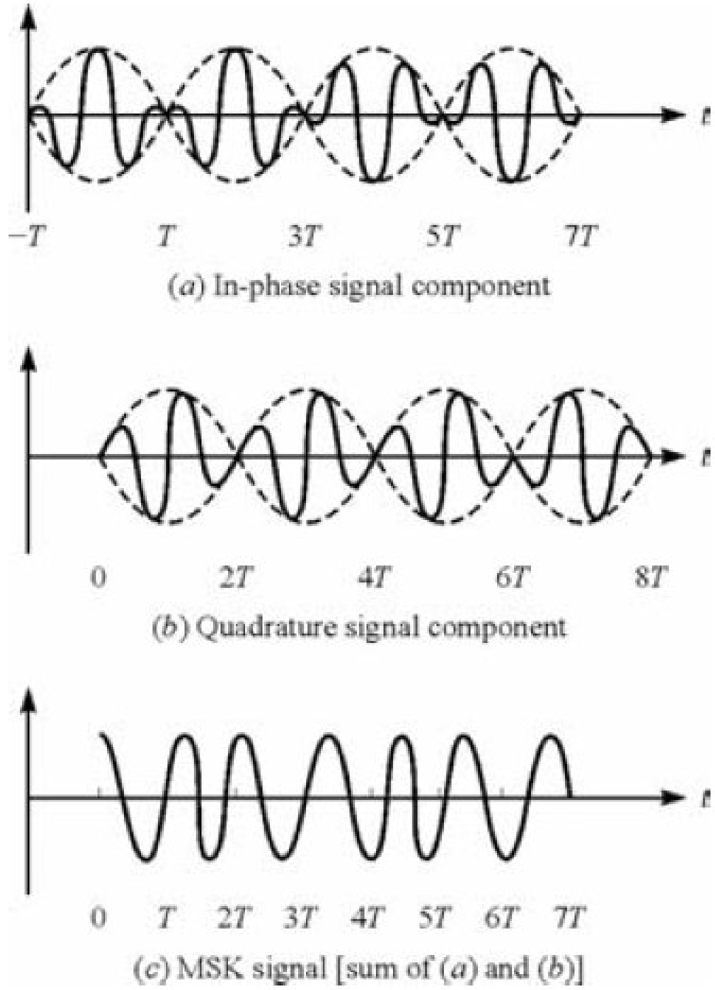
$$A\sum_{n=-\infty}^{\infty}[I_{2n+1}g(t-(2n+1)T)\sin 2\pi f_c t]$$

Quadrature signal component

s(t), a constant amplitude and frequency modulated signal

Frequency at $[nT,(n+1)T)$

$$f_c+\frac{1}{4T}I_n$$



Comparison of MSK and QPSK

MSK

$$s(t) = A\sum_{n=-\infty}^{\infty}\left[I_{2n}g(t-2nT)\cos 2\pi f_c t + I_{2n+1}g(t-(2n+1)T)\sin 2\pi f_c t\right]$$

where

$$g(t) = \begin{cases} \sin\frac{\pi t}{2T}, & t\in[0,2T) \\ 0, & o.w. \end{cases}$$

Continuous phase

Offset QPSK

$$s(t) = A\sum_{n=-\infty}^{\infty}\left[I_{2n}g(t-2nT)\cos 2\pi f_c t + I_{2n+1}g(t-(2n+1)T)\sin 2\pi f_c t\right]$$

where

$$g(t) = \begin{cases} 1, & t\in[9,2T) \\ 0, & o.w. \end{cases}$$

Possibly ±90 degree phase jump at each T

QPSK

$$s(t) = \sum_{n=-\infty}^{\infty}\left[-\frac{I_{2n}+I_{2n+1}}{2}g(t-2nT)\cos 2\pi f_c t + -\frac{I_{2n}-I_{2n+1}}{2}g(t-2nT)\sin 2\pi f_c t\right]$$

where

Possibly ±90 or ±180 degree phase jump at each $2T$

Signal space diagram of CPM

- Continues phase signal can not be represented by discrete points in signal space, like PAM, PSK
- It can be described by the trajectories from one phase state to another

- Here is signal phase trajectories diagram for CPFSK signal for h=1/4,h=1/3,h=1/2, and h=2/3