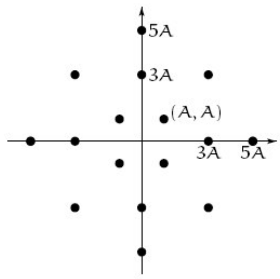
1- Derive the probability of error for the standard 16-QAM modulation technique then compare between its probability of error and that of the following 16-QAM signal space for the same average energy per bit



2- Suppose a digital transmitter produces two dimensional vectors (s_1,s_2) equally likely. After passing by a noisy channel, the received vector (r_1,r_2) is given by

$$r_1 = \sqrt{E}s_1 + n_1$$
$$r_2 = r_1 + \sqrt{E}s_2 + n_2$$

where n_1, n_2 are independent zero mean Gaussian noise with variance $N_0/2$. Find the maximum likelihood decision rule and the expression for its probability of error for the following cases

a- (s_1,s_2) can have values of (1,1), (1,-1), (-1,1), (-1,-1)
b- (s_1,s_2) can have value (-1, -1), (1, -1)

3- Suppose we transmit one of two equally likely signals B, or –B over a noisy channel having the following pdf

$$p(n) = \frac{1}{\sqrt{2\sigma^2}} e^{-\frac{|n|\sqrt{2}}{\sigma}}, \quad -\infty < n < \infty.$$

a- draw the signal space with the noise plotted over it
b- find the decision regions and boundaries
c- what is the optimal decision rule
d- compute the minimum probability of error

4- Consider one of the signals $\{s_i(t)\}_{i=1}^{4}$ being transmitted via an AWGN channel (zero mean, power spectral density of $N_0/2$). These signals are defined as follows:

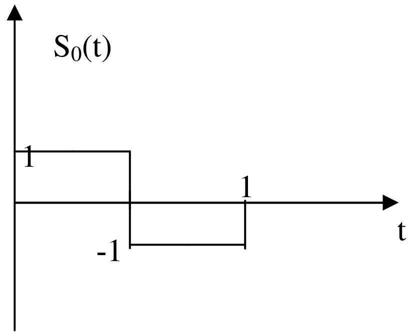
$$s_i(t) = \cos(\omega_c + \frac{2i\pi}{T})t \qquad ;1 \le i \le 4 \qquad where \qquad \omega_c = \frac{2\pi n}{T}$$

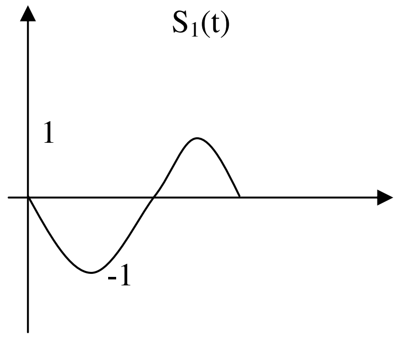
a. Use the Gram-Schmidt orthogonalization procedure to find the orthonormal basis functions for the set of the given signals.
b. Design the maximum likelihood (ML) receiver for the given signals. Assume they all are equally likely transmitted.

5- For the signals shown:

(a) Design a matched-filter detector to decide which of the two signals shown is being received in additive white Gaussian noise (zero-mean, PSD=$N_0/2$).

(b) Find the average error probability of the detector if the noise has PSD of 10^{-6} w/Hz.





6- Suppose that the additive noise $\underline{n}$(t) in the Gaussian channel is white, and the mean which is equal to E { $\underline{n}$(t)} = 0. Let the coefficients of the covariance matrix $\overline{\overline{k}}$ be given by C_{ij} which is given by,

$$C_{ij} = E\{(\underline{n}(t_i) - \mu_i)(\underline{n}(t_j) - \mu_j)\} = E\{(\underline{n}_i - \mu_i)(\underline{n}_j - \mu_j)\} = \frac{N_o}{2}\delta_{ij} = \begin{cases} 0; i \neq j \\ \frac{N_o}{2} \quad ; i = j \end{cases}$$

(c) Find the joint probability density function (pdf) for $\underline{n}$(t).

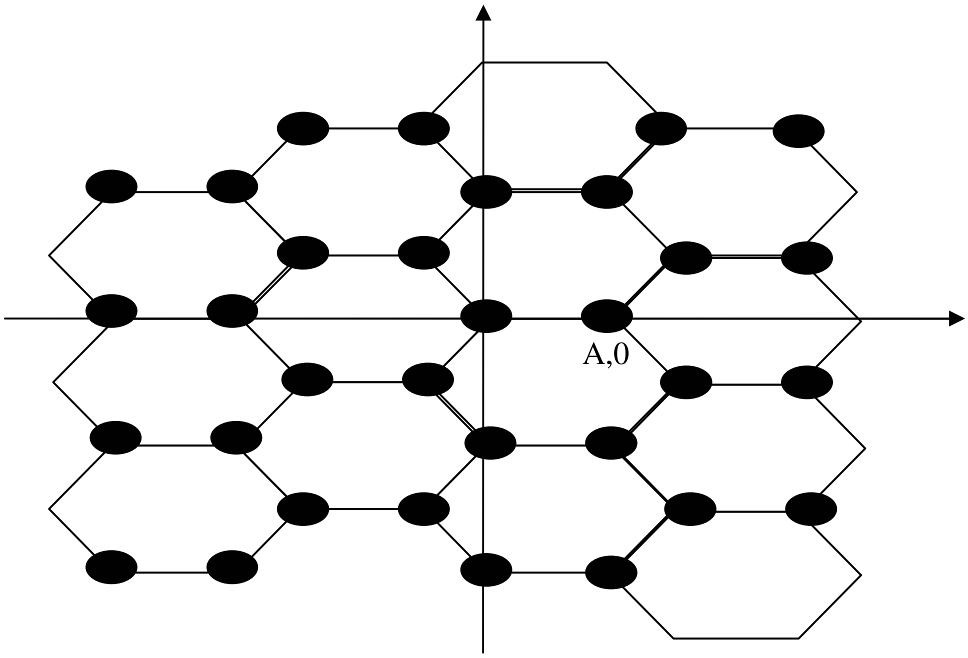
(d) Find the joint pdf of $\underline{y}$(t), where $\underline{y}$(t) is defined by the following equation:

$$\underline{y}(t) = 2\underline{n}(t) + 5\underline{w}(t) + 1$$

Where $\underline{w}$(t) has the same statistics of $\underline{n}$(t) and they both are mutually independent.

7- for the following signal constellation

a. Find the decision boundaries and regions
b. Find the exact and approximate probability of error.
c. Suggest a transmitter and a receiver
d. If the noise is similar to question three, find the exact probability of error
e. Compare between this model and the standard 32-QAM for same average energy per bit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