EC 421 Statistical Communication Theory

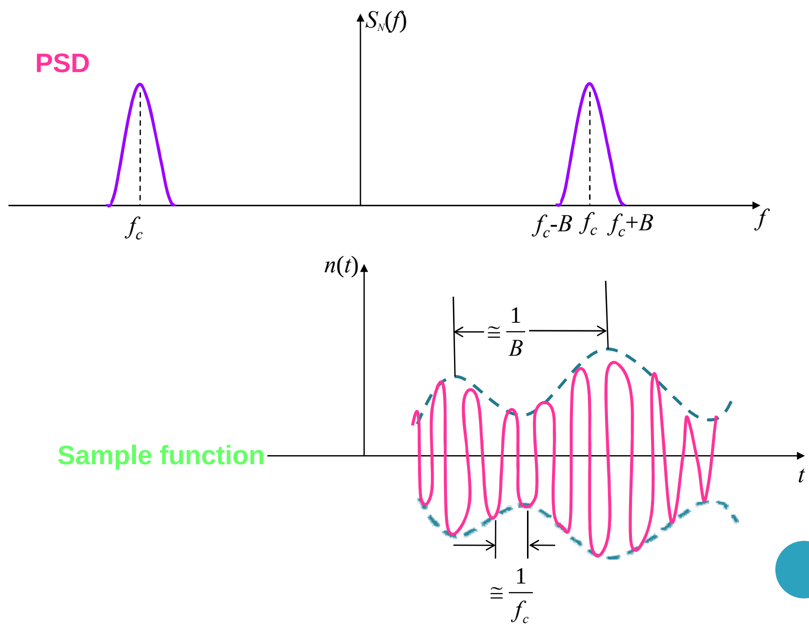
Instructor: Dr. Heba A. Shaban

Lecture # 7

Narrowband Noise (NBN)

- **Filters** at the receiver have **enough bandwidth** to pass the **desired signal** but not too big to pass **excess noise**.
- **Narrowband (NB)**→ center frequency is much bigger that the bandwidth.
- **Noise** at the output of such filters are called **narrowband noise (NBN)**.
- **NBN** has spectral concentrated about some mid-band frequency $\pm f_c$
- The sample function of such NBN $n(t)$ appears as a sine wave of frequency f_c which modulates slowly in amplitude and phase.

$$S_N(f) = \frac{N_0}{2}|H(f)|^2$$



REPRESENTATION OF NBN

- $n(t)$ can be represented by its **pre-envelope** and **complex envelope** as follows:

$$n_+(t) = n(t) + j\,\hat{n}(t)$$
$$\tilde{n}(t) = n_+(t)\exp(-j2\pi f_c t)$$

$$\tilde{n}(t) = n_c(t) + jn_s(t)$$

In phase comp. of NBN: $$n_c(t) = n(t)\cos(2\pi f_c t) + \hat{n}(t)\sin(2\pi f_c t)$$

Quadrature comp. of NBN: $$n_s(t) = \hat{n}(t)\cos(2\pi f_c t) - n(t)\sin(2\pi f_c t)$$

$\therefore$

$$n(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

PROPERTIES OF INPHASE AND QUADRATURE COMPONENTS OF NBN

1. If **$n(t)$ is zero mean** then **$n_c(t)$** and **$n_s(t)$** are also **zero mean**
2. If **$n(t)$ is Gaussian** RP then **$n_c(t)$** and **$n_s(t)$** are **jointly Gaussian**
3. If **$n(t)$ is WSS** then **$n_c(t)$** and **$n_s(t)$** are **jointly WSS**

$$R_{N_c}(\tau) = R_{N_s}(\tau) = R_N(\tau)\cos(2\pi f_c \tau) + \hat{R}_N(\tau)\sin(2\pi f_c \tau)$$

Crosscorrelation

$$R_{N_c N_s}(\tau) = -R_{N_s N_c}(\tau) = R_N(\tau)\sin(2\pi f_c \tau) - \hat{R}_N(\tau)\cos(2\pi f_c \tau)$$

PROPERTIES OF INPHASE AND QUADRATURE COMPONENTS OF NBN

4. **PSD** of **inphase** and **quadrature** components are the **same** and are related to the PSD of the original NBN PSD as follows

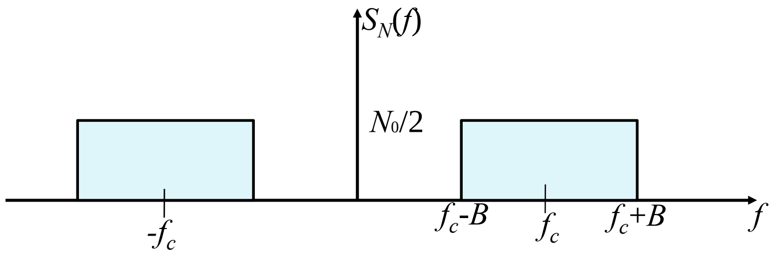
$$S_{N_c}(f) = S_{N_s}(f) = \begin{cases} S_N(f - f_c) + S_N(f + f_c), & -B \le f \le B \\ 0 & otherwise \end{cases}$$

5. If **$n(t)$ is zero mean** then **$n_c(t)$** and **$n_s(t)$** will have the **same variance** as **$n(t)$** itself

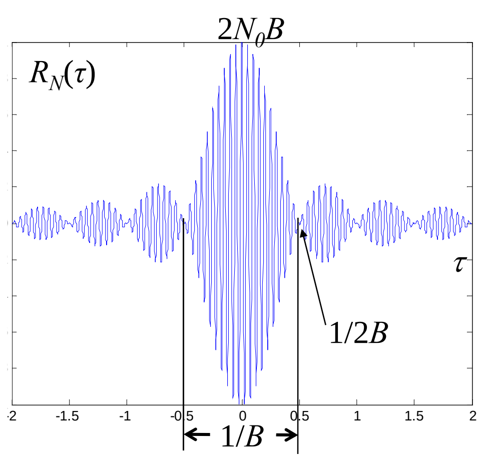
6. If **$n(t)$ is zero mean** Gaussian with symmetric PSD around **f_c**, then **$n_c(t)$** and **$n_s(t)$** are **statistically independent**

IDEAL BPF (IBPF) WHITE NOISE

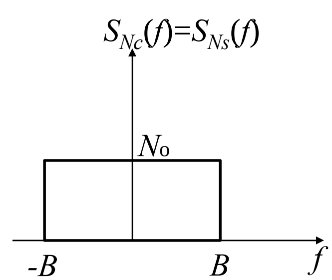
- Consider a WGN of **zero mean** and **PSD $N_0/2$** which passes by an **IBPF** of **unit** amplitude response and **BW = 2*B***.
- The **PSD** of the **filtered noise** $n(t)$ has the **same shape** of the BPF



IDEAL BPF WHITE NOISE



$$R_N(\tau) = 2N_0 B \sin c(2B\tau)\cos(2\pi f_c \tau)$$



$$R_{N_c}(\tau) = R_{Ns}(\tau) = 2N_o B \sin c(2B\tau)$$

REPRESENTATION OF NBN W.R.T ENVELOPE AND PHASE COMPONENTS

- *n*(*t*) can be represented as:

  $$n(t) = r(t)\cos(2\pi f_c t + \theta(t))$$

  where

  $$r(t) = \sqrt{n_c^{\,2}(t) + n_s^{\,2}(t)}$$

  $$f_R(r) = \frac{r}{\sigma^2}\exp(-\frac{r^2}{\sigma^2}) \quad r \geq 0$$

  and

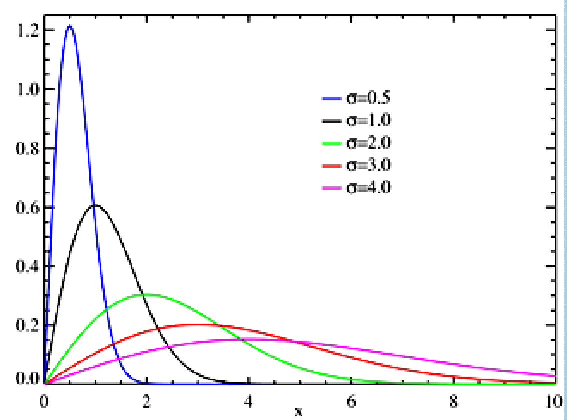
  $$\theta(t) = \tan^{-1}(\frac{n_s(t)}{n_c(t)})$$

- *r*(*t*) is called the **envelope** of ***n*(*t*)** and ***θ*(*t*)** is called the **phase** of *n*(*t*)

- *r*(*t*) will have a **Rayleigh** distribution and ***θ*(*t*)** will have a **uniform** distribution

RAYLEIGH PDF

$$f_R(r) = \frac{r}{\sigma^2}\exp(-\frac{r^2}{\sigma^2}) \quad r \geq 0$$



ENVELOPE OF SINE WAVE PLUS NBN

- Suppose we **add NBN to a sinusoidal signal**:

$$x(t) = A\cos(2\pi f_c t) + n(t)$$

$$x(t) = A\cos(2\pi f_c t) + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

$$x(t) = [A + n_c(t)]\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

- If ***n(t)*** is a **zero mean** σ^2 **variance Gaussian RP**
  - $n_c(t)$ and $n_s(t)$ are G RP and S.I
  - $n_c(t)$ and $n_s(t)$ are zero mean
  - $n_c(t)$ and $n_s(t)$ are σ^2 variance

ENVELOPE OF SINE WAVE PLUS NBN

- The envelope of $x(t)$ will be **Rician distribution**

$$f_R(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2+K^2}{2\sigma^2}\right)I_o\left(\frac{Kr}{\sigma^2}\right)$$

where

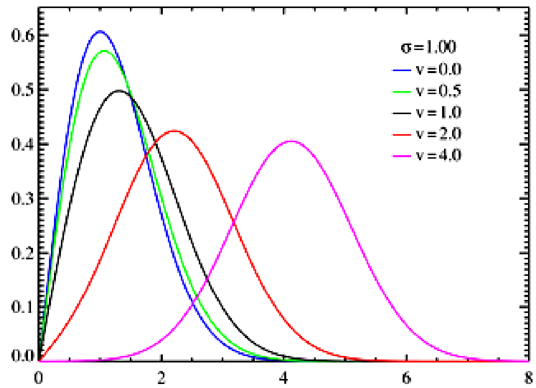
I_o is the modified Bessel function of first kind of zero order

- If K =0, it becomes **Rayleigh** distribution
- If K>>, it becomes approximately **Gaussian**

RICIAN DISTRIBUTION

•**v** in figure refers to *K* factor.

•When **K = 0**, the distribution reduces to **Rayleigh**.



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