

Final Exam (Take home). Only lecture slides and textbooks, No Internet is allowed

Q1. Suppose you have a digital communication system with the following 8-QAM signal

$$s_m(t) = g_T(t) \cos \left(2\pi f_c t + \frac{2\pi m}{8} \right), \quad m = 1, 2, 3, 4,$$

and

$$s_m(t) = 2g_T(t) \cos \left(2\pi f_c t + \frac{2\pi m}{8} \right), \quad m = 5, 6, 7, 8,$$

where f_c is the carrier frequency and $g_T(t)$ is given by

$$g_T(t) = \frac{1}{4T} \left(1 - \cos \left(\frac{\pi t}{T} \right) \right), \quad t \in [0, T],$$

Find the low pass representation of $s(t)$

Q2. Consider the output of the receiver of a DCS is

$$r_m = a_m + 2a_{m-1} + v_m,$$

Where a_m is the data having values $+1$ or -1 . v_m is a Gaussian noise of zero mean and unit variance. If $a_0 = +1$ and the detected outputs at the first three time instants are $r_1 = 1$, $r_2 = -1$ and $r_3 = 0$. Use the Viterbi algorithm to find the optimal maximum likelihood sequence for a_1 based on r_1 , r_2 and r_3 . Draw the trellis and identify the Euclidian distance for each branch.

Q3. A convolutional encoder is shown in figure one:

- What is the rate of this encoder? What is the constraint length of this encoder?
- How many states the state diagram will have? How many transitions per state?
- Draw the state diagram of the encoder.
- Determine the encoder output produced by the message sequence 10111
- Flip the third and eighth bit of the output of part d and show the process of decoding using the trellis diagram.
- Find the transfer function of such encoder and the free distance

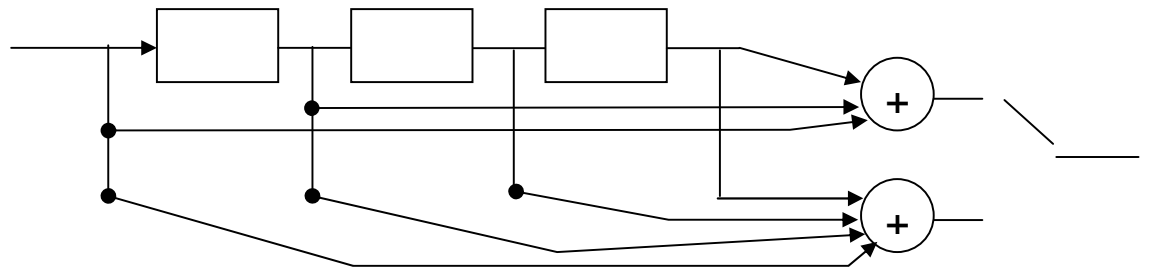


Figure one

Q4. A convolutional encoder having the rate $R=1/5$, constraint length $K=4$ and a generator matrix

$$G = [17 \ 17 \ 13 \ 15 \ 15].$$

- How many shift registers are used in the encoder? Draw the encoder.
- How many states the state diagram will have? Sketch the state diagram.
- Draw one stage of the trellis diagram.

Q5. A linear block code has the following parity matrix

$$P = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

- a- Express the generating matrix G in the form of $[P:I]$ form.
- b- Determine the parity check matrix H
- c- Construct the table of syndromes
- d- Determine the minimum distance of the code.
- e- Show that the codeword corresponding to the information sequence 101 is perpendicular on H