EC 421 Statistical Communication Theory

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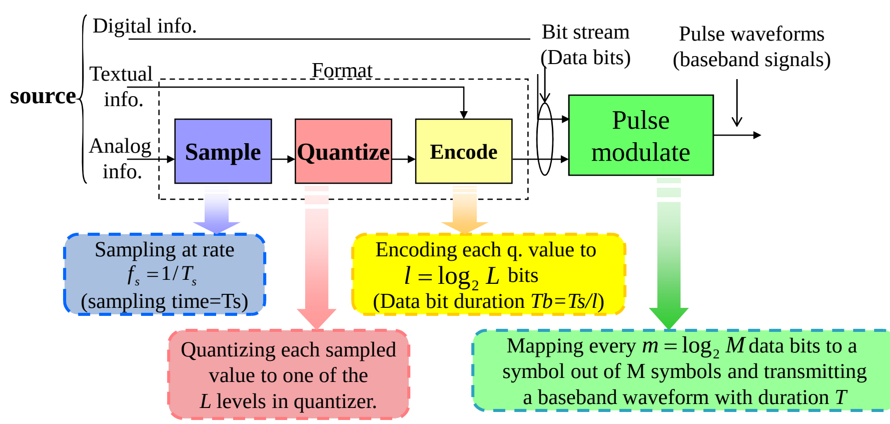
Lecture # 11

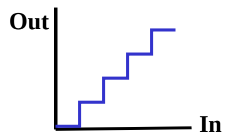
Formatting and transmission of baseband signal

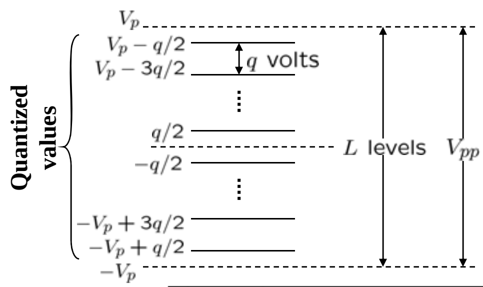


- Information (data) rate: $R_b = 1/T_b$ [bits/sec]
- Symbol rate : $R = 1/T$ [symbols/sec]
  - For real time transmission: $R_b = mR$

QUANTIZATION

- **Amplitude quantizing:** Mapping samples of a continuous amplitude waveform to a finite set of amplitudes.





- Average quantization noise power

$$\sigma^2 = \frac{q^2}{12}$$

- Signal peak power

$$V_p^2 = \frac{L^2 q^2}{4}$$

- Signal power to average quantization noise power

$$\left(\frac{S}{N}\right)_q = \frac{V_p^2}{\sigma^2} = 3L^2$$

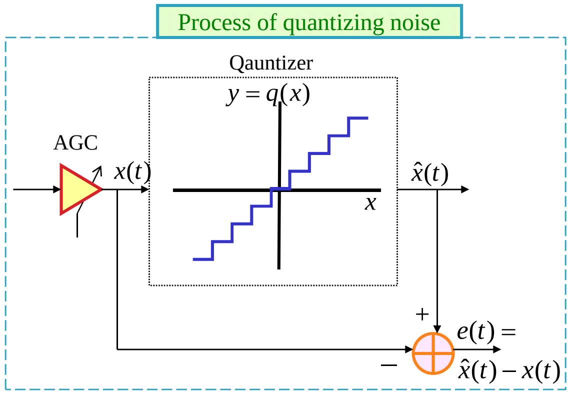
$$(\text{SNR})_{dB} = 6.02\,R + \alpha$$

$\alpha = 4.77$

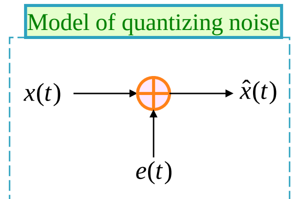
QUANTIZATION ERROR

- **Quantizing error:** The difference between the input and output of a quantizer $\Rightarrow e(t) = \hat{x}(t) - x(t)$

Process of quantizing noise



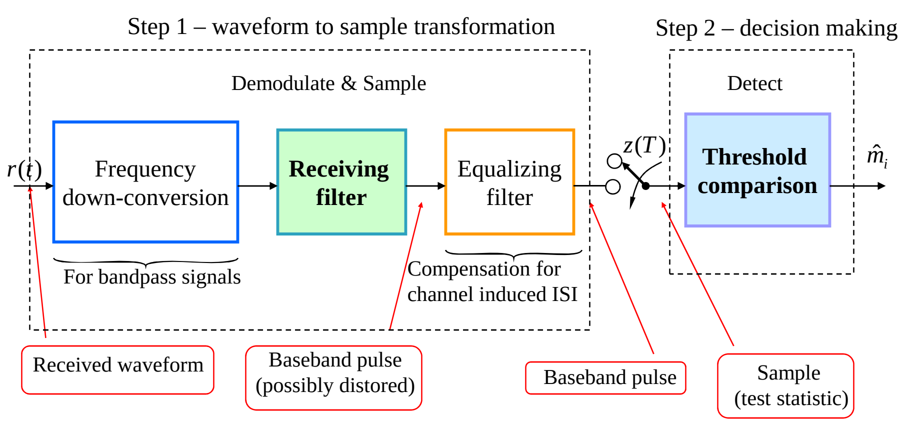
Model of quantizing noise



RECEIVER JOB

- Demodulation and sampling:
  - Waveform recovery and preparing the received signal for detection:
    - Improving the signal power to the noise power (SNR) using matched filter
    - Reducing ISI using equalizer
    - Sampling the recovered waveform
- Detection:
  - Estimate the transmitted symbol based on the received sample

RECEIVER STRUCTURE

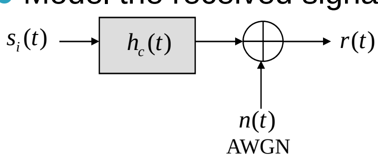


Steps in Designing the Receiver

- Find optimum solution for receiver design with the following goals:
  1. Maximize SNR
  2. Minimize ISI
- Steps in design:
  - Model the received signal
  - Find separate solutions for each of the goals.
- First, we focus on designing a receiver which maximizes the SNR.

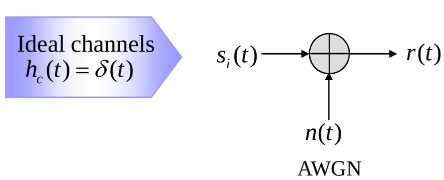
Design the Receiver Filter to Maximize the SNR

- Model the received signal



$$r(t) = s_i(t) * h_c(t) + n(t)$$

- Simplify the model:
  - Received signal in AWGN



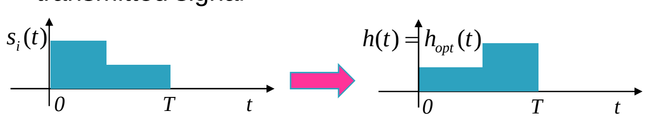
$$r(t) = s_i(t) + n(t)$$

Matched filter receiver

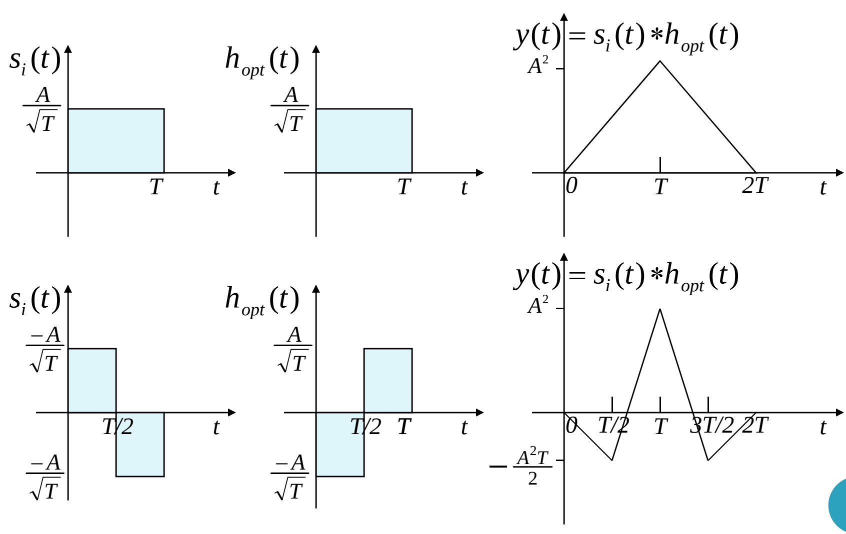
- **Problem:**
  - Design the receiver filter $h(t)$ such that the SNR is maximized at the <u>sampling time</u> when $s_i(t), i = 1, \ldots, M$ is transmitted.
- **Solution:**
  - The optimum filter, is the Matched filter, given by

$$h(t) = h_{opt}(t) = s_i^*(T - t)$$
$$H(f) = H_{opt}(f) = S_i^*(f)\exp(-j2\pi fT)$$

which is the time-reversed and delayed version of the conjugate of the transmitted signal



Example of matched filter



PROPERTIES OF THE MATCHED FILTER

1. The Fourier transform of a matched filter output with the matched signal as input is, except for a time delay factor, proportional to the ESD of the input signal.

$$Z(f) = |S(f)|^2 \exp(-j2\pi fT)$$

2. The output signal of a matched filter is proportional to a shifted version of the autocorrelation function of the input signal to which the filter is matched.

$$z(t) = R_s(t-T) \Rightarrow z(T) = R_s(0) = E_s$$

3. The output SNR of a matched filter depends only on the ratio of the signal energy to the PSD of the white noise at the filter input.

$$\max\left(\frac{S}{N}\right)_T = \frac{E_s}{N_0/2}$$

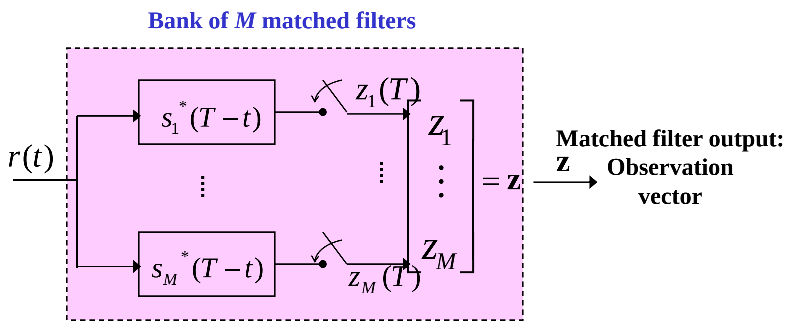
4. Two matching conditions in the matched-filtering operation:
   - spectral phase matching that gives the desired output peak at time T.
   - spectral amplitude matching that gives optimum SNR to the peak value.

CORRELATOR RECEIVER

- The matched filter output at the sampling time, can be realized as the correlator output.

$$z(T) = h_{opt}(T) * r(T)$$
$$= \int_0^T r(\tau) s_i^*(\tau) d\tau = \langle r(t), s(t) \rangle$$

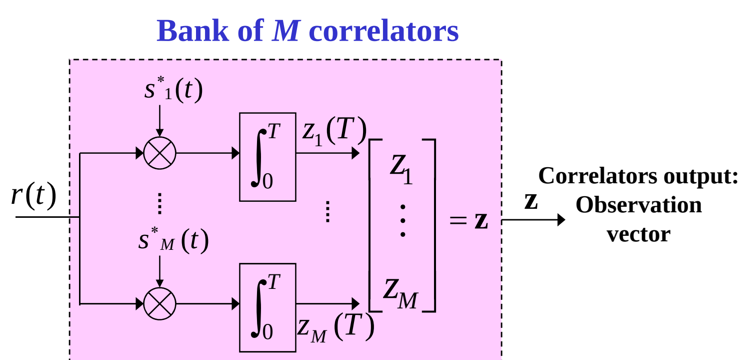
Implementation of Matched Filter Receiver



$$z_i = r(t) * s^*_i(T-t) \quad i = 1,\ldots,M$$

$$\mathbf{z} = (z_1(T), z_2(T), \ldots, z_M(T)) = (z_1, z_2, \ldots, z_M)$$

Implementation of Correlator Receiver



$$\mathbf{z} = (z_1(T), z_2(T), \ldots, z_M(T)) = (z_1, z_2, \ldots, z_M)$$

$$z_i = \int_0^T r(t) s_i(t)\,dt \qquad i = 1,\ldots,M$$